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# JTS Topology Suite Technical Specifications

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Version 1.4



## Document Change Control

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## 1. OVERVIEW

The JTS Topology Suite is a Java API that implements a core set of spatial data operations using an explicit precision model and robust geometric algorithms. JTS is intended to be used in the development of applications that support the validation, cleaning, integration and querying of spatial datasets. This document is the design specification for the classes, methods and algorithms implemented in the JTS Topology Suite.

JTS attempts to implement the OpenGIS Simple Features Specification (SFS) as accurately as possible. In some cases the SFS is unclear or omits a specification; in this case JTS attempts to choose a reasonable and consistent alternative. Differences from and elaborations of the SFS are documented in this specification.

The detailed documentation of the class hierarchy and methods will be presented in the form of JavaDoc for the source code.

## 2. OTHER RESOURCES

- OpenGIS Simple Features Specification For SQL Revision 1.1 (referred to as SFS in this document). This document provides the master specification for the spatial data model and the definitions of the spatial predicates and functions implemented by JTS.

## 3. DESIGN GOALS

The design of JTS is intended to fulfil the following goals:

- The spatial model and method definitions will conform to the OpenGIS Simple Features Specification as accurately as possible, consistent with correct implementation.
- The API design will follow Java conventions wherever possible. For instance:
  - ∅ accessor functions will use the Java getX and setX convention
  - ∅ predicates will use the isX convention
  - ∅ methods will start with a lowercase letter
- JTS functions will support a user-defined precision model. JTS algorithms will be robust under that precision model.
- Methods will return topologically and geometrically correct results within the defined precision model wherever possible.
- Correctness is the highest priority; space and time efficiency is important but secondary.
- JTS will be fast enough to be used in a production environment.
- The algorithms and code used in JTS will be clear and well-structured, to facilitate understanding by other developers.

## 4. TERMINOLOGY

| Term                  | Definition   |
|-----------------------|--|
| Coordinate            | A point in space which is exactly representable under the defined precision model  |
| Exact Computation     | Numerical computation which maintains all digits of numbers through all operations. Usually requires computationally expensive algorithms  |
| Node                  | A point where two lines within the same or different geometries intersect. This point is not necessarily representable by a coordinate, since the output of the computation of the intersection point in general requires greater precision than the input points. |
| Noding (also Noded)   | The process of computing the nodes where one or more geometries intersect.   |
| Non-coordinate        | A point which is not representable as a coordinate   |
| Numerical Stability   | The stability of a numerical algorithm is determined by the maximum bound on the error in its outputs. An algorithm is considered to be stable if this bound is small.   |
| Point                 | An arbitrary point in $R^3$ . In general, not finitely representable.  |
| Proper intersection   | An intersection between two line segments where the intersection is a single point and is internal to both segments  |
| Robust Computation    | Numerical computation which is guaranteed to return the correct answer for all inputs. Usually requires algorithms which are specially designed to handle round-off error.   |
| SFS                   | OGC Simple Features Specification  |
| Unit of Resolution    | The smallest representable distance under the defined precision model.   |
| Vertex (pl. vertices) | A "corner point" of a geometric object. These are the coordinates explicitly stored to locate a geometric object.  |

## 5. NOTATION

- Items in the specification which adhere to the SFS are indicated by referring to the relevant section in the SFS in parentheses: (SFS 1.0)
- Items in the specification which elaborate on or differ from the SFS will be indicated by the term "JTS" in parentheses: (JTS)

## 6. JAVA IMPLEMENTATION

Java coding style is in some cases different to the coding style used in the SFS. Where the two are different in general JTS follows Java conventions. JTS coding style differs from SFS coding style in the following ways:

- the SFS sometimes uses Integer to represent a boolean value. JTS will use a boolean in this case
- method names in the SFS start with an uppercase letter. In JTS all method names start with a lowercase letter
- method names in JTS sometimes have the prefix "get" or "set" added to them, to conform to the conventions for Java Beans.

## 7. COMPUTATIONAL GEOMETRY ISSUES

### 7.1 PRECISION MODEL

All numerical computation takes place under some form of precision model. There are several possible types of precision model:

|          |   |
|----------|---|
| Fixed    | Coordinates are represented as points on a grid with uniform spacing. Computed coordinates are rounded to this grid.  |
| Floating | Coordinates are represented as floating-point numbers. Computed coordinates may have more digits of precision than the input values (up the maximum allowed by the finite floating-point representation).                 |
| Exact    | Coordinates are represented exactly (often as rational numbers with integral numerator and denominator). Implementing this model carries a penalty in space and time performance, which is often considered unacceptable. |

Often the precision model of a computation is not stated explicitly, but is implied by the model used for representing the values (such as floating point or integer). A limitation in this approach is that the user is unable to work in a precision model with lower precision. It is often the case that computed results are of higher precision than the inputs. The higher precision values may not be acceptable either for further computation or for storage in a format with the original (or lower) precision.

JTS deals with this problem by allowing the user to specify an explicit precision model. The precision model allows the client to state how many bits of precision are to be assumed in the input coordinate values, and maintained in any computed coordinates.

In JTS methods input Geometries may have different precision models. In the case of methods which return Geometries, the precision model of the returned result is the maximum of the two input precision models (i.e. the one with largest precision). Note that this only works if the two precision models are compatible. Two precision models are compatible if the scale factor of one is an integer multiple of the scale factor of the other. No attempt is made to reconcile incompatible precision models.

JTS supports two basic types of precision model: Fixed and Floating.

#### 7.1.1 Fixed Precision

In the Fixed precision model, coordinates are assumed to fall exactly on the intersections of a discrete grid. The size of the grid is determined by a scale factor. The grid size is the inverse of the scale factor. The scale factor can also be thought of as determining how many decimal places of precision are maintained. The scale factor may be either greater or less than 1, depending on whether the "precision point" is to the right or left of the decimal point.

Coordinates are made precise according to the following equations:

```
jtsPt.x = round( inputPt.x * scale ) / scale  
jtsPt.y = round( inputPt.y * scale ) / scale
```

Precise coordinates will be represented internally as double-precision values. This is known as the “precise internal representation”. Since Java uses the IEEE-754 floating point standard, this provides 53 bits of precision. (Thus the maximum precisely representable value is 9,007,199,254,740,992).

Input routines are responsible for rounding coordinates to the precision model before creating JTS structures. (The input routines supplied with JTS will perform this rounding automatically.)

### 7.1.2 Floating Precision

There are two types of Floating precision model supported, double and single precision. Both of these are based on the Java floating point model, which in turn is based on the IEEE-754 floating point standard. This provides approximately 16 digits of precision for double precision and 6 digits of precision for single precision.

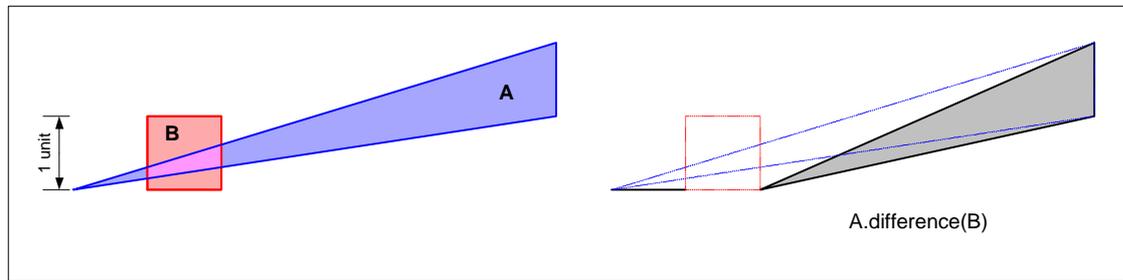
In the Floating Double Precision Model, coordinates can have the full precision available with Java double-precision floating point numbers. Input coordinates are not assumed to be rounded off, and internal operations which compute constructed points do not round off the computed coordinates. Note that this does not mean that constructed points are exact; they are still limited to the precision of double-precision numbers, and hence may still be only an approximation to the exact point.

In the Floating Single Precision Model, computed coordinates are rounded to single precision. This supports situations where the eventual destination of computed geometry is a single-precision format (e.g. such as Java2D).

## 7.2 CONSTRUCTED POINTS AND DIMENSIONAL COLLAPSE

Geometries computed by spatial analysis methods may contain constructed points which are not present in the input Geometries. These new points arise from intersections between line segments in the edges of the input Geometries. In the general case it is not possible to represent constructed points exactly. This is due to the fact that the coordinates of an intersection point may contain as much as twice as many bits of precision as the coordinates of the input line segments. In order to represent these constructed points explicitly, JTS must round them to fit the given Precision Model.

Unfortunately, rounding coordinates moves them slightly. Line segments which would not be coincident in the exact result may become coincident in the truncated representation. For Line-Line combinations, this can produce result Geometries containing points which were not in the interior of the input Geometries. More seriously, for Line-Area combinations, this can lead to dimensional collapses, which are situations where a computed component has a lower dimension than it would in the exact result.



Figure

1 - An example of dimensional collapse

JTS handles dimensional collapses as gracefully as possible, by forming the lower-dimension Geometry resulting from the collapse. For instance, an Area-Area intersection with a dimensional collapse would return a Line or Point Geometry as a component of the result.

