Dissipative Destabilization of External Rossby Waves

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(Manuscript received 24 May 1985, in final form 7 October 1985)

ABSTRACT

External Rossby waves in vertical shear can be destabilized by thermal damping. They can also be destabilized by damping of potential vorticity if this damping is larger in the lower than in the upper troposphere. Results are described in detail for Charney's model. Implications for the effects of diabatic heating and mixing due to smaller scale transients on equivalent barotropic stationary or quasi-stationary long waves are discussed. It is pointed out that energy or potential enstrophy budgets may indicate that transients are damping the long waves while, in fact, their presence is destabilizing these waves.

1. Introduction

In Held et al. (1985) (hereafter referred to as HPP), we examined the dispersion relation for neutral external Rossby waves in vertical shear, the vertical structure of these waves, and their excitation by orographic and thermal forcing. The present paper is an extension of that work to include the effects of dissipation on these waves, with particular emphasis on the possibility that some kinds of "dissipation" amplify rather than damp the otherwise neutral waves. We believe these results may have some implications for theories of low frequency variability in the extratropical troposphere.

The linear tropospheric response to localized orographic or stationary thermal forcing in midlatitudes is dominated far from the source by stationary external Rossby wavetrains. These neutral waves are equivalent barotropic, with maximum geopotential amplitude near the tropopause. The teleconnection patterns of low-frequency variability in midlatitudes have a very similar vertical structure. Whatever the precise connection between teleconnection patterns and simple quasi-stationary external Rossby wavetrains, it seems reasonable to assume that the same linear balances that determine the external mode's vertical structure are also of paramount importance for the vertical structure of the teleconnection patterns. It follows that a discussion of the effects of damping on external Rossby waves in which this vertical structure plays the key role is directly relevant to the question of the maintenance of the teleconnection patterns.

The relationship between the external Rossby wave and the Charney and Green mode instabilities for the Charney model basic state with typical midlatitude parameter settings is shown schematically in Fig. 1. The real part of the phase speed is plotted as a function of horizontal wavenumber k, with the unstable modes indicated by dashed lines. The vertical structure of the streamfunction for an external Rossby wave $\psi_e(z)$ moving slowly westward with respect to the surface wind $\bar{u}(0)$ is also illustrated. As discussed in HPP (p. 871), although the streamfunction amplitude increases with height in the troposphere, it increases less rapidly than does the mean zonal flow $\bar{u}(z)$, a fact that will be of importance in what follows.

It is well known that Ekman pumping can destabilize neutral modes in Phillips' two-layer model (Holopainen, 1961; Wiin-Nielsen et al., 1967). However, as discussed briefly in HPP and reiterated below, Ekman pumping invariably stabilizes the external mode in continuous model atmospheres (see also Card and Barcilon, 1982, for the effects of Ekman pumping in Charney's model). That *thermal* damping can destabilize neutral modes in the two-layer model is discussed by Wiin-Nielsen, et al. (1967), Haltiner (1967), and Pedlosky (1975). White and Clark (1975) use these two-layer results to argue that the perturbed heat fluxes through the ocean surface induced by quasi-stationary long waves could be a mechanism of the excitation of such waves. Based on calculations with continuous models, Geisler (1977) criticizes this claim; however, he considers only the effect of thermal damping on the unstable eastward propagating long waves and not on the neutral external waves that propagate westward with respect to the surface wind. Hendon and Hartmann (1982), using a primitive equation model on the sphere linearized about a flow with realistic vertical and horizontal shear, find that low-level thermal

