NOAA Technical Report NOS 74 NGS 9

Survey of the McDonald Observatory Radial Line Scheme by Relative Lateration Techniques

Rockville, Md.
June 1978

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Survey

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NOAA geodetic publications

Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1974, reprinted 1975, 1976, 12 p. (PB265442). National specifications and tables show the closures required and tolerances permitted for first-, second-, and third-order geodetic control surveys.

Specifications to Support Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1975, reprinted 1976, 30 p. (PB261037). This publication provides the rationale behind the original publications, "Classification, Standards of Accuracy, ...".

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Juanita M. Kreps, Secretary
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SURVEY OF THE McDONALD OBSERVATORY RADIAL LINE SCHEME BY RELATIVE LATERATION TECHNIQUES

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ABSTRACT. During May and June 1977, the National Ocean Survey/National Geodetic Survey (NOS/NGS) performed a special survey in the vicinity of the University of Texas McDonald Observatory. This was the initial phase of an extensive geodetic-geophysical study to detect any secular or episodic motions of the observatory relative to prominent topographic features within a region extending as far as 100 km from the observatory.

An important part of the study plan is the monitoring, by periodic resurveys, of any changes in the lengths of a radial pattern of lines that are as long as 93 km. A method of relative lateration, the "ratio method," using electromagnetic distance measurements is being tested. Independent May and June measurements were consistent to the level of a few parts in 10^7, the largest discrepancy amounting to only 10 mm on a 52-km line, approximately 0.2 parts per million.

This paper contains descriptive information about the methods employed in the collection, reduction, and analysis of the survey data, tabulations of the observational data, and the numerical and interpretive results of our analyses.

INTRODUCTION

During May and June 1977, the National Ocean Survey/National Geodetic Survey (NOS/NGS) performed a special survey in the vicinity of the University of Texas McDonald Observatory. The survey was funded jointly by the National Aeronautics and Space Administration (NASA) and NGS, and was the initial phase of an extensive geodetic-geophysical study to detect any secular or episodic motions of the McDonald Observatory relative to prominent topographical features within a distance of approximately 100 km.

The McDonald Observatory has been utilized regularly, since 1969, for Lunar Ranging Experiment (LURE) observations (Bender et al. 1973). Among the important geodetic-geophysical goals of the LURE program are the detection and measurement of contemporary plate motions. Lunar ranging measurements made at the McDonald Observatory, located on the North American plate, will be combined with similar measurements made at the University of Hawaii LURE Observatory (Carter and Williams 1973) located on Mt. Haleakala, Maui, on the Pacific Plate. If the contemporary plate motions approximate the long-term rates, the two observatories should be moving relative to one another with a velocity of several centimeters per year. Before any detected motion can be ascribed to motions of the plates (continental drift), any local and regional effects must be accounted for (Carter et al. 1977).

McDONALD OBSERVATORY TECTONIC SETTING

The McDonald Observatory is located at an elevation of 2066 m at the summit of Mt. Locke in the Davis Mountains of western Texas. Preliminary studies of the McDonald Observatory tectonic environment by the University of Texas Marine Science Institute, Geophysical Laboratory (Dorman and Latham 1976), found no evidence of active faulting to the east or in the immediate vicinity (5 to 10 km) of the Observatory. However, they did find evidence of moderate seismic activity in one or more active rift zones to the west. The strongest activity appears to be in Mexico, more than 100 km from the observatory, but there is also some evidence of it in the Chispa Valley, within approximately 30 km of the observatory. The Geophysical Laboratory has begun the installation of several geophysical monitoring instruments, including an array of seismometers and tilt meters, to produce a detailed record of the seismic activity and to develop a better understanding of the crustal structure within the region of interest.

RADIAL SURVEY SCHEME

Figure 1 is a schematic depiction of the radial line Electromagnetic Distance Measurement (EDM) survey scheme established to monitor the relative displacements of selected surface features within approximately 100 km of the observatory. The method employs a system of lines radiating from a single central station. Each line serves as a strain gage and provides information about strain or displacement along the line only. Similar patterns have been used by other investigators in other crustal deformation and fault monitoring studies. However, there are important constraints and goals to be considered in this survey:

- Several of the lines are exceptionally long, with the longest reaching 93 km.