### 7.3 ROBUSTNESS

Geometric algorithms involve a combination of combinatorial and numerical computation. As with all numerical computation using finite-precision numbers, the algorithms chosen are susceptible to problems of robustness. A robustness problem occurs when a numerical calculation produces an inexact answer due to round-off errors. Robustness problems are especially serious in geometric computation, since the numerical errors can propagate into the combinatorial computations and result in complete failure of the algorithm. (See [Bri98], [Sch91].)

There are many approaches to dealing with the problem of robustness in geometric computation. Not surprisingly, most robust algorithms are substantially more complex and less performant than the non-robust versions. JTS attempts to deal with the problem of robustness in two ways:

- The important fundamental geometric algorithms (such as Line Orientation, Line Intersection and the Point-In-Polygon test) have been implemented using robust algorithms. In particular, the implementation of several algorithms relies on the robust determinant evaluation presented in [Ava97]).
- The algorithms used to implement the SFS predicates and functions have been developed to eliminate or minimize robustness problems. The binary predicate algorithm is completely robust. The spatial overlay and buffer algorithms are non-robust, but will return correct answers in the majority of cases.

### 7.4 NUMERICAL STABILITY

A desirable feature of numerical algorithms is that they exhibit stability. The stability of a numerical algorithm is determined by the bound on the maximum error in its outputs. An algorithm is considered to be stable if this bound is small.

The primary numerical algorithm used in JTS is the computation of the intersection point between two segments. This algorithm is inherently inexact, since the bits of precision required to represent the intersection point is several times greater than the precision of the inputs. A stable algorithm for this computation will always produce approximate answers that are close to the exact answer. In particular, the computed points should at least lie within the bounding box of the input line segments! Ideally, the computed points will lie within a single precision model grid unit of the exact answer.

One way to increase the stability of numerical algorithms is to condition their inputs. Conditioning inputs involves numerically manipulating them in some way that produces the

same answer while preserving more precision during the calculations. JTS uses a technique of “normalizing” the input line segments to the line intersection computation. Normalized line segments have been translated to be as close to the origin as possible. This has the effect of removing common significant digits from each ordinate, and thus increases the bits of precision available to maintain the accuracy of the line intersection computation.

## 7.5 COMPUTATIONAL PERFORMANCE

Runtime performance is an important consideration for a production-quality implementation of geometric algorithms. The most computationally intensive algorithm used in JTS is intersection detection. Many JTS methods need to determine both all intersection between the line segments in a single Geometry (self-intersection) and all intersections between the line segments of two different Geometries.

The obvious algorithm for intersection detection, that of comparing every segment with every other, has unacceptably slow performance. There is a large literature of efficient algorithms for intersection detection. Unfortunately, many of them involve substantial code complexity. JTS tries to balance code simplicity with performance gains. It uses some special techniques to produce substantial performance gains for common types of input data. These techniques include in-memory spatial indexes of various types, and sophisticated methods for structuring data such as the technique of Monotone Chains.

### 7.5.1 Monotone Chains

JTS uses the technique of “Monotone Chains” to obtain substantial performance improvements with minimal additional code complexity. This technique involves dividing edges into monotone chains of segments. A monotone chain consists of a sequence of segments whose direction vectors all lie in the same quadrant. Monotone chains have two important properties:

**Non-Intersection Property:** the segments within a monotone chain do not intersect.

**Endpoint Envelope Property:** the envelope of any contiguous subset of the segments in a monotone chain is the envelope of the endpoints of the subset.

The Non-Intersection Property means that there is no need to test pairs of segments from within the same monotone chain for intersection. The Endpoint Envelope Property allows binary search to be used to find the intersection points along a monotone chain. In addition, the larger bounding boxes of monotone chains relative to individual segments act as a form of “clustering” of segments, which reduces the overall number of intersection tests required.

For data with a significant percentage of monotone chains, these properties eliminate a large number of segment comparisons. Monotone chains are common in data that has been generated by stream digitizing along natural features. Performance improvements of up to 100 times over the naive algorithm have been observed.

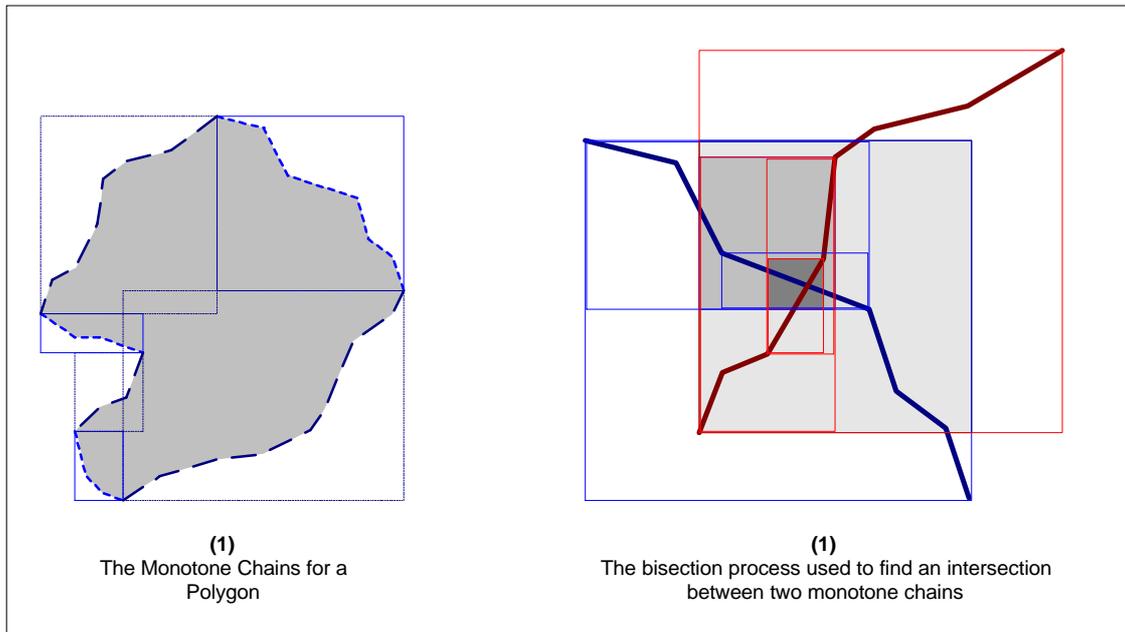


Figure 2 - Monotone Chains

## 8. SPATIAL MODEL

### 8.1 DESIGN DECISIONS FOR SPATIAL MODELS

The SFS is just one of several spatial models in use in existing spatial databases and APIs. These models are for the most part quite similar. Generally, they all support representing 2-dimensional points, lines and polygons. There are some subtle differences between the ways Geometries are represented, however. These differences represent design decisions made by the designers of the spatial API. Some important design choices are listed below (in each case, the choice made in the SFS and JTS is indicated).

|                 |  |
|-----------------|--|
| Design Decision | Repeated Points allowed in Geometries  |
| SFS Choice      | Repeated Points are allowed  |
| JTS Choice      | Same as SFS  |
| Comments        | In general spatial algorithms are not tolerant of repeated points. Allowing repeated points causes a performance and space penalty, since every spatial method must check for repeated points and remove them. JTS does support repeated points, since not doing so is a major point of incompatibility with the OGC model. However, there is a small memory and performance cost to doing so. |

|                 |  |
|-----------------|--|
| Design Decision | Linestrings allowed to self-intersect (i.e. can be non-simple)   |
| SFS Choice      | Linestrings are allowed to self-intersect  |
| JTS Choice      | Same as SFS  |
| Comments        | Allowing non-simple linestrings exacts a small performance penalty, since it means that linestrings must be noded before being used in spatial methods. However, it is desirable to be able to represent non-simple linestrings, so if the LineString class itself is defined to be simple, another class must be introduced to represent non-simple lines (sometimes referred to as "Spaghetti"). |

|                 |  |
|-----------------|--|
| Design Decision | Polygon rings can self-touch at single points.   |
| SFS Choice      | Polygon rings can NOT self-touch at single points  |
| JTS Choice      | Same as SFS  |
| Comments        | <p>This decision arises from the need to support representing polygons containing holes which touch the shell at a single point ("inverted" polygons). It also covers the case of representing a single hole which contains an exterior area which is disconnected (an "exverted" hole). In order to represent inverted polygons and exverted holes, either polygon rings must be allowed to self-touch at a single point OR rings must be allowed to mutually touch at single points.</p> <p>This design decision is a sense the dual of the choice of whether polygon rings can mutually touch at single points</p> <p>Unfortunately, making the choice that polygon rings can NOT self-touch results in slightly more complex algorithms, since the usual polygon-building algorithm results in shells which self-touch. It is necessary to perform a further step to convert the boundaries of the areas isolated by the self-touch into a hole.</p> |

|                 |   |
|-----------------|---|
| Design Decision | Polygon rings can mutually touch at single points   |
| SFS Choice      | Polygon rings can mutually touch at single points   |
| JTS Choice      | Same as SFS   |
| Comments        | This design decision is the dual of the decision about whether polygon rings can self-touch at single points. |

In most cases these design choices are of no consequence to the users of the API, since they do not change the set of Geometries that can be represented. However, they do have implications for the performance and complexity of the algorithms implemented in the API. Also, it is generally non-trivial to convert between the representations of two APIs that have made different design choices (in particular, if two APIs make different choices for whether polygon rings can self-touch, some relatively complex processing is necessary to convert the polygonal representations).

