HYBRID CALORIMETER ALGORITHM DEVELOPMENT FOR PRIMEX EXPERIMENT

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ABSTRACT

The PrimEx Collaboration seeks to measure the lifetime of the π^0 meson (neutral pion) at high precision. The decay rate of the pion is considered to be the most fundamental prediction of low-energy quantum chromodynamics (QCD). Pions will be produced by the Primakoff Effect: a few GeV photon interacts with the coulomb field of a nucleus to produce a pion. The pion then decays almost immediately (~10^{-16} seconds) into two photons. The decay photons will be detected by an electromagnetic hybrid calorimeter (HYCAL), an array of lead tungstate and lead glass crystals. An algorithm is needed to calculate the angular separation of the two decay photons (and thus the invariant mass of the pion) from the energies deposited in HYCAL. A GEANT Monte Carlo simulation of the experiment is used to test and develop the algorithm to achieve the best angular resolution. The development of the algorithm is essential to the PrimEx project.

INTRODUCTION

Agreement between theoretical predictions and experimental results is the essence of a good theory. The modern theories of quantum physics have survived decades of experiments. Textbooks boast that predictions of quantum theory and their corresponding experimental measurements show an amazing amount of agreement. Yet some of these predictions have not yet been tested with high precision. One of these, the lifetime of the π^0 meson (neutral pion), is a fundamental prediction of low-energy quantum chromodynamics (QCD). All previous experiments have measured this value with levels of precision around 10%. With the modern technology, it has become possible to conduct a higher-precision experiment. The goal of the PrimEx Collaboration is to measure the π^0 lifetime to within approximately 1.4%. The PrimEx experiment thus pins down this important piece of particle data.

QCD is the theory of the nuclear strong force and deals with the interaction of quarks. The familiar proton and neutron (examples of baryons) consist of three quarks, whereas mesons, such as the π^0, consist of two. The π^0 is the lightest of all mesons and usually decays into two high-energy photons (gamma rays). It has an extremely short lifetime of about 10^{-16} seconds, and it is therefore difficult and unfeasible to set up a high-precision experiment to directly measure the time of decay. The PrimEx Collaboration will instead make use of the Primakoff Effect (Primakoff, 1951). The Primakoff Effect is a form of photopion production; a high-energy photon interacts with the coulomb field of a nucleus to produce a π^0 meson (Figure 1). This is essentially the reverse action of the $\pi^0 \rightarrow \gamma\gamma$ decay, and the cross-section of the outgoing π^0 angle (θ_π) can be used to extract the lifetime of the π^0 meson.

The PrimEx experiment will take place in Experimental Hall B at the Thomas Jefferson National Accelerator Facility (Jefferson Lab), and will utilize the Hall B photon tagging system. The photon beam hits a Primakoff target (a thin, meticulously prepared sheet of carbon-12, tin-116, or lead-208) to produce the pions, which almost immediately decay into pairs of photons. These pairs of photons hit the PrimEx Hybrid Calorimeter (HYCAL), a matrix of 663 lead glass Cherenkov counters and 480 lead tungstate ($PbWO_4$) scintillating crystals (Figure 2). HYCAL essentially detects the amount of energy that is deposited in each crystal. The incidence positions of the photons are calculated from this data from HYCAL. The cross section of θ_π is then extracted, and the rest mass of the pion is deduced. The pion lifetime can then be calculated from the spectral width (the energy spread) of the pion rest mass. (The

