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The macroturbulence of the troposphere

By ISAAC M. HELD, *Geophysical Fluid Dynamics Laboratory/NOAA, Princeton, NJ 08542, USA*

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ABSTRACT

Eddy length scales, eddy velocity scales, and the amplitude of eddy fluxes in the mid-latitude troposphere are discussed, primarily from the qualitative perspective provided by studies of quasi-geostrophic turbulence. The utility of a diffusive picture for the near surface poleward flux of heat is emphasized, as is the extent to which a full closure theory for the troposphere, including the interior potential vorticity fluxes, must revolve around this theory for the heat flux. A central problem in general circulation theory is then to determine which factors control the horizontal diffusivity near the surface. The baroclinic eddy production problem has distinctive features that make it stand out from other inhomogeneous turbulence problems such as Benard convection and laboratory shear flows, the crucial point being that there can be scale separation between the eddies and the scale of the mean flow inhomogeneity in the direction of the relevant transport. This scale separation makes diffusive closures more compelling. In addition, it allows one to compute diffusivities from models of homogeneous turbulence.

1. Introduction

To acquire some qualitative understanding of the flow in a system as complex as the atmosphere requires one to bury much of the complexity behind rather crude idealizations. An important, and at times maligned, idealization is that of *turbulent diffusion*. In this essay it will be argued that the poleward eddy transport of heat near the surface, the quantity around which any picture of the general circulation ultimately revolves, should be thought of as fundamentally diffusive. A core problem is then to understand the factors that control the value of the diffusivity.

Section 2 begins with a qualitative discussion of the importance for the global climate of the eddy heat flux through mid-latitudes. Quasi-geostrophic theory suggests that it is really only the near-surface poleward eddy heat flux that we should be focusing on, along with the equatorward potential vorticity fluxes in the upper troposphere. The rôle of the near-surface poleward heat flux is best understood by relating it to the surface branch of the mean meridional mass-overturning in the extratropics, as discussed in Section 3. Also in Section 3, we argue that it is useful to focus attention on this surface branch of the overturning, and to think of the poleward flow in the interior, and the associated equatorward potential vorticity flux, as adjusting to the near-surface poleward heat flux. In Section 4, it is argued that the potential for scale separation implies that theories for eddy fluxes due to baroclinic eddy production should be *intrinsically local* in the horizontal, and that diffusive theories are therefore more justifiable in this problem than in many other turbulent flows. The parameters that help control the diffusivity are discussed in Sections 5 and 6. An important limitation to this perspective, concerning theories for "storm tracks" with eddy statistics that are inhomogeneous along mean streamlines, is discussed briefly in Section 7.

Given present-day computational resources, with which we can directly simulate the transports by the energy containing eddies in the extratropical atmosphere, one can legitimately ask if a search for an intuitive understanding of this and other aspects of the general circulation has become less important. It is interesting to speculate how Carl Gustav Rossby, whose work we celebrate at this symposium, would have responded to this question if he were alive today. While his research was clearly motivated in great part by practical concerns, the central goal of simple intuitive explanations for the form of the atmospheric circulation is manifest in many of his most famous papers. I suspect that he would turn the question around, and point out that we no longer have any excuse for our lack of intuitive understanding — given the increasing ease with which we can "experiment" with numerical atmospheric models and test competing theories.

In any case, Rossby's work provides the essential foundation upon which our understanding of the general circulation must be based. The mantra-like recurrence of his name in this essay — Rossby number, Rossby wave, Rossby radius of deformation — may help to reinforce this fact for the uninitiated. Perhaps we should be grateful that potential vorticity does not also carry Rossby's name, as well it might!

2. Diffusive energy balance models

Diffusive energy balance models have played a nontrivial, albeit modest, rôle in climate research. They may play an important pedagogical rôle when we introduce our students to the subject of climate modeling, but then often recede into the background to make place for baroclinic instability theory, potential vorticity dynamics and wave-mean flow interaction in discussions of the general circulation. In the most basic version of such a model, the absorbed solar flux has a pre-determined latitudinal distribution, $S(\theta)$; the outgoing infrared flux is a function of the temperature only, a linear one in the simplest case, $A + BT$; and the meridional flux of heat is represented as a simple diffusion of temperature on a sphere. In equilibrium,

$$0 = \nabla \cdot (cD\nabla T) + S(\theta) - (A + BT). \qquad (1)$$

The factor c, the heat capacity per unit horizontal area, has been included so that D is a kinematic diffusivity. In the absence of transport, $T = T_E \equiv (S(\theta) - A)/B$. If we approximate the absorbed solar flux as a constant plus a part proportional to the second Legendre polynomial $P_2(\theta)$, the simplest form symmetric about the equator, then the pole-to-equator temperature difference in the diffusive model is

$$\Delta T = \frac{\Delta T_E}{1 + 6d}, \qquad (2)$$

where ΔT_E is the temperature difference in the absence of transport, and $d \equiv cD/(Ba^2)$ where a is the radius of the earth. Values used for d in the literature on energy balance models [North (1975), Suarez and Held (1978)] vary over a fairly wide range, depending to some extent on which temperature it is that is being diffused (surface, 500 mb, etc.) and on whether the intention is to include oceanic transport in the diffusive flux, but typically the poleward heat transport reduces the N–S temperature contrast in such a model by a factor of 2–3, corresponding to $d \approx 0.2$–0.35. With a typical value of $B = 2\ \mathrm{W\,m^{-2}\,K^{-1}}$, and using $d = 0.25$ for the atmosphere in isolation, and $c = 10^7\ \mathrm{J\,m^{-2}\,K^{-1}}$, we find a kinematic diffusivity a bit smaller than $\approx 2 \times 10^6\ \mathrm{m^2\,s^{-1}}$.