## 8.2 GEOMETRIC DEFINITIONS

All JTS methods assume that their arguments are valid Geometric objects, according to the definitions given in the SFS.

The following definitions elaborate or clarify the definitions given in the SFS.

## 8.2.1 Geometry

A Precision Model object will be a member of every Geometry object.

According to the SFS Geometry objects can represent only closed sets. This is a reasonable decision which allows for practical implementation. However, there are some implications for the semantics of the spatial analysis methods (see Section 12 Spatial Analysis Methods).

JTS has a simple scheme for adding attributes to a Geometry: applications may set a Geometry's user data field to any object.

## 8.2.2 Empty Geometry

The SFS specifies that objects of each Geometry subclass may be empty. It is sometimes necessary to construct a generic empty object of class Geometry (e.g. if the exact type of the Geometry to be returned is not known). The SFS does not define a specific class or object to represent this generic empty Geometry. JTS uses the convention that an empty GeometryCollection will be returned.

## 8.2.3 GeometryCollection

The dimension of a heterogeneous GeometryCollection is the maximum dimension of its elements.

## 8.2.4 Curve

Curves may not be degenerate. That is, non-empty Curves must have at least 2 points, and no two consecutive points may be equal.

## 8.2.5 MultiCurve

The SFS specifies using a "Mod-2" rule for determining the boundary of a MultiCurve. A point is on the boundary of the MultiCurve iff it is on the boundary of an odd number of elements of the MultiCurve. It should be noted that this leads to cases where the set of points in the SFS boundary is larger than either intuition or point-set topology would indicate. That is, a point with an odd number  $> 1$  of edges incident on it is on the boundary according to the SFS rule, but might not intuitively be considered as part of the boundary. This also is inconsistent with the topological definition of boundary, which is "the set of points which are not contained in any open subset of the set of points in the Geometry". For example, in Figure 3 (3), the point B is in the boundary according to SFS, but is an interior point according to point-set topology.

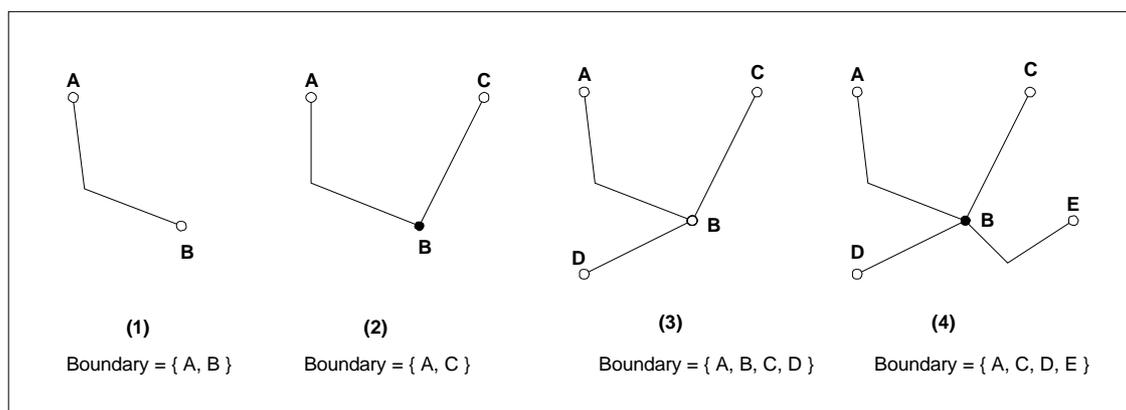


Figure 3 - Effect of the Mod-2 rule in MultiLineStrings

Additional logic is required in JTS to implement the Mod-2 rule.

### 8.2.6 LineString

We are using the definition of LineString given in the OGC SFS. This differs in an important way from some other spatial models (e.g. the one use by ESRI ArcSDE). The difference is that LineStrings may be non-simple. They may self-intersect in points or line segments.

In fact boundary points of a curve (e.g. the endpoints) may intersect the interior of the curve, resulting in a curve that is technically topologically closed but not closed according to the SFS. In this case topologically the point of intersection would not be on the boundary of the curve. However, according to the SFS definition the point is considered to be on the boundary. JTS follows the SFS definition.

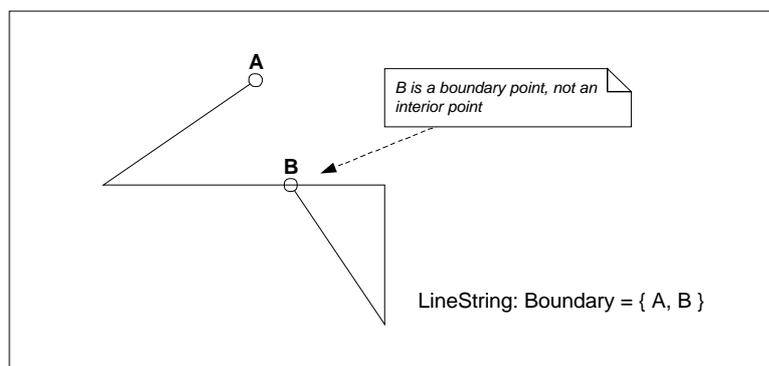


Figure 4 - A LineString with a boundary point intersecting an interior point

### 8.2.7 LinearRing

LinearRings are the fundamental building block for Polygons. LinearRings may not be degenerate; that is, a LinearRing must have at least 3 points. Other non-degeneracy criteria are implied by the requirement that LinearRings be simple. For instance, not all the points may be collinear, and the ring may not self-intersect. The SFS does not specify a requirement on the orientation of a LinearRing. JTS follows this by allowing LinearRings to be oriented either clockwise or counter-clockwise.

### 8.2.8 Polygon

The shell and holes of a Polygon are LinearRings. The SFS definition of Polygon has the following implications:

- The shell and holes cannot self-intersect (this is implied by the fact that they are LinearRings)
- Holes can touch the shell or another hole at a single point only. This means that holes cannot intersect one another at multiple points or in a line segment.
- Polygon interiors must be connected (This is implied by the previous statement).
- There is no requirement that a point where a hole touches the shell be a vertex.

Note that the SFS definition of Polygon differs from that in some other commonly used spatial models. For instance, the ESRI ArcSDE spatial model allows shells to self-intersect at vertices, but does not allow holes to touch the shell. The SFS and the ArcSDE model are equivalent in the sense that they describe exactly the same set of areas. However, they may require different polygon structures to describe the same area.