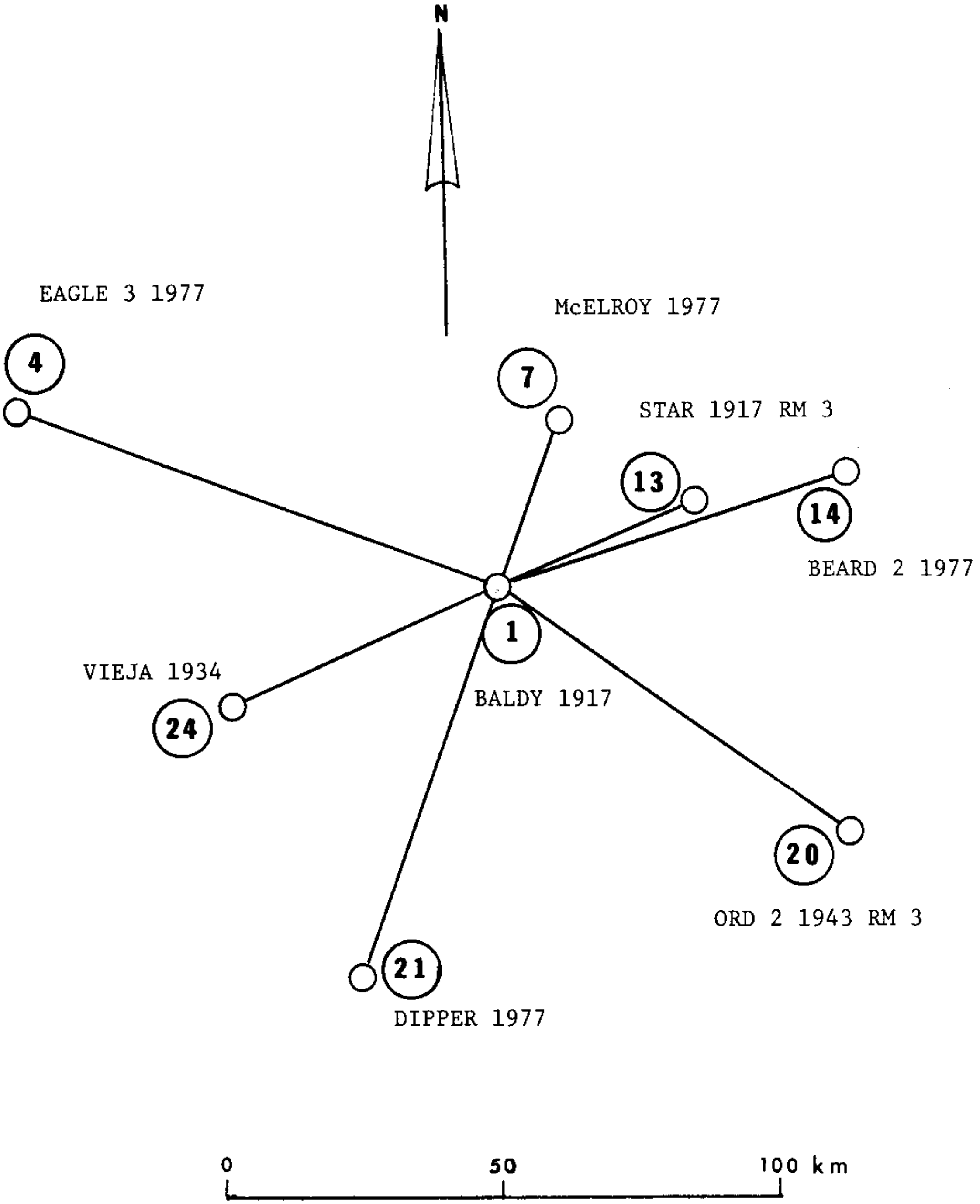

Figure 1.--Radial line electromagnetic distance measurement survey scheme.

- The desired resolution in sensing motions within the scheme is a few centimeters, which corresponds to a few parts in 10^7 for the long lines.

- The survey method should yield the highest possible resolution, with currently operational instrumentation, at the lowest possible cost.

RATIONALE FOR SELECTING THE RATIO METHOD

Several approaches were considered, including the use of balloon-borne meteorological instruments tethered along the lines of sight, the use of geometrically strong closed figures with many repetitions of the measurements, and the use of aircraft to fly meteorological instruments along the lines of sight concurrently with the EDM observations. All these methods were rejected because of projected inadequate resolution or excessive cost. During the period that the various methods were being considered, the Project Manager, W. E. Carter, attended a briefing at Defense Mapping Agency Headquarters, at which K. D. Robertson, of the U.S. Army Engineer Topographic Laboratories (ETL), reviewed some results he had achieved with a relative lateration technique that he referred to as the "Ratio Method" (Robertson 1972, 1975). Robertson's experience suggested that the ratio method could prove to be a powerful method for detecting changes in line lengths, providing improvement over conventional lateration by a factor 3 to 10.

Methods of reducing and analyzing observational data collected by the ratio method were also available (Vincenty 1973, 1974, 1975).

The ratio method is based on the supposition that, while the effective indices of refraction along various lines radiating from a station may be significantly different, the temporal changes in them will tend to be similar, and their ratios will remain relatively constant over the range of atmospheric conditions under which measurements are normally conducted. Repeat observations at different epochs may yield changes in the apparent lengths of lines of a few parts per million (ppm), even after application of the best available refraction corrections. But, if all the lines are observed on time according to an observing schedule that results in the mean times of measurements being nearly equal for all lines, the measured line lengths will all tend to vary proportionally. It is then convenient to think in terms of distance ratios and to regard the scale as a parameter that varies with time.

The McDonald survey project appeared to be an excellent candidate for the application of the ratio method because:

- The regional elevation is high, with the selected stations in the radial scheme ranging from 1532 to 2555 m above sea level.

- Mt. Livermore is an excellent site for the central station. The summit (elevation 2555 m) is a barren knob of granite which affords an unobstructed view of many other peaks in the area.

- There are enough peaks within the area so that the remote ends of the lines could be placed on elevated points, resulting in good ground clearance along even the longest lines.

- The terrain and vegetation are similar throughout the area with no major discontinuities.

The decision was made to use the ratio method during the first survey, but to approach it as an unproven research method that might not be used in subsequent surveys. To provide a basis for evaluating the success or failure of the method, it was decided that at least two observational programs would be performed during the first survey. Initial measurements were made in May. The party then performed other aspects of the McDonald survey and returned to the radial scheme to make a second set of measurements during June. The plan assumed that no detectable motion would occur within the scheme during a period of a few weeks and that any variations between the results of the two sets of measurements would indicate deficiencies in the method or its implementation.

OBSERVATIONAL PROCEDURES

The primary EDM instrument used for this project is a Model 4 Geodimeter that has been modified extensively by George B. Lesley, at the NGS Instrument and Equipment Branch at Corbin, Va. The modifications included the installation of a 10 milliwatt HeNe laser to increase the maximum range of the instrument. Some measurements were also made with Model 8 Geodimeters. The operation of the instruments was carefully checked throughout the survey, and special precautions included the use of an auxiliary frequency standard and counter to verify the internal frequency standards of the instruments during the observation periods.

The observing routine was similar to that used for horizontal direction observations. A typical chronological pattern of the measurements was, for example: lines 7, 13, 21, 23, 23, 21, 13, 7. The advantage of such a schedule is that, assuming a nearly constant observing rate, the mean epochs of the observations will be approximately equal for all lines. In a few cases, field conditions caused deviations from the routine, thus upsetting the symmetry of the round.

Even with the extended range of the modified Geodimeter, there were not enough retroreflectors to measure all the lines during a single observing period. The longest line required 86 reflectors. There were also personnel constraints and the need to limit the time required to complete a round of measurements. The lines were divided into smaller groups, or subsets, to be measured during separate "setups." Four two-man teams were ferried by helicopter to the remote stations of each setup. They tended the retroreflector arrays and measured and recorded meteorological data and vertical angles while the EDM observations were in progress.

The philosophy used in the selection of lines to be included in each group was the following:

- Line 13, BALDY to STAR RM 3, was selected as the primary line because both terminals are located in exposed base rock of the Davis Mountains and the line is expected to remain essentially fixed over the time span of the study. As the primary line, it is included in all groups and is considered to be of invariable length in analyzing the measurements.

- Line 7, BALDY to McELROY, which also lies entirely within the Davis Mountains and is therefore expected to be relatively stable, was chosen as the secondary reference line and also included in all groups. By inclusion of lines 7 and 13 in each group, a check ratio is available for each setup.

- Additional lines were selected for inclusion in groups to maximize the sensitivity of each group for detecting and measuring "expected" motions, based on the preliminary geophysical model of the area.

SUPPORTIVE MEASUREMENTS

In order to aid in evaluating and understanding the results of the ratio method, certain supportive and complementary measurements were made that might not have been made if the method were accepted practice.

The elevations of all the radial line stations were determined by a combination of EDM and reciprocal vertical angle measurements between them and existing elevation bench marks. The pertinent data, formulas, and results are summarized in appendix D.

Reciprocal vertical angles were also observed between BALDY, the central station of the radial scheme, and the remote stations, concurrently with the EDM observations. The purpose of these measurements was to obtain the coefficient of refraction for use with the distance reductions, independently from their determinations from meteorological data alone. The use of the reciprocal vertical angle data is discussed in detail in appendix C. Horizontal directions were also measured.