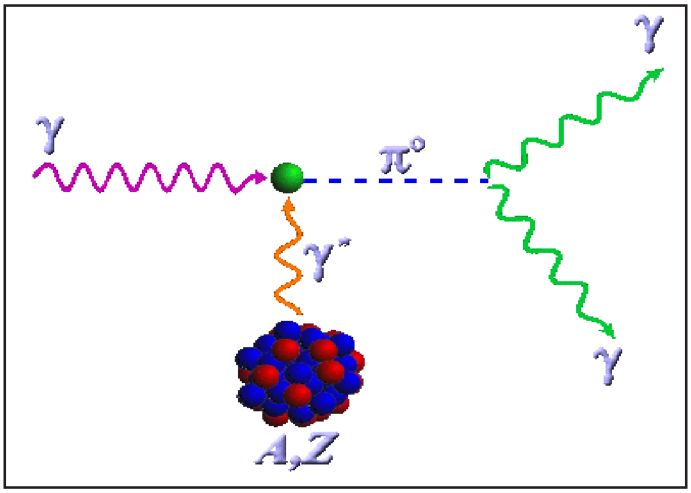

Figure 1. An illustration of the Primakoff Effect
A high-energy photon (γ) interacts with the coulomb field (γ*) of a nucleus (A,Z) to produce a π^0 meson, which then decays into two photons.

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lifetime τ of a decay and the spectral width Γ of the rest mass of the decaying particle are intimately related through the Energy-Time Uncertainty Principle: $\tau \bullet \Gamma = \hbar / 2$.) An algorithm must be developed for the task of reconstructing the incidence positions of the photons on HYCAL from the energy data. Since the goal of the experiment is high precision, it is essential that this algorithm be as accurate as possible.

METHODS

Using GEANT, a physics simulator developed by CERN, a Monte Carlo simulation of the PrimEx experiment was used to test the algorithm for reconstructing the incidence positions of the photons (Figure 3). In the simulation, photons ranging from 0.5 GeV to 6 GeV are directed perpendicularly towards HYCAL. (The x and y directions run along the face of HYCAL while z is the direction of the beam.) In each run, the photon generation point (and thus the photon incidence position) is varied in the x direction from the center of one module to the center of the next module. This is done both for the lead glass counters and for the lead tungstate scintillators. Each run consists of roughly 2000 photon-generation events, and in each event the resulting shower of particles in the detector is tracked down to 0.1 MeV.

When a photon hits the calorimeter, most of the energy is deposited in the modules nearest the position of incidence (Figure 4). Reconstructing the incidence position first requires a cluster-finding algorithm, which identifies the modules whose energies are associated with a particular incident photon. Other members of the PrimEx Collaboration are developing a good cluster-finding algorithm, but a very simple algorithm suffices for this one-photon simulation. The module with the most energy deposited is found, and the cluster consists of that module and the eight directly surrounding it. Thus, in the methods that follow, only a three-by-three matrix of modules is used for the calculations. For simplicity, only the reconstruction of the x-coordinate is shown. Since HYCAL is symmetric along the x and y directions, the y-coordinate is reconstructed in a similar manner.

Two methods of reconstructing the incidence positions of the photons were used in developing the algorithm. The first is the linear method, in which the incidence position is estimated to be the center of gravity of the shower

$$x_{\text{calc}} = \frac{\sum_i w_i x_i}{\sum_i w_i} \qquad (1)$$

where x_i is the x-coordinate of the center of the i-th module and w_i is taken to be E_i, the energy deposited in the i-th module (Awes et al. 1992). The second method uses a logarithmic weighting scheme. The equation is the same as above, but instead of using $w_i = E_i$, the weights to be used are given by the following expression:

$$w_i = \max\left\{0, \left[w_0 + \ln\left(\frac{E_i}{E_{\text{total}}}\right)\right]\right\} \qquad (2)$$

In this equation, E_i is the energy deposited in module i, E_{total} is the total energy in the cluster ($\sum_i E_i$), and w_0 is a free parameter (Awes et al. 1992).

The algorithm developed varies slightly from the equations above. In the calculation of the x-coordinate, for example, the first step is to add the energies of the modules with the same x-value.