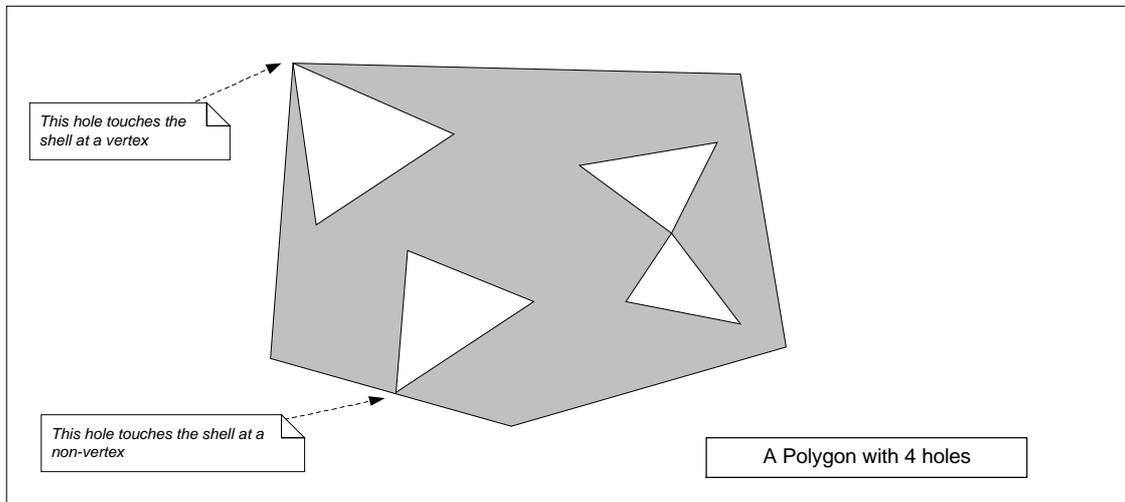


Figure 5 - An example of a Polygon containing holes

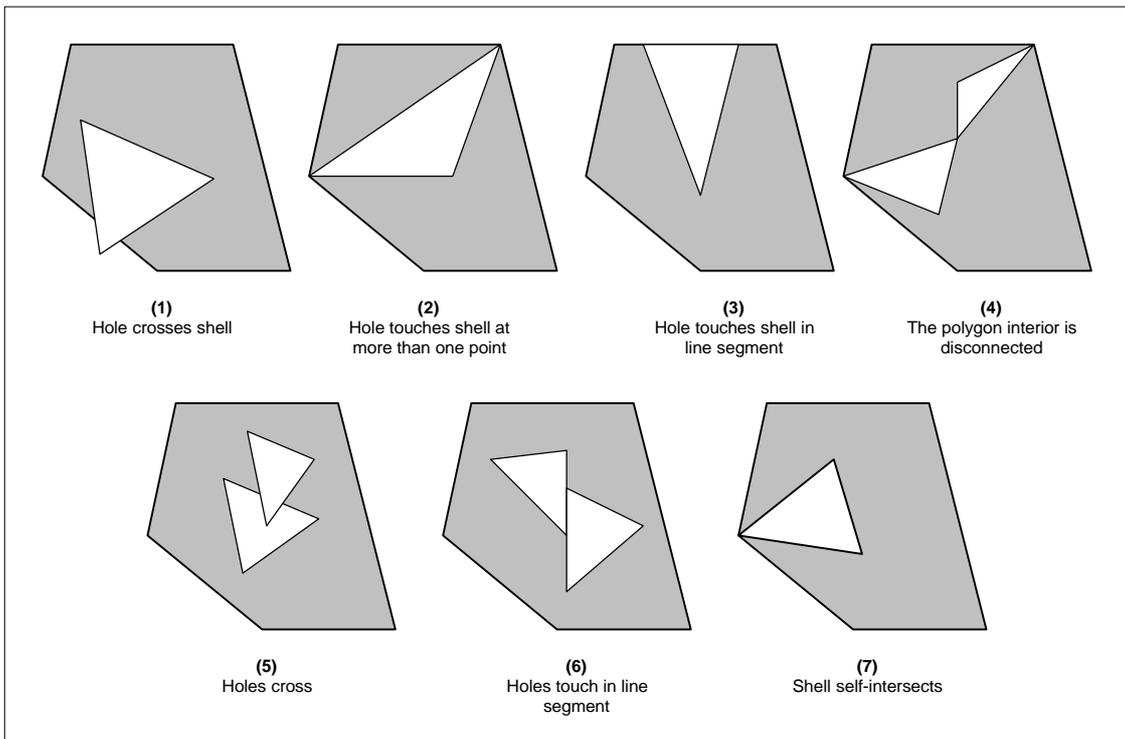


Figure 6 - Examples of objects not representable as polygons

Empty Polygons may not contain holes.

Since the shell and holes of Polygons are LinearRings, there is no requirement on their orientation. They may be oriented either clockwise or counterclockwise.

## 8.2.9 MultiPolygon

The element Polygons in a MultiPolygon may touch at only a finite number of points (e.g. they may not touch in a line segment). The interiors of the elements must be disjoint (e.g. they may not cross). There is no requirement that a point of intersection be a vertex.

## 8.3 SIMPLE FEATURE CLASSES

All Geometry classes allow empty objects to be created, and support the isEmpty method. Empty Geometries will be represented by their internal arrays having zero length.

All Geometry classes support the equalsExact() method, which returns true if two Geometry subclasses are equivalent and have identical sequence(s) of coordinates. Two objects are "equivalent" if their classes are identical. The only exception is LinearRing and LineString, which JTS considers to be equivalent.

All Geometry classes support the clone() method, which will return a deep copy of the object.

### 8.3.1 Geometry

Geometry is non-instantiable and is implemented as an abstract class.

### 8.3.2 GeometryCollection

A GeometryCollection is implemented as an array of Geometry objects.

### 8.3.3 Point

A Point is implemented as a single Coordinate.

### 8.3.4 MultiPoint

A MultiPoint inherits the implementation of GeometryCollection, but contains only Points.

### 8.3.5 Curve

Curve is non-instantiable and is implemented as an interface.

### 8.3.6 LineString

A LineString is implemented as an array of coordinates.

### 8.3.7 Line

JTS does not implement the Line class, since LineString offers equivalent functionality.

### 8.3.8 LinearRing

A LinearRing containing n coordinates is implemented with an array of Coordinates containing n+1 points, and coord[0] = coord[n].

### 8.3.9 MultiCurve

MultiCurve is non-instantiable and is implemented as an interface.

### 8.3.10 MultiLineString

A MultiLineString inherits the implementation of GeometryCollection, but contains only LineStrings.

### 8.3.11 Surface

Surface is non-instantiable and is implemented as an interface.

### 8.3.12 Polygon

A Polygon is implemented as a single LinearRing for the outer shell, and an array of LinearRings for the holes. The outer shell is oriented CW and the holes are oriented CCW.

### 8.3.13 MultiSurface

MultiSurface is non-instantiable and is implemented as an interface.

### 8.3.14 MultiPolygon

A MultiPolygon inherits the implementation of GeometryCollection, but contains only Polygons.

## 8.4 NORMAL FORM FOR GEOMETRY

JTS defines a normal (or canonical) form for representing [Geometries](#). Normal form is a unique representation for [Geometries](#). It can be used to test whether two Geometries are equal in a way that is independent of the ordering of the coordinates within them. Normal form equality is a stronger condition than topological equality, but weaker than pointwise equality.

The definitions for normal form use the standard lexicographical ordering for coordinates. "Sorted in order of coordinates" means the obvious extension of this ordering to sequences of coordinates.

| Geometry Class     | Definition of normal form  |
|--------------------|--|
| Point              | Points are always in normal form   |
| MultiPoint         | Element Points are sorted in order of their coordinates  |
| LineString         | Obeys the following condition:<br>If there is an $i$ such that $\text{coord}[i] \neq \text{coord}[n - i - 1]$<br>then $\text{coord}[i] < \text{coord}[n - i - 1]$  |
| LinearRing         | same as LineString   |
| MultiLineString    | Element LineStrings are in normal form, and are sorted in order of their coordinates   |
| Polygon            | The LinearRings of the Polygon are ordered such that the smallest point is first. The shell is ordered clockwise, and holes are ordered counterclockwise. Holes are sorted in order of their coordinates               |
| MultiPolygon       | Element Polygons are in normal form, and are sorted in order of their coordinates  |
| GeometryCollection | Element <a href="#">Geometries</a> are in normal form.<br>The list of elements is ordered by class (using the order of this list). Within each subsequence of like class, elements are sorted in order of coordinates. |

## 8.5 SUPPORT CLASSES

### 8.5.1 Coordinate

Coordinate is the lightweight class used to store coordinates. It is distinct from Point, which is a subclass of Geometry. Unlike objects of type Point (which contain additional information such as an envelope, a precision model, and spatial reference system information), a Coordinate only contains ordinate values and accessor methods.

Coordinates are two-dimensional points, with an additional z-coordinate. JTS does not support any operations on the z-coordinate except the basic accessor functions. Constructed coordinates will have a z-coordinate of NaN.

Coordinate implements the standard Java interface Comparable. The implementation uses the usual lexicographic comparison. That is,

```
c1.compareTo(c2) =
  -1 : c1.x < c2.x Ú ((c1.x = c2.x) Ú (c1.y < c2.y))
   0 : (c1.x = c2.x) Ú (c1.y = c2.y)
   1 : c1.x > c2.x Ú ((c1.x = c2.x) Ú (c1.y > c2.y))
```

Coordinate implements equals() using the obvious implementation of pointwise comparison.

### 8.5.2 CoordinateSequence

A CoordinateSequence is the internal representation of a list of Coordinates inside a Geometry. Because it is an interface, it is possible to create alternatives to the default implementation (an array of Coordinates). For example, one may choose to store the data as an array of some entirely different coordinate class, or as an array of x's and an array of y's. Note that non-Coordinate-array implementations will pay a performance penalty when the #toArray method is called.

### 8.5.3 Envelope

A concrete class containing a maximum and minimum x and y value.

### 8.5.4 IntersectionMatrix

An implementation of the Dimensionally Extended 9-Intersection Model (DE-9IM) matrix. The class can be used to represent both actual instances of a DE-9IM matrix as well as patterns for matching them. Methods are provided to:

- set and query the elements of the matrix in a convenient fashion
- convert to and from the standard string representation (specified in SFS Section 2.1.13.2).
- test to see if a matrix matches a given pattern string.

### 8.5.5 GeometryFactory

A GeometryFactory supplies a set of utility methods for building Geometry objects from lists of Coordinates.

### 8.5.6 CoordinateFilter

GeometryImpl classes support the concept of applying a coordinate filter to every coordinate in the Geometry. A coordinate filter can either record information about each coordinate or change the coordinate in some way. Coordinate filters implement the interface CoordinateFilter. (CoordinateFilter is an example of the Gang-of-Four Visitor pattern). Coordinate filters can be used to implement such things as coordinate transformations, centroid and envelope computation, and many other functions.

### 8.5.7 GeometryFilter

GeometryImpl classes support the concept of applying a Geometry filter to the Geometry. In the case of GeometryCollection subclasses, the filter is applied to every element Geometry. A Geometry filter can either record information about the Geometry or change the Geometry in some way. Geometry filters implement the interface GeometryFilter. (GeometryFilter is an example of the Gang-of-Four Visitor pattern.)

## 8.6 SPATIAL REFERENCE SYSTEM

JTS will support Spatial Reference System information in the simple way defined in the SFS. A Spatial Reference System ID (SRID) will be present in each Geometry object. Geometry will provide basic accessor operations for this field, but no others. The SRID will be represented as an integer.

The SRID of constructed objects will be copied from the SRID of one of the input objects if possible, or will be 0.

## 9. BASIC GEOMETRIC ALGORITHMS AND STRUCTURES

### 9.1 POINT-LINE ORIENTATION TEST

This function is fundamental to operations such as ordering edges around a node. Since it is essentially a geometric calculation, it is susceptible to robustness problems unless implemented using robust algorithms. JTS implements this method using a robust algorithm which returns the correct result for all input values. The algorithm used is based on the robust method of evaluating signs of determinants developed by Avanim et. al. ([Ava97]).

