Interactive Educational Tool for Classical Airfoil Theory

Thomas J. Benson*
NASA Lewis Research Center
Cleveland, Ohio 44135

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ABSTRACT

A workstation-based interactive educational tool has been developed to aid in the teaching and understanding of classical airfoil theory. The tool solves for the flow conditions around a plate, an elliptical airfoil, or a Joukowski airfoil by performing a conformal mapping from flow around a circular cylinder. The effects of airfoil camber, thickness and angle of attack can be studied interactively. As the student varies design conditions through a graphical user interface, the new flowfield is calculated and displayed immediately. The student can vary conditions in either the generating plane of the cylinder or the mapped plane of the airfoil to explore the effects of flow circulation and the Kutta condition. An interactive plotter is used to record flow variations, and a flow probe can be used to explore the flowfield around the airfoil. This paper details the numerical methods used in the program and shows several examples of how the package can be used in the classroom. The tool employs X-based graphics and has been tested on a variety of workstations using the Unix operating system.

*Research Engineer

INTRODUCTION

The use of personal computers and workstations in undergraduate aerospace engineering is finding wider acceptance in the educational community. Tedious numerical computations associated with many interesting flow problems can now be performed on the computer, giving the student time to explore the fluid dynamics aspects of these problems while minimizing the time spent in arithmetical drudgery. The interactive, graphical nature of computer output can provide the student with information in ways which are not possible with standard textbooks, lectures, or other visual aids. As a result, modern textbooks (Refs 1-3) are often accompanied with computer software to be used on assignments. Some professors develop their own software tools to be used by students in fluids courses (Refs 4-7). while others employ computational fluid dynamics (CFD) codes to promote further understanding of basic fluids problems (Refs 8-9). Recently, special interactive software has been developed in industry, Ref. 10, and government (Refs. 11-13) to enhance the teaching of fluid mechanics.

This paper will describe a new software package named VU-FOIL which has been developed at NASA Lewis Research Center to interactively study classical airfoil theory. The package is intended for use in an undergraduate course in mechanical or aerospace engineering as a supplement to formal course work. The package is based on previous computer codes developed principally for aeropropulsion and described in Ref. 14. While the earlier packages were chiefly concerned with performance aspects of various propulsion components, the new code is designed to display some fundamental concepts in a new way. The new code models the classic Joukowski conformal mapping of ideal flow around a cylinder onto an airfoil. Through the use of multiple display windows the student can see how the mapping works and by interactively varying the conditions of the problem can explore both the aerodynamics and the mathematics involved in the analysis. The effect of circulation on the flow about the cylinder can be interactively studied and the Kutta condition for flow at the trailing edge can be demonstrated. A guiding principal in the development of VU-FOIL is to involve the student in the learning process; to make the student work with the package to achieve a result, not just present the answer to a problem. Some special features to promote this student interaction are included in this tool and described in this paper.

ANALYSIS

The analysis of ideal flow past an airfoil can be found in most undergraduate fluid mechanics textbooks such as Panton, Ref. 15, White, Ref. 16, or Milne-Thompson, Ref. 17. The reader will find the details of the equation derivations in these texts but some of the important equations have been collected here for future reference. An irrotational, inviscid, incompressible, two dimensional flow past a circular cylinder of radius a is shown in Fig. 1. This flow

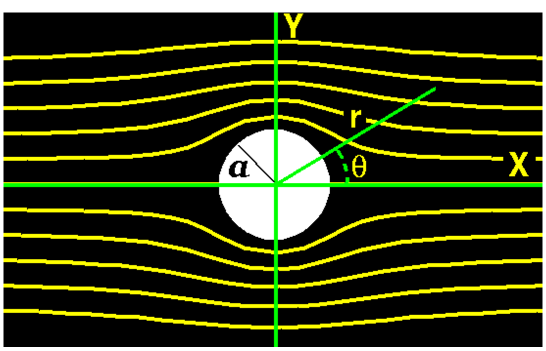

Figure 1: Flow past a cylinder

is generated by the superposition of a uniform flow and the flow around a doublet at the center of the cylinder. The stream function, Ψ, for this flow is given by