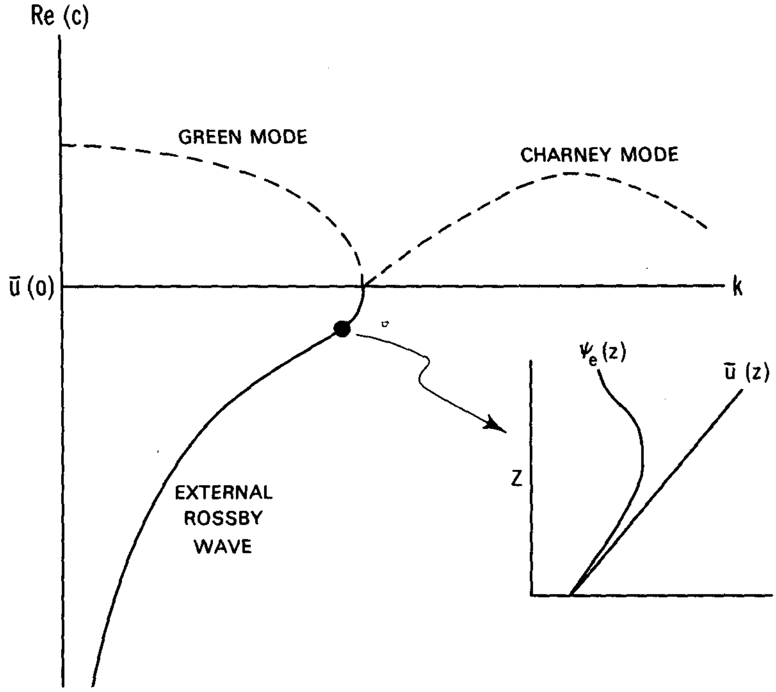

FIG. 1. Schematic depicting the real part of the phase speed of the external Rossby wave, the unstable Charney mode, and the unstable Green mode for Charney's model, using typical midlatitude parameters. The bifurcation occurs precisely at the point where $c_R = u(0)$ if one ignores the small "non-Doppler" term in the lower boundary condition, as we have in this paper. Also shown in the insert is a comparison of the vertical structure of the external mode streamfunction $\psi_e(z)$ and the vertical structure of the mean zonal flow $\bar{u}(z)$.

damping decreases the amplitude of the far-field response to orographic or stationary thermal forcing in midlatitudes, but that deep thermal damping extending into the upper troposphere increases these amplitudes. One of our motivations in this work is to relate these results and reexamine the plausibility of the thermal damping mechanism for the excitation of external waves.

It is also well-known that the transient eddy heat fluxes tend to damp the deviations from zonal symmetry in the climatological temperature field, particularly at low levels. This is evident from energy analyses of the sort first performed by Holopainen (1970) and the local budgets discussed by Lau (1979). However, it is not simply the eddy heat flux, but rather the full potential vorticity flux (in addition to the heat flux at the ground), which is needed to determine the effects of the cyclone-scale transients on the time-mean flow. In this regard, it is interesting that Youngblut and Sasamori (1980) and Holopainen et al. (1982) find that the transient eddy potential vorticity fluxes damp the stationary eddy potential enstrophy. Holopainen et al. estimate an *e*-folding time of 4–5 days averaged over the extratropical troposphere. One is tempted to conclude that the stationary eddies would be larger if the transients were not present. While this may in fact be true (as also argued by Vallis and Roads, 1984) based on calculations with an idealized two-layer model), the effects of potential vorticity damping on simple external Rossby waves warn us not to jump to this conclusion on the basis of the potential enstrophy budget alone.

We begin in section 2 by considering the effect of various damping mechanisms on the external mode in the limit of very small damping rates. We consider both the temporal growth (or decay) of modes with real horizontal wavenumber, and the spatial growth of modes with real (in fact, zero) frequency, the latter being relevant for the structure of forced stationary wavetrains. A straightforward perturbation theory relates the growth or decay to the vertical structure of the unperturbed neutral mode. This perturbation theory also makes clear that the most natural way of understanding the destabilization is in terms of the vertically integrated "wave activity" or pseudomomentum budget.

Whether or not a small amount of damping results in destabilization, one still expects that sufficiently large damping may be stabilizing. We therefore present numerical results for large damping in section 3 to indicate the range of damping coefficients that result in destabilization. Also described are calculations of the effects of thermal and potential vorticity damping in the presence of finite Ekman pumping. We restrict consideration to Charney's model. The substantial differences between the effects of damping in the two-layer model and in continuous models are discussed elsewhere.

