The Treasury High Quality Market Corporate Bond Yield Curve
For the Pension Protection Act of 2006
October 9, 2007

This note describes the methodology used to construct the Treasury High Quality Market (HQM) Corporate Bond Yield Curve, which provides discount rates for the Pension Protection Act of 2006 (PPA). The description applies to the methodology in use at present; the methodology may be changed in the future because of changes in market conditions or for other reasons. Any significant changes in this methodology will be announced by notice.

The yield curve shows yields at different maturities from a particular category of fixed-income securities. The HQM yield curve corresponds to U.S corporate bonds whose credit quality is a market-weighted average of the top three qualities – AAA, AA, and A.

There are different types of yields and yield curves. For the purposes of the PPA, the relevant type of yield is the spot yield, which is the yield on a bond with a single payment at a future date, that is, a zero-coupon bond.

The yield curve pertains to a specific point in time, usually late in a business day, and is computed from the prices at that time on a set of bonds. The first step in constructing the curve is to assemble an appropriate set of bonds.

The HQM methodology currently uses a set of U.S. corporate bonds that covers the AAA, AA, and A markets. The maximum maturity of the bonds is 30 years, and currently, the par amount outstanding of each bond is at least $250 million. The part of the curve at lower maturities is filled in with AA commercial paper rates from the Board of Governors of the Federal Reserve System. The bond prices are bid prices.

The bonds currently chosen for the bond set pay fixed nominal semiannual coupons and the principal amount at maturity. Bonds with other characteristics, such as convertible bonds or bonds with floating coupons, are generally omitted. Bonds with embedded options are also excluded at present, although bonds with make-whole call provisions are included. All bonds are issued by U.S. corporations; asset-backed bonds are excluded.

With the set of bonds selected, the next step is to determine the prices of the bonds, including accrued interest, and the amounts and timings of the cash flows from the bonds. For precision, the HQM methodology moves forward the dates of cash flows that are scheduled to be received on weekends or holidays to the next business day.

The yield curve construction proceeds by fitting a statistical model that relates the bond prices to their cash flows, and computing the spot yield curve rates from this model. The model has two parts. The first part is based on the concept of a discount function, labeled $d(t)$, which gives the present value of $1 to be received at maturity t years in the future with credit quality equal to the market-weighted average of the top three qualities. The present value of the cash flows from a bond is obtained by applying $d(t)$ to the flows.

However, the price of the bond can deviate from the present value because the credit quality of the bond may not be average high quality. Therefore, in the second part of the model, adjustment factors are added to the present value to compensate. The adjustment factors are specified later in this note.

Writing the bond price model symbolically, the price p of a bond is given as:

$$p = \sum_{i=1}^{n} d(t_i)\, c_i + \sum_{j=1}^{m} b_j\, x_j + e$$

where the c_i for $i=1,...,n$ are the n cash flows from the bond to be received at times t_i, the $b_j x_j$ for $j=1,...,m$ are the adjustment factors, made up of the m coefficients b_j on the m adjustment variables x_j, and e is the random disturbance for this bond. The b_j are fixed across the bonds, while the values of the x_j variables change with the bonds.

The computation of the yield curve is done by estimating the $d(t)$ function and the b_j coefficients from the bond data set of prices and cash flows. Given $d(t)$, the spot rate $y(t)$ at maturity t in percent (for maturities of ½ year or greater) is directly calculated by the following formula, which follows market convention including semiannual compounding:

$$y(t) = 200 \left(\frac{1}{d(t)^{\frac{1}{2t}}} - 1 \right)$$

In order to estimate the bond price model, a specific functional form must be chosen for $d(t)$. This is done by writing $d(t)$ in terms of the forward rate $f(t)$, which is the short-term interest rate on a forward contract t years in the future, and applying the functional form to $f(t)$. The following formula writes $d(t)$ in terms of $f(t)$ as the exponential of the negative of the integral of $f(t)$:

$$d(t) = \exp\left(-\int_0^t f(z)dz \right)$$

To determine the functional form for $f(t)$, the entire maturity span of the bonds from zero to 30 years maturity is divided into maturity ranges. Currently, the HQM methodology uses five ranges, which are delineated by the maturity points 0, 1.5, 3, 7, 15, and 30 years.