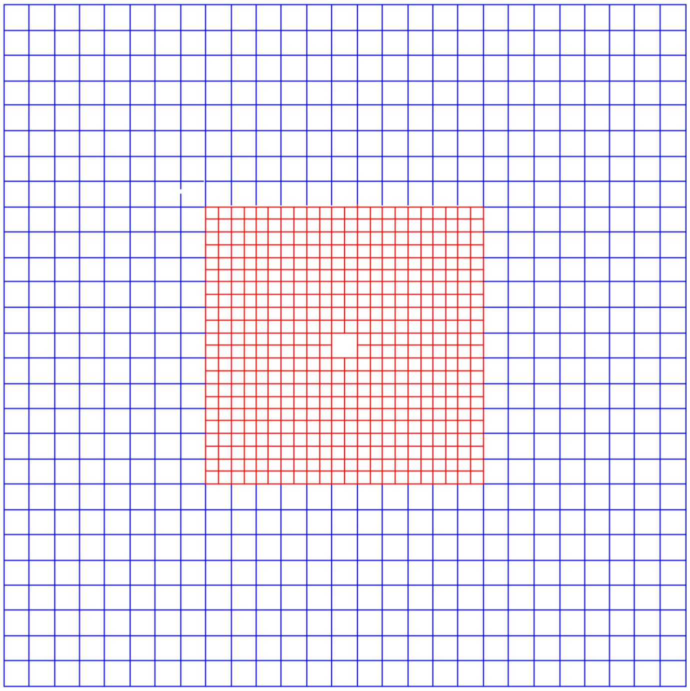

Figure 2. An illustration of the Hybrid Calorimeter (HYCAL)
The front face of HYCAL. The outer modules are lead glass; the inner modules are lead tungstate. The square in the center is the beam hole.

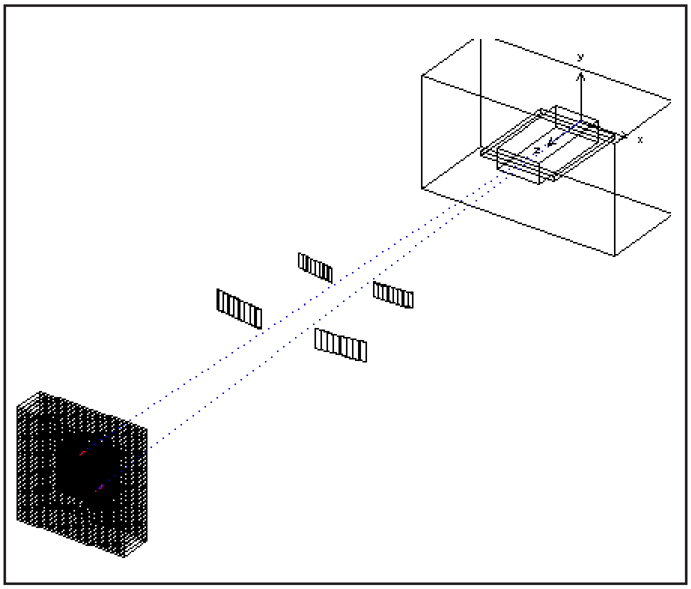

Figure 3. The GEANT setup of the PrimEx experiment.
The target sits at the origin of the axes shown. HYCAL is shown at the bottom left. A typical trajectory of a pair of photons is also shown. The large box near the target is a dipole magnet, which deflects the electron-positron pairs that are produced. Between the dipole and HYCAL are the pair spectrometers which detect the electron-positron pairs and thus monitor the photon beam flux.

Since the cluster (in this study) is a three-by-three matrix, the result is three values of energy: E_{left}, E_{center}, and E_{right}. Equation 1 is then applied to these values. In the linear version, the end result is exactly the same (the two approaches are mathematically the same). The logarithmic version yields an answer that is slightly different, but the algorithm is more efficient: in this case three logarithmic computations instead of nine. It was found that this slightly altered logarithmic method had a negligible effect on the results.

RESULTS

When the calculated position is plotted against the true incidence position, the result for the linear method is a well-known S-curve (Figure 5). The S-shape arises because, in reality, the energy of the particle shower formed in the modules drops off exponentially with distances. The result is that there is not enough weight given to the energies of the outer modules, and the reconstructed value does not change much as the incident position is varied. When the incident position approaches the edge between two crystals, the reconstructed value suddenly approaches the true value (since now there are two crystals with approximately equal energies). In the figure, the units for the axes are half-module lengths away from the center of the module (i.e. –1 is the left edge, 0 is the center, and 1 is the right edge). The logarithmic method yields an S-curve situated nearer the ideal 45-degree line (for which the reconstructed value equals the true value) (Figure 6).

