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AN INVESTIGATION OF THERMAL TIDAL OSCILLATIONS IN THE EARTH'S ATMOSPHERE USING A GENERAL CIRCULATION MODEL

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ABSTRACT

An investigation of tidal oscillations in the earth's atmosphere has been made using an 18 vertical level, hemispheric general circulation model. This approach permitted these tides to be investigated without resorting to linearization of the governing differential equations, as is required by the conventional approach. In addition, it allows the tides to be studied in relation to a realistic atmosphere, and thus in their actual roles as small perturbations, at least in the lower atmosphere, on the basic meteorological fields. Day-to-day surface pressure variations in good agreement with observation were produced by the model, the diurnal and semidiurnal pressure amplitudes and phases also being close to the observed values. An investigation into the excitation mechanism of the oscillation gave results supporting previous work in attributing the dominant cause of the tides to absorption of solar radiation by water vapor and ozone in the atmosphere. Contrary to previous studies, water vapor was found to be of primary importance in exciting both the diurnal and semidiurnal oscillations in the model atmosphere.

Generally speaking, the tidal wind and temperature variations obtained were also in agreement with observation and other theoretical work.

1. INTRODUCTION

Since the time of Laplace many attempts have been made to explain the small, but regular, surface pressure tidal oscillations that are observed in the earth's atmosphere. Despite the development of elaborate theoretical frameworks to account for these oscillations, it is a fair statement to say that even at the present time their properties are not completely understood. These tides are very small and are only readily observed in the Tropics, where the maximum amplitude of about 1.5 mb. is found, and pressure variations due to meteorological causes are minimal. The principal tides in the atmosphere have frequencies corresponding to the solar diurnal, semidiurnal, and terdiurnal periods, with the semidiurnal oscillation predominating, as is well known, with maxima at about 10 a.m. and 10 p.m. local time. The dominance of the semidiurnal component would be expected if the atmospheric tides were gravitationally induced, but in that case the lunar period would also be expected, as observed in the oceans, rather than the solar period. Laplace therefore attributed the tides in the atmosphere to solar heating, and Kelvin suggested that the reason for the magnitude of the semidiurnal oscillation, compared to that of the diurnal, might be that the atmosphere had a natural period which coincided more closely with the former than the latter. This gave rise to the famous resonance theory which has been developed by many workers, and which was able to account for many of the observed tidal effects. This theory has lately fallen into disfavor, as the measured temperature profiles in the atmosphere do not agree with those which are required to produce the resonance phenomenon. The case for this theory has been documented by Wilkes [19].

The situation up to 1961 regarding tidal oscillations of the atmosphere has been extremely well presented by Siebert [17], who not only discusses various failings of the resonance theory but advances an alternative approach to the problem. He showed that at least part of the tidal amplitude could be accounted for directly, by the heating associated with the absorption of solar radiation in the troposphere by water vapor, without recourse to amplification by resonance. In addition Siebert states that the semidiurnal tide is no more favored by resonance than the other higher frequency tides. He therefore raised the pertinent point that the problem is to explain why the diurnal tide is not larger, rather than to account for the magnitude of the semidiurnal tide, which was the case with the resonance theory. Siebert's explanation is that

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although a diurnal thermal-forcing term is present, which is larger than the corresponding semidiurnal term, the propagation of the diurnal tide is suppressed by the atmosphere. According to Butler and Small [2] the suppression of the diurnal tide is associated with its small equivalent depths, and therefore short wavelengths, which lead to interference and thus destruction of the tide. Further insight into this problem has been provided by Green [3], who showed that the small equivalent depths of the diurnal tide result from the Coriolis effect owing to the earth's rotation. Green also developed an expression for the equivalent depths in a compressible atmosphere, and found that the (1, 1) mode of the diurnal tide and the (2, 2) mode of the semidiurnal tide were associated with depths of 4 and 70 km., respectively. From this he concluded that the diurnal tide would respond only locally to radiative heating, but that the semidiurnal tide would be able to propagate, and that it would represent the integrated effects over a considerable depth.

Siebert [17] also suggested that better agreement would be obtained with the observed semidiurnal pressure oscillation if the absorption of solar radiation by ozone was incorporated in the calculations. Butler and Small [2] have in fact performed such calculations, and obtained agreement with several features of the surface pressure oscillations produced by the diurnal, semidiurnal, and terdiurnal tides, if they included the contribution due to water vapor given by Siebert. On the other hand, Harris et al. [5] recently concluded from a study of the semidiurnal tide based on upper air observations that not only were absorption by ozone and water vapor of importance, but that both eddy transfer of heat and also momentum had to be taken into consideration, all three mechanisms being of about the same magnitude.

Recently interest in the diurnal tide in the atmosphere has increased, largely stimulated by the theoretical work of Lindzen [10]. He has shown that the diurnal surface pressure oscillation is to a large extent due to the absorption of solar radiation by water vapor. In addition he computed the variation with altitude of the tidal winds and temperature and obtained satisfactory agreement with observation. The important development in tidal theory which produces the good agreement with observation in the case of the diurnal tide is the realization by Kato [8] and Lindzen [11] that negative equivalent depths are important for the numerical evaluation of this tide.

In spite of the apparent adequacy of the current state of tidal theory, it is still desirable to investigate tidal oscillations with a more realistic model, in which all of the various forcing functions are included simultaneously, and in which most of the approximations required to make the problem analytically tractable have been removed. Such an approach is possible using a general circulation model of the atmosphere, as this automatically incorporates wind and temperature variations, frictional and momentum effects, and overcomes the analytic problems by numerically integrating the governing equations. The use of a general circulation model also permits the tidal oscillations to be viewed against the general meteorological situation, as well as simultaneously studied over a range of latitudes and longitudes. Atmospheric tides have been obtained previously with such models by Mintz [16] and Leith. Mintz computed a semidiurnal tide at the Equator of ± 1-mb. amplitude, and approximately the correct phase, using a two-level model with a highly parameterized scheme for computing the radiative heating. Leith also obtained a semidiurnal oscillation with another general circulation model, in which the radiative heating due to the absorption of insolation by water vapor was calculated, but has only "published" his results in the form of a short, but beautifully made, documentary film. The model used in the present study has higher vertical resolution, and extends to greater heights than those of Mintz and Leith; in addition it also has a more complete radiative transfer scheme. It was therefore hoped that it would permit the contributions of the various excitation mechanisms of the tides to be isolated more accurately than was possible previously. In addition, since models developed at this Laboratory have not so far incorporated diurnal effects, but have been based on a radiative state corresponding to a mean solar zenith angle for a 24-hr. period, it is of interest to see what differences are obtained when the diurnal variation is permitted.

However, because of the complexity of general circulation models it is not considered that they would ever replace the conventional analytic approach, although they do provide a needed check on the approximations inherent in that approach. Moreover, it is recognized that particular problems, such as the explanation of the relative magnitude of the diurnal and semidiurnal surface pressure amplitudes, and the corresponding phase of the semidiurnal tide, are more easily interpreted by the conventional analytic approach.

2. DESCRIPTION OF THE MODEL

The model used in the present study is essentially that described previously by Manabe and Hunt [13], but some of the relevant details will be repeated here. The model consisted of a hemisphere with a nonconducting, frictionless wall at the Equator, the sphericity of the earth being retained by mapping onto a polar stereographic projection. This projection was converted into the computational space mesh required for representing the finite difference analogs of the governing equations, by dividing the Pole to Equator distance into 20 equal parts, thus giving approximately 1,200 points per level. The model had 18 vertical levels defined by means of a normalized p coordinate system, the lowest being at 914 mb. (0.85 km.), the highest at 4 mb. (37.5 km.); see table 1.

No mountains were included in the model, but the mass of the actual atmosphere was still used; hence this resulted in the surface pressures being somewhat lower than those observed. Also no land-sea contrast was included in the model, and no allowance was made for the transport of

Table 1.—*Heights and pressures of the model layers*

Level	Height (km.)	Pressure (mb.)
1	37.5	4.0
2	29.5	12.9
3	25.6	23.4
4	22.8	36.1
5	20.5	51.2
6	18.5	69.4
7	16.8	91.1
8	15.2	117.1
9	13.7	148.2
10	12.3	185.5
11	10.9	230.1
12	9.55	283.6
13	8.20	347.5
14	6.75	424.1
15	5.35	515.8
16	3.9	625.6
17	2.4	757.0
18	0.85	914.3

latent heat by the atmosphere or for the effects of large-scale condensation. The moist convective anjustment procedure developed by Manabe and Strickler [15] was incorporated in order to obtain a realistic static stability. This procedure, of course, does not reproduce the diurnal variation of convection in the troposphere, but it should adequately represent its mean effects. Of particular importance for a tidal study is the use in the present model of a complete radiative transfer scheme, a feature conspicuously absent from previous tidal computations. This scheme was originally developed in order to permit diabatic heating to be included in the general circulation study as part of the calculation of the temperature tendency, and includes both long and shortwave absorption of radiation by water vapor, carbon dioxide, and ozone. Climatological distributions of ozone and water vapor corresponding to annual mean conditions were used in the radiative calculations.