Within each of these five ranges, $f(t)$ is given by a cubic equation, which is an equation that can have terms raised to the third power, and the five cubic equations are smoothly joined together across the ranges, with continuous first and second derivatives. A set of cubic equations connected like this is called a spline, so $f(t)$ is represented by a cubic spline.

Three constraints are placed on the cubic spline for $f(t)$. First, for smoothness, $f(t)$ is constrained to be approximately a line at short maturities; specifically, the second derivative of $f(t)$ is set to zero at maturity zero.

Second, the value of $f(t)$ at the furthest maturity – 30 years – is constrained to equal its average value from 15 years to 30 years, and $f(t)$ is kept constant at this average for all maturities beyond 30 years. Third, $f(t)$ is made to approach this constant value smoothly; specifically, the derivative of $f(t)$ is set to zero at maturity 30 years. The constant value of $f(t)$ beyond 30 years is used to project $d(t)$ and the spot yields beyond 30 years maturity.

It can be demonstrated that the cubic spline for $f(t)$ as set out here is determined by five parameters. These parameters are estimated from the bond data.

In addition, the adjustment factors in the second part of the bond price model must also be specified and estimated. The HQM methodology currently includes two adjustment variables x_1 and x_2, which adjust the bond prices so that $d(t)$ and the spot rates represent market-weighted average credit quality of the top three qualities – AAA, AA, and A.

To define the variables, let w_1 be the total par amount outstanding of the AA bonds in the bond set relative to total amount outstanding of AAA and AA bonds. Then the first adjustment variable x_1 is defined as w_1 times the maturity for AAA bonds, w_1-1 times the maturity for AA bonds, and zero for A bonds. Larger values of w_1 adjust AAA bonds more and move $d(t)$ nearer to a AA discount function relative to AAA.

Analogously, let w_2 be the total par amount outstanding for A bonds relative to the total for all bonds in the bond set, which are rated AAA, AA, or A. Then the second adjustment variable x_2 is defined as w_2 times the maturity for AAA and AA bonds, and w_2-1 times the maturity for A bonds. Larger values of w_2 adjust AAA and AA bonds more, and move $d(t)$ nearer to a discount function at the A level.

These variables imply that the amount of the adjustment to each bond's price relative to the present value of the bond's cash flows given by $d(t)$ is greater when the quality class of the bond is a smaller proportion of the high-quality market. Consequently, the resulting estimate of $d(t)$ is more representative of the quality classes with larger shares and pertains to the market-weighted average of the top three quality classes.

In sum, the seven parameters, comprising five parameters in the cubic spline for $f(t)$ and the two adjustment coefficients b_j on the adjustment variables, are estimated from the bond price data. The estimation is done by nonlinear least squares, that is, the seven parameter estimates are chosen to minimize the sum of the squared differences between the actual bond prices and the prices given by the bond price model. In the estimation, the adjustment factors can be regarded as linear regression terms added to the present value part of the model.

Before the estimation is carried out, the bond data are weighted. The weighting consists of two stages.

In the first stage, equal weights are assigned to the commercial paper rates at the short end of the curve, and the par amounts outstanding of all the bonds are rescaled so that their sum equals the sum of the weights for commercial paper. Then the squared price difference for each bond is multiplied by the bond's rescaled par amount outstanding, and the squared difference for each commercial paper rate is multiplied by the commercial paper weight.

In the second stage, for bonds with duration greater than unity, the weighted squared price difference for each bond from the first stage is divided by duration. Commercial paper durations are less than unity.

The nonlinear least squares estimation is carried out after weighting. The estimation produces values for the five cubic spline parameters and the two adjustment variable coefficients. The spline parameters are used to generate $f(t)$, which, in turn, produces $d(t)$ and the spot rates.

The HQM methodology is applied to the bond data for each business day, and spot rates are computed for maturities at half-year intervals from ½ year through 100 years. The monthly average spot rates are the arithmetic means of the spot rates at these maturities for all the business days of the month.