2. Small damping

Consider the quasi-geostrophic thermodynamic and vorticity equations linearized about a zonal flow, in the presence of linear thermal damping $\gamma_T(z)$ and mechanical damping $\gamma_M(z)$:

$$\psi'_{zt} + \bar{u}\psi'_{zx} - v'\bar{u}_z + N^2 f_0^{-1} w' = -\gamma_T(z)\psi'_z, \quad (1a)$$

$$\zeta'_t + \bar{u}\zeta'_x + \beta v' - f_0\rho_0^{-1}(\rho_0 w')_z = -\gamma_M(z)\zeta'. \quad (1b)$$

Our notation follows that in HPP. The corresponding potential vorticity equation reads

$$q'_t + \bar{u}q'_x + \bar{q}_y v' = D', \quad (2a)$$

$$D' \equiv -\gamma_M\zeta' - \rho_0^{-1}(\rho_0\epsilon\gamma_T\psi'_z)_z, \quad (2b)$$

where $\epsilon(z) \equiv (f_0/N)^2$. If we also include Ekman pumping, the lower boundary condition over a flat surface becomes

$$\psi'_{zt} + \bar{u}\psi'_{zx} - v'\bar{u}_z = D'_0, \quad (3a)$$

$$D'_0 \equiv -\gamma_T\psi'_z - HN^2 f_0^{-2}\gamma_{\text{EK}}\zeta', \quad (3b)$$

at $z = 0$.

Multiplying (2a) by $\bar{q}_y^{-1}q'$, integrating in the vertical, and combining with (3a) multiplied by $\bar{u}_z^{-1}\psi'_z$, one can derive the following conservation law:

$$\partial_t[\{(2\bar{q}_y)^{-1}\overline{q'^2}\} - \epsilon(2H\bar{u}_z)^{-1}\overline{\psi_z'^2}|_{z=0}]$$
$$= \partial_y\{\overline{u'v'}\} + \{\bar{q}_y^{-1}\overline{q'D'}\} - \epsilon(H\bar{u}_z)^{-1}\overline{\psi'_z D'_0}|_{z=0}, \quad (4)$$

where an overbar refers to the zonal mean and curly brackets denote a vertical average:

$$\{A\} \equiv H^{-1}\int_0^\infty \rho_0 A dz.$$

We refer to (4) interchangeably as the vertically averaged "wave activity" conservation law, following Edmon et al. (1980), or as a vertically averaged pseudomomentum conservation law, in the spirit of Andrews and McIntyre (1978), since it is a direct consequence of the translational invariance of the basic state. We note, however, that the pseudomomentum defined by Andrews and McIntyre differs from that in (4) by the addition of a term whose integral over the domain vanishes and by an overall minus sign, as discussed for the barotropic case in Held (1985). In this context, it is more convenient to work with a quantity which is positive definite for external Rossby waves.

We are interested in how a neutral external Rossby wave is modified by the addition of small amounts of damping. We begin by considering the standard modal instability problem for real (k, l), and set $\psi' = \psi \exp[i(kx + ly - \omega t)] + (\text{c.c})$, where

$$\psi = \psi_e + \delta\psi, \quad \omega \equiv \omega_R + i\omega_I = kc_e + \delta\omega. \quad (5)$$

The subscript e refers to the unperturbed external mode. For convenience, we choose our reference frame so that $c_e = 0$. The mode under consideration is assumed to be propagating westward with respect to the flow at all levels, so that $\bar{u} > 0$ in this reference frame. The momentum flux convergence on the right-hand side of (4) is identically zero for these normal mode solutions. To lowest order in the damping, the left-hand side of (4) becomes $2\omega_I P$, where P is proportional to the wave activity or pseudomomentum of the unperturbed mode:

$$P = \{\bar{q}_y^{-1}q_e^2\} - \epsilon(H\bar{u}_z)^{-1}\psi_{e,z}^2|_{z=0}. \quad (6)$$