It is easy enough to criticize such a model. The picture of diffusive transport is certainly inappropriate for the large-scale oceanic circulation. It is also inappropriate for the tropical atmosphere. It is only in the extratropical atmosphere, in fact, that we might consider the possibility of temperature, near the surface at least, as being diffused in some rough approximate sense by the energy-containing cyclones and anticyclones. Yet it is precisely this atmospheric heat transport through mid-latitudes by large-scale eddies that is *the* central element controlling the temperature distribution on our earth.

Let us pause for a moment to evaluate this claim. Atmospheric transport is larger than oceanic transport in middle and higher latitudes, but the case for atmospheric dominance in the extratropics can be made more strongly. It is useful to conceive of an extreme model, in which the atmospheric diffusivity is essentially zero below some critical value of the temperature gradient, and rises very steeply once above this value, to the extent that the temperature gradient cannot rise appreciably higher. The result is a version of what is often termed *baroclinic adjustment* (Stone, 1978), in which mid-latitude eddies simply set the mid-latitude temperature gradient, independently of any oceanic flux. In a less exteme view, consistent with the scaling arguments discussed below, the atmospheric transport increases quite rapidly as the temperature gradient increases, so the atmosphere's rôle in setting this gradient is magnified beyond that indicated by its share of the poleward flux, as long as the oceanic flux does not have a similar level of sensitivity.

Within the tropics, on the other hand, the oceans clearly transport more heat away from the equator than does the atmosphere. How can one argue that mid-latitude eddies are a central player here as well? One can improve our simple diffusive model (1) somewhat by allowing the diffusivity to approach zero before reaching the equator, thereby sucking heat out of the subtropics, mimicking the eddy energy transport in the atmosphere. This creates a large temperature difference between the equator and the subtropics. But substantial temperature contrasts that extend through a significant depth of the troposphere cannot be sustained in the tropics; otherwise thermal wind balance implies physically impossible upper tropospheric winds (Held and Hou, 1980; Plumb and Hou, 1992; Emanuel, 1995). One can then add to the equation a *Hadley adjustment* (Lindzen and Farrell, 1980) that transports enough heat to more or less eliminate the temperature differences between the equator and the subtropics, representing transport by the tropical overturning circulations in *both* ocean and atmosphere. This simple picture makes clear that it is the export of heat out of the subtropics by atmospheric eddies that determines tropical temperatures and the radiative deficit at the top of the tropical atmosphere, to first approximation, despite the fact that it is primarily the ocean that is transporting this heat out of the equatorial zone!

So energy balance models point to the thermal diffusivity arising from the *macroturbulence* of the mid-latitude troposphere as the central concern of any theory for the climatic distribution of temperature. Yet quasi-geostrophic (QG) theory provides a somewhat more complex picture. QG theory points to the equatorward potential vorticity fluxes in the upper troposphere, in addition to the near surface poleward eddy heat flux, as key ingredients in the eddy driving of the mean circulation. The explanation for the distinctive rôle of the near-surface poleward heat flux is best appreciated in isentropic coordinates.

3. Mean meridional overturning

If we denote the pressure thickness of the layer between the potential temperatures Θ and $\Theta + \delta\Theta$ as $H\delta\Theta$, then mass conservation within this layer, averaged over time, reads

$$0 = -\nabla \cdot (\overline{vH}) - \frac{\partial Q}{\partial \Theta}, \qquad (3)$$

where Q represents diabatic heating. Zonally averaged, we can define a mass transport streamfunction from $\overline{vH}$ and Q, which will look like that in Fig. 1, which happens to have been obtained from an atmospheric model (Held and Schneider, 1998). The meridional mass transport can be divided into mean and eddy components, with the mean dominant in the tropics and the eddy transport dominant elsewhere. Some of the equatorward return flow near the surface occurs in layers that are colder than the mean surface temperature, as illustrated in the figure. The use of moist rather than dry entropy would change the picture dramatically in low latitudes, but the changes would be only qualitative in the Northern extratropics during winter.

Held and Schneider (1998) argue that the poleward flow tends to be confined to isentropic layers that are typically uninterrupted by the surface at the latitude in question, while the equatorward flow occurs in layers that are often interrupted. Focusing on a particular latitude, let Θ_I be the isentrope that separates these two distinctive tropospheric regions, hereafter referred to as the *surface layer* and the *tropospheric interior*. If the magnitude of the total mass transport in each of these regions is V and the characteristic potential temperature difference between them $\Delta_V\Theta$, then the energy transport by the circulation (ignoring the distinction between Θ and static energy) is proportional to $V\Delta_V\Theta$.