In both the linear and logarithmic cases, a polynomial curve fit was used to straighten out the S-shape, to complete algorithms with the best possible agreement between reconstructed position and true incident positions of the photons. Since we must eventually consider energies other than 4.0 GeV, the final version of the algorithm will most likely have an energy-dependent empirical formula for the curve fit. The polynomial fit is as follows:

$$x_{\text{fit}} = a + b(x_{\text{calc}} - 0.5) + (2 - 4a)(x_{\text{calc}} - 0.5)^2 + (4 - 4b)(x_{\text{calc}} - 0.5)^3 \quad (3)$$

In this equation, x_{fit} is the calculated x-coordinate after the curve fit, x_{calc} is the direct result of the linear or logarithmic methods, and a and b are the two parameters of the fit. These values are for the positive values of x (in the units mentioned above), and it is assumed that the negative values are symmetrical. Simply put, the equation is a third-order polynomial fixed at (0,0) and (1,1) with a as the value of x_{fit} at 0.5 (halfway between the center and the edge) and b as the slope of the curve at this same point. Figure 7 shows the curve fit being applied, and Figure 8 shows the results of straightening out the S-curve.

The linear method and the logarithmic method (using a few different values for w_0) were tested for the resolution of incidence position. In each case, the curve fit was applied and a histogram of $x_{\text{calc}} - x_{\text{true}}$ was taken (Figure 9). When tested on the lead tungstate crystals, the standard deviation for $w_0 = 3.25$ was s = 0.114 (in the relative units) while $\sigma = 0.128$ for the linear method. Since the lead tungstate crystals are 2.125 cm wide, the resolution is approximately 0.12 cm on the face of the calorimeter. HYCAL is 700 cm away from the target, so this position resolution translates to an angular resolution of 0.00017 radians or 0.01 degrees.

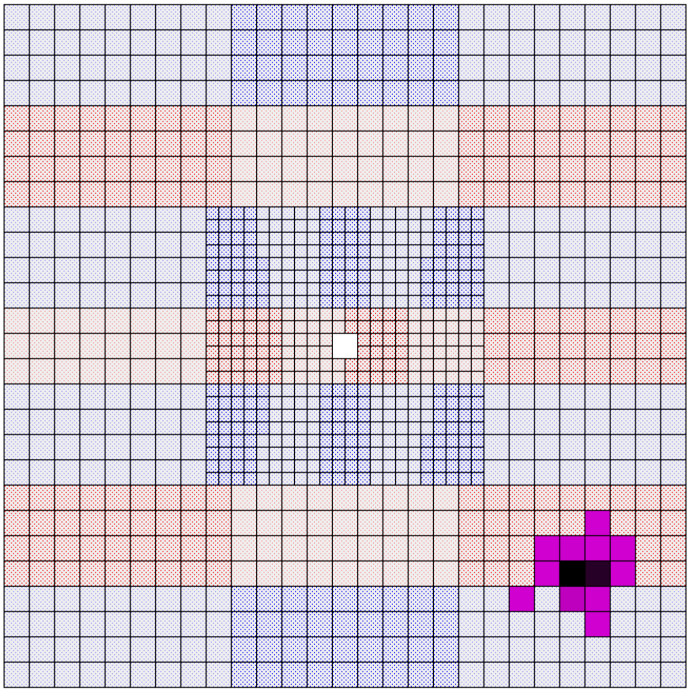

Figure 4. An example of a cluster of deposited energies from a Monte Carlo simulation.
A 4.0 GeV photon was directed at lead glass module (5,5). The modules are numbered starting at the bottom right of each part of the calorimeter.