DATA REDUCTION AND ANALYSIS

The observed data were processed through the routine field and Horizontal Network Branch computer coding, checking, and computation procedures, which resulted in the formation of a computer readable data set containing a minimum of errors. This basic data set was used, along with supplemental data not used in the routine reduction and analysis of lateration observations, to generate another data set with the correct form and content for use with NGS program HAVAGO (Horizontal And Vertical Adjustment of Geodetic Observations). Program HAVAGO was developed by T. Vincenty for the analysis of special purpose surveys in which it is desirable to combine horizontal, vertical, astronomic, and EDM observations in a three-dimensional adjustment.

Individual observations, arranged by groups, are listed in appendix A. Appendix B gives the mean values used as input in the adjustments. Appendix C contains explanatory information and formulas used to reduce the observed EDM data to measured line lengths.

Since no astronomic data were available for the stations in the radial line scheme, the adjustments are not rigorous within the meaning of three-dimensional geodesy. The astronomic latitudes and longitudes were set equal to the geodetic values. The published geodetic latitude and longitude of station BALDY were held fixed. A previously determined geodetic azimuth from BALDY to STAR RM 3 was used for directional orientation. Bench mark elevations were assumed to represent heights above the ellipsoid and were held fixed. None of these assumptions affect the validity of the adjustments for the purpose of this project.

Each distance measurement was given an a priori standard error of ±0.015 m ±0.4 ppm, which was divided by $\sqrt{n}$ when n measurements were meaned to form a determination. The ratio concept is introduced into the adjustments by assigning a scale unknown to each group of distance observations and adjusting the groups to a common scale which may be supplied from an independent source or determined in the adjustment.

Several adjustments were performed during the analytical phase of this project. An initial adjustment was made using the entire set of distances reduced using coefficients of refraction computed from meteorological data. The distance from BALDY to STAR RM 3, line 13, was then fixed for all subsequent adjustments and thus defines the scale. Individual adjustments for the May and June surveys were performed in order to determine the repeatability of the measurements. Tables 1 and 2 contain a summary of the results.

Combined and separate adjustments were also made of the set of distances reduced using coefficients of refraction computed from the vertical angle data. The results are given in tables 3 and 4.

Table 1.--Adjusted positions (coefficient of refraction by atmospheric observations)

Station	Latitude	Longitude	Height	
1 BALDY 1917	30°38'07".61900	104°10'23".61100	2554.72	±0.10
4 EAGLE 3 1977	30 55 16.35537	105 05 05.80801	2283.78	0.07
7 MC ELROY 1977	30 54 32.76664	104 03 46.53811	1991.26	0.15
13 STAR 1917 RM 3	30 46 39.06742	103 47 44.06772	1933.56	0.06
14 BEARD 2 1977	30 49 39.11128	103 31 11.46253	1531.68	0.21
20 ORD 2 1943 RM 3	30 14 23.62659	103 30 54.85369	2060.43	0.04
21 DIPPER 1977	30 00 16.87132	104 26 07.79627	1861.47	0.36
24 VIEJA 1934	30 27 09.22151	104 40 40.30080	1982.14	0.12

Table 2.--Adjusted distances (coefficient of refraction by atmospheric observations)

To	Slope Distance (m)		May	June	Difference m	Difference ppm
4	92882.041 ±	0.009	~~0~~1.044	~~0~~1.033	-0.011	-0.1
7	32138.982	0.003	.981	.984	0.003	0.1
13	39476.328	0.003	*	*		
14	66128.125	0.012	.127	.125	-0.002	0.0
20	76957.135	0.013	.131	.138	0.007	0.1
21	74361.946	0.012	.945	.949	0.004	0.1
24	52518.349	0.010	.354	.344	-0.010	-0.2

*Fixed line.

Table 3.--Adjusted positions (coefficient of refraction by vertical angles)

Station	Latitude	Longitude
1 BALDY 1917	30°38'07".61900	104°10'23".61100
4 EAGLE 3 1977	30 55 16.35528	105 05 05.80771
7 MC ELROY 1977	30 54 32.76674	104 03 46.53807
13 STAR 1917 RM 3	30 46 39.06742	103 47 44.06772
14 BEARD 2 1977	30 49 39.11139	103 31 11.46217
20 ORD 2 1943 RM 3	30 14 23.62670	103 30 54.85387
21 DIPPER 1977	30 00 16.87134	104 26 07.79626
24 VIEJA 1934	30 27 09.22156	104 40 40.30066

Table 4.--Adjusted distances (coefficient of refraction by vertical angles)

To	Slope Distance (m)		May	June	Difference	
					m	ppm
4	92882.032	± 0.011	~~0~~.025	~~0~~.044	0.019	0.2
7	32138.985	0.004	.982	.990	0.008	0.2
13	39476.328	0.000	*	*		
14	66128.136	0.013	.136	.136	0.000	0.0
20	76957.129	0.014	.130	.141	0.021	0.3
21	74361.946	0.013	.939	.955	0.016	0.2
24	52518.345	0.011	.347	.343	-0.004	-0.1

*Fixed line

LINE LENGTHS, RATIOS, AND SCALE

When the means of all distance measurements are formed for each line, they agree closely with the lengths obtained by the adjustments. This is not surprising because:

- The scale derived by the adjustments is an average one as determined by a least-squares adjustment.
- The observing schedules and numbers of repetitions resulted in a reasonably balanced sampling of observing conditions for all lines.

EDM observations made at night tend to yield shorter line lengths than those obtained in the daytime, the differences sometimes amounting to several parts per million. This is evident from table 5 where, e.g., group 6, observed at night, and group 7, during the day, differ in scale by more than 2 ppm. A closer inspection of table 5 reveals that, in general, the daytime measurements received large negative changes in scale, and the nighttime measurements received somewhat smaller positive changes in scale. The differences in the corrections do not mean that the scale of the night observations is better, but result only because most of the measurements were made at night.

A typical example of the rates of change of scale experienced during the McDonald survey is given in table 6. The table contains the results of repeated measurements of a group of three lines.

Table 5.--Scale corrections (ppm)

Group	Day	Time	k by atmospheric observations		k by vertical angles	
1	136	2400	0.77	± 0.18	0.66	± 0.19
2	137	0200	0.59	0.19	0.62	0.21
3	138	1800	-1.38	0.17	-1.38	0.19
4	138	2015	-0.64	0.17	-0.65	0.19
5	138	2215	0.55	0.16	0.50	0.18
6	138	2400	0.84	0.22	0.64	0.24
7	139	1800	-1.46	0.18	-1.42	0.20
8	139	2030	-0.17	0.16	-0.19	0.18
9	139	2300	0.48	0.16	0.49	0.17
10	140	1930	-0.60	0.38	-0.42	0.39
11	140	2330	0.04	0.19	0.16	0.20
12	141	0200	0.10	0.17	0.15	0.18
13	164	2215	0.31	0.16	0.30	0.18
14	164	2400	0.69	0.19	0.66	0.20
15	165	2115	-0.18	0.38	-0.31	0.39
16	166	1500	-1.22	0.26	-0.96	0.27
17	166	2300	0.42	0.18	0.38	0.19
18	167	0130	0.79	0.15	0.76	0.16
19	167	1945	-0.42	0.19	-0.23	0.20
20	167	2030	0.23	0.20	0.48	0.21
21	167	2200	0.26	0.25	0.36	0.27

Notice that the line lengths changed by about 2 ppm in 4 hours. However, during the same period of time, the ratios remained nearly constant, changing by only a few parts in 10^7. The tabulated changes in ratios have been right-justified 6 digits for convenience of presentation.