[diagram of point-line orientation]

### 9.2 LINE INTERSECTION TEST

This function tests whether two line segments intersect. It uses the robust Point-Line Orientation function specified above. It does not actually compute the point of intersection, and thus returns an exact answer. The function computes full information about the topology of the intersection, including the following data:

|                      |  |
|----------------------|--|
| HasIntersection()    | True if the line segments intersect  |
| getIntersectionNum() | The number of intersection points found (0, 1, or 2)                                 |
| IsProper()           | True if the intersection point is proper (i.e. is not equal to one of the endpoints) |

### 9.3 LINE INTERSECTION COMPUTATION

This function computes the intersection of two line segments. Two line segments may intersect in a single point, a line segment, or not at all. If the intersection is representable with coordinates in the Precision Model, it will be computed exactly. Otherwise, an approximation will be computed.

Intersections which are line segments will always be representable with coordinates, since each endpoint of the intersection segment must be equal to an endpoint of one of the input segments. Obviously, null intersections can also be computed exactly (although the intersection test must be performed with robust code to be correct). Intersections which are points may or may not be representable, since in general computed intersections require greater precision than the input points, and will not necessarily fall exactly on the precision model grid.

An important property of the line intersection algorithm is that it is numerically stable. Computed approximate points should be within the Precision Model tolerance of the exact intersection point.

In addition to the information computed by the Line Intersection test, the Line Intersection Computation computes information about the actual points of intersection:

|                        |  |
|------------------------|--|
| GetIntersection(int i) | The coordinate for the I'th intersection point |
|------------------------|--|

Determining the edge graph requires further information about the precise order of intersection points along each line segment. The Line Intersection class provides other functions to determine the order of intersection points along each segment, and to compute the (approximate) distance of a given intersection point along a segment.

## 9.4 POINT-IN-RING TEST

The Point-In-Ring predicate is implemented in a robust fashion by using the usual stabbing-line algorithm and making use of the robust Line Intersection Test.

In some cases it is necessary to test for the inclusion of multiple points in a given ring (e.g. in the IsValid predicate to test for the correct inclusion of holes). In this case performance can be gained by using a spatial index for the line segments of the ring. JTS implements a 1-dimensional Interval Tree to speed up the intersection tests made in the stabbing-line algorithm.

## 9.5 RING ORIENTATION TEST

This test returns true if a ring of coordinates is oriented in a clockwise direction. The test is used to determine on which side of the rings of the shell and holes of a Polygon the interior and exterior of the Polygon lie.

# 10. TOPOLOGICAL COMPUTATION

## 10.1 TOPOLOGY GRAPHS

The computation of the Intersection Matrix relies on the use of a structure called a "topology graph". The topology graph contains nodes and edges corresponding to the nodes and line segments of a Geometry. Each node and edge in the graph is labeled with its topological location relative to the source geometry.

Note that there is no requirement that points of self-intersection be a vertex. Thus to obtain a correct topology graph, Geometries must be self-noded before constructing their graphs.

Two fundamental operations are supported by topology graphs:

- Computing the intersections between all the edges and nodes of a single graph
- Computing the intersections between the edges and nodes of two different graphs

## 10.2 LABELS

Topology graphs support the concept of labeling nodes and edges in the graph. The label of a node or edge specifies its topological relationship to one or more geometries. (In fact, since JTS operations have only two arguments labels are required for only two geometries). A label for a node or edge has one or two elements, depending on whether the node or edge occurs in one or both of the input Geometries. Elements contain attributes which categorize the topological location of the node or edge relative to the parent Geometry; that is, whether the node or edge is in the interior, boundary or exterior of the Geometry. Attributes have a value from the set {Interior, Boundary, Exterior}. In a node each

element has a single attribute <On>. For an edge each element has a triplet of attributes <Left, On, Right>.

#### Example 1

If A and B are simple polygons and A contains B, the labels on their edges are:

|   |  |
|---|--|
| A | A: < Left = Exterior, On = Boundary, Right = Interior ><br>B: < Left = Exterior, On = Exterior, Right = Exterior > |
| B | A: < Left = Interior, On = Interior, Right = Interior ><br>B: < Left = Exterior, On = Boundary, Right = Interior > |

### 10.3 COMPUTING THE INTERSECTION MATRIX FROM A LABELING

The Intersection Matrix (IM) for an overlay graph is computed from the labeling of nodes and edges in the graph. To compute the IM, we sum the contributions to the IM of each node and edge whose label contains elements for both Geometries. The IM contribution for a node is  $dim \geq 0$  for the IM entry corresponding to the topological location of the node in the parent Geometries. (For example, a node which is in the Interior of Geometry A and in the Boundary of Geometry B would have  $label[0][On] = Interior$  and  $label[1][On] = Boundary$ , and  $IM(Interior, Boundary) = 0$ .) The IM contribution for an edge is  $dim \geq 1$  for the IM entry corresponding to the topological location of the edge itself in the parent Geometries, and  $dim \geq 2$  for the entries corresponding to the topological locations of the areas on the left and right sides of the edge.

The algorithmic expression of these rules is:

```
function Node.computeIM(im : IntersectionMatrix)
    if (label[0] != null and label[1] != null) then
        im.setAtLeast(label[0][On], label[1][On], 0)
    end if
end function

function Edge.computeIM(im : IntersectionMatrix)
    if (label[0] != null and label[1] != null) then
        im.setAtLeast(label[0][On], label[1][On], 1)
        im.setAtLeast(label[0][Left], label[1][Left], 2)
        im.setAtLeast(label[0][Right], label[1][Right], 2)
    end if
end function
```

For each combination of Geometries there is a maximum possible IM value. For efficiency this maximum value can be tested after each IM summation and the computation terminated if the value is obtained.

It is always the case that  $dim(Ext(A) \cap Ext(B)) = 2$ .

#### Example 2

Using the labels in Example 1 we have

for the labeling of the edge of A  
 $IM(\text{Boundary}, \text{Exterior}) = 1$   
 $IM(\text{Exterior}, \text{Exterior}) = 2$   
 $IM(\text{Interior}, \text{Exterior}) = 2$

for the labeling of the edge of B  
 $IM(\text{Interior}, \text{Boundary}) = 1$   
 $IM(\text{Interior}, \text{Exterior}) = 2$   
 $IM(\text{Interior}, \text{Interior}) = 2$

The full IM is:

|   |   |   |
|---|---|---|
| 2 | 1 | 2 |
| F | F | 1 |
| F | F | 2 |

## 10.4 THE RELATE ALGORITHM

The relate algorithm computes the Intersection Matrix describing the relationship of two Geometries. The algorithm for computing relate uses the intersection operations supported by topology graphs. Although the relate result depends on the resultant graph formed by the computed intersections, there is no need to explicitly compute the entire graph. Instead the structure of the graph is computed locally at each intersection node.

The relate algorithm is robust, by virtue of the robustness of the underlying operations. It is not subject to dimensional collapse problems, since it avoids calculating intersection points which might not lie on precise coordinates.

The algorithm to compute relate has the following steps:

1. Build topology graphs of the two input geometries. For each geometry all self-intersection nodes are computed and added to the graph.
2. Compute nodes for all intersections between edges and nodes of the graphs.
3. Compute the labeling for the computed nodes by merging the labels from the input graphs.
4. Compute the labeling for isolated components of the graph (see below)
5. Compute the Intersection Matrix from the labels on the nodes and edges.

### 10.4.1 Labeling isolated components

Isolated components are components (edges or nodes) of an input Geometry which do not contain any intersections with the other input Geometry. The topological relationship of these components to the other input Geometry must be computed in order to determine the complete labeling of the component. This can be done by testing whether the component lies in the interior or exterior of the other Geometry. If the other Geometry is 1-dimensional, the isolated component must lie in the exterior (since otherwise it would have an intersection with an edge of the Geometry). If the other Geometry is 2-dimensional, a Point-In-Polygon test can be used to determine whether the isolated component is in the interior or exterior.

## 10.5 THE OVERLAY ALGORITHM

The Overlay Algorithm is used in spatial analysis methods for computing set-theoretic operations (boolean combinations) of input Geometries. The algorithm for computing the overlay uses the intersection operations supported by topology graphs. To compute an

overlay it is necessary to explicitly compute the resultant graph formed by the computed intersections.

The algorithm to compute a set-theoretic spatial analysis method has the following steps:

1. Build topology graphs of the two input geometries. For each geometry all self-intersection nodes are computed and added to the graph.
2. Compute nodes for all intersections between edges and nodes of the graphs.
3. Compute the labeling for the computed nodes by merging the labels from the input graphs.
4. Compute new edges between the compute intersection nodes. Label the edges appropriately
5. Build the resultant graph from the new nodes and edges.
6. Compute the labeling for isolated components of the graph. Add the isolated components to the resultant graph.
7. Compute the result of the boolean combination by selecting the node and edges with the appropriate labels. Polygonize areas and sew linear geometries together.

## 11. BINARY PREDICATES

### 11.1 GENERAL DISCUSSION

The binary predicates can be completely specified in terms of an Intersection Matrix pattern. In fact, their implementation is simply a call to relate with the appropriate pattern.