Since the lowest level of the model was situated at 0.85 km. the vertical structure of the planetary boundary layer was not resolved. The exchange of heat and momentum between the earth's surface and the atmosphere was therefore computed by evaluating these terms at the anemometer level. The anemometer velocity was assumed to have a magnitude of 0.6 of that of the corresponding total velocity at the lowest level, which was taken to be representative of conditions in the free atmosphere, and a direction counterclockwise to it by 20°. Because of the depth of the layer adjacent to the surface, the momentum and heat fluxes were incorporated into the equations defining the temperature and velocity components at level 18 only. Any subsequent redistribution of heat and momentum upwards was then carried out by the large-scale dynamics of the atmosphere. Both of these terms represent possible forcing agents for the tidal oscillations. Dissipation due to horizontal diffusion was incorporated at all levels in the model.

For details of the system of equations and the finite difference scheme used in the computations, the interested reader is referred to the 9-level model of Smagorinsky et al. [18], from which the present model was developed.

The only modification made in order to convert the steady state model into a diurnal model was to include the diurnal variation of solar radiation, which was done rather crudely in the present study. The model was run for a fixed solar declination ($\delta=0$) corresponding to the time of the equinox, and the radiative calculation in the model was made at hourly intervals for the time corresponding to the middle of the interval. The atmospheric pathlength of the solar radiation was assumed to vary as the secant of the solar zenith angle, a rather poor approximation for angles greater than 70°, and the solar radiation was cut off for angles greater than 84°. The longwave radiation calculation was, of course, performed both at night and day regardless of the variation of the solar radiation. No allowance was made for heat storage in the earth; this and the very approximate representation of the planetary boundary layer are probably the poorest features of the diurnal model.

No consideration was given to the gravitational excitation of tides in the model, so that the results obtained are free from complications associated with this possible forcing term.

One final point concerns the question as to whether "lids" on numerical models can cause errors, since Lindzen et al. [12] concluded that such lids would generate spurious resonances in tidal studies. The lid on the present model, i.e. the height at which $dp/dt=0$ where p is the pressure, is at infinity. Lindzen et al. found that the equivalent depths associated with these resonances were normally less than 1 km.; thus they could well annihilate one another by interference, assuming they were excited. In addition, since a number of drastic simplifications were made in their analytic study, their results might not be valid for a more realistic atmosphere incorporating friction.

3. INITIAL CONDITIONS AND TIME INTEGRATION

As mentioned previously the diurnal model was developed from another general circulation model, which had a radiative state corresponding to a mean daily zenith angle. This model was converted into the diurnal model at an arbitrary time when the former had reached its stage of quasi-equilibrium; hence the diurnal model had fully developed temperature and wind fields etc. from its initiation. It might be mentioned here that the nondiurnal model gave, on the whole, a quite satisfactory representation of the atmosphere, except for a general tendency for the major variables to be displaced somewhat equatorwards; see Manabe and Hunt [13]. This model was closest to winter conditions in the actual atmosphere in many of its features; despite this the diurnal model was run for the time of the equinox because of the ease of calculating the solar zenith angles.

The original plan of this study was to run the diurnal model to a quasi-steady state, which it was hoped would reproduce the amplitude and phase of the observed tidal surface pressure oscillations of the actual atmosphere. The importance of the absorption of solar radiation by ozone and water vapor could then be investigated by removing their individual diurnal contributions from the thermal forcing function, and observing what happened to these oscillations. Unfortunately this ideal situation did not quite work out as planned, and considerably more effort than expected was necessary to obtain the desired results. The details of the various stages of experimentation are given briefly in the following sections.

It will be clear from the results subsequently presented that the amplitude and phase of the tides at higher latitudes were not satisfactorily established, because of the high level of meteorological noise which was present. A very much longer period of running would have been required for each model to have improved the signal-to-noise ratio at these latitudes, but because of the enormous amount of computer time involved this was not feasible. Hence, this study should be regarded as a preliminary investigation, and subsequently more refined experiments may be carried out.

4. DIURNAL EXPERIMENTS

Because a variety of slightly different diurnal models are discussed in this and the subsequent section, it is convenient at this stage to summarize them in table 2. Models 1 to 4 will be presented in this section and models 5 and 6 in section 5.

MODEL 1

This model was started from the nondiurnal model and was integrated until a stable pressure oscillation had developed. Figure 1 shows the surface pressure variation at a near equatorial point, which is seen to have a semidiurnal period with the correct phase but a quite remarkable amplitude. The noise associated with the pressure variation was caused by short period gravity waves at the model surface. These gravity waves had a very short wavelength and in no way interacted with the tidal motions; as will be seen later, they were only of importance in the Tropics. Since in the atmosphere the double amplitude of the tidal oscillations at the Equator is of the order of 3 mb., it is clear that this model grossly overestimated the tidal effects.

TABLE 2.—*Details of models*

Model	Starting Point	Surface Convective Activity	Thermal Forcing in Atmosphere
1	Nondiurnal model	Unrestricted	Diurnal heating by H_2O, CO_2 and O_3
2	"	Surface temperature fixed	"
3	Model 1 termination	Suppressed by using steady state solar insolation at surface	"
4	Nondiurnal model	"	"
5	Model 4 steady state	"	Diurnal heating by CO_2 and O_3 only
6	"	"	Diurnal heating by CO_2 and H_2O only

The reason for this is not difficult to locate when conditions at the surface of the earth and the model are compared. The entire model surface is "land," which underwent a marked diurnal temperature variation resulting in a strong diurnal variation of convection, which caused the tides to be amplified. This marked surface temperature variation can be attributed to the omission of heat storage capacity in the lower boundary of the model, which meant that it had to respond fully to the current incoming solar radiation absorbed at the surface. Now, since 70 percent of the earth's surface is covered with water, and it is an experimental fact that the sea surface temperature has a diurnal range of the order of 0.1°C., the convective forcing of the tides obtained in the model is largely missing in the atmosphere, and a smaller amplitude results. A further difference is that in the actual atmosphere most of the vertical energy flux from the earth's surface consists of latent energy. This part of the energy flux does not always condense and cause a temperature change locally, whereas in the model the moist convective adjustment is based on the simplifying assumption that all the latent energy is converted into sensible energy as soon as evaporation occurs.

The model results indicate that convection from the surface can be a mechanism of considerable importance in exciting tidal oscillations, a feature which could be of importance in other planetary atmospheres. In fact, when man finally attains his ultimate goal of concreting over the whole surface of the globe for car-parking space, one should expect tidal phenomena to play a more important part in day-to-day affairs than at present.

MODEL 2

This model was started from the same point of the nondiurnal model as model 1, but in order to suppress the

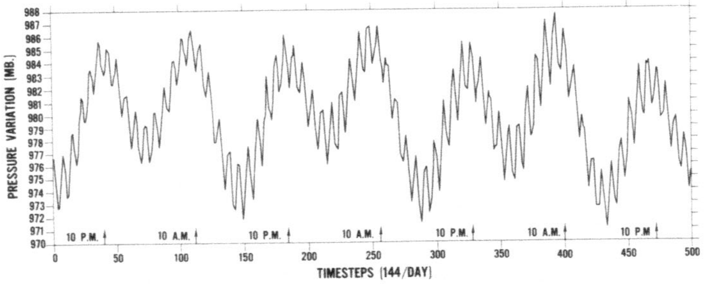

FIGURE 1.—The variation of the surface pressure oscillation in the Tropics for model 1 illustrating the large amplitude produced by the convective activity.

convection arising from the diurnal variation of the surface temperature, the latter was set permanently to the values of the nondiurnal model. This was rather a drastic change and caused the model to blow up after a few tens of timesteps. This trouble was eventually resolved by reducing the timestep a factor of two to 300 sec., which produced stable integrations. The reason for this behavior is unknown. However, a fixed surface temperature, although used in other general circulation models, is undesirable as it tends to destroy the baroclinicity of the atmosphere. This results from the moist convective adjustment destroying the eddy available potential energy in the model troposphere. Nevertheless, as shown in figure 2, the desired result was obtained, as the amplitude of the pressure oscillation in the Tropics was reduced to a value in satisfactory agreement with observation. The pressure variation in this figure was obtained from the superposition of 3 days' data in order to remove some higher frequency noise.

Models 1 and 2 can be considered to represent extreme conditions corresponding to a totally land or a totally sea-covered earth respectively, with conditions for the actual earth corresponding much more closely to model 2. Despite the fact that model 2 appeared to reproduce satisfactorily the tidal oscillations of the atmosphere, it was decided not to continue this run because of the restrictive boundary condition of a fixed surface temperature. The model, however, served a useful purpose as it revealed that by suitably adjusting conditions at the surface the desired results should be obtained.

MODEL 3

Based on this knowledge, a fresh approach was made to the problem. This consisted of starting the current model from where model 1 was terminated, but reducing the marked diurnal surface temperature variation of that model by replacing the diurnal solar radiation absorption at the earth's surface by a mean value, averaged over the day in the manner used in the nondiurnal model. This meant that the surface temperature was still allowed to vary with time, but was now computed for the mean value of the solar radiation absorption at the surface. The surface received the same total solar radiation in a 24-hr. period as in the case of model 1, so that the same energy input was maintained. For all practical purposes the diurnal variation of the intensity of convection from the surface was suppressed by this procedure. The atmosphere, of course, still experienced the complete diurnal variation of the solar radiation.