Focusing further on the surface layer only, the region with $\Theta < \Theta_I$, we can try to relate the poleward eddy heat flux near the surface to the equatorward mass transport within this layer. We denote time-averages by an overbar and deviations from this average by a prime. If the thickness (the mass per unit horizontal area) of this layer is not

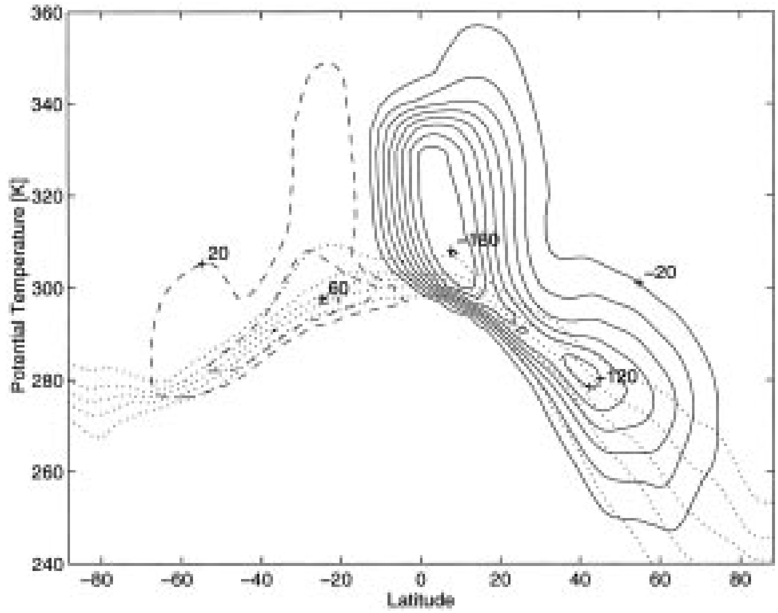

Fig. 1. The mean meridional mass transport in isentropic coordinates, as obtained from the January climate of an R30, 14 level atmospheric general circulation model. The contour interval for the streamfunction is 2×10^{10} kg/s. The five dotted lines represent the probability distribution (the 10%, 30%, 50%, 70%, and 90% isolines) for the surface potential temperature. (Surface potential temperature is above the 10% isoline 10% of the time, etc.).

too large, a Taylor's expansion provides a relation between the eddy thickness perturbations and the near-surface eddy potential temperature Θ_S, given the mean static stability of the lower troposphere (note that Θ_I is a constant by definition, and has no eddy component):

$$\Theta_S' - \Theta_I' = \Theta_S' \approx (p_S' - p_I') \Big/ \frac{\partial \bar{p}}{\partial \Theta}, \tag{4}$$

and the eddy mass flux is

$$V = \overline{v'(p_S' - p_I')}/g \approx \overline{v'\Theta_S'} \frac{\partial \bar{p}}{\partial \Theta} \Big/ g. \tag{5}$$

After multiplication by the Coriolis parameter, the RHS of (5) is also the vertical component of the QG Eliassen-Palm flux. This isentropic perspective provides a simple physical interpretation of the fact that the δ-function equatorward mass flux along the surface in the Transformed Eulerian Mean equations of Andrews and McIntyre (1976) is proportional to the eddy heat flux.

Assuming that the eddy contribution dominates the mass transport in mid-latitudes, and that the static stability does not vary too much in the vertical, the total energy transport is then simply proportional to $V\Delta_V\Theta \approx \overline{v'\Theta_S'}\Delta p/g$ where

$$\Delta p = (\Delta_V \Theta) \frac{\partial \bar{p}}{\partial \Theta} \tag{6}$$

is proportional to the mass per unit horizontal area of the troposphere. Therefore, if we obtain this near-surface eddy flux from a diffusive theory, we regain (and justify) the simple diffusive energy balance model. We need not think of temperature as being diffused throughout the atmosphere, but only near the surface. In the interior, QG theory tells us that it is potential vorticity (PV) that we should think of as being mixed by the turbulent flow, and that the poleward mass flux can be thought of as controlled by this PV flux, but at the level of the energy budget of the troposphere as a whole this need not be made explicit. The interior PV fluxes are simply setting (or adjusting themselves to) the depth of the troposphere.

Since the equatorward flow must balance the poleward flow, while one is controlled by the surface heat flux and the other by the interior PV flux, these two fluxes are obviously closely related.

Ignoring horizontal eddy momentum fluxes, the quasi-geostrophic form of this relationship is, (Pedlosky, 1987, p. 398),

$$\int \overline{vq}\rho \, \mathrm{d}z = -\frac{f\rho}{N^2}\,\overline{vb}|_{z=0}. \tag{7}$$

Here ρ is the reference density, q is the QG potential vorticity, and $b \equiv g\Theta/\Theta_0$ is the buoyancy. (In the discussion above, we effectively ignored horizontal eddy momentum fluxes at the point that we assumed that the contribution of the mean circulation to the mass transport in the surface layer was negligible.) It seems evident that one should try to treat the upper and lower branches of this circulation on an equal footing, but there are, in fact, reasons why one might want to focus attention preferentially on the surface branch.

