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USE OF CONSTRAINED FITS TO IMPROVE MASS RESOLUTION IN PARTICLE PHYSICS

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ABSTRACT

The Hall B Physics Division at Thomas Jefferson National Accelerator Facility (JLab) uses the CEBAF Large Acceptance Spectrometer (CLAS) and offline techniques to identify reactions by detecting all outgoing particles but one; this last one is identified by the missing mass of the reaction. For our purposes, we specifically examined the reaction $e\ p \rightarrow e'\ K^+\ \Lambda(1405)$, where $\Lambda(1405) \rightarrow \Sigma^-\ \pi^+$ and $\Sigma^- \rightarrow (n)\ \pi^-$. Our analysis requires that there is good resolution on the missing mass to minimize background and we achieve a better resolution if a formula is used in which one variable is constrained to the accepted value; for the aforementioned reaction, this would be the neutron mass. We proved this by using two different methods of calculating the reaction's Σ^- mass, either using the experimental neutron mass or the known neutron mass. For both methods, we plotted histograms of the calculations, as well as deriving the propagated error equations. From our methods and analysis, we conclude that there is a smaller experimental error on the Σ^- mass which uses the accepted neutron mass, rather than the experimentally determined neutron mass. In general, this is a technique that can be applied to other physics analyses and shows that a fit constrained by external information yields a better result than an unconstrained fit.

INTRODUCTION

Thomas Jefferson National Accelerator Facility (JLab) in Newport News, Virginia conducts scattering experiments to understand the structure of the sub-nuclear world. Each experimental hall at JLab is equipped with spectrometers to study a wide variety of areas pertaining to particle physics. Hall B uses a large acceptance spectrometer called CLAS, the acronym for CEBAF Large Acceptance Spectrometer. The main difference between CLAS and the other spectrometers at JLab is that CLAS's solid angle is 4π radians, about 2π steradians, and much larger than the solid angles of the other spectrometers. The advantage of a larger solid angle is that multi-particle final states typical of reactions, where excited mesons and baryons are produced, are more easily seen [1]. Such observations and studies are the focus of Hall B's experiments.

CLAS is a magnetic spectrometer; its magnet is comprised of six superconducting coils arranged azimuthally around the electron beamline. The magnetic field inside CLAS is calculated directly from the current flow in the coils. This design is ideal for good momentum resolution over a large solid angle in covering large areas of charged particles in polarized-target experiments. Electronics provide CLAS with its trigger system and data acquisition (DAQ) capabilities. The detectors used in CLAS are arranged circularly around the beamline in successive layers, each of which plays a role in tracking a particle's path [1].

The main use of CLAS is for measuring cross sections for several strange final states; for this project, we studied the specific reaction $e\,p \rightarrow e'\,K^+\ \Lambda(1405)$, where $\Lambda(1405) \rightarrow \Sigma^-\,\pi^+$ and $\Sigma^- \rightarrow (n)\,\pi^-$. The parentheses around the neutron symbol indicate that the neutron is not detected directly, whereas the e', K^+, π^+ and π^- 4-momenta are detected and their energies computed. Detection of hyperons in similar reactions is done by means of the missing mass of the reaction, which increases our statistics and ensures that our acceptance is smooth and easily calculable. In addition, there exists background to the missing mass, which is a common experimental situation, so the goal is to eliminate as much of the background as possible. Graphically, we want to make the missing mass peak as narrow as possible.

One way to improve the resolution on the missing mass is to use a constrained calculation, where the missing energy and momentum of an intermediate state are constrained as per the known missing mass from the Particle Data Group's (PDG) tables. In this calculation, there are two methods used: the first uses the missing momentum and missing energy from the event as the momentum and energy of the neutron; the second uses the event's missing momentum, but uses the accepted value of the neutron's mass instead of the event's missing energy [2]. In using the accepted value for the neutron's mass, we place a mass constraint on the data, thus using our prior knowledge of the neutron's accepted mass.