It is important to note that binary predicates are topological operations rather than pointwise operations. Even for apparently straightforward predicates such as Equals it is easy to find cases where a pointwise comparison does not produce the same result as a topological comparison. (for instance: A and B are MultiPoints with the same point repeated different numbers of times; A is a LineString with two collinear line segments and B is a single line segment with the same start and endpoints; A and B are rings with identical sets of points but which start at different points). The algorithm used for the relate method is a topology-based algorithm which produces a topologically correct result.

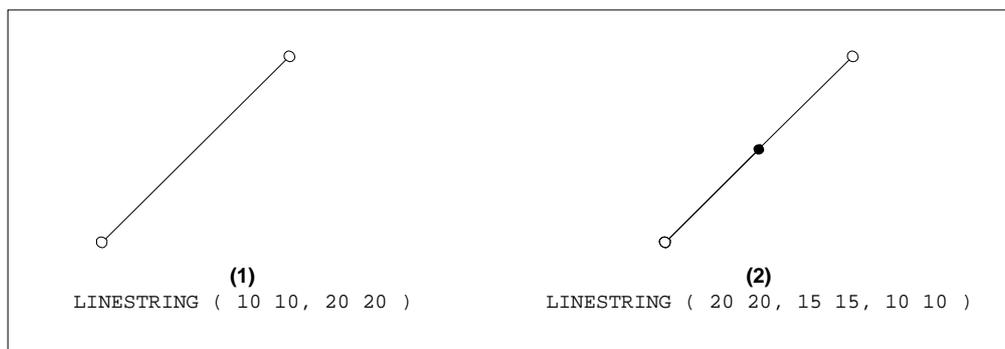


Figure 7 - Two Geometries that are pointwise unequal but topologically equal

As in the SFS, the term P is used to refer to 0-dimensional Geometries (Point and MultiPoint), L to 1-dimensional Geometries (LineString, and MultiLineString), and A to 2-

dimensional Geometries (Polygon and MultiPolygon). The dimension of a GeometryCollection is equal to the maximum dimension of its components.

In the SFS some binary predicates are stated to be undefined for some combinations of dimensions (e.g. touches is undefined for P/P). In the interests of simplifying the API, combinations of argument Geometries which are not in the domain of a predicate will return false (e.g. touches(Point, Point) => false).

If either argument to a predicate is an empty Geometry the predicate will return false.

Because it is not clear at this time what semantics for spatial analysis methods involving GeometryCollections would be useful, GeometryCollections are not supported as arguments to binary predicates or the relate method.

## 11.2 METHOD SPECIFICATIONS

Binary predicates are implemented as calls to relate, with the appropriate pattern supplied for the input Geometries. The specifications for most of the binary predicates are well described in the SFS, and are here simply specified by their relate pattern(s). Equals is not described in the SFS, however, so it is specified symbolically as well.

### 11.2.1 Equals

The Equals relation applies to all combinations of Geometries. Two Geometries are topologically equal iff their interiors intersect and no part of the interior or boundary of one Geometry intersects the exterior of the other. Symbolically,

$$a.equals(b) \iff I(a) \cap I(b) \neq \emptyset \wedge (I(a) \cap B(b) = \emptyset \wedge I(b) \cap B(a) = \emptyset) \iff a.relate(b, "T**F**FFF**")$$

Equals() is a topological relationship, and does not imply that the Geometries have the same points or even that they are of the same class. (This more restrictive form of equality is implemented in the equalsExact() method.)

| Argument Dimensions | Relate Pattern |
|---------------------|----------------|
| all                 | T**F**FFF*     |

### 11.2.2 Disjoint

| Argument Dimensions | Relate Pattern |
|---------------------|----------------|
| all                 | FF**FF*****    |

### 11.2.3 Intersects

$$A.intersects(B) = ! A.disjoint(B)$$

### 11.2.4 Touches

| Argument Dimensions     | Relate Pattern                           |
|-------------------------|--|
| P/L, P/A, L/L, L/A, A/A | FT*****<br>or F**T*****<br>or F***T***** |
| P/P                     | undefined                                |

### 11.2.5 Crosses

| Argument Dimensions | Relate Pattern |
|---------------------|----------------|
| P/L, P/A, L/A       | T*T*****       |
| L/L                 | O*****         |
| P/P, A/A            | undefined      |

### 11.2.6 Within

| Argument Dimensions | Relate Pattern |
|---------------------|----------------|
| all                 | T*F**F***      |

### 11.2.7 Contains

A.contains(B) = A.within(B)

### 11.2.8 Overlaps

| Argument Dimensions | Relate Pattern |
|---------------------|----------------|
| P/P, A/A            | T*T***T**      |
| L/L                 | 1*T***T**      |
| P/L, P/A, L/A       | undefined      |

## 12. SPATIAL ANALYSIS METHODS

### 12.1 GENERAL DISCUSSION

The SFS lists a number of spatial analysis methods including both constructive operations (buffer, convex hull) and set-theoretic operations (intersection, union, difference, symmetric difference).

#### 1.1.1 Representation of Computed Geometries

The SFS states that the result of a set-theoretic method is the “point-set” result of the usual set-theoretic definition of the operation (SFS 3.2.21.1). However, there are sometimes many ways of representing a point set as a Geometry.

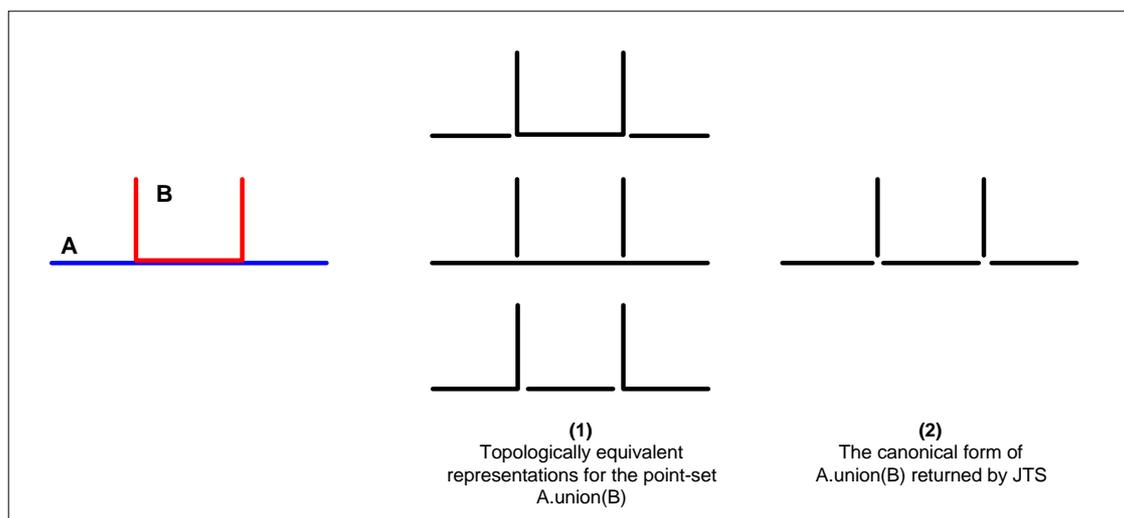


Figure 8 - Representation of computed Geometries

The SFS does not specify an unambiguous representation for point sets returned from a spatial analysis method. One goal of JTS is to make this specification precise and unambiguous. JTS uses a canonical form for Geometries returned from spatial analysis methods. The canonical form is a Geometry which is simple and noded:

- Simple means that the Geometry returned will be simple according to the definition in Section 13.1.3
- Noded applies only to overlays involving LineStrings. It means that all intersection points between the argument LineStrings will be present as endpoints of LineStrings in the result.

This definition implies that for non-simple geometries which are arguments to spatial analysis methods, a line-dissolve process is performed on them to ensure that the results are simple.

## 12.2 CONSTRUCTIVE METHODS

Because the `convexHull()` method does not introduce any new coordinates, it is guaranteed to return a precisely correct result. Since it is not possible to represent curved arcs exactly in JTS, the `buffer()` method returns a (close) approximation to the correct answer.

GeometryCollections are supported as arguments to the `convexHull()` method, but not to the `buffer()` method.

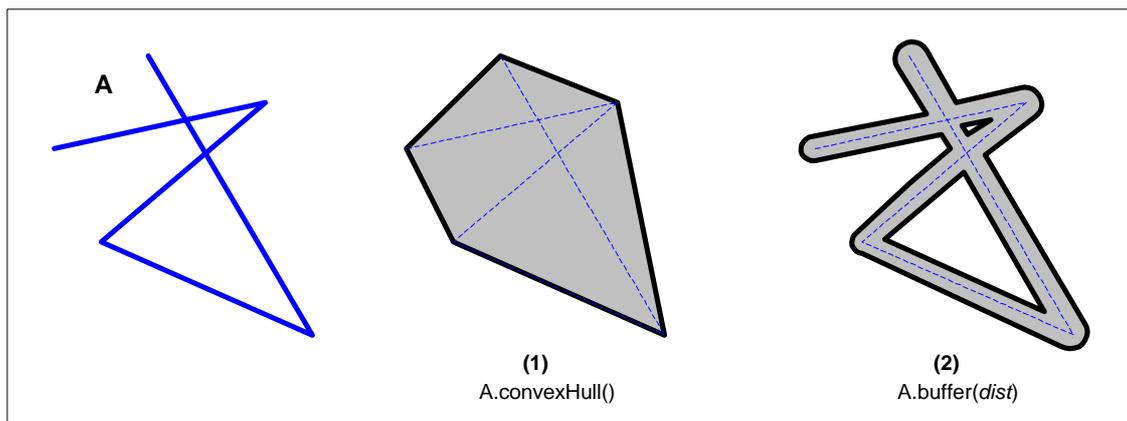


Figure 9 - The constructive spatial analysis methods

## 12.3 SET-THEORETIC METHODS

The spatial analysis methods will return the most specific class possible to represent the result. If the result is homogeneous, a Point, LineString, or Polygon will be returned if the result contains a single element; otherwise, a MultiPoint, MultiLineString, or MultiPolygon will be returned. If the result is heterogeneous a GeometryCollection will be returned.