One one level, it is just helpful to cut the chain of cause and effect at some point in order to have a starting point for the discussion. But in making this cut it is helpful to isolate relatively simple parts of the system. Near-surface heat fluxes are simpler than the upper tropospheric PV fluxes for several reasons. Perhaps most importantly, the mean temperature gradient near the surface is more strongly forced than the upper tropospheric PV distribution. The north–south temperature distribution is monotonic most of the time; therefore eddies typically see a well-defined background temperature gradient. In contrast, in the upper troposphere the eddies are too successful in distorting the PV distribution, complicating matters. Also, most of the meridional wave propagation occurs in the upper troposphere, so the relation between the source of the wave activity and resulting mean flow modification is fuzzier. Therefore, the argument goes, first develop some understanding of the near-surface heat flux and the associated mass transport. Then think of the upper tropospheric dynamics as adjusting itself to generate the required poleward mass flux, determining the distribution of this mass flux and related properties of the circulation, such as the height of the tropopause and the extratropical static stability, in the process. The theory for the surface flux will depend, in turn, on some of these aspects of the upper tropospheric flow, closing the circle. From this perspective, there is a superficial resemblance to wind-driven ocean circulation theory in which the interior ocean flow is forced by the divergence of the Ekman mass transport in the mixed layer.

But why talk in terms of a diffusivity, dividing the flux by a mean gradient? On one level, one can think of this simply as a way of normalizing the flux so that it has units of velocity times length; one can then try to relate this normalized flux to eddy length and time scales. One can also more easily relate fluxes of different quantities to each other. However, the case for thinking in terms of diffusivities is stronger than this.

While this essay is focused primarily on the zonal mean fluxes, it is instructive at this point to glance at observations of the local, time-averaged, horizontal fluxes of temperature near the surface. The lower panel in Fig. 2 displays the Northern Hemisphere wintertime fluxes at 850 mb from the NCEP/NCAR reanalysis (courtesy of P. Kushner). Marshall and Shutts (1981) and Illari and Marshall (1983) have shown that the rotational part of tracer fluxes cannot be expected to bear any systematic relation to the mean gradients — rather this component of the flux tends to be aligned with the isolines of tracer variance. The rotational part of the flux, which has no effect on the temperature tendencies, has been removed from Fig. 2, although this adjustment is smaller in the case of lower tropospheric heat fluxes than for upper tropospheric PV fluxes. Also shown in this same figure are the fluxes predicted by a simple isotropic diffusivity, with the diffusion coefficient having the structure shown in the upper panel, following Kushner and Held (1998). (Justification for this particular choice of local diffusivity is postponed until Section 7.) The simple diffusive theory evidently does a rather good job at mimicking these fluxes.

The empirical evidence seems sufficient to justify focusing on diffusivity as a meaningful quantity for heat near the surface. Why is this the case? We expect that a necessary condition for local down-gradient diffusion being an appropriate picture of the eddy transport is a scale separation between the mean flow and the eddies.

4. Scale separation

The starting point for any theory of eddy activity in the mid-latitude troposphere is the classic theory of baroclinic instability first described by