For about the first ½ day after the start of model 3 its behavior, as judged by the surface pressure variation at an equatorial point, essentially represented a continuation of that of model 1. The model then became rather unstable and produced gravity wave pressure oscillations of a quite outstanding amplitude. With continued integration the amplitude damped down to more reasonable values of 3 to 4 mb., but it was found that periodically the oscillation virtually disappeared into noise produced by short period gravity waves. Figure 3 illustrates a transition region from predominantly tidal oscillations to short period gravity waves, although a longer interval is really required to indicate the full extent of this phenomenon. Because further integration resulted in a growth of the importance of the short period gravity waves relative to that of the tides, this model was terminated.

What appears to have happened was that in changing from model 1 to model 3 a planetary inertia-gravity wave of period 11.5 hr. was excited. It is known from the nondiurnal model (see figure 2 of Manabe and Hunt [13]), that the model can excite such an inertia-gravity wave, and that it takes 40 to 50 days to die out naturally. This wave then seems to have produced a beat phenomenon with the semidiurnal oscillation about every 12 days, which accounts for the periodic disappearance of the tidal oscillations. Rather than try to overcome this trouble by forcefully damping the inertia-gravity wave it was decided to use the approach given in the following section for model 4.

MODEL 4

Since the basic idea of model 3 seemed to be sound, the present model made use of this approach but was started from the nondiurnal model rather than model 1. This was the only difference between models 3 and 4. Using this approach it was hoped that the beat phenomenon would not be excited; this expectation proved to be correct and model 4 gave stable tidal oscillations and also appeared to behave satisfactorily meteorologically. Hence the tidal surface pressure oscillations for this model will be analyzed and presented here, before discussing further experiments designed to isolate the excitation mechanism of the tides.

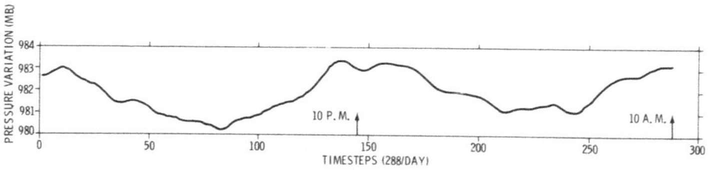

Figure 2.—The variation of the surface pressure oscillation in the Tropics for model 2 time averaged over a 3-day period. The surface temperature was fixed in this model.

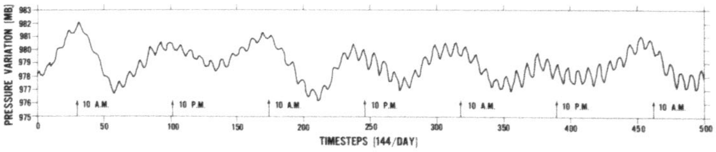

Figure 3.—The variation of the surface pressure oscillation in the Tropics for model 3 illustrating part of the decay of the diurnal oscillation owing to the beat phenomenon.

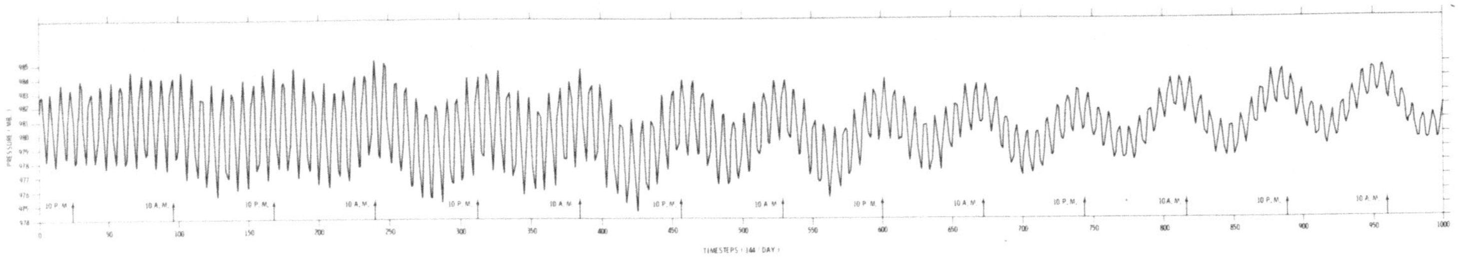

Figure 4.—The variation of the surface pressure oscillation in the Tropics for model 4. The growth of the predominantly semidiurnal tide and the concurrent decay of the external gravity wave over a 6-day period from the start of the model is shown.

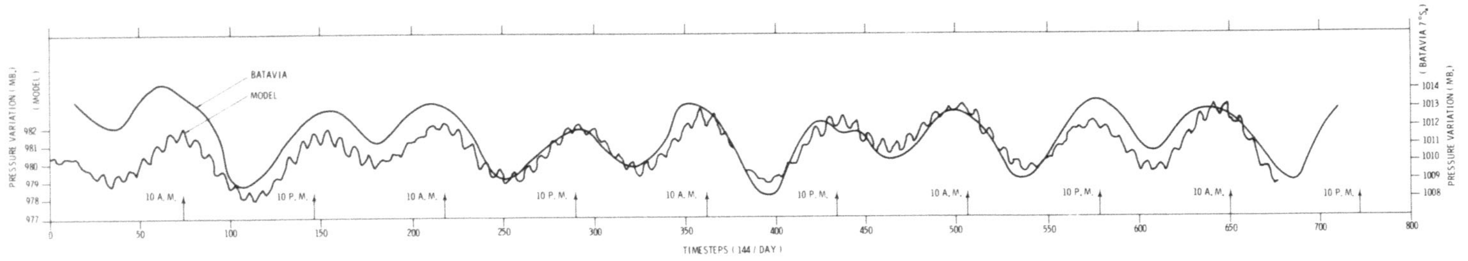

Figure 5.—A comparison of the observed surface pressure oscillation at Batavia, 6°S., with the computed surface pressure oscillation for the same point in model 4 as in the previous figure.

The growth of the surface pressure oscillation in model 4 is shown in figure 4 for a tropical point. The rather large amplitude external gravity wave which exists at the surface in the model at these latitudes gradually damped as the semidiurnal oscillation developed over a period of about 3 days. With continued integration this gravity wave was damped even more, as indicated in figure 5, and only in the Tropics did it really survive. Presumably this is because the convective adjustment mechanism was most active in the Tropics, and also because the Coriolis parameter is very small there.

In figure 5 the surface pressure variation for the same tropical point as figure 4, but for conditions starting 12 days after the initation of the model, is compared with the observed variation at Batavia (Djakarta), 6°S., for January 1924. Overall, the agreement is very satisfactory, as the model exhibits the same form of interdiurnal variability as the atmosphere, and also has a close correspondence in both phase and amplitude. The magnitude of the surface pressure in the model is much less than in the actual atmosphere for reasons given previously. The interdiurnal variability can be attributed to meteorological effects associated with the large-scale dynamics, which can either affect the pressure itself directly, or may influence the propagation of the tidal oscillations from their source regions, and thus affect the pressure variation itself directly. In view of the agreement of the model results with those for Batavia, it would appear rather unlikely that they are unduly influenced by spurious resonances, or other perturbations, associated with the finite difference formulation required for numerical studies.

The diurnal variation of the surface pressure at different latitudes in the model is illustrated in figure 6 for a particular line of longitude, the results presented being values time averaged over a 4-day period in order to suppress the small-scale gravity wave components. The mean pressure at each latitude follows from the normal latitudinal variation of the surface pressure, except that the subtropical High appears to have been destroyed in the diurnal model. The temperate latitude low pressure belt which is usually at about 60° lat. is displaced equatorwards in the model. This was also a feature of the nondiurnal model which was observed in other fields; see Manabe and Hunt [13] for further details. The decrease of the amplitude of the tidal oscillation with increasing latitude is apparent in figure 6, and north of about 50° it disappears into the meteorological background. At the high latitudes large-scale pressure variations were apparently taking place during the 4-day period considered here.

