Long tail of Bit-Error-Rate distribution in long haul optical transmission

Optical fibers are widely used for transmission of information. In an ideal case, information carried by pulses would be transmitted non-damaged. In reality, however, various impairments lead to the information loss. Noise generated by optical amplifiers and fiber birefringence are the two major impairments in high-speed fiber communications. The amplifier noise is short-correlated in time, while the birefringence varies significantly along the optical line and is practically frozen in time, since the characteristic temporal scale of such variations is long compared to the signal propagation time through the entire fiber line. Coexistence of two different sources of randomness characterized by two well-separated time scales is common in statistical physics of disordered systems. A classical example is the glassy behavior driven by short-correlated thermal noise in a system with frozen structural disorder. Complete statistical description of a system, with both disorder and noise present, requires the two step averaging, and formally introducing a so-called "higher"-order statistics, i.e. a probability distribution function (second-step averaging over the disorder) of a quantity defined in terms of another probability distribution function (first-step averaging over the noise). Extreme non-Gaussianity of the "higher"-order statistics is an important feature of the disordered systems. We show that, first, the disordered system approach is appropriate for optical fiber systems, and second, we report emergence of an extremely non-Gaussian tail in the optical fiber system "higher"-order statistics [1, 2, 3, 4].

Birefringent disorder is caused by weak random ellipticity of the fiber cross section. Birefringence splits the pulse into two polarization components and also leads to pulse broadening. This effect known as polarization mode dispersion (PMD) have been extensively studied experimentally and theoretically . PMD is usually characterized by the so-called PMD vector that was found to obey Gaussian statistics. It was also shown that first-order PMD compensation corresponding to cancellation of the PMD vector on the carrier frequency is experimentally implementable. Higher-order generalizations of the PMD vector (introduced to resolve a complex frequency dependence of the PMD phenomenon with higher accuracy) as well as a suggestion on how to compensate for PMD in higher orders have been also discussed and implemented experimentally. Common wisdom hiding behind the standard approach says that one should start with evaluating effects of PMD and amplifier noise separately and then estimate the joint effect taking the impairments on equal footing. In [1, 2, 3, 4] we challenged this equal-footing approach. We showed that the overall effects of temporal noise and structural disorder may not be separated since BER strongly depends on a realization of birefringent disorder. Thus, the PDF of BER and especially its tail corresponding to large values of BER are the objects of prime interest and practical importance for describing the probability of the system outage.

The envelope of the electromagnetic (optical) field propagating through optical fiber in the linear regime (i.e. at relatively low pulse intensity), which is subject to PMD distortion and amplifier noise, satisfies the following equation

$$\partial_z \boldsymbol{\Psi} - \hat{m}(z)\partial_t \boldsymbol{\Psi} = \boldsymbol{\xi}(z,t), \qquad (1)$$

z, t and $\boldsymbol{\xi}$ being the position along the fiber, the retarded time (measured in the reference frame moving with the optical signal), the amplifier noise. (Here we discuss a simple model situation. See [1, 2, 3, 4] for description of steps leading to Eq.(1.) The envelope $\boldsymbol{\Psi}$ is a two-component complex field where the components stand for two polarization states of the optical signal. We assume the optical system length Z to be much larger than the distance between the nearest amplifier stations. Coarse-graining on the inter-amplifier scale allows treating amplification in the continuous limit. Zero in average additive noise $\boldsymbol{\xi}$ is the amplification leftover. The amplifier noise has Gaussian statistics and its correlation time is much shorter then the pulse width. Therefore, the statistics of ξ is fully determined by its pair correlation function $\langle \xi_\alpha(z_1,t_1)\xi^*_\beta(z_2,t_2)\rangle = D_\xi \delta_{\alpha\beta}\delta(z_1-z_2)\delta(t_1-t_2)$, and the coefficient D_ξ characterizes the noise strength. Averaging over birefringent disorder is of different nature: Statistics here is collected over different fibers or, alternatively, over different states of the same fiber collected over time. The birefringence 2×2 matrix $\hat{m}$ can be expanded in the Pauli matrices $\hat{m}(z) = h_j(z)\hat{\sigma}_j$ where h_j is a real three-component field. The field is zero in average and short-correlated in z $\langle h_i(z_1)h_j(z_2)\rangle = D_m\delta_{ij}\delta(z_1 - z_2)$, where D_m characterizes the disorder strength.