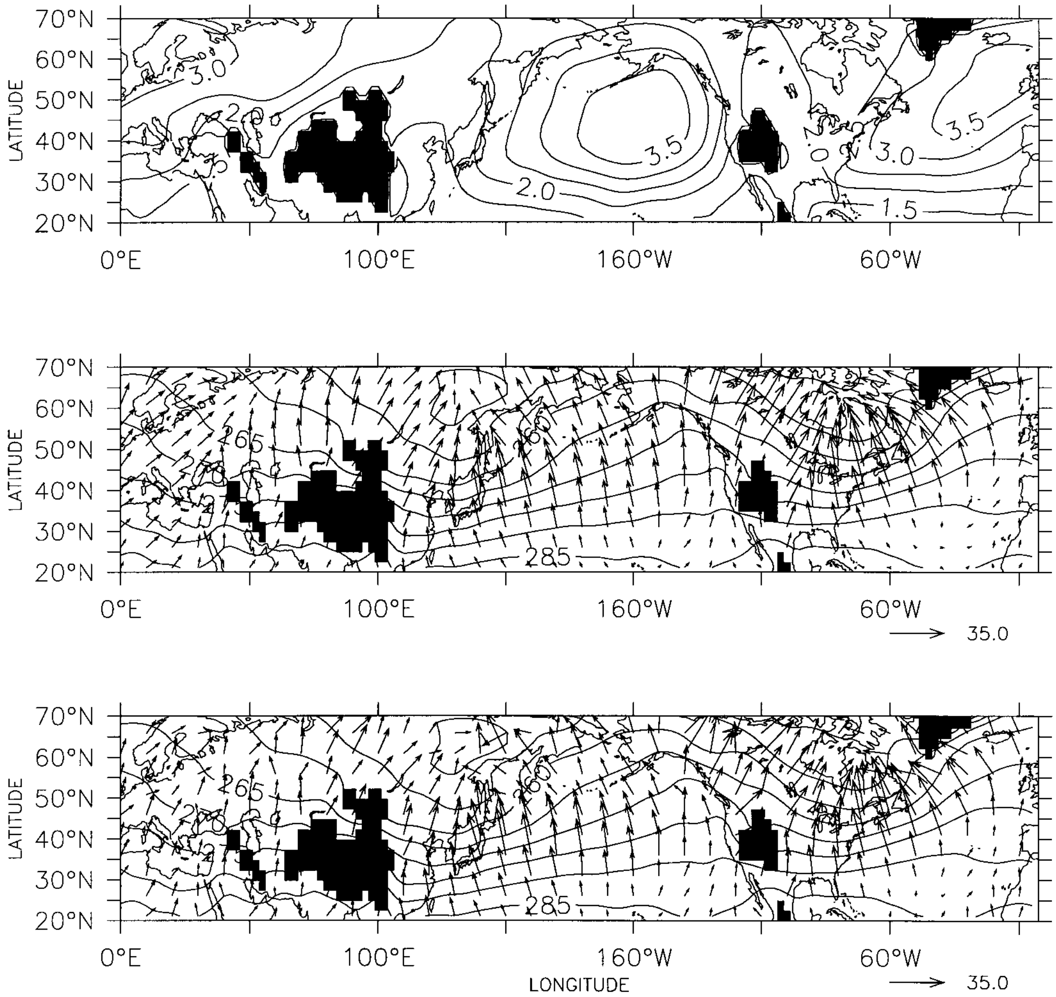

Fig. 2. Lower panel: the divergent part of the 850 mb transient eddy temperature flux in January, as obtained from the NCEP/NCAR reanalysis. Middle panel: eddy flux predicted from the mean temperature gradient at 850 mb and the diffusivity shown in the top panel. Top panel: standard deviation of eddy streamfunction for January at 850 mb in the NCEP/NCAR reanalysis, multiplied by 0.34, a proportionality constant that gives the best fit to the heat flux [see Kushner and Held (1998) for details].

Charney and Eady. In the years immediately following the development of this basic theory, one often encountered comparisons between baroclinic instability and convection associated with gravitational instability. Indeed, the phrase slantwise convection came into use in some circles. Today, it is clear that baroclinic instability is best understood as a shear instability associated with counter-propagating Rossby waves, and that the analogy with convection is generally unhelpful. A much closer and more useful analogy exists instead between linear baroclinic instability and the simplest barotropic shear instability in a homogeneous fluid. In this analogy, the vertical shear of a balanced flow plays the rôle of the horizontal shear in the barotropic problem, and potential vorticity plays the rôle of vorticity.

When the flow becomes nonlinear, the barotropic analogy breaks down. Finite amplitude baroclinic instability, and the resulting macroturbulence of the troposphere, has certain distinctive features that make it stand apart from mixing in simple shear flows or any other turbulence problems of which I am aware. This distinctiveness arises from the central rôle played by the Rossby radius of deformation in controlling the horizontal scale of the dominant eddies in the flow.

Consider classic laboratory turbulence problems such as flow through a pipe or Benard convection between parallel plates. In the pipe problem we are concerned with predicting the profile of the mean along-pipe velocity and, therefore, the eddy fluxes that mix momentum across the pipe, from the center to the walls. The dominant eddies are presumed to have the scale of the width of the pipe, there being no other length scale in this problem in the limit of very large Reynolds number. As a result, there can be no scale separation between mean flow and eddies: the cross-pipe mean flow variations occur on the same scale as the dominant eddies themselves. Both are simply set by the size of the pipe.

The situation is broadly analogous in the Benard convection problem, in which one is interested in the heat flux from one plate to the other and the temperature profile between the plates. The distance between the two parallel plates determines the scale of the dominant eddies. Once again there is no hope of scale separation between the scale of the eddies and scale of the mean flow inhomogeneity in the direction of the eddy flux of interest. This absence of scale separation is common to all familiar laboratory turbulent flows; turbulent eddies never, it seems, find themselves in a more or less homogeneous environment. Simple ideas of mixing lengths and turbulent diffusion have at best a limited heuristic value in such strongly inhomogeneous flows, and this is one of the main sources of complexity in theories of turbulent mixing.

Why might we think that baroclinically unstable flows are fundamentally different in this respect? There is a distinctive feature in the baroclinic problem, the rôle played by the Rossby radius of deformation in helping to set the characteristic eddy scale. The radius of deformation for eddies of vertical scale H is $\lambda = NH/f$, the vertical scale normalized by the Prandtl ratio N/f. The radius of deformation and the Prandtl ratio enter discussions of baroclinic eddies in a variety of ways. QG theory informs us that a balanced flow at one height will interact strongly with the flow at another height separated by the distance H only if the horizontal scale of these flows is larger than NH/f. Partly as a consequence of this fact, classic baroclinic instability theory of zonal flows invariably predicts that the most unstable waves have this ratio of zonal to vertical scale.