The curves shown in figure 6 were harmonically analyzed to obtain the phase and amplitude of the diurnal and semidiurnal components. The latitudinal variation of these terms for the semidiurnal component is compared in figure 7 with that for the actual atmosphere taken from the figures given by Haurwitz [6]. His values for 80°W. long. were selected, although the choice of longitude is only really of importance in the Tropics, where Haurwitz shows that the largest amplitudes tend to occur over the land areas. This may reflect the influence of convection associated with the diurnal variation of the surface temperature over the land, or possibly, that the observations are more accurate at land stations. The computed

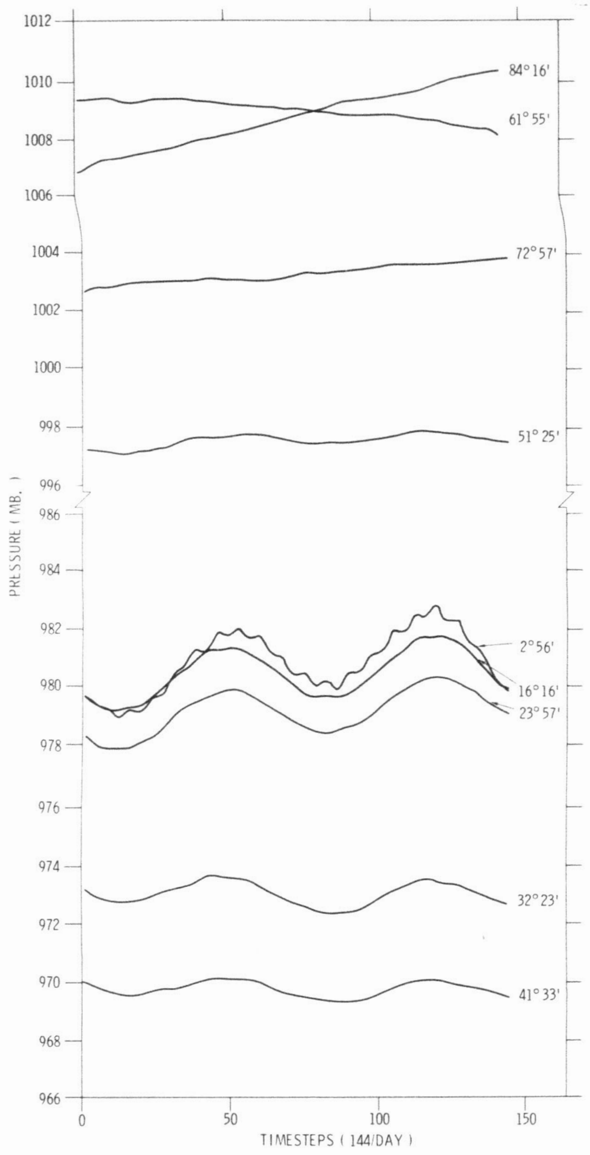

Figure 6.—The diurnal variation of the surface pressure at various latitudes in model 4 time averaged over a period of 4 days.

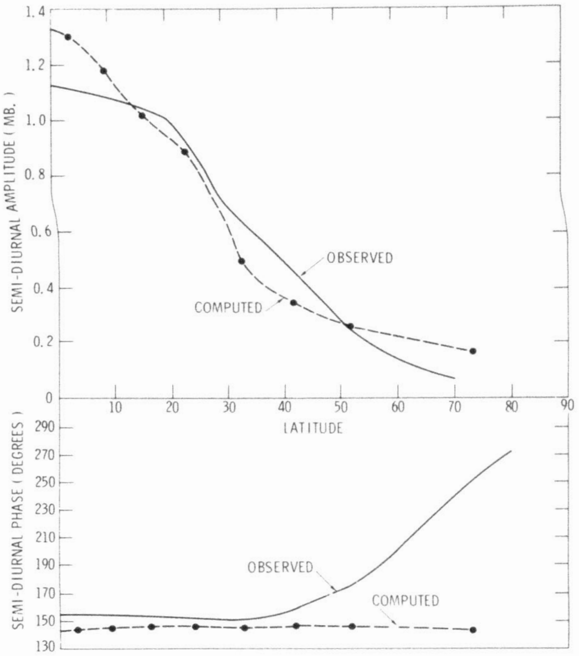

Figure 7.—Comparison of the semidiurnal amplitudes and phases of the surface pressure oscillation as a function of latitude for model 4 and observed values given by Haurwitz [6]. The computed values are the average of results at two longitudes 180° apart. The phases illustrated are for the time of the maximum.

results in figure 7 are the average of values at two longitudes 180° apart (data were only stored for two longitudes), the average being shown as it produced a somewhat smoother distribution. Some of the results at the higher latitudes could not be used as the harmonic analysis gave answers which were not meaningful, because of the predominance of meteorological rather than tidal effects at these latitudes. Despite these limitations the agreement between the observed and computed semidiurnal amplitude in figure 7 is satisfactory except at high latitudes. The model phase only agrees with observation up to about 40°, and presumably the variation of the observed phase with latitude is caused by the earth's topography. The semidiurnal pressure oscillation is largely determined by the polar vibration at high latitudes (see Wilkes [19], p. 10) and according to Siebert [17] the land-sea distribution is important for this tidal component. Thus the omission of topography from the model may have resulted in the invariance of the computed phase with latitude. In figure 8 the corresponding diurnal terms are compared with an empirical expression given by Haurwitz [7], the values for both longitudes being given separately in order to illustrate clearly the different behavior of the phases. The computed and observed amplitudes are fairly similar up to about 50° lat., but the phases are only in agreement south of about 30°. Since only 4 days of data were used in the harmonic analysis, and no attempt was made to remove meteorological effects from these data, it is considered that the agreement is as good as can be expected.

5. EXCITATION MECHANISMS OF THE TIDAL OSCILLATIONS

The work of Siebert [17], Butler and Small [2], and Lindzen [10], based on the conventional analytic approach, clearly indicates that the tides in the atmosphere are primarily excited by the diurnal heating resulting from the absorption of solar radiation by ozone and water vapor. In the case of the amplitude of the semidiurnal surface pressure oscillation, when using the combined results of Siebert and Butler and Small, one finds the individual contributions of water vapor and ozone to be about 30 and 50 percent respectively of the total amplitude. The corresponding values for the diurnal amplitude according to Lindzen are about 50 and 15 percent respectively. These results, although generally accepted, are rather heterogeneous, since each worker used a different model and examined a particular component of the tide for a particular excitation mechanism. In addition,

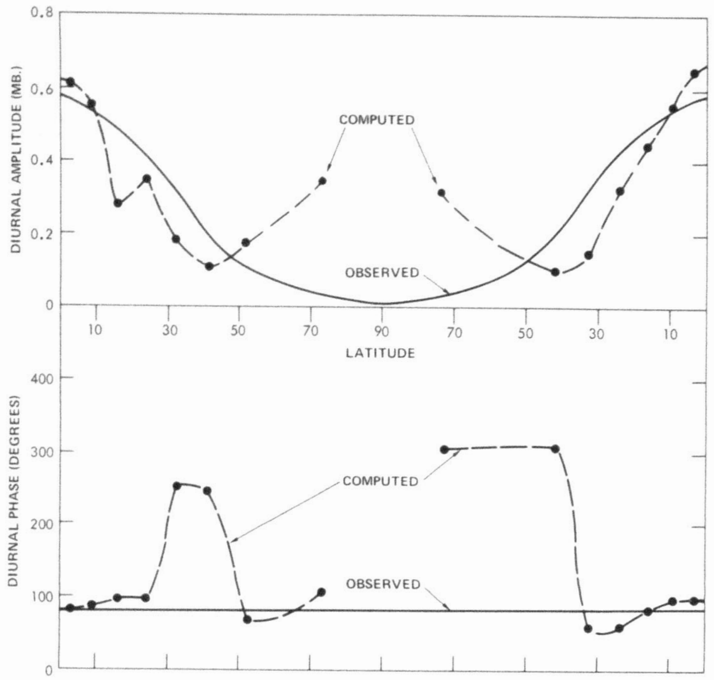

FIGURE 8.—Comparison of the diurnal amplitudes and phases of the surface pressure oscillation as a function of latitude for model 4 and an empirical expression given by Haurwitz [7]. The computed values are given for two longitudes 180° apart. The phases illustrated are for the time of the maximum.

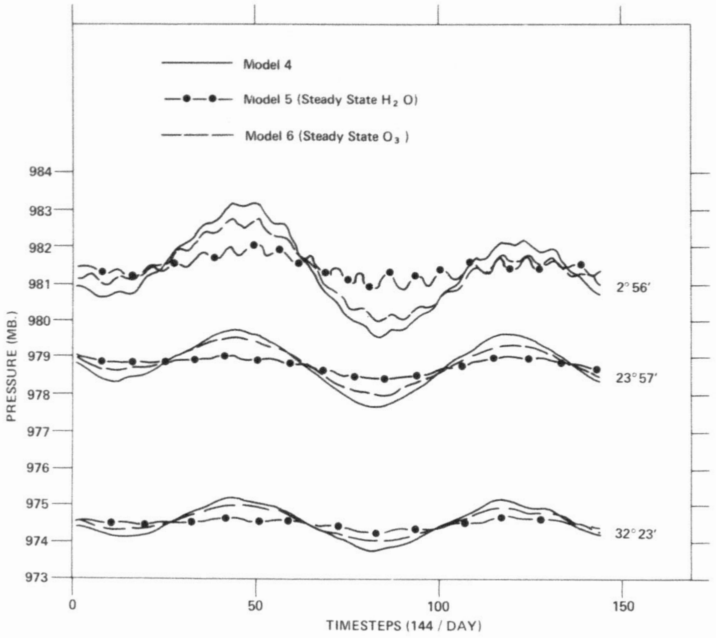

FIGURE 9.—A comparison of the surface pressure oscillations at selected latitudes for models 4, 5, and 6, illustrating the effects of removing the diurnal heating owing to the absorption of solar radiation by ozone or water vapor. Values shown have been time averaged over a 4-day period.

rather limited radiative models were used, which consisted essentially of estimates of the heating rates owing to the absorption of solar radiation. Longwave radiative cooling was assumed to take place at a uniform rate over a 24-hr. period, and was not computed explicitly. It is obviously desirable that a more consistent evaluation of the contributions of ozone and water vapor to the excitation of the atmospheric tides should be made, and this is possible with the present model.