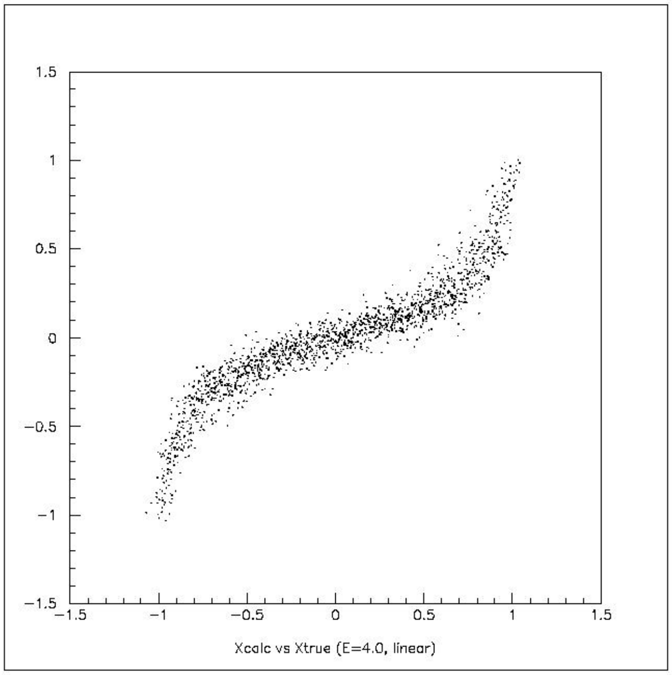

Figure 5. The linear method of incidence position reconstruction. 4.0 GeV photons on lead tungstanate. The units are half-module lengths away from the center of the module (i.e., –1 is the left edge, 0 is the center, and 1 is the right edge).

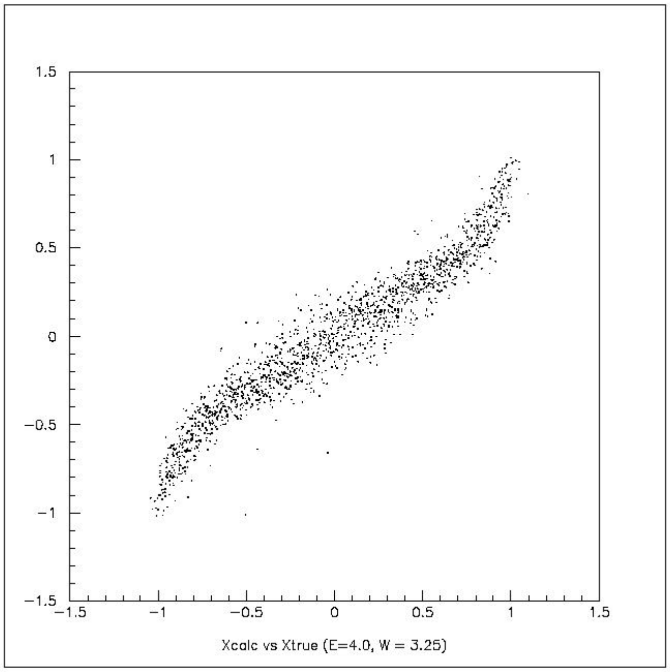

Figure 6. The logarithmic method of incidence position reconstruction. 4.0 GeV photons on lead tungstate. Logarithmic method with w_0= 3.25.