Since the ratios are not affected by uniform changes of scale, the choice of scale is unimportant for this survey. The scale adopted is an average one determined by a least-squares adjustment, and the reader is again reminded that the tabulated line lengths may be systematically in error by as much as 1 or 2 ppm.

Table 6.--Example of change of scale with time

Time	Line	Distance *	Change m	Change ppm	Ratio	Change Ratio	Change ppm
1800	7	32139.038			1.000 000 00		
	13	39476.377			1.228 299 90		
	21	74362.057			2.313 761 13		
2020	7	32139.002	-0.036	-1.1	1.000 000 00		
	13	39476.342	-0.035	-0.9	1.228 300 18	0.28	0.23
	21	74361.972	-0.085	-1.1	2.313 761 08	-0.05	-0.02
2210	7	32138.979	-0.059	-1.8	1.000 000 00		
	13	39476.296	-0.081	-2.1	1.228 299 63	-0.27	-0.22
	21	74361.917	-0.140	-1.9	2.313 761 02	-0.11	-0.05

*Mean of two measurements.

RESULTS AND CONCLUSIONS

In this first NGS operational field test of the ratio method, the results have equaled or exceeded our most optimistic expectations. The largest discrepancy occurred on line 24 and amounted to only 10 mm or 0.2 ppm. If these results prove to be at all typical of the ratio method, it is indeed a powerful tool that will find wide application.

Some interesting results concerning specific facets of the project also deserve comments. Atmospheric refraction corrections computed from meteorological data and an atmospheric model gave slightly better results than those computed from vertical angle data. This finding should not be generalized to conclude that the vertical angle method will not prove useful in other surveys.

The proportional component of the standard error of a measurement with a Geodimeter in the relative mode appears to be smaller than 0.4 ppm. The average standard error of the scale corrections listed in table 6 is only 0.2 ppm. The quadratic means of the residuals for the lines vary between 10 and 20 mm.

RECOMMENDATIONS

It is recommended that the ratio method be used during the next resurvey of the McDonald radial line scheme, presently planned for completion during the spring of 1979.

In preparation for future surveys at McDonald and other similar projects, the NGS should develop standard operating procedures for all phases of the ratio method, including appropriate recording and computation forms.

APPENDIX A. LISTING OF INDIVIDUAL EDM DATA

DISTANCE MEASUREMENTS

1 MEASUREMENT NUMBER.

2 FOREPOINT.

3 GROUP.

4 DAY.

5 TIME.

6 DISTANCE CORRECTED FOR REFRACTIVE INDEX AND REDUCED TO MARKS.

7 SUM OF BEAM CURVATURE AND SECOND VELOCITY CORRECTIONS (FITTED K).

8 INDEX RATE CORRECTION (FITTED K).

9 CORRECTED DISTANCE (SUM OF 6, 7, AND 8).

10 DISTANCE CORRECTED FOR BEAM CURVATURE AND SECOND VELOCITY USING A STANDARD VALUE OF K = 0.18 (FOR COMPARISON ONLY).

1	2	3	4	5	6	7	8	9	10
1	4	1	136	2312	92882.197	-0.184	0.001	92882.014	92881.927
2	4	1	136	2326	92882.161	-0.184	0.001	92881.978	92881.891
3	7	1	136	2353	32138.962	-0.008	0.001	32138.955	32138.951
4	13	1	137	0017	39476.297	-0.015	0.002	39476.284	39476.276
5	13	1	137	0031	39476.284	-0.015	0.002	39476.271	39476.263
6	7	1	137	0045	32138.955	-0.008	0.002	32138.949	32138.944
7	4	1	137	0104	92882.180	-0.191	0.001	92881.990	92881.910
8	4	2	137	0127	92882.121	-0.188	0.001	92881.934	92881.851
9	7	2	137	0146	32138.980	-0.008	0.001	32138.973	32138.969
10	13	2	137	0157	39476.311	-0.015	0.002	39476.298	39476.290
11	13	2	137	0204	39476.336	-0.015	0.002	39476.323	39476.315
12	7	2	137	0213	32138.980	-0.008	0.001	32138.973	32138.969
13	4	2	137	0231	92882.173	-0.185	0.001	92881.989	92881.903
14	21	3	138	1653	74362.128	-0.084	0.004	74362.048	74361.990
15	13	3	138	1727	39476.397	-0.013	0.002	39476.386	39476.376
16	7	3	138	1739	32139.039	-0.007	0.001	32139.033	32139.028
17	4	3	138	1756	92882.311	-0.162	0.001	92882.150	92882.041
18	4	3	138	1812	92882.310	-0.156	0.001	92882.155	92882.040
19	7	3	138	1833	32139.047	-0.006	0.001	32139.042	32139.036
20	13	3	138	1844	39476.379	-0.012	0.001	39476.368	39476.358
21	21	3	138	1854	74362.143	-0.081	0.003	74362.065	74362.005
22	21	4	138	1917	74362.086	-0.082	0.004	74362.008	74361.948
23	13	4	138	1938	39476.374	-0.013	0.002	39476.363	39476.353
24	7	4	138	1945	32139.014	-0.006	0.001	32139.009	32139.003
25	4	4	138	2009	92882.368	-0.159	0.001	92882.210	92882.098
26	4	4	138	2047	92882.233	-0.161	0.001	92882.073	92881.963
27	7	4	138	2054	32139.000	-0.007	0.001	32138.994	32138.989
28	13	4	138	2101	39476.331	-0.013	0.002	39476.320	39476.310
29	21	4	138	2112	74362.017	-0.083	0.004	74361.938	74361.879
30	21	5	138	2119	74362.021	-0.082	0.004	74361.943	74361.883
31	13	5	138	2132	39476.327	-0.013	0.002	39476.316	39476.306
32	7	5	138	2140	32138.989	-0.006	0.001	32138.984	32138.978
33	4	5	138	2153	92882.152	-0.158	0.001	92881.995	92881.882
34	4	5	138	2217	92882.129	-0.176	0.001	92881.954	92881.859
35	7	5	138	2239	32138.980	-0.007	0.001	32138.974	32138.969
36	13	5	138	2255	39476.288	-0.014	0.002	39476.276	39476.267
37	21	5	138	2307	74361.978	-0.091	0.004	74361.891	74361.840
38	21	5	138	2323	74361.988	-0.093	0.005	74361.900	74361.850
39	13	6	138	2331	39476.292	-0.014	0.002	39476.280	39476.271
40	7	6	138	2339	32138.982	-0.007	0.001	32138.976	32138.971
41	4	6	138	2350	92882.145	-0.182	0.001	92881.964	92881.875
42	7	6	139	0023	32138.965	-0.007	0.001	32138.959	32138.954
43	7	6	139	0035	32138.965	-0.007	0.001	32138.959	32138.954

