Reply

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Rennó and Ingersoll (1996) and Emanuel and Bister (1996) propose very closely related theories for the strength and intermittency of moist convection based on the entropy budget of an atmosphere in radiative–convective equilibrium. These theories are based on two fundamental assumptions: 1) the atmosphere behaves more or less as a perfect heat engine, and 2) frictional dissipation in the atmosphere is the result of a turbulent energy cascade from the convective scales to the smaller scales at which viscosity can act. The first assumption, whether the tropical atmosphere does or does not act like a perfect heat engine, is discussed in depth in the thesis of Pauluis (1999; see also Pauluis and Held 2001a,b, manuscripts submitted to *J. Atmos. Sci.*), where it is demonstrated that irreversible phase changes and diffusion of water vapor result in a severe reduction of the thermodynamic efficiency of moist convection. Pauluis et al. (2000) focus on the second assumption, and argue that during moist convection a significant amount of frictional dissipation occurs instead in the shear zones surrounding falling hydrometeors. Scaling arguments indicate that the amount of this precipitation-induced dissipation in the Tropics is surprisingly large (≈2–4 W m^{-2}). Numerical simulations with a cloud ensemble model confirm this estimate and show that this precipitation-induced dissipation is significantly larger than the dissipation associated with the turbulent energy cascade from the convective scales to the smaller scales in the model.

In his comment, Rennó (2001) argues that our estimate of this precipitation-related dissipation is incorrect. We do not agree with this criticism, as described below. He also provides a number of arguments that convince him that the low thermodynamic efficiency in our cloud ensemble model cannot be representative of atmospheric phenomena on a range of scales from waterspouts to the Hadley cell. We choose not to reply to the second set of arguments here. The entropy budgets of many of the phenomena described involve a variety of subtle issues that we do not see as directly relevant to the much simpler topic analyzed in our paper.

Rennó points out that the aerodynamical drag on a falling hydrometeor can be decomposed into a pressure drag, due to the pressure variations at the particle surface, and a frictional drag, associated with the viscous stress at the particle surface. While this is true, we reiterate that the loss of mechanical energy due to drag does not depend on the decomposition of this drag between its pressure and frictional parts. Mechanical energy is lost whenever the drag force acts to reduce the velocity of a falling particle relative to ambient air, independently of the mechanisms producing the drag. The lost mechanical energy is converted into internal energy either through viscous dissipation in the shear zone around the hydrometeor or through turbulent dissipation in the wake of the hydrometeor.

We repeat the simple force balance argument. Consider a hydrometeor moving at velocity $\mathbf{V} + \mathbf{V}_T$ in ambient air whose velocity is $\mathbf{V}$. The drag performs the work $\mathbf{F} \cdot (\mathbf{V} + \mathbf{V}_T)$ on the hydrometeor. For a falling hydrometeor [with $(\mathbf{V} + \mathbf{V}_T) \cdot \mathbf{k} \leq 0$, where $\mathbf{k}$ is the vertical unit vector], this work is negative: the drag reduces the mechanical energy of the hydrometeor. By the equality of action and reaction, an opposite force acts on the ambient air. The work performed on the air is thus $-\mathbf{F} \cdot \mathbf{V}$. This is the work transferred to the air when a downdraft is accelerated (for $\mathbf{V} \cdot \mathbf{k} \leq 0$) or the work extracted in the process of decelerating an updraft

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(for $\mathbf{V} \cdot \mathbf{k} \geq 0$). The sum of the work on the hydrometeor and ambient air gives the total rate of change in mechanical energy: $\mathbf{F} \cdot (\mathbf{V} + \mathbf{V}_T) - \mathbf{F} \cdot \mathbf{V} = \mathbf{F} \cdot \mathbf{V}_T$. As the drag points in the opposite direction to the relative velocity, the scalar product $\mathbf{F} \cdot \mathbf{V}_T$ is negative, and the total mechanical energy of the system is reduced. This mechanical energy is transferred to the small-scale flow around the falling hydrometeor. As there is no mechanism allowing for this energy to be transferred to larger scales (such as an inverse energy cascade) or to be stored in another form (such as electrostatic potential), the mechanical energy must be converted into internal energy through dissipation. This dissipation occurs either through viscosity in the shear zone surrounding the hydrometeor, or through turbulence in the wake of the hydrometeor. Rennó argues that some of the drag is used to accelerate the downdrafts. This aspect is fully accounted for in the previous analysis, in the work performed on the ambient air $-\mathbf{F} \cdot \mathbf{V}$.

Rennó attributes the numerical results of Pauluis et al. (2000) to an incorrect treatment of the drag in numerical models. However, the model equations used are entirely standard; the sole impact of condensed water on the equations of motion is through the effect of water loading (see Houze 1993).

Hydrometeors can be treated as having reached their terminal velocity, so the drag on the hydrometeors balances the gravitation acceleration. The total dissipation occurring in the microscopic shear zones (or turbulent wakes) around the hydrometeors can be estimated by the integral

$$W_p = \int_\Omega g\rho_c \upsilon_T, \tag{1}$$

where the integral is over the whole atmospheric domain, ρ_c is the mass of falling precipitation per unit volume, and $\upsilon_T = \mathbf{V}_T \cdot \mathbf{k}$ is the terminal velocity of the hydrometeors. An alternative formulation for this dissipation term can be derived from the conservation of total water:

$$W_p = \int_\Omega g\rho_t w, \tag{2}$$

with ρ_t the total mass of water (in all phases) per unit volume, and w the vertical velocity of the air. It is this expression that provides the starting point for the estimates offered by Pauluis et al. (2000), which suggest that the average precipitation-induced dissipation in the Tropics should be between 2 and 4 W m^{-2}. The expression (2) is also used to compute the precipitation-related dissipation in our cloud ensemble model. This computation confirms the qualitative estimates. Rennó (2001) feels that the numerical model cannot possibly provide a robust estimate of this dissipation. None of his arguements provide any reason for us to doubt the model results. If it is the case that the value of W_p in radiative–convective equilibrium is sensitive to the details of the numerical model of deep moist convection, this will be of interest, but we do not feel it will be for any of the reasons outlined by Rennó.

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