The motivation for using successive mass constraints is based on eliminating as many background signals as possible and narrowing down the amount of data that fits all the cut requirements. Any signals that do not fulfill all the constraints are cut out of the sample, whereas signals that meet the prerequisites are kept. Therefore, a narrower peak width will have less background contamination than a wider peak width [2].

For this reason, our error analysis sought to prove that more constraints on the data will ultimately generate smaller error than data that does not have as many or no constraints in effect. Specifically, we focused on the calculation of the missing neutron mass and the invariant $n\pi^-$ mass resulting from $\Lambda(1405)$'s decay, although this analysis can apply to any physics analysis. In order to prove our conjecture, we decided to analyze the missing neutron mass, the invariant $n\pi^-$ mass without constraining the neutron mass and the invariant $n\pi^-$ mass after using a neutron mass constraint, both graphically and deriving the propagated error equations.

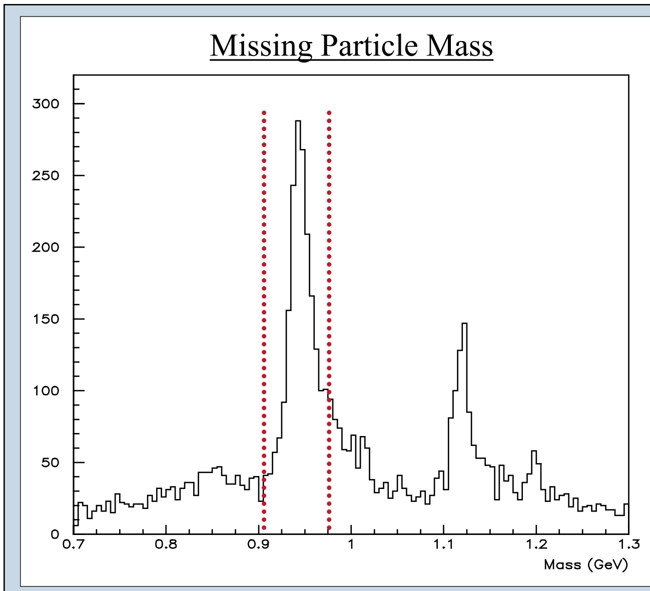

Figure 1. Missing particle mass of the reaction, with the neutron cut marked at 0.915 and 0.98.

METHODS

The first step in finding the invariant $n\pi^-$ mass is to make a cut on the identification of the neutron. To do this, we calculate the missing mass of the reaction from:

$$M^2 = E^2 - P^2 \qquad (1)$$

where M is the missing mass of the reaction, E is the missing energy and P is the missing momentum.

Next, we create a plot of this missing mass to determine what the reaction's missing particle is. As can be seen in Figure 1 [3], the peak at about 0.94 GeV is statistically identified as a neutron, so we place a cut at 0.915 and 0.98 for further analysis.

Since we want to exclusively study the $n\pi^-$ in this project, we define the invariant $n\pi^-$ mass in terms of 4-momenta as:

$$|M_{\Sigma^-}|^2 = |\vec{P}_{\Sigma^-}|^2 = (\vec{P}_{\pi^-} + \vec{P}_n)^2 \qquad (2)$$

where we introduce the shorthand for the Σ^-, π^- and neutron 4-momenta.

In turn, the 4-momentum is defined as:

$$\vec{P} = [E,P] = (E,P_x,P_y,P_z) \qquad (3)$$

where P is the 3-momentum vector and P_x, P_y and P_z are the x-, y- and z- components of the 3-momentum vector respectively.

The most important point to remember is as follows; the 4-momentum of the neutron candidate, called so because it is only statistically identified as a neutron, can be calculated two ways, with and without the mass constraint. These calculations are done only for events with e', K^+, π^+ and π^- detected and the event's missing mass approximately that of a neutron. For our purposes, we found the error in terms of the 4-momentum of each detected particle. We used histograms that were previously made of the missing neutron mass and the two invariant $n\pi^-$ masses for the reaction [3].