In passing, we note that linear instability theory does not provide any justification for thinking of the meridional scale of the eddies as being set in the same way; in fact, if the meridional scale of the unstable region is much wider than λ, the most unstable linear modes have meridional scales much larger than λ as well. But these anisotropic eddies do not survive when the flow is nonlinear, in which case the eddies tend towards being more isotropic. Due to this nonlinear isotropization, we assume that whatever mechanism sets the zonal eddy scale sets the meridional mixing length as well. It is actually a serious oversimplification to assume that λ sets the characteristic scale of the eddies in a fully developed nonlinear flow, due to the potential existence of an energy cascade to larger horizontal scales, a point that we will return to in some detail below. But let us agree to ignore this issue for the moment, and accept λ as the eddy scale to see where this takes us.

Rather than say that the horizontal scale is set by the Rossby radius of deformation, we could instead say that the vertical scale H and the static stability N have been set by external factors, that is, by factors that we prefer not to discuss at this point. Multiplication of H by the Prandtl ratio then determines the horizontal scale. One could try to create an analogous convective problem by constraining the flow to occupy a narrow chimney. One might then expect a sufficiently turbulent flow to be dominated by eddies with characteristic scale set by the width of the chimney and not its height. i.e., *the eddy scale would be determined by the scale of the inhomogeneity perpendicular to the direction of the heat flux, rather than the scale of the inhomogeneity in the direction of the flux.* Unfortunately, this is not a good analog for the baroclinic problem, in that much of the heat transport would be carried by a mean circulation, there being no counterpart to the constraint that the mean meridional flow in the baroclinic problem is ageostrophic and therefore weak. There appear to be few if any systems discussed in the turbulence literature that are comparable to this baroclinic flow, with its potential for scale separation between the eddy length scale and the scale of the mean flow inhomogeneity in the direction of eddy transport.

Supposing that the radius deformation does set the length scale, what sets the velocity scale V? Given the Prandtl ratio between horizontal and vertical eddy scales, and the thermal wind equation $f\partial_z u = -\partial_y b$ we have

$$V \approx b/N. \tag{8}$$

Equivalently, there is rough equipartition between eddy kinetic energy and eddy available potential energy: $V^2 \approx b^2/N^2$. A standard mixing length argument for the transport of buoyancy along the surface provides us with the temperature scale

$$b \approx \lambda \frac{\partial b}{\partial y} \approx \lambda f \frac{\partial u}{\partial z} \tag{9}$$

leading to the estimate,

$$V \approx (\lambda/N) \frac{\partial b}{\partial y} \approx H \frac{\partial u}{\partial z}. \tag{10}$$

Since the mean flow is weak near the surface, this is equivalent to the claim that eddy velocities are of the order of the mean zonal flow in the upper troposphere, that is, a rough equipartition between eddy and zonal kinetic energy. Equivalently, the eddy kinetic energy is comparable to the mean available potential energy contained within a region of meridional width equal to the Rossby radius of deformation. We could also think in terms of a characteristic eddy time scale,

$$\tau = \lambda/V \approx \frac{N}{f\partial u/\partial z}, \tag{11}$$

which, to within a constant, happens to be the growth rate in both the Eady and Charney models of baroclinic instability.

Using these scales, we estimate the eddy diffusivity as

$$D \approx V\lambda \approx (H^2 N/f) \frac{\partial u}{\partial z}. \tag{12}$$

By this reckoning, the diffusivity itself is proportional to the vertical shear, or the temperature gradient, so the the heat flux is proportional to the square of the temperature gradient. This particular form was first suggested by Stone (1972).

How do we test such a theory? One approach has been inspection of the seasonal cycle of the atmospheric eddy heat flux and the mean zonal temperature gradient, which does suggest a diffusivity that increases with increasing temperature gradient (Stone and Miller, 1980). This is not an ideal test, partly because the importance of latent heating for the eddy dynamics changes as the mean temperature changes. Trying to use the longitudinal structure in the eddy flux is even more fraught with problems, as is briefly addressed in Section 7.

5. Computing diffusivities

If we really believe that there is an intrinsic diffusivity that is a function of some environmental parameters such as horizontal and vertical potential temperature gradients, we should be able to design numerical experiments to measure this diffusivity in a clean way. Consider the analogy of laboratory measurements of electrical resistivity. We place a voltage across the material and simply measure the current passing through the material. Of course, it is essential that the size of the sample is much larger than the effective mean free path of the electrons, otherwise one would be creating an accelerator, and a short curcuit. Once the sample size is large enough, one does not expect the resistance to depend on the size of the sample.

One can perform analogous numerical experiments with baroclinic flows, at least within QG theory, by imposing a horizontally uniform temperature gradient, or, more generally, a temperature gradient at the surface and potential vorticity gradients in the interior, across a domain much larger than the radius of deformation. By making the assumption that the eddies are doubly periodic, the resulting eddy statistics are horizontally homogeneous. The doubly periodic assumption allows solutions that grow without bound, but this is as it should be, as such solutions are analogous to the runaway acceleration and resulting short circuit in the resistivity experiment. It is a test of the idea of a well-defined mixing length that these solutions, which are coherent across the entire domain, play no rôle in the final statistically steady state. The results from such experiments are unambiguous; the solution does not run away, and the eddy flux is independent of the size of the domain if the domain is large enough (Haidvogel and Held, 1980). We *can* create a numerical apparatus for measuring diffusivity through a baroclinically unstable system.