The approach used consisted of treating model 4 as the standard model and carrying out two experiments, in which the diurnal absorption of solar radiation by either ozone or water vapor was suppressed separately. In each case the diurnal absorption was replaced by a steady state value corresponding to the mean value obtained by averaging over a 24-hr. period, in the same way as used in a nondiurnal model (see Smagorinsky et al. [18]). By this procedure the same total amount of solar radiation was absorbed by the atmosphere in 24 hr. as in the normal diurnal model, this being required to make the various models energetically consistent. However, the ability of the solar radiation absorbed by the particular gas to excite a tidal oscillation was removed. The total hemispheric integral of kinetic energy varied by only about 0.5 percent between the various experiments as a result of these modifications. The model in which the diurnal absorption of solar radiation by water vapor was suppressed will be referred to subsequently as model 5, the corresponding case for ozone being model 6. Both of these models were initiated from model 4 at the same point in time, about 10 days after the start of that model so that the tidal oscillation was well defined. They were each run for a period of 7 days which overlapped with model 4; the results to be presented are based on data time averaged over the last 4 days of the experiments, in order to make them correspond to those described in the previous section for model 4. The tidal oscillation reacted quite fast to the suppression of the diurnal absorption of the solar radiation, and it is therefore thought that the results obtained are fairly representative of the actual contributions of ozone and water vapor to the maintenance of these oscillations.

In figure 9 the surface pressure variation averaged over this 4-day period is compared for the three models for three different latitudes. At all latitudes the results indicate that removing the diurnal solar heating associated with the water vapor resulted in a marked reduction of the amplitude of the tidal oscillation, whereas the corresponding change for ozone produced a much smaller effect. A more quantitative estimate of the relative importance of the ozone and water vapor excitation mechanisms is given in figure 10, where the variation of the amplitude of the semidiurnal component obtained by harmonically analyzing the data for all latitudes is shown for the three models. The results presented are again the average of values at two longitudes 180° apart, with the same data being rejected as for model 4 because of the distortion of the tides by meteorological variations. In addition, it was also necessary to reject data for model 5 at one of the longitudes for latitudes of approximately 30° and 40°, because the semidiurnal phase and the diurnal amplitude were greatly changed from the corresponding values of model 4.

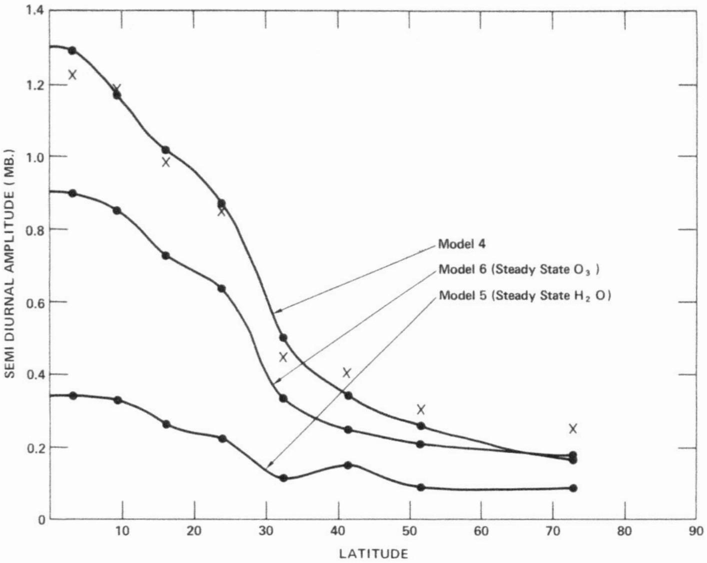

FIGURE 10.—The semidiurnal amplitudes of the surface pressure oscillations as a function of latitude are compared for models 4, 5, and 6. The difference between the amplitude of model 4 and that for either model 5 or model 6 indicates the contribution of water vapor or ozone respectively to the actual amplitude of the semidiurnal tide. The crosses give the values obtained by summing the amplitudes of models 5 and 6.

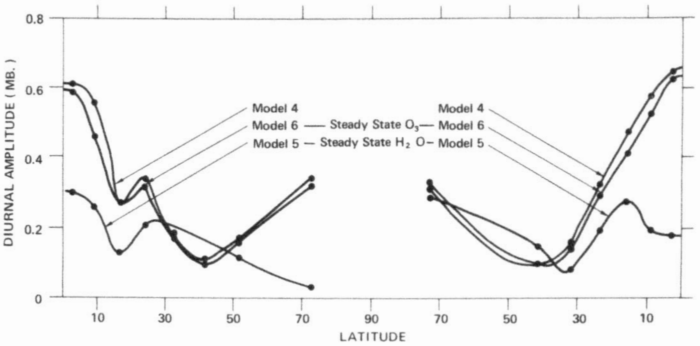

FIGURE 11.—The diurnal amplitudes of the surface pressure oscillations as a function of latitude are compared for models 4, 5, and 6. The difference between the amplitude of model 4 and that for either model 5 or model 6 indicates the contribution of water vapor or ozone respectively to the actual amplitude of the diurnal tide.

An examination of the pressure variation at these latitudes during the 4-day period of interest revealed that a sizable cyclone passed through, which did not appear to have greatly influenced the larger tidal oscillations associated with models 4 and 6, but was sufficient to distort the rather residual oscillation of model 5. This same cyclone may be responsible for the sudden change in the phase of the diurnal tidal component between 30° and 40° lat. shown on the left hand side of figure 8. The *difference* between the amplitude of model 4 and that of either models 5 and 6 at any given latitude indicates how much of the amplitude can be attributed to the water vapor or ozone excitation mechanisms respectively. Thus in the Tropics the water vapor is responsible for about 75 percent of the tide, with ozone accounting for virtually all of the remaining 25 percent. At higher latitudes the results are not considered to be sufficiently accurate to make definitive statements, but it appears that north of about 50° lat. the contribution of ozone to the tidal oscillation becomes negligible, while the water vapor still accounts for about 50 percent of the amplitude.

Since the phases of the semidiurnal components for the data shown in figure 10 were in fairly close agreement for all three models, normally within 10°–15°, the amplitudes can be added numerically rather than vectorially for the purposes of comparison. The resulting combined amplitude of models 5 and 6 is given by the crosses in figure 10, and it can be seen that they agree almost perfectly with the amplitude of model 4 up to about 30° lat. This is taken to indicate that, for this latitudinal range, the amplitude of the semidiurnal component can be entirely attributed to the diurnal heating produced by the absorption of solar radiation by ozone and water vapor, no other mechanism apparently being required. At higher latitudes the combined amplitude is greater than model 4, and assuming that the model results are meaningful at these latitudes, this reveals that some additional mechanism to those considered here is operative. This could possibly be the so-called polar vibration, but because of the small amplitudes involved it is not thought that the results are accurate enough to justify a discussion.

The comparison of the diurnal components for the three models is given in figure 11, the same data being rejected as for figure 10. The results leave little doubt that ozone heating due to absorption of solar radiation is of minor importance in the model at all latitudes as regards the generation of the diurnal tide. The corresponding water vapor heating appears to be of considerable importance, and accounts for between ½ and ⅔ of the amplitude up to about 30° latitude. The results at higher latitudes do not justify a discussion. The model behavior is at least consistent in attributing the greater part of the amplitude of both the diurnal and semidiurnal tides to the water vapor excitation mechanism. Apart from finding a somewhat larger contribution due to ozone heating, the analysis of the diurnal tide by Lindzen [10] produced similar results, as mentioned previously. He also could only account for approximately ⅔ of the diurnal amplitude at low latitudes in terms of absorption of solar radiation, but north of 45° found that this thermal drive could explain the complete amplitude. This suggests that some additional mechanism is required to account for part of the diurnal tide at low latitudes. Surface convective activity is suspected because of the latitudinal region involved, as the residual convection permitted in the model might have been capable of producing the small amplitude required (0.3 mb.), since it operated in the densest part of the atmosphere. Also, because the convective activity would have a primarily diurnal component, this might explain why convection does not seem to influence the semidiurnal pressure oscillation. Various attempts have been made to assign a larger contribution to surface