We can also explicitly derive the propagated error equations for each method and compare in order to determine which error is smaller, starting with the 4-momentum conservation equation of the situation:

$$\vec{P}_{\Sigma^-} = \vec{P}_{\pi^-} + \vec{P}_n \qquad (4)$$

where we have the 4-momenta of Σ^-, π^- and the neutron.
Next, we square the equation, which gives us:

$$\vec{P}_{\Sigma^-}^{\,2} = (\vec{P}_{\pi^-} + \vec{P}_n)^2 = \vec{P}_{\pi^-}^{\,2} + 2\vec{P}_{\pi^-}\vec{P}_n + \vec{P}_n^{\,2} \qquad (5)$$

Therefore, the general error propagation equation is:

$$\delta(M^2_{\Sigma-}) = 2M_{\Sigma-}\delta M_{\Sigma-} = 2M_{\Sigma-}\sqrt{\sum_i\left(\frac{\partial(M^2_{\Sigma-})}{\partial(P_i)}\delta(P_i)\right)^2} \tag{7}$$

where we have the sum of the squares of all the partial derivatives.

Given the following unconstrained 4-momentum:

$$\vec{P}_{n,unconstr} = \vec{P}_{e-} - \vec{P}_{K+} - \vec{P}_{\pi+} + \vec{P}_{\pi-}$$

we can substitute into Equation 4:

$$\vec{P}_{\Sigma-,unconstr} = \vec{P}_{\pi-} + \left(\vec{P}_{e-} - \vec{P}_{K+} - \vec{P}_{\pi+} - \vec{P}_{\pi-}\right) = \vec{P}_{e-} - \vec{P}_{K+} - \vec{P}_{\pi+} \tag{8}$$

Therefore, squaring Equation 8 will yield $M^2_{\Sigma-}$:

$$\begin{aligned} M^2_{\Sigma-,unconstr} &= \vec{P}^2_{\Sigma-,unconstr} = \left(\vec{P}_{e-} - \vec{P}_{K+} - \vec{P}_{\pi+}\right)^2 \\ M^2_{\Sigma-,unconstr} &= \vec{P}^2_{e-} + \vec{P}^2_{K+} + \vec{P}^2_{\pi+} - 2\vec{P}_{e-}\vec{P}_{K+} - 2\vec{P}_{e-}\vec{P}_{\pi+} + 2\vec{P}_{K+}\vec{P}_{\pi+} \end{aligned} \tag{9}$$

The error on the unconstrained $M^2_{\Sigma-}$ will be the partial derivatives with respect to each momentum term, multiplied by the error on it. These partial derivatives are:

$$\frac{\partial(M^2_{\Sigma-,unconstr})}{\partial(\vec{P}_{e-})}\delta(\vec{P}_{e-}) = \left(2\vec{P}_{e-} - 2\vec{P}_{K+} - 2\vec{P}_{\pi+}\right)\delta(\vec{P}_{e-}) \tag{10a}$$

$$\frac{\partial(M^2_{\Sigma-,unconstr})}{\partial(\vec{P}_{K+})}\delta(\vec{P}_{K+}) = \left(2\vec{P}_{K+} - 2\vec{P}_{e-} + 2\vec{P}_{\pi+}\right)\delta(\vec{P}_{K+}) \tag{10b}$$

$$\frac{\partial(M^2_{\Sigma-,unconstr})}{\partial(\vec{P}_{\pi+})}\delta(\vec{P}_{\pi+}) = \left(2\vec{P}_{\pi+} - 2\vec{P}_{e-} + 2\vec{P}_{K+}\right)\delta(\vec{P}_{\pi+}) \tag{10c}$$