One also has to test the relevance of these diffusivities for inhomogeneous flows of interest, the simplest being zonally symmetric baroclinically unstable jets. Pavan and Held (1996) describe such a test for the simple two-layer QG model on a β-plane. In this test, one first performs a series of homogeneous simulations with a wide variety of imposed PV gradients, generating a theory for the diffusivity by fitting smooth curves to these experiments. One then uses this diffusive "theory" in place of the eddy fluxes to predict the statistically steady states for flows that generate baroclinically unstable jets, and one compares against the eddy-resolving solutions. For the parameter settings discussed in Pavan and Held, the results are in excellent agreement when the baroclinic zone is wider than a few Rossby radii. One expect this diffusive theory to fail quantitatively for jets that are sufficiently narrow. Yet even for baroclinic zones 2 Rossby radii wide, the results are still qualitatively useful. In addition, the departures of the eddy-resolving solutions from the predictions of the diffusive theory are systematic, implying that one might be able to use the local diffusive limit as a starting point for a more accurate theory. While this work has been performed to date only with the two-layer model, there is no obvious reason to expect the flux/gradient relations obtained from homogeneous models to be any less useful in flows with more complex vertical structure (although the relations themselves might be strongly dependent on this structure).

There also exist parameter regimes in which the local diffusive theory will not do as well. For example, Lee (1997) describes two-layer simulations in which eddy fluxes vary non-monotonically as a function of the meridional scale of the thermal forcing, behavior that cannot be explained by a simple diffusivity dependent on the local baroclinicity. In these calculations, the flow is only weakly supercritical and the surface frictional stresses are weak enough that large eddy-driven barotropic jets emerge. The scale of these jets is simply the energy-containing eddy scale itself, as seen in homogeneous turbulence simulations (Panetta, 1993). The non-monotonic behavior is seen most clearly as the system makes the transition from a single-jet to a two-jet configuration. This is clearly a finite-size effect that cannot be captured with a diffusive theory based on large-domain homogeneous turbulence simulations, even though similar jets are generated in the homogeneous model.

The possibility of having an apparatus for computing diffusivities, rather than simply examining the fluxes that are produced in various inhomogeneous flows, is important. It allows one to divide the problem of eddy flux closure into two elements: a theory of homogeneous turbulence that predicts the dependence of the diffusivity on the mean gradients; and study of the relevance of, or departures from, this local theory in inhomogeneous flows of interest. Few researchers have found this approach appealing as yet. I suspect this is due to an unwarranted suspicion that homogeneous models can have little to do with real inhomogeneous flows.

The diffusivity that is obtained in these two-level models increases more rapidly with increasing temperature gradient than expected from the Stone scaling described above. This issue is discussed by Larichev and Held (1995) and Held and Larichev (1996). This sensitivity is clearly related in the models to the inverse energy cascade which results in eddy scales that are larger than the Rossby radius. Surprisingly, perhaps, a rough equipartition between eddy kinetic and eddy available potential energy continues to hold even in the presence of a substantial inverse energy cascade. Since the temperature perturbations increase in size proportionally to the increase in eddy length scale, and the eddy velocities increase likewise, from equipartition, the diffusivity increases as the square of the length scale:

$$b \approx Lf \frac{\partial u}{\partial z}, \tag{13}$$

$$V \approx \frac{L}{\lambda}\left(H \frac{\partial u}{\partial z}\right), \tag{14}$$

$$D \approx \frac{L^2 f}{N} \frac{\partial u}{\partial z}. \tag{15}$$

Note that the eddy time scale is unchanged, since the velocity and length scales increase proportionally.

That a rough equipartition continues to hold in these models in the presence of an inverse energy cascade is not self-evident. If scales are much larger than the Rossby radius, then the eddy kinetic energy in the baroclinic component of the flow will be much smaller than the eddy available potential energy. The eddy kinetic energy of the barotropic component of the flow takes up the slack. Larichev and Held provide an explanantion for this behavior based on the relationship between the direct cascade of available potential energy on large scales and the inverse kinetic energy cascade.

What if there is no room for a cascade to larger scales, particularly larger meridional mixing lengths? If we assume that the mixing length is basically fixed, that the cascade reaches a limit set by the geometry, then the diffusivity (15) is still proportional to the horizontal temperature gradient. This form of the diffusivity was first discussed by Green (1970). Note that the diffusivities given by (12) and (15) are quite different in their dependence on rotation rate and static stability. If there is no cascade beyond the radius of deformation NH/f, then Stone's diffusivity is appropriate; if the cascade reaches its maximum extent, Green's form is more relevant; if there is a cascade but it is stopped before the eddies fill the entire unstable region, then an intermediate value is called for.

6. Stopping the inverse energy cascade

It is interesting to contrast two distinct possibilities for stopping the inverse energy cascade, other than the size of the domain: surface friction, and the beta effect.