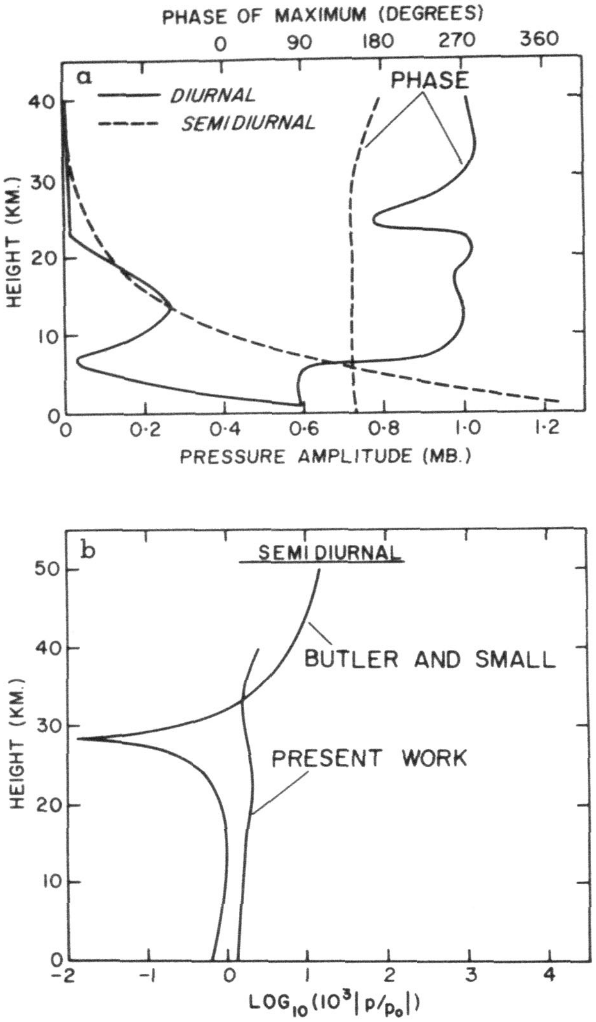

FIGURE 12.—(a) The variation with altitude of the amplitudes and phases of the pressure oscillations of the diurnal and semidiurnal tides is shown. The results given are for model 4 and have been time averaged over a 4-day period. (b) The ratio of the local pressure amplitude to the local static pressure for the semidiurnal tide is given as a function of height for the general circulation model and the model of Butler and Small [2]. All data in (a) and (b) are for a near-equatorial point.

convection in the tidal excitation mechanism. However, over the oceans convection should probably be relatively unimportant owing to the invariance of the sea surface temperature, and, since the oceans dominate the surface of the globe, it would appear that this particular mechanism is of limited importance in the atmosphere.

Some insight into the reasons for the different behavior of these tides is provided in figure 12a. The results presented there were obtained by harmonically analyzing the data of model 4 time averaged over a 4-day period, the data being interpolated to give results at intervals of 1 km. from the surface to 40 km. The variation with height of the amplitude and phase of the pressure oscillation is seen to be completely different for the two tidal components. The relatively constant value of the semidiurnal phase with height shown in figure 12a indicates that the individual pressure oscillations produced by the local heating mechanisms at the different altitudes act in unison; hence the amplitude of the semidiurnal surface pressure oscillation is effectively the integral of the individual oscillations. On the other hand the diurnal component has two distinct regions of activity which act independently. According to figure 12a, in the Tropics the diurnal surface pressure oscillation in the model is produced by only the lowest 6 km. of the atmosphere. The quite large diurnal pressure oscillations that are produced around 15 km. do not propagate to the surface, because they are 180° out of phase with the corresponding oscillations in the lower troposphere, with the result that the oscillations cancel. This produces the minimum observed in the diurnal amplitude at 6 km. As discussed previously the low value of the surface pressure amplitude of the diurnal tide is associated with its small equivalent depths. Figure 12a reveals that in practice the effect of these equivalent depths must be to produce phase variations with altitude; these then result in the cancellation of the tidal oscillation in certain height ranges. Why the phases have their particular values is not clear, but these differ from the phases of the tidal temperature oscillations given in the following section, and as suggested by Harris et al. [5] friction may be responsible for these phase differences.

It is also possible, with the help of figure 12a, to understand the importance of the water vapor absorption in the tide producing mechanism. Manabe and Möller [14] have computed the radiative temperature tendencies due to the absorption of solar radiation by water vapor, carbon dioxide, and ozone, and have shown that in the troposphere water vapor is overwhelmingly dominant. The largest temperature tendencies of the water vapor were in the lower troposphere, and were found to be almost invariant with height up to 5 km. Because of this invariance the largest pressure response would be expected to be where the density is greatest, i.e. near the surface, which explains why the pressure amplitudes in figure 12a increase so rapidly with decreasing altitude in the lower troposphere. Now, although the ozone radiative temperature tendencies are very large in the stratosphere above about 20 km., the ozone heating does not dominate the semidiurnal oscillation. This is because its effects are confined to a low density region, and the magnitudes of the pressure oscillations produced are rather small. To express this another way, it is simply because the density decreases faster with altitude than the radiative temperature tendency increases, that ozone is less effective than water vapor in exciting the tides. It is therefore difficult to understand how Butler and Small [2] obtained such a large contribution to the amplitude of the semidiurnal surface pressure oscillation from ozone heating. They computed the maximum absorption of solar radiation by ozone in the Tropics to be at 40 km., where the static pressure is only 3 mb. Hence, if this altitude range is to contribute significantly to the surface pressure oscillation, very large percentage variations in the masses of these levels are required. In the present model a fairly constant variation of 0.1 percent of the static mass was computed to occur at all levels due to the semidiurnal tide; see figure 12b.

The negligible contribution of ozone to the amplitude of the diurnal surface pressure oscillation illustrated in figure 11 can also be explained. This result is obtained because the ozone heating is essentially confined to the stratosphere, and the local pressure oscillations produced are in an altitude region from which propagation to the surface is not permitted for this tide. In the case of the semidiurnal tide no such restriction on propagation exists, and ozone heating makes a contribution to the surface pressure oscillation; see figure 10.

Harris et al. [4] have attempted to deduce the variation with altitude of the phase and amplitude of the pressure components of these tides, from balloon observations over the Azores (38°N.). The observations available to them were not entirely suitable for this analysis, and they obtained rather different results depending upon whether they computed the pressures directly from the observations, or indirectly from the winds. There were some regions of agreement between their results and those of the model, but further experimental work is required in order to assess whether the distributions given in figure 12a are realistic.

Finally, mention should be made of the variation with altitude of the semidiurnal pressure amplitude expressed as a fraction of the local static pressure, which is shown in figure 12b. Also given is the corresponding ratio due to ozone excitation only, computed by Butler and Small [2]. Their distribution differs considerably from that of the general circulation model as they predict a sharp node at about 28 km. This node is thought to arise from the simplifications made in the conventional analytic approach which was used by Butler and Small. It seems rather unlikely that such a node would be maintained in the actual atmosphere in the presence of wind fluctuations and dissipation, hence its absence from the general circulation model is to be expected. Lettau [9] has attempted, inconclusively, to verify the existence of the node experimentally. Butler and Small also predict a phase change of 180° at the height of the node, a feature also in disagreement with the present model. However, there appears to be a phase difference of 180° between the semidiurnal tide in the ionosphere and that at the surface according to Wilkes [19]. Lettau's analysis suggests that the phase change occurs gradually starting at about 45 km., so that the observations are not inconsistent with the general circulation model.

6. METEOROLOGICAL ASPECTS OF THE DIURNAL VARIATION

It is interesting to contrast model 4 with its nondiurnal counterpart in order to see what differences, other than those described so far, were produced by the incorporation of the diurnal solar variation, especially since only nondiurnal models have been used previously at this Laboratory. One of the terms of importance in this regard is the change of the vertically integrated hemispheric kinetic energy of the model. It was found that both models experienced the same form of variation of kinetic energy with time, with the diurnal model having an increase of about 2.5 percent compared with the nondiurnal model. A similar increase occurred in the hemispheric integral of dissipation for the model, with practically all of the increase being confined to the level of the tropospheric jet.

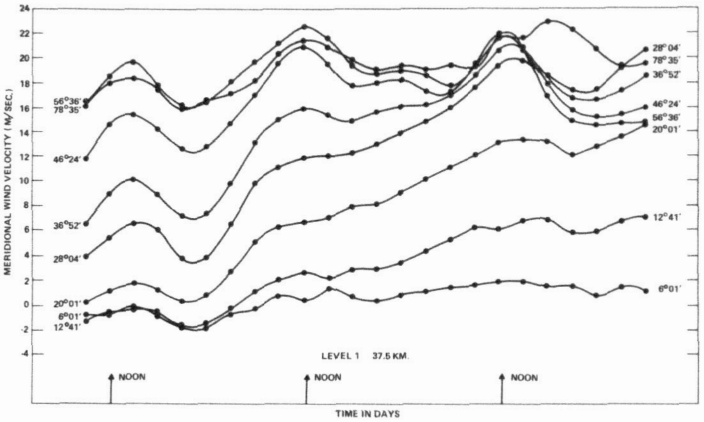

FIGURE 13.—The variation over a 3-day period of the meridional wind component at various latitudes in the top level of model 4.

A comparison was also made of hemispheric maps giving the instantaneous distribution of the temperature and geopotential height fields, and the zonal and meridional winds, for the top two levels of the diurnal and nondiurnal models for the same time. Although some differences existed, these were not sufficient to justify presenting these maps; in addition, no diurnal variation could be observed by a cursory inspection of a time sequence of these maps for model 4 made at 3-hr. intervals. These findings, together with the relatively minor changes obtained in the kinetic energy and dissipation integrals, indicate that at least up to 4 mb. no great distortion of the basic meteorological properties occurs in restricting a general circulation model to nondiurnal conditions. This is a matter of some practical, as well as scientific, interest, since a noticeable increase in computation time results from converting the normal model to a diurnal model, owing to the greater frequency of the radiation calculation.