The relations

$$\left.\begin{aligned} -\bar{q}_y^{-1}q_e &= \psi_e\bar{u}^{-1} \equiv \eta_e, & z \geq 0 \\ \bar{u}_z^{-1}\psi_{e,z} &= \psi_e\bar{u}^{-1}, & z = 0 \end{aligned}\right\}, \quad (7)$$

where $\eta_e(z)$ is the meridional particle displacement in the external wave, lead to the following alternative forms for P:

$$P = \begin{cases} \{\bar{q}_y\eta_e^2\} - \epsilon H^{-1}\bar{u}_z\eta_e^2|_{z=0}, & (8a) \\ K^2\{\psi_e^2\bar{u}^{-1}\} + \{\epsilon\psi_{e,z}(\psi_e\bar{u}^{-1})_z\}, & (8b) \end{cases}$$

where $K^2 \equiv k^2 + l^2$. P is positive for all of the neutral modes discussed in this paper, a point we return to below. To lowest order in the damping, the right-hand side of (4) can also be evaluated using the neutral external mode eigenfunction. Substituting the expressions (2b) and (3b) for D' and D'_0 into (4), manipulating the right-hand side using (7), and then dividing by $2P$, one obtains

$$-\omega_I = \frac{K^2\{\gamma_M\psi_e^2\bar{u}^{-1}\} + \gamma_{\text{EK}}K^2\psi_e^2\bar{u}^{-1}|_{z=0} + \{\gamma_T\epsilon\psi_{e,z}(\psi_e\bar{u}^{-1})_z\}}{K^2\{\psi_e^2\bar{u}^{-1}\} + \{\epsilon\psi_{e,z}(\psi_e\bar{u}^{-1})_z\}}. \quad (9)$$

If $\gamma_M = \gamma_T = \gamma$, a constant independent of z, and if $\gamma_{\text{EK}} = 0$, then $\omega_I = -\gamma$ as required.

We refer to the numerator in (9) by the symbol D_p, so that $\omega_I = -D_p/P$. The sign of D_p, that is, the sign of the vertically averaged dissipation of wave activity, determines whether the neutral wave is stabilized or destabilized by the weak damping. If D_p is negative, the wave is destabilized. Since $\bar{u}$ is assumed to be positive, (9) implies that weak mechanical damping, $\gamma_M > 0$, or Ekman pumping, $\gamma_{\text{EK}} > 0$, cannot destabilize the wave. However, weak thermal damping can destabilize the wave if

$$\psi_{e,z}(\psi_e\bar{u}^{-1})_z < 0. \quad (10)$$

As indicated in Fig. 1 and discussed in HPP, this is generally the case: the external mode streamfunction increases less rapidly with height than does the mean zonal wind.

We use Charney's model ($\bar{u} = \Lambda z$, Λ = const, N^2 = const) to illustrate this destabilization. Throughout our discussion of this model here and in section 3, we nondimensionalize vertical distance by H (the scale height), horizontal distances by NH/f, and velocities by $H\Lambda$. After computing the vertical structure and horizontal wavenumber of the external mode as a function of its phase speed c, we compute the effect of uniform thermal damping ($\gamma_T = \gamma$ = const; $\gamma_M = \gamma_{EK} = 0$) using (9). The resulting growth rate is proportional to γ. The dashed line in Fig. 2a is a plot of ω_I/γ as a function of c, with the nondimensional β-parameter in Charney's model, $r = \beta N^2 H/(f^2\Lambda)$, set equal to 0.5. [One obtains $r \approx 0.5$ by choosing $N \approx 10^{-2}$ s^{-1}, $H \approx 8$ km, and $\Lambda \approx 2 \times 10^{-3}$ s^{-1} at 45° latitude; our unit of time is then $N/(f\Lambda) \approx 5 \times 10^4$ s.] All of these waves are destabilized, with the largest growth rates (for a given small damping) for those waves with small phase speeds. The ω_I are large for the modes with small c because these modes have very little vertically averaged pseudomomentum. Indeed, P vanishes as $c \rightarrow 0-$, as the external mode approaches its point of bifurcation with the unstable Charney and Green modes. The range of c shown in Fig. 2a corresponds to a rather small range of wavenumbers ($K = 0.56$ at $c = 0$; $K = 0.52$ at $c = -0.5$). Results for a larger range of wavenumbers are described in section 3.