Hence, the total error on the unconstrained invariant $n\pi^-$ mass is:

$$\delta(M^2_{\Sigma-,\,unconstr}) = 2M_{\Sigma-,\,unconstr}\delta M_{\Sigma-,\,unconstr}$$

$$\delta(M^2_{\Sigma-}) = 2M_{\Sigma-,unconstr}\sqrt{\begin{array}{l}\left\{\left(2\vec{P}_{e-} - 2\vec{P}_{K+} - 2\vec{P}_{\pi+}\right)\delta\vec{P}_{e-}\right\}^2 + \\ \left\{\left(2\vec{P}_{K+} - 2\vec{P}_{e-} + 2\vec{P}_{\pi+}\right)\delta\vec{P}_{K+}\right\}^2 + \\ \left\{\left(2\vec{P}_{\pi+} - 2\vec{P}_{e-} + 2\vec{P}_{K+}\right)\delta\vec{P}_{\pi+}\right\}^2\end{array}} \tag{11}$$

where:

$$M_{\Sigma-,\,unconstr} = (\vec{P}_{e-} - \vec{P}_{K+} - \vec{P}_{\pi+})$$

The uncertainty in $M^2_{\Sigma-}$ when the neutron mass is constrained to its known value is given by:

$$M^2_{\Sigma-,\,constr} = (M_n + \vec{P}_{\pi-})^2$$

$$M^2_{\Sigma-,\,constr} = M_n^2 + 2M_n\vec{P}_{\pi-} + \vec{P}^2_{\pi-} \tag{12}$$

Therefore, the partial derivative gives the following:

$$\frac{\partial\left(M^2_{\Sigma-,constr}\right)}{\partial\left(\vec{P}_{\pi-}\right)}\delta\left(\vec{P}_{\pi-}\right) = \left(2M_n + 2\vec{P}_{\pi-}\right)\delta\left(\vec{P}_{\pi-}\right) \tag{13}$$

Hence, the total error is:

$$\begin{aligned} \delta(M^2_{\Sigma-,constr}) &= 2M_{\Sigma-,constr}\sqrt{\left\{\left(2M_n + 2\vec{P}_{\pi-}\right)\delta\left(\vec{P}_{\pi-}\right)\right\}^2} \\ &= 2M_{\Sigma-,constr}\left(2M_n\delta\vec{P}_{\pi-} + 2\vec{P}_{\pi-}\delta\vec{P}_{\pi-}\right) \end{aligned} \tag{14}$$

where:
$M_{\Sigma-,\,constr} = (M_n + \vec{P}_{\pi-})$

RESULTS

Figure 2 illustrates the invariant $n\pi^-$ mass without the neutron mass constraint applied, marking the width of the peak between two dotted lines and Figure 3 shows the invariant $n\pi^-$ mass with the neutron mass constraint applied.

Equation 11 from above is the error on the unconstrained invariant $n\pi^-$ mass:

$$\delta(M^2_{\Sigma-}) = 2M_{\Sigma-,unconstr}\sqrt{\begin{array}{l}\left\{\left(2\vec{P}_{e-} - 2\vec{P}_{K+} - 2\vec{P}_{\pi+}\right)\delta\vec{P}_{e-}\right\}^2 + \\ \left\{\left(2\vec{P}_{K+} - 2\vec{P}_{e-} + 2\vec{P}_{\pi+}\right)\delta\vec{P}_{K+}\right\}^2 + \\ \left\{\left(2\vec{P}_{\pi+} - 2\vec{P}_{e-} + 2\vec{P}_{K+}\right)\delta\vec{P}_{\pi+}\right\}^2\end{array}}$$

where:

$$M_{\Sigma-,\,unconstr} = (\vec{P}_{e-} - \vec{P}_{K+} - \vec{P}_{\pi+})$$

and Equation 14 is the error on the constrained invariant $n\pi^-$ mass:

$$\delta(M^2_{\Sigma-,constr}) = 2M_{\Sigma-,constr}\left(2M_n\delta\vec{P}_{\pi-} + 2\vec{P}_{\pi-}\delta\vec{P}_{\pi-}\right)$$

where:
$M_{\Sigma-,constr} = (M_n + \vec{P}_{\pi-})$

DISCUSSION & CONCLUSIONS

In analyzing the aforementioned results, it is important to remember the purpose of the project. Intuitively, we know that using the neutron mass's known value, rather than the experimental value, should give us less error.