Although no diurnal variation was observed in an examination of the hemispheric maps, this does not mean that such a variation did not exist in model 4. In figure 13 the change of the meridional wind speed with time, obtained by reading values from these maps, is illustrated for the top level of this model for a series of latitudes at one particular longitude. Some form of predominantly diurnal variation appears to be superposed on the basic wind distribution, the maximum wind being obtained at about noon. The higher latitudes seem to be more responsive to the diurnal drive, and an amplitude of about 2 m./sec. may exist there, which would represent about 10 percent of the basic meridional speed. Because

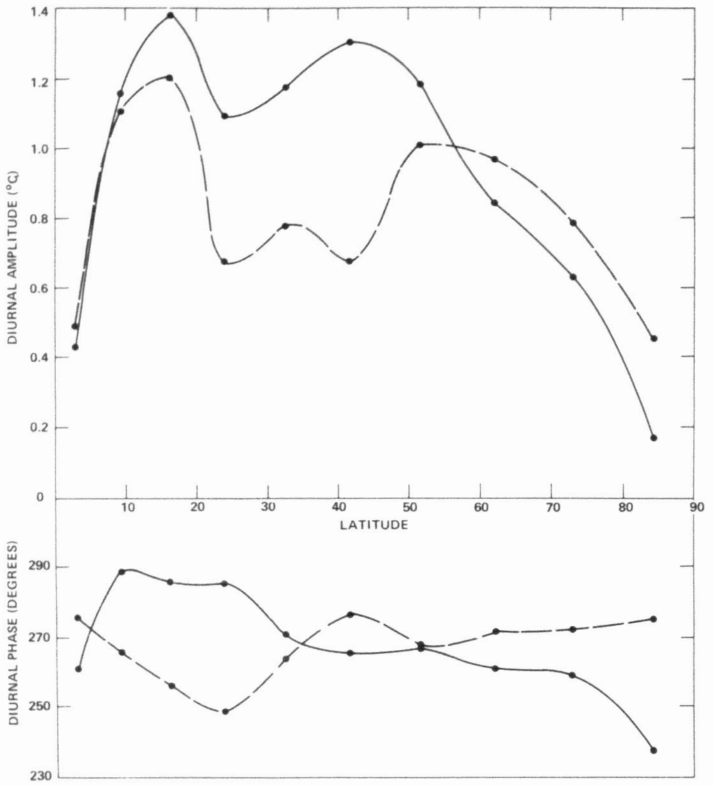

FIGURE 14.—The variation with latitude of the diurnal amplitude and phase of the temperature in the top level of model 4. Results for two longitudes 180° apart are indicated by the full and dashed lines in the figure. The phases illustrated are for the time of the maximum.

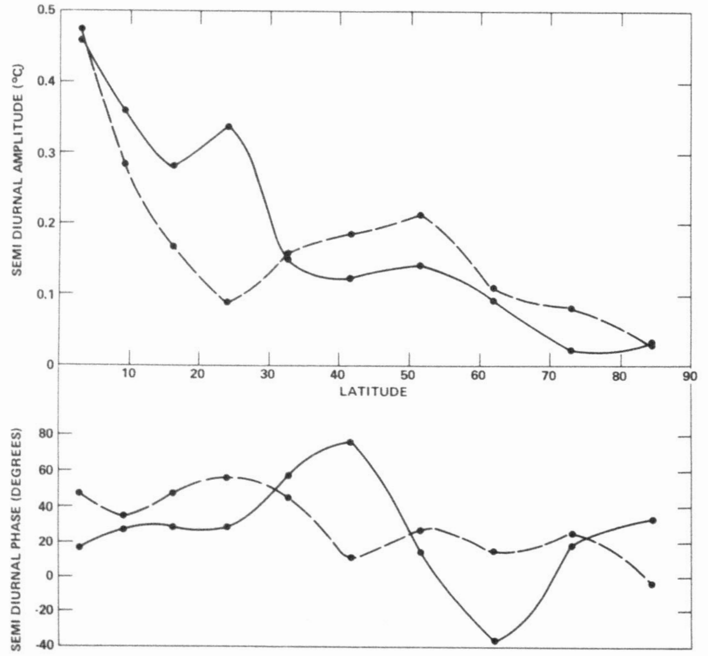

FIGURE 15.—The variation with latitude of the semidiurnal amplitude and phase of the temperature in the top level of model 4. Results for two longitudes 180° apart are indicated by the full and dashed lines in the figure.

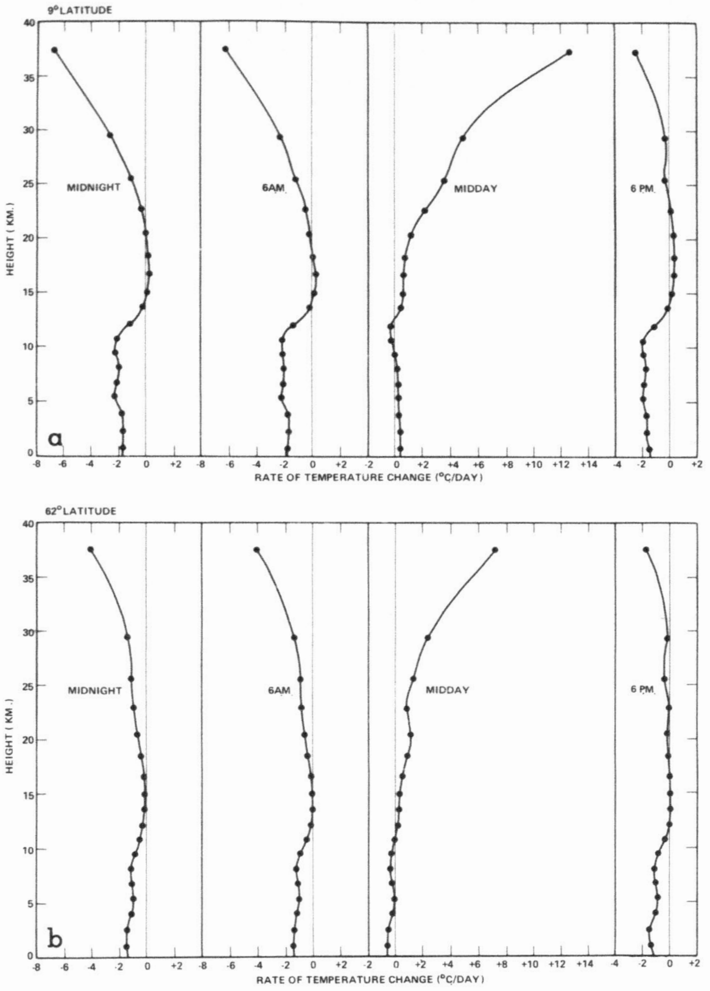

FIGURE 16.—(a) The net radiative temperature tendency for 9° lat. is given as a function of height for four different times of the day. (b) Similar results for 62° lat. are given. All results are for model 1.

only a rather limited data sample was available, the meridional speeds were not harmonically analyzed. The analytic calculations of Lindzen [10] indicate that the diurnal tide should have an amplitude of the order of 1–2 m./sec. in this altitude range (37.5 km.). The phase computed by Lindzen differs from that of the model, as he predicts, at least at midlatitudes, that the maximum amplitude should occur around midnight. The zonal wind in the top level also varied in a similar fashion, except that the correspondence in phase was not so apparent. Tidal oscillations in the winds at the second model level were obtained as well, although the noise level was much higher.

Another quantity which was investigated in somewhat more detail was the diurnal temperature variation in the upper levels of model 4. In figure 14 the diurnal amplitudes and phases are given for level 1 (4 mb.), for two locations differing by 180° in long., based on temperatures averaged over a 4-day period. The amplitudes in the Tropics are low, and this may have been caused by adiabatic effects associated with the vertical motion near the wall at the

Equator in the model, although no obvious correlation could be detected. Apart from this there is a general tendency for the amplitude to decrease with increasing latitude, the maximum amplitude being approximately 1.3°C. at about 15° lat. The phase distribution in the lower part of figure 14 indicates that the time of the maximum temperature should be at about 6 p.m. for all latitudes. Both of these results are in agreement with Lindzen's [10] calculations, but there is a shortage of observations for comparison with the theory. Measurements at 30°N. by Beyers and Miers [1] indicate that a diurnal temperature range of perhaps 5°C. might be expected at 40 km., which is noticeably larger than the 3°C. value given by the model. At higher altitudes they give very much larger temperature ranges, which may indicate errors in their measurements. The diurnal amplitudes at levels 2 and 3 corresponded quite closely at most latitudes, but the magnitude of the amplitudes was only about one-third of that for level 1.

The corresponding semidiurnal components of the temperature variation are given in figure 15. As would be expected the semidiurnal amplitudes are much smaller, being only about one-third of the diurnal amplitudes, and have a fairly similar decrease in magnitude with increasing latitude. The phase is rather irregular at high latitudes but a value of about 30° is approximately representative, hence maximum amplitudes are to be expected at 2 a.m. or 2 p.m.