Consider the energetics of these amplifying waves. The thermal damping is destroying available potential energy, but since the wave is growing this damping must be overcompensated by a generation of available potential energy due to a downgradient heat flux. Also, the growth of kinetic energy must be due to a positive generation: $w'\psi'_z > 0$. The downgradient heat flux and eddy kinetic energy generation are created by the thermal damping. A standard energy analysis that does not take this into account would lead to the misleading conclusion that the thermal damping is destroying the wave. In contrast, the pseudomomentum budget yields the correct picture. The key point is that the pseudomomentum equation is a conservation law, whereas the eddy energy equation includes an eddy-mean flow conversion term whose modification by the damping must be included to compute the effect of the dissipation on the wave energetics.

The phase relationships in the neutral external wave may help one to understand the energetics intuitively. Since the wave amplitude increases with height in the troposphere, the highs are warm and the lows cold. The steady thermodynamic equation can be rewritten as

$$N^2 f^{-1} w_e = -\bar{u}^2(v_e \bar{u}^{-1})_z, \tag{11}$$

where $v_e = \psi_{e,x}$. Since $v_{e,z}$ and v_e are of the same sign in the troposphere, (10) implies that v_e and w_e are in phase. As one moves eastward into a region of rising motion, the air becomes warmer in spite of the adiabatic cooling, the meridional advection of warm air being dominant. (This phase relation between vertical motion and temperature perturbations is just the opposite of that in a stationary gravity wave.) The addition of thermal damping can be thought of as displacing the temperatures upstream, moving the warm air into the region of upward motion and thereby generating eddy kinetic energy.

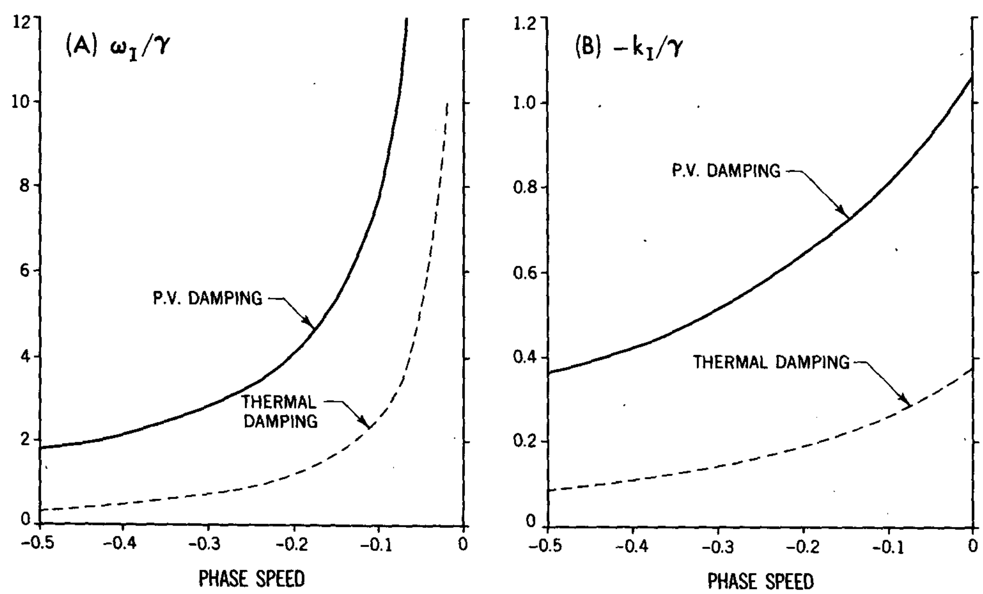

FIG. 2. (a) Temporal growth rate for the external mode in Charney's model (with $r = 0.5$), perturbed by uniform thermal damping of strength γ (dashed line) and potential vorticity damping of strength $\gamma e^{-z/H}$ (solid line); ω_I/γ is plotted against the nondimensional phase speed of the unperturbed mode. (b) The corresponding nondimensional spatial growth rate, $-k_I/\gamma$.