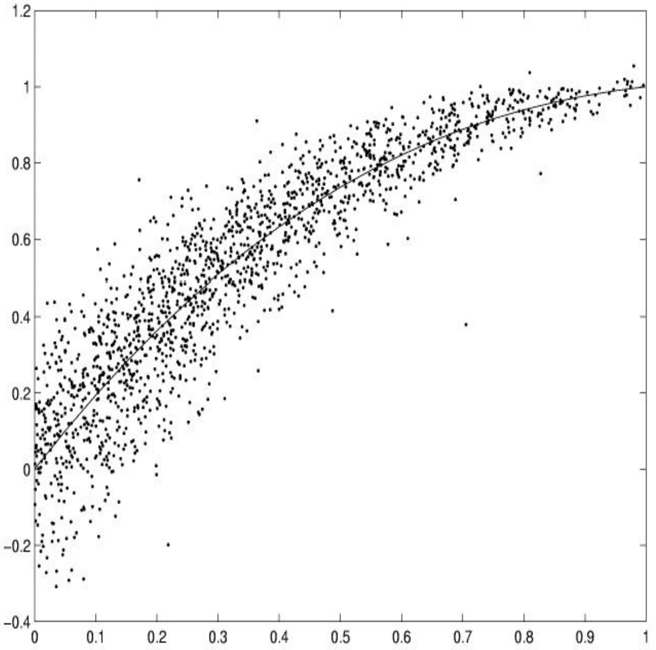

Figure 7: The curve fit applied to the results of the logarithmic method. 6.0 GeV photons on lead glass. Logarithmic method with w_0= 3.25.

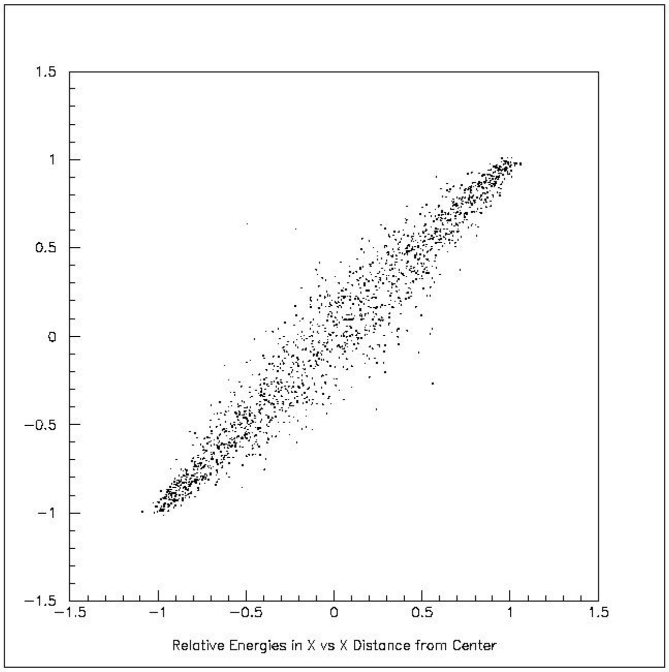

Figure 8. The results of applying the curve fit to the logarithmic method 4.0 GeV photons on lead tungstate. Logarithmic method with w_0= 3.25.

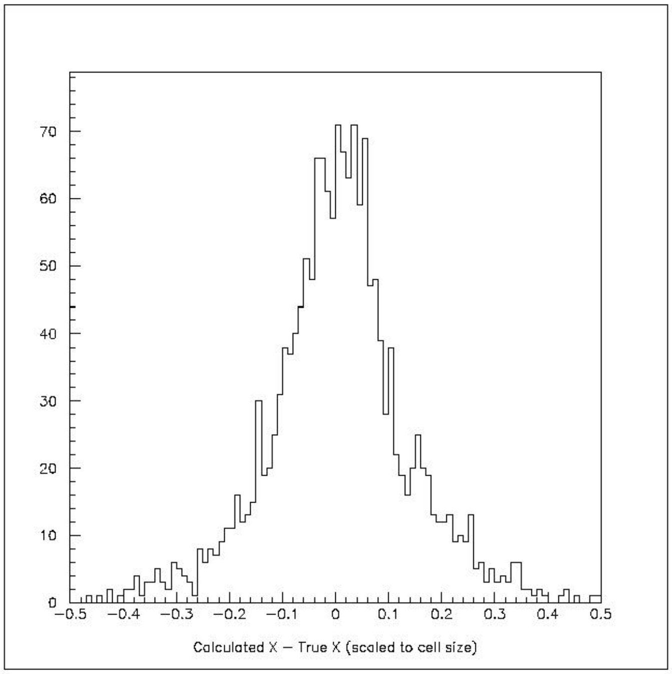

Figure 9. The histogram for $x_{calc} - x_{true}$ after applying the curve fit. 4.0 GeV photons on lead tungstate. Logarithmic method with w_0= 3.25.