We consider the return-to-zero (RZ) modulation format when pulses in a given frequency channel are well separated in t. Detection of a pulse at the fiber output corresponding to $z = Z$ requires a measurement of the pulse intensity $I = \int \mathrm{d}t\, G(t)\, |\mathcal{K}\boldsymbol{\Psi}(Z,t)|^2$, where the function $G(t)$ is a convolution of the electrical (current) filter function with the sampling window function (limiting the information slot). The linear operator $\mathcal{K}$ stands for an optical filter and may also incorporate a variety of engineering "tricks" applied to the output signal $\boldsymbol{\Psi}(Z,t)$. Ideally, I takes a distinct value if the bit encodes "1" and is negligible if the bit encodes "0". Both the noise and disorder enforce I to deviate from its ideal value. One declares the output signal to encode 0 or 1 if the value of I is less or larger than the decision threshold I_0. The information is lost if the output value of the bit differs from the input one. The probability of such event should be small (this is a mandatory condition for a successful fiber line performance) i.e. both impairments typically cause only small distortion of a pulse. Formally, this means: $D_\xi Z \ll 1$, $D_m Z \ll 1$, where the initial signal width and its amplitude are both re-scaled to one.

Here, we skip details of calculations, that can be found in [1, 2, 3, 4], presenting bare results. We found that the remote tail , $\mathcal{B} \gg \mathcal{B}_0$, of the PDF of $\mathcal{B}$ in the "setting the clock" case is

$$\mathcal{S}(B)\,\mathrm{d}B \sim \frac{B_0^\alpha\,\mathrm{d}B}{B^{1+\alpha}}, \qquad \alpha = \frac{D_\xi}{2\mu_2 D_m}. \quad (2)$$

If no compensation is applied Eq.(2) transforms into, $\ln\mathcal{S} \approx -D\xi^2 Z/(2D_m\mu_1^2)$, while in the first-order compensation case the asymptotic result for the PDF tail (2) remains valid with μ_2 replaced by another constant μ_2'/π. The dependence of the PDF on BER is illustrated in Fig. 1. One also finds from Eq. (2) that the outage probability, $\mathcal{O} \equiv \int_{B_*}^1 \mathrm{d}B\,\mathcal{S}(B)$, where B_* is some fixed value taken to be much larger than B_0, is estimated as $\ln\mathcal{O} \sim (D_\xi/D_m)\ln(B_0/B_*)$.

Eq. (2) describes our major result: The PDF of BER has a long algebraic tail. The exponent α of the algebraic decay is proportional to the ratio of the amplifier noise variance, D_ξ, to the birefringent disorder variance, D_m. This statement clearly shows that effects of noise and disorder are, actually, inseparable. Another interesting feature of Eq. (2) is that the exponent α is Z-independent. The only Z-dependent factor in the final result (2) is the overall normalization factor B_0^α. Note also that some numerical results, consistent with Eq. (2) are already available that shows a linear relation between $\ln S$ and $\ln B$, i.e. just the algebraic decay predicted by Eq. (2).

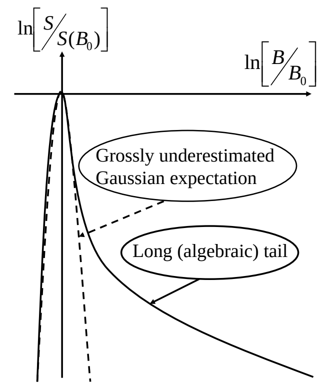

Figure 1: Schematic log-log plot of the PDF of Bit-Error-Rate.

References

[1] V. Chernyak, M. Chertkov, I. Kolokolov, and V. Lebedev, Probability of anomalously large Bit-Error-Rate in long haul optical transmission, to appear in Phys.Rev.E **68**, 066619 (2003).

[2] V. Chernyak, M. Chertkov, I. Kolokolov, and V. Lebedev, "Compensation for Extreme Outages caused by Polarization Mode Dispersion and Amplifier noise", Optics. Express. **11**, 1607 (2003).

[3] V. Chernyak, M. Chertkov, I. Kolokolov, and V. Lebedev, "Periodic and Quasi-Periodic Compensation Strategies of Extreme Outages caused by Polarization Mode Dispersion and Amplifier Noise", Pis'ma v ZhETF. **78**(4), 234-237 (2003).

[4] V. Chernyak, M. Chertkov, I. Gabitov, I. Kolokolov, and V. Lebedev, "PMD induced fluctuations of Bit-Error-Rate in optical fiber systems", Journal of Lightware Technology **22**, 1155 (2004).

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