1	2	3	4	5	6	7	8	9	10
44	20	7	139	1657	76957.359	-0.090	0.002	76957.271	76957.206
45	13	7	139	1713	39476.406	-0.013	0.002	39476.395	39476.385
46	7	7	139	1724	32139.029	-0.006	0.001	32139.024	32139.018
47	7	7	139	1751	32139.024	-0.007	0.001	32139.018	32139.013
48	13	7	139	1758	39476.399	-0.013	0.002	39476.388	39476.378
49	20	7	139	1809	76957.353	-0.091	0.002	76957.264	76957.200
50	13	7	139	1823	39476.385	-0.013	0.002	39476.374	39476.364
51	7	7	139	1836	32139.016	-0.007	0.001	32139.010	32139.005
53	7	8	139	1913	32139.014	-0.007	0.001	32139.008	32139.003
54	13	8	139	1919	39476.376	-0.013	0.002	39476.365	39476.355
55	20	8	139	1931	76957.256	-0.093	0.002	76957.165	76957.103
56	20	8	139	2037	76957.217	-0.104	0.002	76957.115	76957.064
57	20	8	139	2049	76957.205	-0.104	0.002	76957.103	76957.052
58	4	8	139	2055	92882.250	-0.182	0.001	92882.069	92881.980
59	4	8	139	2104	92882.246	-0.182	0.001	92882.065	92881.976
60	4	8	139	2113	92882.237	-0.165	0.001	92882.073	92881.967
61	4	8	139	2128	92882.206	-0.165	0.001	92882.042	92881.936
62	7	8	139	2138	32138.975	-0.007	0.001	32138.969	32138.964
63	13	9	139	2155	39476.340	-0.013	0.002	39476.329	39476.319
64	13	9	139	2213	39476.338	-0.014	0.002	39476.326	39476.317
65	7	9	139	2224	32138.958	-0.007	0.001	32138.952	32138.947
66	13	9	139	2312	39476.318	-0.014	0.002	39476.306	39476.297
67	13	9	139	2325	39476.317	-0.014	0.002	39476.305	39476.296
68	7	9	139	2333	32138.973	-0.007	0.001	32138.967	32138.962
69	4	9	139	2337	92882.200	-0.180	0.001	92882.021	92881.930
70	4	9	139	2352	92882.143	-0.180	0.001	92881.964	92881.873
71	4	9	140	0005	92882.165	-0.180	0.001	92881.986	92881.895
74	7	10	140	1918	32138.991	-0.007	0.001	32138.985	32138.980
75	13	10	140	1935	39476.380	-0.014	0.002	39476.368	39476.359
76	24	11	140	2222	52518.399	-0.033	0.002	52518.368	52518.350
77	7	11	140	2246	32138.984	-0.007	0.001	32138.978	32138.973
78	13	11	140	2302	39476.345	-0.014	0.002	39476.333	39476.324
79	13	11	140	2325	39476.343	-0.014	0.002	39476.331	39476.322
80	14	11	140	2336	66128.193	-0.067	0.009	66128.135	66128.096
81	14	11	141	0012	66128.172	-0.068	0.009	66128.113	66128.075
82	7	11	141	0032	32138.956	-0.008	0.001	32138.949	32138.945
83	24	11	141	0046	52518.375	-0.034	0.002	52518.343	52518.326
84	24	12	141	0058	52518.377	-0.035	0.002	52518.344	52518.328
85	7	12	141	0107	32138.988	-0.008	0.001	32138.981	32138.977
86	13	12	141	0117	39476.328	-0.015	0.002	39476.315	39476.307
87	14	12	141	0123	66128.186	-0.070	0.010	66128.126	66128.089
88	14	12	141	0128	66128.167	-0.070	0.010	66128.107	66128.070
89	13	12	141	0136	39476.329	-0.015	0.002	39476.316	39476.308
90	7	12	141	0143	32138.990	-0.008	0.001	32138.983	32138.979
91	24	12	141	0154	52518.369	-0.035	0.002	52518.336	52518.320
92	24	12	141	0232	52518.376	-0.032	0.002	52518.346	52518.327
93	7	12	141	0152	32138.973	-0.007	0.001	32138.967	32138.962
94	13	12	141	0303	39476.363	-0.014	0.002	39476.351	39476.342

1	2	3	4	5	6	7	8	9	10
155	7	13	164	2120	32138.986	-0.008	0.001	32138.979	32138.975
156	13	13	164	2134	39476.324	-0.015	0.002	39476.311	39476.303
157	20	13	164	2144	76957.226	-0.108	0.003	76957.121	76957.073
158	4	13	164	2209	92882.187	-0.189	0.001	92881.999	92881.917
159	4	13	164	2221	92882.167	-0.194	0.001	92881.974	92881.897
160	20	13	164	2245	76957.241	-0.111	0.003	76957.133	76957.088
161	13	13	164	2255	39476.338	-0.015	0.002	39476.325	39476.317
162	7	13	164	2304	32138.976	-0.008	0.002	32138.970	32138.965
163	7	13	164	2310	32138.978	-0.008	0.002	32138.972	32138.967
164	13	14	164	2317	39476.311	-0.015	0.002	39476.298	39476.290
165	20	14	164	2334	76957.225	-0.110	0.003	76957.118	76957.072
166	4	14	164	2352	92882.187	-0.192	0.001	92881.996	92881.917
167	4	14	164	2359	92882.119	-0.192	0.001	92881.928	92881.849
168	20	14	165	2426	76957.138	-0.112	0.003	76957.029	76956.985
169	13	14	165	0040	39476.349	-0.016	0.002	39476.335	39476.328
171	7	15	165	2114	32138.982	-0.007	0.001	32138.976	32138.971
172	13	15	165	2127	39476.359	-0.015	0.002	39476.346	39476.338
173	13	16	166	1414	39476.360	-0.013	0.002	39476.349	39476.339
174	13	16	166	1438	39476.362	-0.013	0.002	39476.351	39476.341
175	7	16	166	1531	32139.063	-0.007	0.001	32139.057	32139.052
176	7	16	166	1545	32139.048	-0.007	0.001	32139.042	32139.037
177	7	17	166	2159	32138.980	-0.008	0.002	32138.974	32138.969
178	13	17	166	2217	39476.316	-0.015	0.002	39476.303	39476.295
179	21	17	166	2239	74362.002	-0.100	0.005	74361.907	74361.864
180	4	17	166	2301	92882.311	-0.193	0.001	92882.119 R	92882.041
181	4	17	166	2321	92882.216	-0.188	0.001	92882.029	92881.946
182	21	17	166	2324	74362.005	-0.097	0.005	74361.913	74361.867
183	21	17	166	2335	74362.003	-0.097	0.005	74361.911	74361.865
184	13	17	166	2350	39476.331	-0.015	0.002	39476.318	39476.310
185	7	17	167	0000	32138.966	-0.008	0.001	32138.959	32138.955
186	7	18	167	0008	32138.963	-0.008	0.002	32138.957	32138.952
187	13	18	167	0025	39476.317	-0.016	0.002	39476.303	39476.296
188	21	18	167	0034	74362.008	-0.102	0.006	74361.912	74361.870
189	4	18	167	0048	92882.182	-0.196	0.001	92881.987	92881.912
190	4	18	167	0055	92882.160	-0.209	0.001	92881.952	92881.890
191	13	18	167	0120	39476.329	-0.017	0.003	39476.315	39476.308
192	7	18	167	0129	32138.950	-0.009	0.002	32138.943	32138.939
193	7	18	167	0231	32138.963	-0.008	0.002	32138.957	32138.952
194	13	18	167	0250	39476.279	-0.015	0.002	39476.266	39476.258
195	7	19	167	1920	32138.990	-0.007	0.001	32138.984	32138.979
196	13	19	167	1930	39476.359	-0.013	0.002	39476.348	39476.338
197	14	19	167	1935	66128.217	-0.061	0.008	66128.164	66128.120
198	24	19	167	1943	52518.387	-0.031	0.002	52518.358	52518.338
199	24	19	167	1950	52518.403	-0.031	0.002	52518.374	52518.354
200	14	19	167	1955	66128.247	-0.062	0.008	66128.193	66128.150
201	13	19	167	2005	39476.353	-0.013	0.002	39476.342	39476.332
202	7	19	167	2006	32138.981	-0.007	0.001	32138.975	32138.970