$$\Psi = U_0 r sin\theta \left(1 - \frac{a^2}{r^2}\right) \qquad (1)$$

where U_0 is the free stream velocity, r and θ are the radius and angle of a cylindrical coordinate system centered on the cylinder. Similarly, the velocity potential, Φ, is given by

$$\Phi = U_0 r cos\theta \left(1 + \frac{a^2}{r^2}\right) \qquad (2)$$

Because the stream function and velocity potential satisfy the Cauchy-Riemann equations, we may introduce a complex variable coordinate system, Z, and a complex potential function F

$$Z \;=\; re^{i\theta} = X + iY \qquad (3)$$

$$F \;=\; \Phi + i\Psi \qquad (4)$$

The complex potential of the flow past a cylinder is then described by the equation

$$F = U_0 \left(Z + \frac{a^2}{Z}\right) \qquad (5)$$

and the velocity components at any point in the flow can be found by taking the appropriate derivatives of the complex potential.

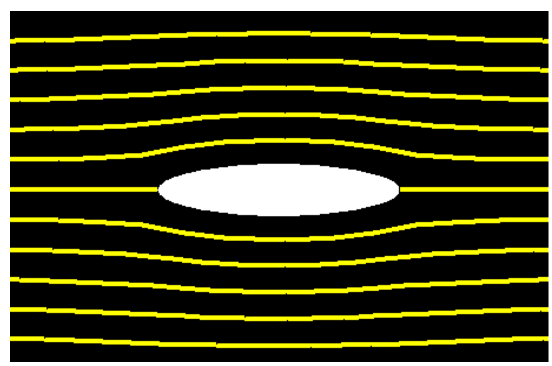

Figure 2: Flow past an ellipse

The flow around a circular cylinder can be mapped into the flow about another body by using conformal mapping with the complex potential preserved. Joukowski proposed the transformation

$$z = Z + \frac{1}{Z} \qquad (6)$$

This transformation maps the cylinder of radius a into an ellipse with semi major axis equal to $a + 1/a$

and semi minor axis of $a - 1/a$. The resulting flowfield in the z plane is shown in Fig. 2. For a cylinder with radius equal to one, the ellipse collapses into a flat plate. The poles of the Joukowski transformation occur at $Z = (-1, 0)$ and $Z = (1, 0)$ and the cylinder must enclose these poles to produce a closed surface in the mapped plane. If the center of the generating cylinder is moved from $Z = (0, 0)$ along the X axis to the point $Z = (-x, 0)$, and the radius of the cylinder is set to $a = 1+x$, a symmetric airfoil is generated in the mapped plane. One such airfoil is shown in Fig. 3 for a cylinder centered at

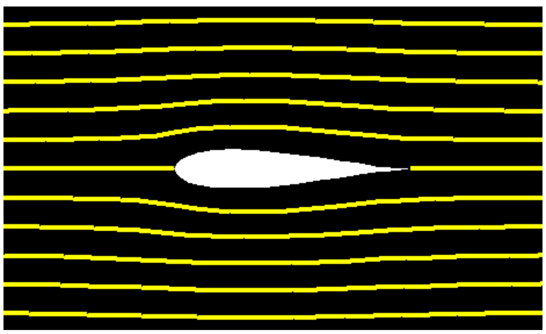

Figure 3: Symmetric Joukowski airfoil

$Z = (-.15, 0)$ and $a = 1.15$. If the center of the cylinder is moved parallel to the Y axis, camber is introduced into the foil in the z plane as shown in Fig. 4.