Generally, the accepted rule of a physics analysis is to use all of the information known in order to make a better decision on which direction to proceed in the analysis. Sometimes, this may not help, because it is possible for data to conflict with one another, thus making it difficult to decide which data is correct.

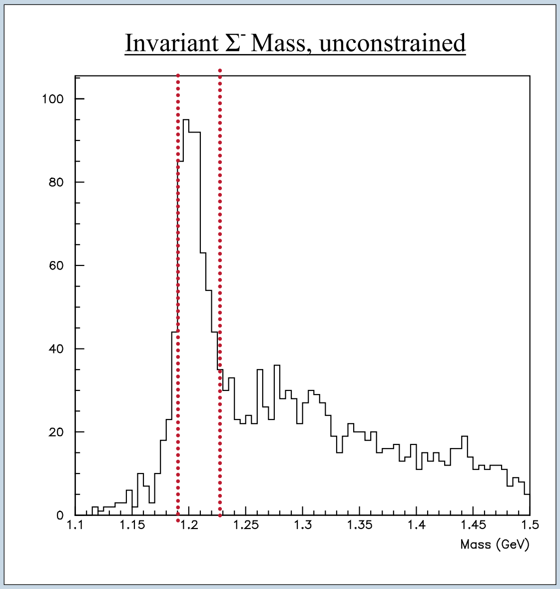

Figure 2. Peak width (1.179 < width < 1.235) of unconstrained invariant $n\pi^-$ mass (between the dotted lines).

Nonetheless, all the prior knowledge is used, since knowing more information about a particular reaction should help to decrease the amount of error.

In comparing Figure 2 and Figure 3, it appears that the constrained invariant mass yields a narrower peak width than the unconstrained invariant mass. In comparing the background of each, it is seen that the constrained invariant $n\pi^-$ has a smaller background, hence it has a better experimental resolution than the unconstrained case.

Although we can show this graphically, it is necessary to prove it mathematically by propagating the error for each situation; our derivations show that the smaller error is indeed on the constrained invariant mass. This is a specific example of a general finding that using external data improves the final results.

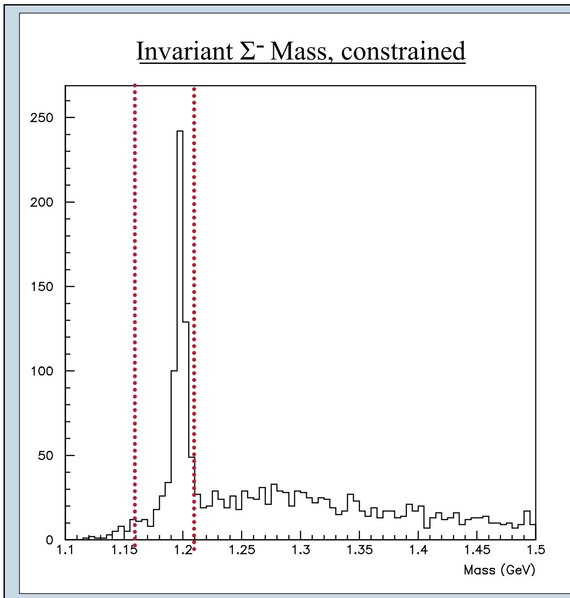

Figure 3. Peak width (1.179 < width < 1.235) of constrained invariant $n\pi^-$ mass (between the dotted lines).

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