1	2	3	4	5	6	7	8	9		10
203	7	20	167	2011	32138.987	-0.007	0.001	32138.981		32138,976
204	13	20	167	2020	39476.348	-0.013	0.002	39476.337		39476,327
205	14	20	167	2025	66128.210	-0.062	0.008	66128.156		66128,113
206	24	20	167	2029	52518.375	-0.031	0.002	52518.346		52518,326
207	24	20	167	2033	52518.195	-0.031	0.002	52518.166	R	52518,146
208	14	20	167	2039	66128.100	-0.062	0.008	66128.046		66128,003
209	13	20	167	2048	39476.320	-0.013	0.002	39476.309		39476,299
210	7	20	167	2054	32138.974	-0.007	0.001	32138.968		32138,963
211	7	21	167	2133	32139.005	-0.007	0.001	32138.999		32138,994
212	13	21	167	2152	39476.345	-0.014	0.002	39476.333		39476,324
213	14	21	167	2204	66128.178	-0.066	0.009	66128.121		66128,081
214	24	21	167	2225	52518.367	-0.033	0.002	52518.336		52518,318

APPENDIX B. LISTING OF INPUT DATA FOR THE ADJUSTMENTS

MEANED OBSERVATIONS

1 OBSERVATION NUMBER.

2 FOREPOINT.

3 GROUP.

4 DAY.

5 START.

6 END.

7 NUMBER OF MEASUREMENTS FORMING THE MEAN.

8 DISTANCE CORRECTED USING K FROM ATMOSPHERIC OBSERVATIONS.

9 DISTANCE CORRECTED USING K FROM VERTICAL ANGLES.

10 DIFFERENCE (9 - 8).

1	2	3	4	5	6	7	8	9	10
1	4	1	136	2310	2500	3	92881.994	92881.997	0.003
2	7	1				2	32138.952	32138.954	0.002
3	13	1				2	39476.278	39476.285	0.007
4	4	2	137	0130	0230	2	92881.962	92881.951	-0.011
5	7	2				2	32138.974	32138.975	0.001
6	13	2				2	39476.310	39476.311	0.001
7	4	3	138	1650	1850	2	92882.152	92882.136	-0.016
8	7	3				2	32139.038	32139.038	0.0
9	13	3				2	39476.377	39476.381	0.004
10	21	3				2	74362.057	74362.061	0.004
11	4	4	138	1920	2110	2	92882.142	92882.126	-0.016
12	7	4				2	32139.002	32139.006	0.004
13	13	4				2	39476.342	39476.350	0.008
14	21	4				2	74361.972	74361.969	-0.003
15	4	5	138	2120	2320	2	92881.974	92881.960	-0.014
16	7	5				2	32138.979	32138.987	0.008
17	13	5				2	39476.296	39476.303	0.007
18	21	5				3	74361.911	74361.913	0.002
19	4	6	138	2330	2440	1	92881.964	92881.964	0.0
20	7	6				3	32138.959	32138.970	0.011
21	13	6				1	39476.280	39476.292	0.012
22	7	7	139	1700	1840	3	32139.018	32139.018	0.0
23	13	7				3	39476.386	39476.387	0.001
24	20	7				2	76957.268	76957.259	-0.009
25	4	8	139	1910	2140	4	92882.062	92882.055	-0.007
26	7	8				2	32138.988	32138.992	0.004
27	13	8				1	39476.365	39476.366	0.001
28	20	8				3	76957.128	76957.125	-0.003
29	4	9	139	2200	2400	3	92881.990	92881.977	-0.013
30	7	9				2	32138.960	32138.967	0.007
31	13	9				4	39476.316	39476.315	-0.001
32	7	10	140	1920	1940	1	32138.985	32138.985	0.0
33	13	10				1	39476.368	39476.358	-0.010
34	7	11	140	2220	2450	2	32138.964	32138.965	0.001
35	13	11				2	39476.332	39476.321	-0.011
36	14	11				2	66128.124	66128.133	0.009
37	24	11				2	52518.356	52518.346	-0.010
38	7	12	141	0100	0300	3	32138.978	32138.976	-0.002
39	13	12				3	39476.327	39476.333	0.006
40	14	12				2	66128.116	66128.120	0.004
41	24	12				3	52518.343	52518.334	-0.009
42	4	13	164	2120	2310	2	92881.986	92881.995	0.009
43	7	13				3	32138.974	32138.975	0.001
44	13	13				2	39476.318	39476.316	-0.002
45	20	13				2	76957.127	76957.117	-0.010
46	4	14	164	2320	2440	2	92881.962	92881.959	-0.003
47	13	14				2	39476.317	39476.313	-0.004
48	20	14				2	76957.074	76957.075	0.001