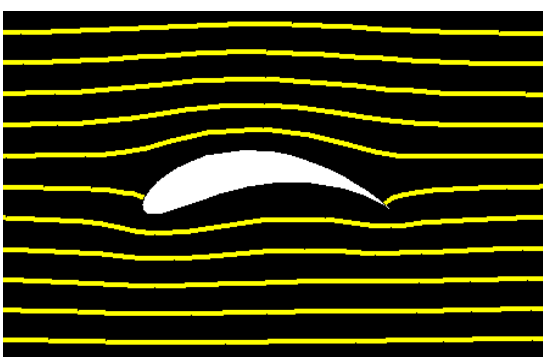

Figure 4: Cambered Joukowski airfoil

Returning to the Z plane, a vortex flow centered on the cylinder is now superimposed on the uniform flow past a cylinder. The resulting flowfield, shown in Fig. 5, corresponds to uniform flow past a rotating cylinder. An additional term describing the

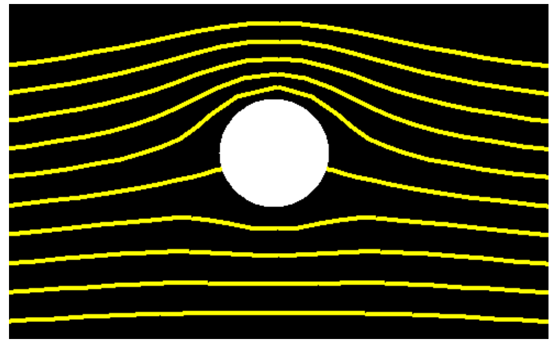

Figure 5: Cylinder flow with circulation.

vortex must be added to the complex potential. The potential then becomes

$$F = U_0 \left(Z + \frac{a^2}{Z} \right) - i\Gamma ln Z \tag{7}$$

where Γ is a constant related to the strength of the vortex. When this flowfield is mapped into the z plane using the Joukowski transformation, the flowfield of Fig. 4 is changed to Fig. 6. Depending on the strength and the sense of circulation the stagnation points will move on the airfoil and the streamlines around the airfoil will be modified. The Kutta

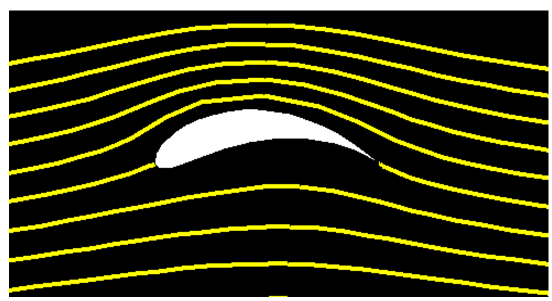

Figure 6: Airfoil with circulation

condition states that of all possible values of circulation about an airfoil the one that actually occurs causes the aft stagnation point to move to the trailing edge with the flow slope bisecting the angle of the trailing edge, as shown in Fig. 6. The Kutta-Joukowski theorem then gives the lift of the airfoil to be

$$L = \rho U_0 \Gamma \tag{8}$$

where ρ is the fluid density. With the complex velocity potential given, the local velocity components