Because it is not clear at this time what semantics for set-theoretic methods involving GeometryCollections would be useful, GeometryCollections are not supported as arguments to the set-theoretic methods.

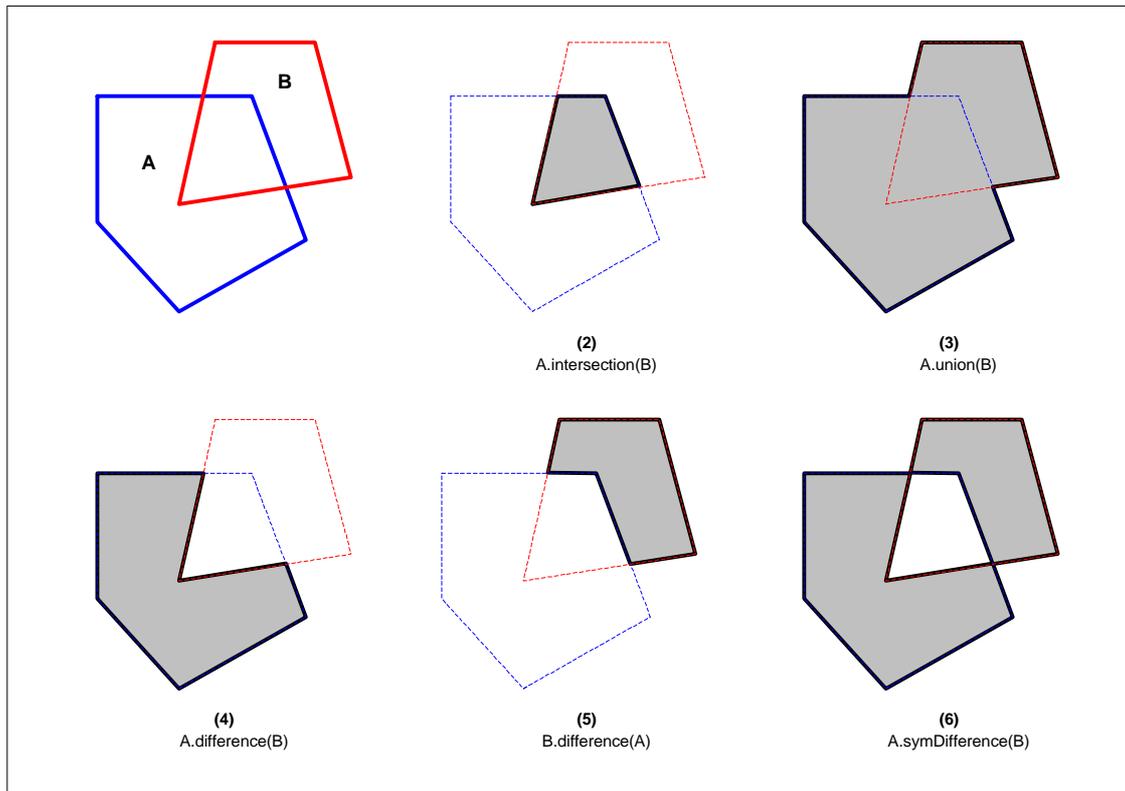


Figure 10 - The set-theoretic spatial analysis methods

For certain inputs, the Difference and SymDifference methods may compute non-closed sets. This can happen when the arguments overlap and have different dimensions. Since JTS Geometry objects can represent only closed sets, the spatial analysis methods are specified to return the closure of the point-set-theoretic result.

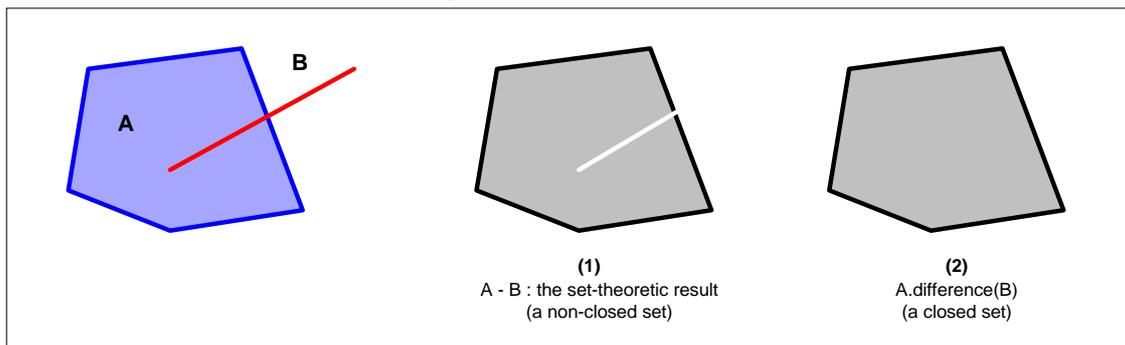


Figure 11 - JTS always returns closed Geometries

## 12.4 METHOD SPECIFICATIONS

### 12.4.1 Buffer

The buffer of a Geometry at a distance  $d$  is the Polygon or MultiPolygon which contains all points within a distance  $d$  of the Geometry. The distance  $d$  is interpreted according to the Precision Model of the Geometry. Both positive and negative distances are supported.

$a.buffer(d) =$

$$d > 0 : \{ x \in \mathbb{R}^2 \mid dist(x, a) \leq d \}$$

$$d < 0 : \{ x \in \mathbb{R}^2 \mid x \in a \cup dist(x, boundary(a)) > d \}$$

In mathematical terms, buffering is defined as taking the Minkowski sum or difference of the Geometry with a disc of radius equal to the absolute value of the buffer distance. Positive and negative buffering is also referred to as dilation or erosion. In CAD/CAM terms, buffering is referred to as computing an offset curve.

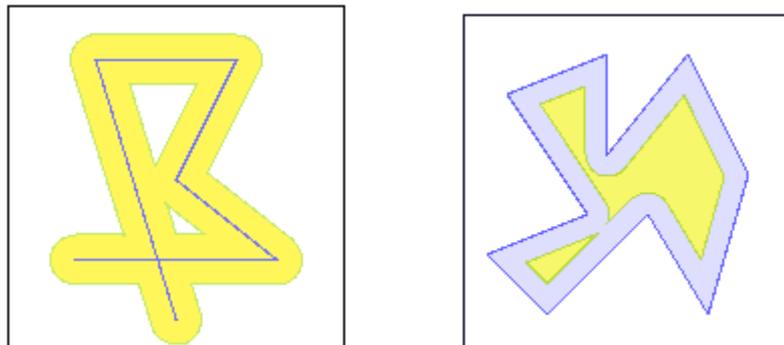
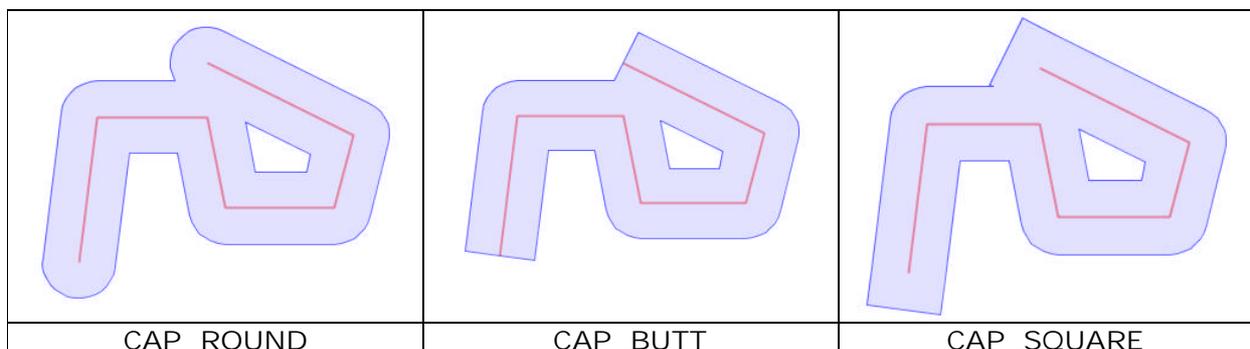


Figure 12 – Positive and Negative buffers

JTS allows specifying different end cap styles for buffers of lines. The end cap style is available when using the `BufferOp` class directly. The following end cap styles are supported:

| Style Name | Description  |
|------------|--|
| CAP_ROUND  | The usual round end caps   |
| CAP_BUTT   | End caps are truncated flat at the line ends                         |
| CAP_SQUARE | End caps are squared off at the buffer distance beyond the line ends |

The following diagrams illustrate the effects of specifying different end cap styles:



## 12.4.2 ConvexHull

The convex hull of a Geometry is the smallest convex Polygon that contains all the points in the Geometry. If the convex hull contains fewer than 3 points, a lower dimension Geometry is returned, specified as follows:

| Number of Points in convex hull | Geometry Class of result |
|---------------------------------|--------------------------|
| 0                               | empty GeometryCollection |
| 1                               | Point                    |
| 2                               | LineString               |
| 3 or more                       | Polygon                  |

JTS will return a Geometry with the minimal number of points needed to represent the convex hull. In particular, no more than two consecutive points will be collinear.