1	2	3	4	5	6	7	8	9	10
49	7	15	165	2110	2130	1	32138.976	32138.983	0.007
50	13	15				1	39476.347	39476.352	0.005
51	7	16	166	1410	1550	2	32139.050	32139.042	-0.008
52	13	16				2	39476.349	39476.342	-0.007
53	4	17	166	2200	2400	1	92882.029	92882.033	0.004
54	7	17				2	32138.967	32138.972	0.005
55	13	17				2	39476.310	39476.305	-0.005
56	21	17				3	74361.910	74361.915	0.005
57	4	18	167	0010	0250	2	92881.970	92881.974	0.004
58	7	18				3	32138.952	32138.964	0.012
59	13	18				3	39476.295	39476.289	-0.006
60	21	18				1	74361.912	74361.905	-0.007
61	7	19	167	1920	2010	2	32138.980	32138.975	-0.005
62	13	19				2	39476.344	39476.336	-0.008
63	14	19				2	66128.178	66128.176	-0.002
64	24	19				2	52518.366	52518.355	-0.011
65	7	20	167	2010	2100	2	32138.974	32138.971	-0.003
66	13	20				2	39476.322	39476.311	-0.011
67	14	20				2	66128.101	66128.093	-0.008
68	24	20				1	52518.346	52518.333	-0.013
69	7	21	167	2130	2230	1	32138.999	32139.000	0.001
70	13	21				1	39476.333	39476.335	0.002
71	14	21				1	66128.081	66128.081	0.0
72	24	21				1	52518.318	52518.306	-0.012

APPENDIX C. EXPLANATORY INFORMATION AND FORMULAS USED TO REDUCE THE EDM DATA

Corrections Applied to the Measured Distances

The distances were reduced to the marks on the ground and corrected for the mean of refractive indices n_1 and n_2 obtained from meteorological measurements at the ends of each line. Further, corrections were applied for the effect of the coefficient of refraction

$$k \simeq - R \frac{dn}{dh} \tag{1}$$

where R is the approximate radius of the Earth, and h is elevation above sea level. For future applications, we also define $k_m = (k_1 + k_2)/2$ and $\Delta k = k_2 - k_1$. It should be noted that k_1 and k_2 as used here are the values at the ends of the line, not at one-third and two-thirds of the way between them.

The coefficient of refraction enters into the computation of three corrections.

1) Beam curvature correction. This small correction is given by

$$c_1 = - {k_m}^2 S^3 /(24R^2) \tag{2}$$

where S is slope distance.

2) Second velocity correction (Saastamoinen 1962, Höpcke 1966). This correction makes allowance for the fact that the midpoint of a nearly horizontal line dips into lower (and warmer) layers of the atmosphere. Its value is

$$c_2 = k_m(1-k_m) S^3/(12R^2). \tag{3}$$

3) Index rate correction (Saastamoinen, 1962, 1975). This correction is expressed by

$$c_3 = - \Delta k \ \Delta h \ S/(12R) \tag{4}$$

where $\Delta h = h_2 - h_1$.

The corrections to distances for the coefficient of refraction are not at all negligible over long lines. The sum of beam curvature and second velocity corrections is

$$c_1 + c_2 = - k_m(2 - k_m)S^3/(24R^2) \tag{5}$$

If $k_m = 0.12$, this correction amounts to -0.15 m or -0.37 ppm over a 40-km line and to -0.119 m or -1.48 ppm over an 80-km line. But k_m can be several times larger or it can become negative. Therefore, the use of a standard value of k_m over long lines is not advisable.

The value of Δk (and consequently the size of the index rate correction) can be unpredictable, especially at night. Meade (1969) gives an example of a 28-km line with Δh of 824 meters in which the corrections for the coefficient of refraction, as determined by vertical angles, changed by more than 3 ppm during the time before sunset and midnight.

Coefficient of Refraction from Meteorological Measurements

From equation (1) we get

$$k_m \simeq - R \frac{\Delta n}{\Delta h} \tag{6}$$

which will give sufficiently accurate results if elevation differences are fairly large, as they are in this survey, but does not give any information about the values of Δk. Fortunately, by equation (4) Δk is needed only when the elevation difference is large, in which case a least-squares solution for k by some model will be strong, and it will also be accurate if the model fits the reality.

The solutions for the coefficient of refraction were obtained from the values of n determined at three or more stations within a short time, generally less than an hour. A parabolic and an exponential model were tried.

In the parabolic model

$$N = A + Bh + Ch^2,$$

where $N = (n - 1)\cdot 10^6$. (7)

A least-squares solution produces the coefficients A, B and C, and we have for any point

$$k_i = -R(B + 2Ch_i). \tag{8}$$

This model gave realistic values for k_m but erratic and often implausible values for Δk, and was, therefore, abandoned.

The exponential model (Pfeifer 1970)

$$N = Ae^{Bh} \tag{9}$$

was found to be much more satisfactory. Taking natural logarithms of its both sides,

$$\mathrm{Ln}\ N = \mathrm{Ln}\ A + Bh. \tag{10}$$

This observation equation was used in least-squares solutions to obtain the coefficients A and B. Then we have

$$k_i = -\ R\ B\ N_i\cdot 10^{-6}. \tag{11}$$

This model gave the values of k_m ranging from 0.10 to 0.14, and Δk values which the average amounted to about -0.01 per 1000 m of Δh. Thus the index rate corrections were very small.

Spot checks were performed by the approximate formula (Bomford 1971)

$$k_i = 672(P_i/T_i^{\,2})(0.0342 + dT/dh) \tag{12}$$

in which P is barometric pressure in mm Hg and T is in $^{\circ}$K. The value dT/dh was taken as ΔT/Δh over the steepest line. This formula gave very nearly the same results as the exponential model.

Analysis of the initial adjustment results suggested that the barometric pressures recorded at station EAGLE during the May observational period were most likely grossly in error. A review of the observational data revealed a large discrepancy between the aneroid and electronic barometer measurements, with the aneroid values being obviously in error. The same aneroid barometer was used at several other stations during the survey, with no obvious malfunctioning. This leads us to suspect that the problem was caused by the observer's consistently misreading the instrument. The aneroid barometer data were rejected, and only the electronic barometer data were used in subsequent adjustments.

After correcting the EAGLE data, the residuals of the refractive indices with respect to the exponential model were generally within 1 ppm, which is compatible with the accuracies that are obtainable in temperature measurements, considering that the temperatures were constantly changing at all points by some amounts, not necessarily equal. This was an indication that <u>in this survey</u> the exponential formula approximated the actual conditions very well and that the determination of k by vertical angles (to be treated later) would not improve the results by much beyond what had already been achieved, if at all.

COEFFICIENT OF REFRACTION BY VERTICAL ANGLES

The mean coefficient of refraction (k_m) can be computed from simultaneous reciprocal vertical angle measurements without knowing the elevations of the end points of the line. With k_m, the beam curvature (c_1) and second velocity (c_2) corrections can be computed. However, for the computation of the index rate correction (c_3), Δk is required, which in turn requires the difference in elevation (Δh) between the end points to be known. In fact, Δh should be the difference in heights above the reference ellipsoid, and the vertical angles should be corrected for the deflection of the vertical, which means that astronomic latitude and longitude must be observed at each station. The astronomic data are seldom available, particularly in situations where they are most important, that is in mountainous areas. Therefore, it is usually necessary to assume that the deflections are zero and that the vertical angles produçe differences in heights above sea level.