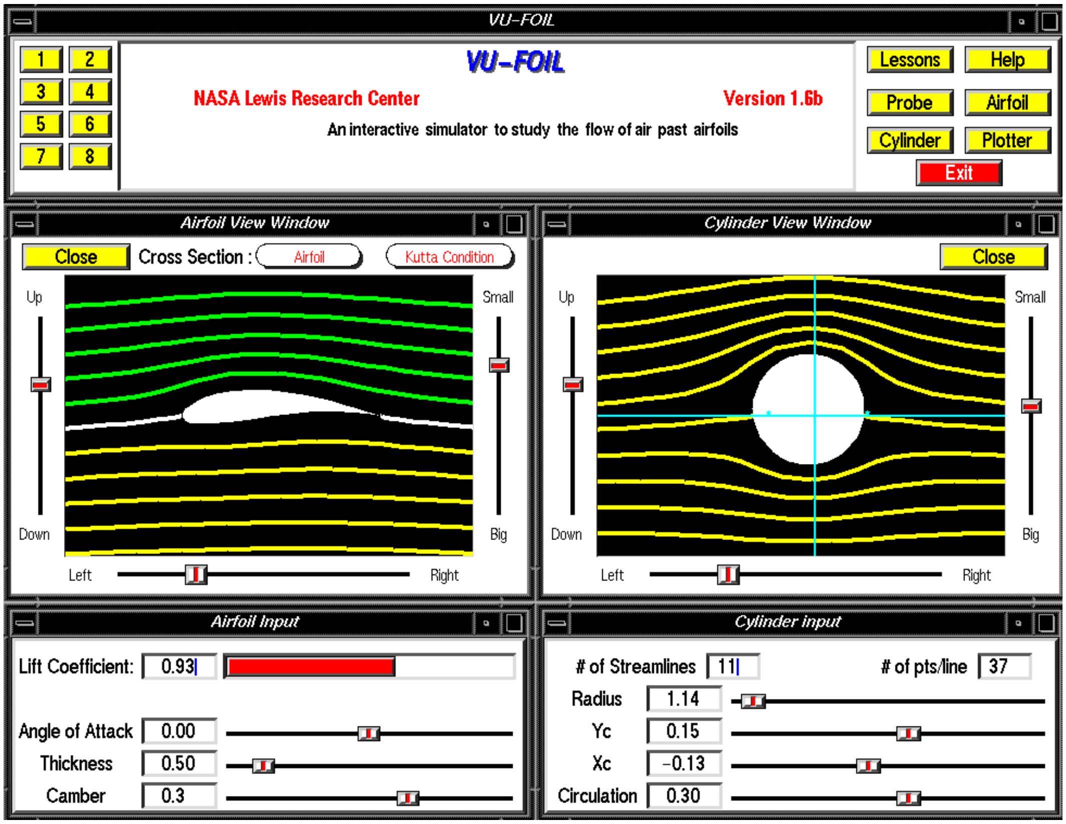

Figure 7: VU-FOIL graphical user interface

can be computed by taking the partial derivative of the potential. Knowing the velocity, the local pressure can be computed from Bernoulli's equation along a streamline. Finally, if the incoming flow is inclined at some angle α, the complex potential is again modified to

$$F = U_0 \left(Ze^{-i\alpha} + \frac{a^2 e^{i\alpha}}{Z} \right) \tag{9}$$

DESCRIPTION OF SIMULATOR

The flow simulator solves the equations presented above; computing the flow around the cylinder and then mapping the flowfield into the airfoil plane using input parameters provided by the student. The input parameters are specified in either the cylinder plane or the airfoil plane by using a graphical user interface (GUI) and the results of the calculations are immediately presented to the student graphically and numerically. Some examples of the GUI's for VU-FOIL are given in Figs. 7-9. Each of these figures have been captured from a Silicon Graphics workstation in color and reproduced here in black and white. Fig. 7 shows the GUI for the VU-FOIL program at program initiation as it would appear on the computer screen. There are five X-graphics windows displayed in Fig. 7. The top window is the main control window which includes a browser and a variety of buttons used to invoke additional windows. Below the main window are two windows to the left to display and provide input for the mapped airfoil while on the right are two windows to display and provide input for the generating cylinder. The student can resize and move the display within the view windows using the sliders located around the edge.

Beginning with the cylinder windows on the right, the upper window shows a flow schematic, while the lower window contains the input devices for the calculations. In the schematic window, the flow is from left to right, the cylinder appears as the filled circle, and the streamlines around the cylinder are noted. The axes of the Joukowski mapping are drawn and the location of the poles are noted at $Z = (-1, 0)$ and $Z = (1, 0)$ by small dots. The student can use the sliders and input boxes in the lower window to translate the cylinder relative to these fixed axes and to change the size of the cylinder. As mentioned above, moving the origin of the cylinder will produce airfoils of different camber and thickness distribution. As the student changes the input, the program computes the new airfoil shape and streamlines and immediately displays the results in the left view window. The student can pick the number of streamlines to display and the number of points in a streamline by using the input boxes at the top of the cylinder input panel. The student can also vary the circulation around the cylinder and note the changes in the streamlines in both the cylinder and airfoil view windows. Introducing circulation will produce lift in the airfoil which is immediately displayed on the bar graph on the airfoil input panel.