In the former case, one can assume that the cascade is halted when the frictional time scale becomes comparable to the eddy time scale. Using a quadratic drag law, in which the surface stress is given by a non-dimensional drag coefficient C multiplied by the surface wind squared, the frictional time scale is $\approx H/(CV)$ where H is the scale height. Using the velocity estimate (14), and setting this time scale equal to $N/(f\partial_z u)$, we obtain

$$\frac{L}{\lambda} \approx \frac{f/N}{C}. \tag{16}$$

The result is a length scale set by a ratio of two non-dimensional parameters, the Prandtl ratio and the drag coefficient. Using 10^{-2} for the former and 10^{-3} for the latter (characteristic of a smooth ocean surface), we find that friction would allow the length scale to expand by an order of magnitude, given a large enough planet. This rough approximation should not be taken too seriously, of course, but rather as indicative that friction is close to becoming a player in stopping the cascade under earth-like conditions.

More likely to be dominant is the rôle of the β-effect in arresting the cascade. As discussed by Rhines (1975), we can expect the cascade to be halted at the length scale at which Rossby wave phase speeds are comparable to rms velocities

$$V \approx \beta L^2. \tag{17}$$

Accepting the fact that the energy-containing scale in the troposphere is constrained by this mechanism, this has the important consequence that the effect of β on mid-latitude eddies will be O(1). The energy-containing eddies will have Rossby wave-like characteristics and be somewhat linear. One can take this as a justification for linear theories for the horizontal and vertical structure of these energy-containing eddies, such as the promising stochastically-forced linear theories of Farrell and Ioannou (1995) and Whittaker and Sardeshmukh (1998). If the inverse cascade is halted in this way, we cannot expect the atmosphere to be dominated by isolated vortices as in the turbulent flows that can be generated in the absence of β.

Using $V \approx (L/\lambda)U$ as above, where $U = H\partial_z u$, we have

$$L/\lambda \approx \frac{U}{\beta\lambda^2}. \tag{18}$$

The resulting length scale is proportional to the north–south temperature gradient. The RHS of this equation is familiar from studies of two-layer models of baroclinic instability — it is a measure of the supercriticality of a flow in the two-layer model when the difference in mean velocity between the two layers is equal to U.

The non-dimensional number $U/(\beta\lambda^2)$ has another important physical interpretation. Rearranging,

$$\frac{U}{\beta\lambda^2} \approx \frac{f}{\beta H} I \approx \frac{a}{H} I, \tag{19}$$

where I is the isentropic slope and the last approximation assumes that, averaged over mid-latitudes, $f/\beta \approx a$, the radius of the earth. If the ratio aI/H is equal to unity, then the isentrope that starts near the surface in low latitudes reaches the tropopause in high latitudes. The scaling arguments summarized above imply that slopes must be larger than this value if there is to be a significant inverse cascade. In the extratropical atmosphere, we do in fact observe $al/H \approx 1$. Given these scaling arguments, this is consistent with the fact that the inverse energy cascade is quite modest.

7. Diffusion and zonally asymmetric stormtracks

Finally, let us return to Fig. 2, which shows the divergent part of the near-surface eddy heat flux. This figure clearly suggests that we should be looking for diffusive theories for this flux. The diffusivity in the upper panel, which provides a useful fit to the observed flux, is not, in fact, constructed from a theory, but is simply the observed standard deviation of the eddy geostrophic streamfunction at this level (Kushner and Held, 1998). The streamfunction has units of length multiplied by time, so it provides a simple way of estimating just that product of eddy length and time scales needeed for estimating the diffusivity. This point has been made by Holloway (1986) in the oceanic context, where it has the imporant implication that we can estimate near-surface horizontal diffusivities from satelite altimetry, which provides direct measurements of the surface geostrophic streamfunction.

Note that this effective diffusivity is not largest where the surface temperature gradients are largest, near the east coasts of North America and Asia, but rather in the oceanic jet exit regions in the east Atlantic and east Pacific. It is likely that this structure in the streamfunction variance is more closely related to eddy dynamics in the upper troposphere than to low-level baroclinicity. We cannot expect a local relationship between fluxes and gradients when eddies are advected and radiate long distances so that the regions of eddy production and dissipation are well-separated. It is only because eddy variance is not redistributed meridionally to any great extent, so that eddies are produced and dissipated at more or less the same latitude, that we can hope for a flux-gradient relation for the zonally averaged flow (at least in the case of a zonally symmetric climate).

A heuristic picture of storm track dynamics can be based on the idea of an upper tropospheric waveguide and reservoir of wave activity, within which eddies propagate zonally, while being deformed both reversibly and irreversibly. The source of wave activity is proportional to the low-level eddy heat flux, while the sink is ireversible wave breaking and/or leakage from the waveguide due to Rossby wave radiation. Upper tropospheric dynamics controls the zonal structure in the streamfunction variance in both upper and lower troposphere, and thus provides the diffusivity with which one can compute the near-surface heat fluxes and the associated wave activity source. Thus, the fact that lower tropospheric eddy heat fluxes are esentially diffusive should enter as one aspect of our qualitative picture of the zonally asymmetric structure of stormtracks, but coupled to a non-local picture of the upper tropospheric wave activity reservoir that controls the zonal structure of the streamfunction variance and diffusivity.

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