An error of 1 meter in Δh introduces an error of $-\Delta h/S$ meters in the index rate correction, which for the McDonald radial line scheme would result in a maximum error in the measured length of any line of less than 20 mm.

An error of 1 second in the vertical angle results in an error in c_3 of -2.4 mm per 1000 meters of Δh. Both the systematic and random error components of the vertical angles will obviously be reflected in the computed values of c_3, and values of c_3 based on individual angle measurements will be fairly "noisy."

If Δh is known, the coefficient of refraction at one-third of the way between points P_1 and P_2 is given by

$$k_{1/3} = 1 - 2R(\Delta h - S \cos z_1)/S^2 \qquad (13)$$

and correspondingly

$$k_{2/3} = 1 + 2R(\Delta h + \cos z_2)/S^2, \qquad (14)$$

from which follow:

$$k_1 = 2k_{1/3} - k_{2/3} \qquad (15)$$

$$k_2 = 2k_{2/3} - k_{1/3} \qquad (16)$$

$$k_m = (k_{1/3} + k_{2/3})/2 = (k_1 + k_2)/2. \qquad (17)$$

The angles used for the determination of k_m and Δk were means of several (generally four to ten) measurements recorded some time before and after the time of distance measurements. Vertical eccentricity corrections were applied. Horizontal eccentricity corrections were found to be negligible and were ignored. In four (out of 148) cases the corrections to distances were interpolated.

APPENDIX D. EXPLANATORY INFORMATION, FORMULAS, AND DATA USED TO COMPUTE STATION ELEVATIONS

ELEVATIONS BY VERTICAL ANGLES

Elevations were established by a combination of EDM observations and reciprocal vertical angles from six bench marks. The measurements were conducted on several days in May and June in the afternoon hours, except for one line over which the angles were measured after sunset.

In the computation of elevation difference by the formula

$$\Delta h = S(\cos z_1 - \cos z_2)/2 \qquad (18)$$

the actual value of k_m, however large, is immaterial because it cancels in subtraction. It is assumed that $\Delta k = 0$, but this is seldom quite true. If Δk is known or can be estimated with sufficient accuracy, then the formula

$$\Delta h = S(\cos z_1 - \cos z_2)/2 + \Delta k\ S^2/(12R) \qquad (19)$$

can be used to give an improved result. Equivalently, we can use equation (18) after the correction

$$\delta = \Delta k\ S/(12R) = 0.0027'' \Delta k\ S \qquad (20)$$

has been subtracted from z_1 and added to z_2.

On the basis of previous adjustments of the refractive index values to the exponential formula it was found that Δk stayed very nearly the same in May and in June, with only slight diurnal variations. During daytime measurements of distances its value was found to be about -0.008 per 1000 m of Δh. At the time of vertical angle measurements no temperature or pressure readings were taken, and the assumption was made that the same value of $\Delta k/\Delta h$ could safely be used. Therefore, this correction was set to

$$\delta = -1.05\ S\ \Delta h\ .10^{-13}, \tag{21}$$

which is -0.022" per kilometer of S and Δh.

Admittedly, this refinement is only as good as the determination of Δk from meteorological data, but nothing in this survey suggests that it is seriously wrong.

The reciprocal angles were corrected for δ and later used together with horizontal directions and distances in a combined adjustment in three dimensions.

The EDM vertical angle and direction data used to compute the station elevations are listed in tables 7, 8, and 9, respectively.

Table 7.--Distance to bench marks

From Sta. No.	To Sta. No.	Designation	Slope Distance	Elevation of B. M.
1	51	BM D 1118	26169.62	1402.28
4	46	BM B 23	12508.39	1318.52
13	44	BM E 1115	7110.58	1386.30
14	47	BM Z 1115	30342.46	993.52
20	54	BM U 706	5084.06	1575.53
24	49	BM P 730	20860.87	1346.42

Table 8.--Reciprocal vertical angles for establishing elevations

FROM	TO	Z(1)	T-O	Z(2)	T-O
1	4	90 32 21.2	-0.16	90 12 11.9	0.46
1	7	91 8 6.8	-0.20	89 7 36.0	0.13
1	7	91 8 8.0	-0.20	89 7 31.4	0.13
1	7	91 8 8.0	-0.20	89 7 36.1	0.28
1	7	91 8 19.6	1.38	89 7 39.1	0.17
1	7	91 8 8.2	-0.20	89 7 37.1	0.28
1	7	91 8 5.9	-0.21	89 7 42.0	0.47
1	13	91 3 43.0	-0.23	89 15 28.1	0.07
1	13	91 3 43.3	-0.23	89 15 32.8	0.07
1	13	91 3 43.8	-0.23	89 15 35.2	0.07
1	13	91 3 43.0	-0.24	89 15 39.2	0.06
1	13	91 3 46.1	-0.23	89 15 33.2	0.07
1	14	91 8 54.6	-0.32	89 22 29.6	0.26
1	14	91 9 4.9	-0.08	89 22 41.1	-0.13
1	20	90 40 36.2	-0.32	89 56 31.5	-0.10
1	21	90 49 50.0	-0.06	89 45 43.9	-0.20
1	21	90 49 38.9	-0.05	89 45 29.3	-0.30
1	21	90 50 0.1	-0.06	89 45 50.1	-0.20
1	24	90 50 1.1	0.08	89 35 13.3	0.03
51	1	87 34 44.8	-0.12	92 37 44.0	0.26
51	1	87 34 45.0	-0.12	92 37 45.1	0.26
46	4	85 37 10.6	-0.88	94 28 55.9	1.19
46	4	85 37 20.5	-0.88	94 28 56.6	1.19
44	13	85 36 49.0	-0.29	94 26 59.0	0.71
47	14	89 6 19.6	-0.30	91 8 22.2	0.41
54	20	84 32 44.5	-0.25	95 30 3.4	0.58
49	24	88 20 25.3	-0.13	91 50 3.5	0.43
49	24	88 20 19.8	-0.13	91 49 59.7	0.43

Table 9.--Directions

To	Direction	To	Direction
7	0 0 0.0	7	0 0 0.0
13	47 12 56.42	13	47 12 54.91
20	105 25 17.19	20	105 25 14.68
4	271 1 15.86	4	271 1 15.22
7	0 0 0.0	7	0 0 0.0
13	47 12 55.14	13	47 12 57.74
21	180 44 25.85	21	180 44 27.24
		4	271 1 13.39
7	0 0 0.0	7	0 0 0.0
13	47 12 56.41	13	47 12 56.70
21	180 44 27.80	14	51 52 39.08
4	271 1 13.31	24	228 14 50.01
7	0 0 0.0	7	0 0 0.0
13	47 12 56.46	13	47 12 56.32
14	51 52 39.33	14	51 52 38.66
24	228 14 51.20	24	228 14 49.71
		7	0 0 0.0
		14	51 52 38.91

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