Moving to the airfoil windows on the left, the upper window shows a flow schematic while the lower window contains input and output devices. On the airfoil view window there are two oval choice buttons at the top which are used to select the type of airfoil for study and control the Kutta condition at the airfoil trailing edge. The student can display three types of airfoils using the oval choice button labeled "Cross Section". A thin plate, an elliptical foil, or a Joukowski airfoil may be chosen. The student can use the oval choice button labeled "Kutta Condition" to either impose this condition or ignore it as described below. The schematic drawing shows the airfoil and streamlines with the flow moving from left to right. On the computer screen, the lines passing over the top of the foil are colored green, those below the foil are yellow, and the stagnation streamlines are white. This color coding is used in connection with the plotter as described below.

The lower panel on the left is used to control input to the calculations and to display the calculated lift coefficient. Using the three sliders and input boxes at the bottom of the input panel, the student can vary the airfoil angle of attack, maximum thickness, and camber. In the cylinder window, the student will see the generating cylinder move and change size as the airfoil thickness and camber are varied. Above the airfoil input sliders is an output of the airfoil lift coefficient. This information is presented both numerically and as a moving bar chart so that the student can sense in which direction and by how much lift changes with a change in input.

The main control panel sits above the view windows. In the middle of the main control panel is a browser which is used to display text information. The student can choose to display help information or lessons in basic airfoil theory by using the two buttons at the upper right. Several different help screens or lessons can be selected by then using the numbered buttons at the left. The help screens describe the operation of all the sliders and input boxes in the package, give some examples of how to generate plots, and generally serves as an on-line user's manual for the program. The lessons are simple ASCII data files which are displayed in the browser window. Prepared by the teacher, these lessons can contain problems to be solved, or narratives on using the package to demonstrate various aspects of classical airfoil theory. On the right side of the main control panel are four buttons which can be used to bring up additional windows. Fig. 8 shows the result of pushing the "Help" button to load the browser and pushing the "Plotter" button to invoke the interactive plotter.

The interactive plotter is used to record changes in airfoil performance and to provide the student with another form of output results. This plotter is similar to one developed in Ref. 13, and the reader should consult this paper for details of the plotter operation. For implementation in this package the plotter has been modified to display lift coefficient versus angle of attack, thickness or camber as well as dynamically display the airfoil surface velocity or pressure distribution. The student picks which type of plot to display by using the radio buttons at the top of the plotter view window. A small yellow light notes the chosen plot. If airfoil lift is chosen as the dependent variable, the student must select an independent variable (angle of attack, thickness, or camber) using the radio buttons which now appear on the airfoil input panel. The student then uses the package to generate plots as shown in Fig. 8. Here the student is plotting lift coefficient versus angle of attack as the camber is varied. The student sets the flow conditions using the input devices on the airfoil input panel and records the data using the