To conclude this section some results will be presented for the variation with time of the net (i.e., longwave + shortwave) radiative temperature tendencies, as these have never been presented previously. The results were taken from model 1 since this was the only model with these data readily available; the large amplitude surface pressure oscillation in this model should have only a rather minor influence on the radiative results. In the nondiurnal model the net radiative temperature tendency is negative at all altitudes and latitudes apart from a region of the tropical stratosphere (see Manabe and Hunt [13]), but there is a considerable departure from this situation in the diurnal model. In figure 16 the radiative tendencies are presented as a function of height for tropical and high latitudes for four times representative of extreme conditions during the day. The low latitudes are obviously more responsive to the variation of the insolation, as would be expected, and the radiative tendency in level 1 in the Tropics changed dramatically during the day, from a minimum of about −7°C./day at midnight to a maximum of nearly 13°C./day at midday. This should be compared with a mean value of 0.5°C./day obtained in the nondiurnal model. The least responsive height range appeared to be the low temperature region around the tropical tropopause, which at all times maintained a very small heating rate, the heating at night being produced by longwave radiation; see the discussion by Manabe and Hunt [13]. On the other hand the region just below the tropical tropopause, 10–12 km., invariably had a small net cooling rate, and this was the only region in the Tropics where this situation existed. The tropical troposphere generally had a net radiative cooling rate of between −1 and −2°C./day; however, near midday a rather minor net heating rate did occur. Convection from the surface counterbalanced the radiative cooling in the troposphere.

At higher latitudes the radiative tendencies were fairly similar to those of the nondiurnal model in the troposphere, since there was a net cooling at all times as shown in figure 16b. The stratosphere was more active and a heating rate of about 7°C./day was reached at midday in the top level. This was somewhat larger than the maximum cooling rate of −4°C./day, which indicates why the rather small cooling rate of about −0.3°C./day was obtained in this region in the nondiurnal model.

7. CONCLUSIONS

The use of a general circulation model to study tidal oscillations in the earth's atmosphere appears to be entirely feasible, and to produce realistic results as far as the present model is concerned. Since the model used had a nonresonant atmosphere and gravitational excitation was excluded, the tidal oscillations obtained can be considered to support strongly Siebert's [17] claim that they are excited by the absorption of solar radiation in the atmosphere.

The model reproduced quite adequately the general features of the observed surface pressure oscillations, which are the most extensively studied feature of the tides. Particularly satisfying was the ability of the model to simulate the phase, amplitude, and interdiurnal variability of the surface pressure in the Tropics. Both the diurnal and semidiurnal amplitudes of the surface pressures were also found to be in agreement with observation at most latitudes. However, the associated phases were rather poor in temperate and higher latitudes, and this is thought to reflect the perturbations produced by the meteorological systems in these regions. The model provides no information concerning why the observed phases have their particular values, although it does reveal that the diurnal surface pressure amplitude is less than the semidiurnal component, because of the variation with altitude of the phase of the diurnal pressure oscillation.

Contrary to the conclusions of Butler and Small [2], the absorption of solar radiation by water vapor was found to be of greater importance in the present model, in exciting the semidiurnal tide, than the corresponding absorption by ozone. Up to about 30° lat. water vapor accounted for about 75 percent of the amplitude and ozone for the remainder. No other excitation source seems to be required, in disagreement with the conclusions of Harris et al. [5]. In the case of the diurnal surface pressure amplitude, ozone was found to be of negligible importance at all latitudes, because propagation from the ozone excitation region to the surface was not permitted. At low latitudes water vapor absorption accounted for perhaps two-thirds of the diurnal amplitude. The remaining part of the amplitude is thought to be excited by convection from the surface, although more experiments are needed to verify this supposition.

As regards the more meteorological aspects of the model it was concluded that, at least up to a height of 4 mb., most features of interest are adequately represented by using the simpler nondiurnal model. Diurnal variations were shown to occur in the temperature and meridional wind distributions in the top layer of the model, but these represented rather small perturbations on the basic distributions.

Finally it is concluded that spurious resonances or other deleterious effects associated with using a model with a "lid" do not appear to be present in the model. Although such resonances may be excited, they presumably do not propagate in this model because of the presence of dissipation. It is also considered that the present results provide some justification for the use of linearized equations in the classical approach to tidal theory, since the more complete equations used here, in conjunction with a realistic atmosphere, produce results in general agreement with this approach.

Additional experiments are required to clarify a number of matters. A model incorporating the hydrologic cycle would permit the investigation of a possible contribution from moist convective processes, and the resulting heat of condensation, to the excitation of tides in the atmosphere. Experiments designed to provide information on the phase distribution of the various terms in the model, and their interrelationships, would also be very valuable. Further experiments of interest would be to investigate the effect on the tides of dropping the friction and nonlinear terms from the equations of motion. This is not possible with the present model as the tides are essentially perturbations superimposed on the basic meteorological fields, and dropping these terms for the tides also means dropping them from the general circulation model, which would result in an unrealistic atmosphere. This particular problem would perhaps best be approached by developing a simplified model specifically to study these features.

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REFERENCES

1. N. J. Beyers and B. T. Miers, "Diurnal Temperature Change in the Atmosphere Between 30 and 60 Km. Over White Sands Missile Range," *Journal of the Atmospheric Sciences*, Vol. 22, No. 3, May 1965, pp. 262–266.
2. S. T. Butler and K. A. Small, "The Excitation of Atmospheric Oscillations," *Proceedings of the Royal Society of London*, Ser. A, Vol. 274, No. 1356, June 1963, pp. 91–121.
3. J. S. A. Green, "Atmospheric Tidal Oscillations: An Analysis of the Mechanics," *Proceedings of the Royal Society of London*, Ser. A, Vol. 288, No. 1415, Nov. 1965, pp. 564–574.
4. M. F. Harris, F. G. Finger, and S. Teweles, "Diurnal Variation of Wind, Pressure and Temperature in the Troposphere and Stratosphere Over the Azores," *Journal of the Atmospheric Sciences*, Vol. 19, No. 2, Mar. 1962, pp. 136–149.
5. M. F. Harris, F. G. Finger, and S. Teweles, "Frictional and Thermal Influences in the Solar Semidiurnal Tide," *Monthly Weather Review*, Vol. 94, No. 7, July 1966, pp. 427–447.
6. B. Haurwitz, "The Geographical Distribution of the Solar Semi-Diurnal Pressure Oscillations," *Meteorological Papers*, College of Engineering, New York University, Vol. 2, No. 5, Dec. 1956, 36 pp.
7. B. Haurwitz, "The Diurnal Surface-Pressure Oscillation," *Archiv für Meteorologie, Geophysik und Bioklimatologie*, Ser. A, Vol. 14, No. 4, Vienna, June 1965, pp. 361–379 (see p. 361).
8. S. Kato, "Diurnal Atmospheric Oscillation: I. Eigenvalues and Hough Functions," *Journal of Geophysical Research*, Vol. 71, No. 13, July 1966, pp. 3201–3209.
9. B. Lettau, "Tidal Motions in the High Atmosphere," *Journal of Geophysical Research*, Vol. 70, No. 14, July 1965, pp. 3255–3263.
10. R. S. Lindzen, "Thermally Driven Diurnal Tide in the Atmosphere," *Quarterly Journal of the Royal Meteorological Society*, Vol. 93, No. 395, Jan. 1967, pp. 18–42.
11. R. S. Lindzen, "On the Theory of the Diurnal Tide," *Monthly Weather Review*, Vol. 94, No. 5, May 1966, pp. 295–301.
12. R. S. Lindzen, E. S. Batten, and J.-W. Kim, "Oscillations in Atmospheres With Tops," *Monthly Weather Review*, Vol. 96, No. 3, Mar. 1968, pp. 133–140.
13. S. Manabe and B. G. Hunt, "Experiments With a Stratospheric General Circulation Model: I. Radiative and Dynamic Aspects," *Monthly Weather Review*, Vol. 96, No. 8, Aug. 1968, pp. 477–502.
14. S. Manabe and F. Möller, "On the Radiative Equilibrium and Heat Balance of the Atmosphere," *Monthly Weather Review*, Vol. 89, No. 12, Dec. 1961, pp. 503–532.
15. S. Manabe and R. F. Strickler, "Thermal Equilibrium of the Atmosphere With a Convective Adjustment," *Journal of the Atmospheric Sciences*, Vol. 21, No. 4, July 1964, pp. 361–385.
16. Y. Mintz, "Very Long-Term Global Integration of the Primitive Equations of Atmospheric Motion," *World Meteorological Organization Technical Note* No. 66, Geneva, 1965, pp. 141–167.
17. M. Siebert, "Atmospheric Tides," *Advances in Geophysics*, Vol. 7, Academic Press, New York, 1961, pp. 105–187.
18. J. Smagorinsky, S. Manabe, and J. L. Holloway, Jr., "Numerical Results From a Nine-Level General Circulation Model of the Atmosphere," *Monthly Weather Review*, Vol. 93, No. 12, Dec. 1965, pp. 727–768.
19. M. V. Wilkes, *Oscillations of the Earth's Atmosphere*, Cambridge University Press, England, 1949, 74 pp.

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