## 12.4.3 Intersection

The intersection of two Geometries A and B is the set of all points which lie in both A and B.

$$a.intersection(b) = \{ x \in \mathbb{R}^2 \mid x \in a \cap x \in b \}$$

## 12.4.4 Union

The union of two Geometries A and B is the set of all points which lie in A or B.

$$a.union(b) = \{ x \in \mathbb{R}^2 \mid x \in a \cup x \in b \}$$

## 12.4.5 Difference

The difference between two Geometries A and B is the set of all points which lie in A but not in B. This method returns the closure of the resultant Geometry.

$$a.difference(b) = \text{closure}(\{ x \in \mathbb{R}^2 \mid x \in a \setminus x \in b \})$$

## 12.4.6 SymDifference

The symmetric difference of two Geometries A and B is the set of all points which lie in either A or B but not both. This method returns the closure of the resultant Geometry.

$$a.symDifference(b) = \text{closure}(\{ x \in \mathbb{R}^2 \mid (x \in a \setminus x \in b) \cup (x \in b \setminus x \in a) \})$$

# 13. OTHER METHODS

## 13.1.1 Boundary

As stated in SFS Section 2.1.13.1, "the boundary of a Geometry is a set of Geometries of the next lower dimension." JTS uses GeometryCollections to represent sets of Geometries.

For all empty [Geometries](#),  $\text{boundary}(G) = \text{empty GeometryCollection (JTS)}$ .

For non-empty Geometries, the boundaries are defined as follows:

| Geometry Class     | Definition of boundary()  |
|--------------------|---|
| Point              | empty GeometryCollection  |
| MultiPoint         | empty GeometryCollection  |
| LineString         | if closed: empty MultiPoint<br>if not closed: MultiPoint containing the two endpoints.  |
| LinearRing         | empty MultiPoint  |
| MultiLineString    | MultiPoint obtained by applying the Mod-2 rule to the boundaries of the element LineStrings   |
| Polygon            | MultiLineString containing the LinearRings of the shell and holes, in that order (SFS 2.1.10)   |
| MultiPolygon       | MultiLineString containing the LinearRings for the boundaries of the element polygons, in the same order as they occur in the MultiPolygon (SFS 2.1.12/JTS)   |
| GeometryCollection | (SFS Section 2.1.13.1) "The boundary of an arbitrary collection of geometries whose interiors are disjoint consist of geometries drawn from the boundaries of the element geometries by application of the Mod-2 rule." |

### 13.1.2 IsClosed

The SFS meaning of "closed" is different to the topological meaning of closed. The SFS "isClosed" method applies to Curves only. It tests whether the start point and end point of the Curve are the same point. In contrast, topological closure depends on whether a geometry contains its boundary. As discussed earlier, all instances of SFS geometry classes are topologically closed by definition.

For empty Curves, isClosed is defined to have the value false.

### 13.1.3 IsSimple

In general, the SFS specifications of simplicity seem to follow the rule:

A Geometry is simple if and only if the only self-intersections are at boundary points.

For Point, MultiPolygon and GeometryCollection the SFS does not provide a specification for simplicity. JTS provides a specification for these Geometry types based on the above rule.

For all empty [Geometries](#), isSimple = true. (JTS)

| Geometry Class  | Definition of isSimple()   |
|-----------------|--|
| Point           | true (JTS)   |
| MultiPoint      | true if no two Points in the MultiPoint are equal (SFS 2.1.4)  |
| LineString      | true if the curve does not pass through the same point twice (excepting the endpoints, which may be identical) (SFS 2.1.5)   |
| LinearRing      | true (SFS 2.1.6)   |
| MultiLineString | true iff all of its element LineStrings are simple and the only intersections between any two elements occur at points that are on the boundaries of both LineStrings. |

|                    |  |
|--------------------|--|
|                    | (SFS 2.1.7)  |
| Polygon            | true (SFS 2.1.10)  |
| MultiPolygon       | true (JTS)   |
| GeometryCollection | true if all its elements are simple and the only intersections between any two elements occur at points that are on the boundaries of both elements. (JTS) |

#### 13.1.4 IsValid

Since JTS Geometry objects are constructed out of user-supplied point sequences, it is possible that a Geometry object does not in fact specify a topologically valid Geometry according to the SFS. JTS does not validate Geometries when they are constructed, for reasons of efficiency. The `isValid()` method is provided to test whether a Geometry is valid according to the SFS spec.

The validation rules checked are as follows:

| Rule                           | Description  | Applies To |
|--------------------------------|--|------------|
| Topologically Consistent Nodes | Nodes are topologically consistent if they are surrounded by a correctly alternating interior and exterior areas.        | A          |
| No Duplicate Rings             | Rings within an area must not be duplicated. Duplicate rings are rings which have identical point sequences up to order. | A          |
| No Self-Intersecting Rings     | Rings must not self-intersect.   | A          |
| Holes Contained In Shell       | Holes must be contained within their parent shell.   | A          |
| Holes Not Nested               | Holes must not be nested.  | A          |
| Shells Not Nested              | Shells must not be nested.   | mA         |
| Interiors Connected            | The interior of a Polygon must be connected.   | A          |

JTS also provides the `IsValidOp` class, which performs the same checks as `isValid` but which returns the exact nature and location of a validation failure.

## 14. WELL-KNOWN TEXT INPUT/OUTPUT

The Well-Known Text format for SFS Features is defined in SFS Section 3.2.5. The Well-Known Text Reader and Writer will parse and output this format.

Note that there is an inconsistency in the SFS. The WKT grammar states that MultiPoints are represented by "MULTIPOINT ( ( x y), (x y) )", but the examples show MultiPoints as "MULTIPOINT ( x y, x y)". Other implementations follow the latter syntax, so JTS will adopt it as well.

The SFS does not define a WKT representation for Linear Rings. JTS has extended the WKT syntax to support these, using the keyword LINEARRING.

d

### 14.1 SYNTAX FOR WELL-KNOWN TEXT

The syntax for the Well-known Text representation of Geometry is defined below.

The notation  $\{ \}^*$  denotes 0 or more repetitions of the tokens within the braces. The braces do not appear in the output token list.

```

<Geometry Tagged Text> :=
  <Point Tagged Text>
  | <LineString Tagged Text>
  | <LinearRing Tagged Text>
  | <Polygon Tagged Text>
  | <MultiPoint Tagged Text>
  | <MultiLineString Tagged Text>
  | <MultiPolygon Tagged Text>
  | <GeometryCollection Tagged Text>

<Point Tagged Text> :=
  POINT <Point Text>

<LineString Tagged Text> :=
  LINESTRING <LineString Text>

<LinearRing Tagged Text> :=
  LINEARRING <LineString Text>

<Polygon Tagged Text> :=
  POLYGON <Polygon Text>

<MultiPoint Tagged Text> :=
  MULTIPOINT <Multipoint Text>

<MultiLineString Tagged Text> :=
  MULTILINESTRING <MultiLineString Text>

<MultiPolygon Tagged Text> :=
  MULTIPOLYGON <MultiPolygon Text>

<GeometryCollection Tagged Text> :=
  GEOMETRYCOLLECTION <GeometryCollection Text>

<Point Text> := EMPTY | ( <Point> )

<Point> := <x> <y>

<x> := double precision literal

<y> := double precision literal

<LineString Text> := EMPTY
  | ( <Point> { , <Point> }^* )

<Polygon Text> := EMPTY
  | ( <LineString Text> { , <LineString Text> }^* )

<Multipoint Text> := EMPTY
  | ( <Point > { , <Point > }^* )

```

```
<MultiLineString Text> := EMPTY
| ( <LineString Text> { , <LineString Text> }* )

<MultiPolygon Text> := EMPTY
| ( <Polygon Text> { , <Polygon Text> }* )

<GeometryCollection Text> := EMPTY
| ( <Geometry Tagged Text>
```

## 14.2 WELL-KNOWN TEXT READER

The Well-Known Text reader (WKTRReader) is designed to allow extracting Geometry objects from either input streams or internal strings. This allows it to function as a parser to read Geometry objects from text blocks embedded in other data formats (e.g. XML).

A WKTRReader is parameterized by a GeometryFactory, to allow it to create Geometry objects of the appropriate implementation. In particular, the GeometryFactory will determine the PrecisionModel and SRID that is used.

The WKTRReader will convert the input numbers to the precise internal representation.

## 14.3 WELL-KNOWN TEXT WRITER

The Well-Known Text writer outputs the textual representation of a Geometry object to a Java Writer.

The WKTRWriter will output coordinates rounded to the precision model. No more than the maximum number of necessary decimal places will be output.

## 15. REFERENCES

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