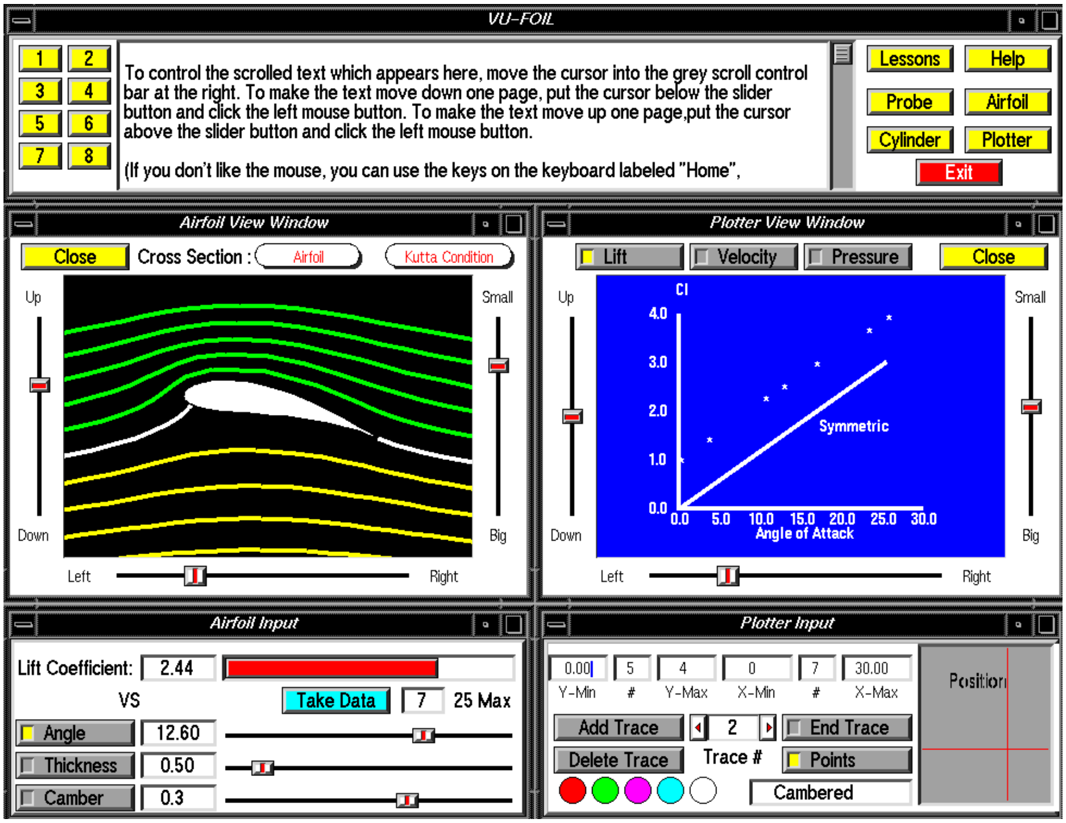

Figure 8: Interactive plotter

"Take Data" button which appears there. As data is recorded, it appears on the plot at the right as a small dot. The final appearance of the plot is then modified by the various buttons and input devices on the plotter input panel at the lower right. Details of the plot generation can be found in Ref. 13 and is described on-line in the help screens. The procedure of changing conditions and recording data to generate the plot is meant to mimic the process encountered in the laboratory. If surface velocity or pressure are chosen for plotting, the graph is automatically displayed as shown in Fig. 9. This plot appears on the computer screen with the velocity or pressure on the upper surface colored green and the value on the lower surface colored yellow corresponding to the streamline colors in the airfoil view window. As the student varies the airfoil input parameters, the graph is instantly modified to reflect the new distribution.

An interactive flow probe is also included in the VU-FOIL package to allow the student to explore the flow field around the airfoil. The probe control panel is invoked from the main control panel and is shown at the top of Fig. 9. The probe panel can be placed anywhere on the screen by the student and be closed or re-opened at any time. The probe, which appears in the airfoil view window, is moved through the flowfield using the sliders at the bottom and left of the probe panel. The student can choose to measure either velocity or pressure using the oval choice button in the middle of the panel. The value of that flow variable at the location of the probe is then displayed graphically on the bar chart at the right and numerically in the box in the middle of the panel. Again, the appearance and use of the flow probe is similar to what the student would encounter in the laboratory.