DISCUSSION AND CONCLUSION

Based on the results, the final algorithm should have the following steps: A good cluster-finding algorithm should be followed by adding the “columns” or “rows” of energies for the calculations of the x-coordinate and y-coordinate respectively; the logarithmic method of reconstructing the incidence position should be implemented with an appropriate value of w_0; and an empirical equation based on the results of curve-fit testing should be applied to give the final reconstructed incidence positions. From these positions, the rest mass of the π^0 will be determined, which will be used to extract the θ_π cross-section.

The algorithm is by no means in its finished state. Many more tests will need to be done to further develop the algorithm in order to yield the best resolution for position reconstruction. First, the best values for w_0 must be found for both lead tungstate and lead glass modules. The value of 3.25 seems to be relatively good for the lead tungstate, but more testing needs to be done. Second, the empirical formula to “straighten out” the results from the logarithmic method must be realized. The formula must be energy-dependent and will differ for the lead tungstate and lead glass cases. The empirical formula will be based on the two parameters (*a* and *b*) mentioned above, and so far there seems to be a smooth dependence of these parameters on the energy (Figure 10). After these tests are done, an actual Monte Carlo simulation of the Primakoff Effect will be performed to determine the resolution of HYCAL.

There are yet other factors that must be taken into consideration. In the current simulation, light attenuation within the crystals is not yet taken into account. Cherenkov light produced at the plane of the detector, for example, will actually yield a smaller signal than the same amount of light produced near the end of the module (nearer the photomultiplier tubes). The effect of light attenuation on the resolution should be determined. It is also uncertain, as yet, whether the plane of the lead tungstate insertion array will be flush with the lead glass array. It should be determined whether having the lead tungstate array recessed by a few centimeters will affect or even improve the resolution.

Finally, since the GEANT simulation cannot match reality perfectly, the detector must actually be tested in the line of the photon beam to see what adjustments must be made to the position-reconstruction algorithm.

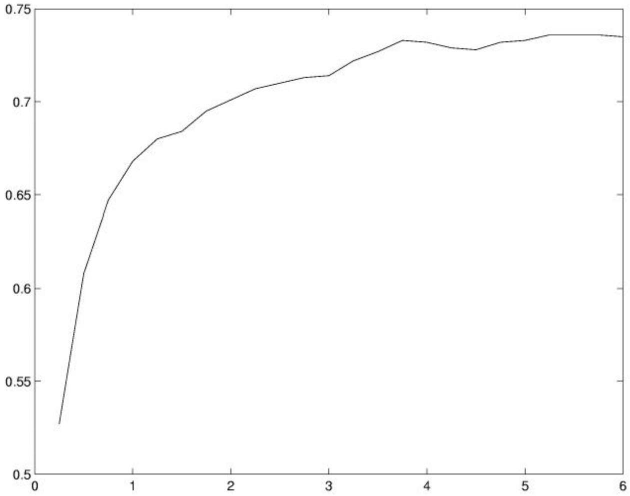

Figure 10. Energy dependence for parameter *a* in the empirical formula. Value of parameter *a* versus photon energy (in GeV). Results are for lead glass.

ACKNOWLEDGMENTS

My thanks go to Ashot Gasparian, my mentor at Jefferson Lab, and Aron Bernstein, my research advisor at MIT. I would also like to thank Tong-Uk Lee at MIT and David Lawrence at Jefferson Lab for their help. Thanks also to Janet Tyler, Stacy Ring, Leigh Ann Garza, and Steve Gagnon for their help during the summer at Jefferson Lab. Lastly I thank the Office of Science at the United States Department of Energy for giving me the opportunity to participate in the Energy Research Undergraduate Laboratory Fellowship program and the National Science Foundation for help in the organization of the program.

The research in this paper was conducted at the Thomas Jefferson National Accelerator Facility under the United States Department of Energy’s Energy Research Undergraduate Laboratory Fellowship program and at the Massachusetts Institute of Technology under its Undergraduate Research Opportunities Program.

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