The probe can also be used to explore the mathematics involved in the generation of the airfoil flowfield. The probe can be displayed with the plotter,

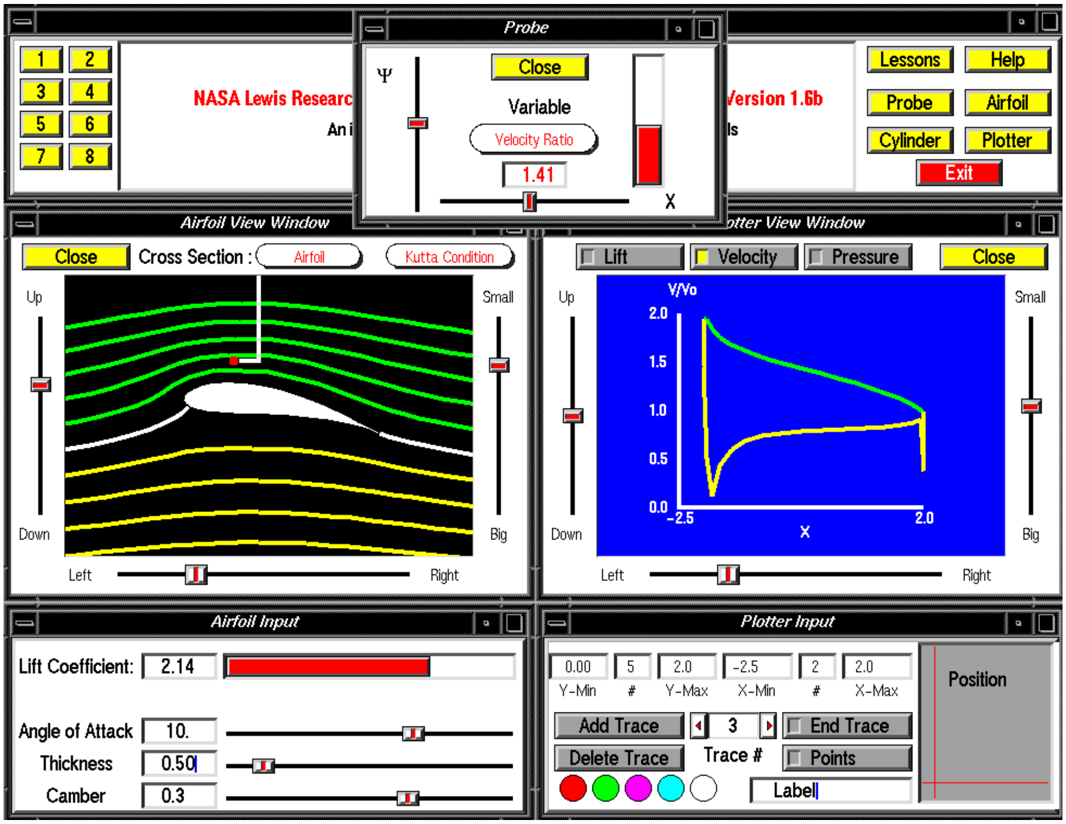

Figure 9: Flow probe

as in Fig. 9, or with the generating cylinder panels of Fig. 10. If the cylinder panel is displayed, the location of the probe in the cylinder plane is also shown by a red dot. In this way, the student can interactively investigate how points in the Z (cylinder) plane are mapped to corresponding points in the z (airfoil) plane through the Joukowski transformation. This is a most unique way to study conformal mapping and could be used for other mappings in addition to the Joukowski transformation.

The student can use the oval choice button labeled "Kutta Condition " on the airfoil view window to perform an interesting experiment. This button cycles between imposing and ignoring the Kutta condition at the trailing edge of the airfoil. By simply cycling this button, the student should notice the lift change from some computed value to zero and back again. Removing the Kutta condition removes the circulation generated by the airfoil and, from Eq. 8, will cause the lift to be zero. The student should monitor the movement of the rear stagnation point and the change in streamlines in the airfoil plane as the Kutta condition is removed. Moving over to the cylinder plane of Fig. 10, with the Kutta condition turned off, the student can vary the circulation and try to set the condition manually, again monitoring the motion of the rear stagnation point on the airfoil. Returning to the airfoil window and turning the Kutta condition back on will show the student how well the condition was set.

SUMMARY

A workstation-based, highly interactive educational tool for classical airfoil theory has been developed. The underlying assumptions and equations which form the basis for this tool have been presented in this paper. The tool employs two basic ideas to help undergraduates better understand airfoil theory. First, the student is required to interact

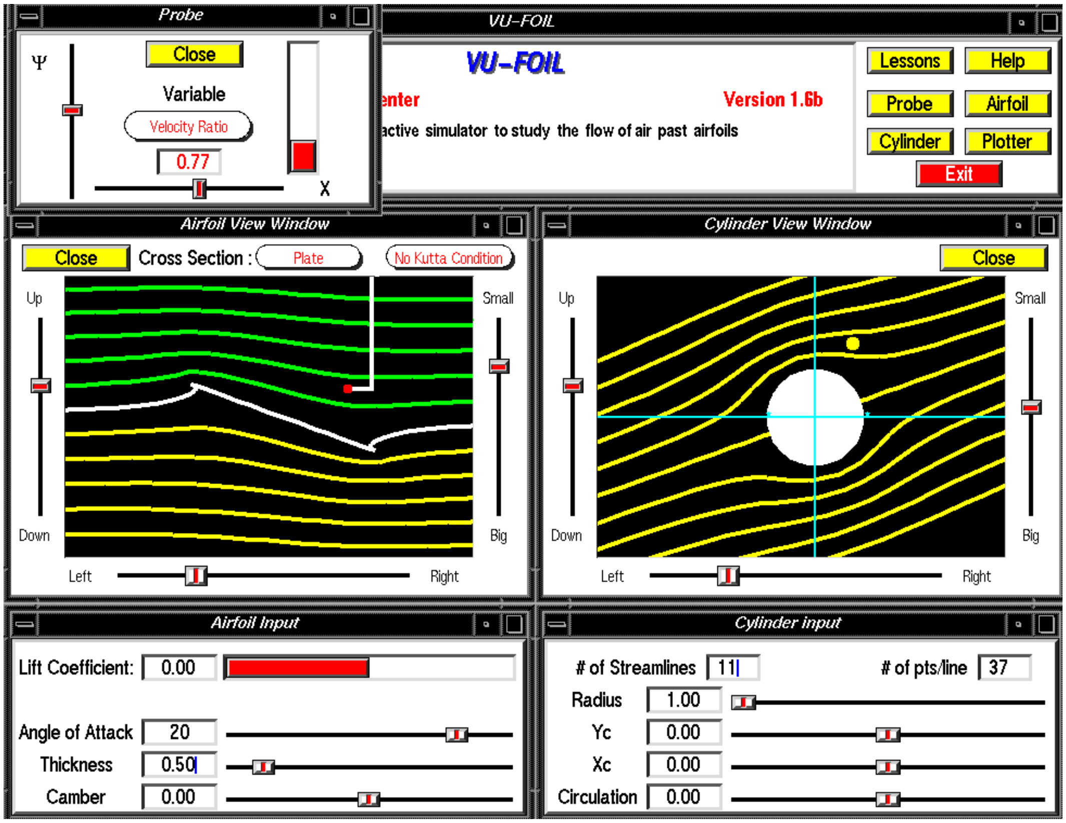

Figure 10: Flat plate without Kutta condition

with the tool to produce results. The student is in control of the parameters defining the problem and must perform several operations to produce the output. Second, the output results are presented to the student in a variety of forms. For a given flow problem the student can see a schematic drawing of the airfoil and its generating cylinder, tabulated numbers for the airfoil results, moving bar charts for the parameters, and plotted data. Further extensions of this technology into other flow regimes and other physics problems are planned.

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