Suppose now that it is the potential vorticity itself that is damped, rather than the momentum or temperature, with the strength of the damping dependent on height; suppose also that $\gamma_{\rm EK} = 0$. Equation (2b) is then replaced with $D' = -\gamma_Q q'$. $\psi'_z(0)$ satisfies the same equation as does $q'(0)$, and any irreversible horizontal mixing that damps $q'(0)$ can be expected to damp $\psi'_z(0)$ as well. Therefore, we also replace (3b) with $D'_0 = -\gamma_Q(0)\psi'_z$. Wherever we refer to the effects of "potential vorticity damping" in the following, it is to be understood that this surface temperature damping is also included. Note that height-dependent potential vorticity damping is not equivalent to (local in the vertical) momentum and thermal damping, for any choices of $\gamma_M(z)$ and $\gamma_T(z)$. Instead of (9), we now have

$$-\omega_I = \frac{\{\gamma_Q \bar{q}_y \eta_e{}^2\} - \epsilon H^{-1}\bar{u}_z \gamma_Q \eta_e{}^2|_{z=0}}{\{\bar{q}_y \eta_e{}^2\} - \epsilon H^{-1}\bar{u}_z \eta_e{}^2|_{z=0}}. \quad (12)$$

The alternative expression (8a) for P has been used to obtain this result. If γ_Q is independent of height, then $\omega_I = -\gamma_Q$, and the wave is damped.

Once again, the denominator in (12) is positive definite for the waves of interest here. However, the numerator can be negative if the second term dominates over the first. From the form of this expression, it is clear that this can occur when the damping is larger at low levels. Since the denominator is a relatively small difference between two large terms for quasi-stationary waves, it is very easy to obtain destabilization in this way. As an example, we have repeated the calculation for the external mode in Charney's model with $r = 0.5$, setting $\gamma_Q = \gamma \exp(-z/H)$. The resulting ω_I/γ is plotted as the solid line in Fig. 2a. The destabilization is consistently stronger than in the case of thermal damping with strength γ.

When analyzing the stationary planetary wave response to localized forcing in a dissipative atmosphere, one is interested not in the temporal growth rates of normal modes, but rather the spatial growth or decay of a stationary wave. Given the dispersion relation $\omega = \Omega(\mathbf{k}, \gamma_i)$, where γ_i represents all the relevant damping mechanisms, the perturbation to the frequency at fixed real $\mathbf{k}$ due to small damping is $\delta\omega = \gamma_i \partial\Omega/\partial\gamma_i$, as computed above. The perturbation to the wavenumber $\mathbf{k}$ at fixed real frequency due to small dissipation is obtained from the relation $0 = \delta\mathbf{k}\cdot\partial\Omega/\partial\mathbf{k} + \gamma_i\partial\Omega/\partial\gamma_i$. Therefore, $\delta\omega = -\delta\mathbf{k}\cdot\mathbf{G}$, where $\mathbf{G} = (G_x, G_y)$ is the horizontal group velocity of the unperturbed external mode plane wave. In a frame of reference moving with the phase speed of the wave, $\mathbf{G}$ is parallel to $\mathbf{k}$. For example, in the case of a zonally directed group velocity ($l = 0$), the spatial growth or decay, $k_I = \mathrm{Im}(\delta k)$, can be obtained directly from the temporal growth or decay, $\omega_I = \mathrm{Im}(\delta\omega)$: $k_I = -\omega_I/G_x$. Thus, *in the presence of weak thermal damping or low-level potential vorticity damping, a stationary external wave will amplify in the direction of its group velocity.*

An expression for k_I that does not require explicit evaluation of the group velocity $\mathbf{G}$ of the stationary wave can be obtained by using the relationship between $\mathbf{G}$ and the vertically averaged wave activity P. Consider first the more familiar result for the simplest barotropic Rossby wave $\psi \exp[ik(x - ct) + ily]$ in a uniform flow $\bar{u}$, for which $c = \bar{u} - \beta K^{-2}$ and

$$\mathbf{G}|_{c=0} = 2\beta k K^{-4}\mathbf{k} = 2\bar{u}kK^{-2}\mathbf{k}. \quad (13)$$

For a stationary barotropic wave, the wave activity P is

$$P = \beta^{-1}\zeta^2 = \beta^{-1}K^4\psi^2 = \bar{u}^{-1}K^2\psi^2. \quad (14)$$

(To within a constant, P is equal to the wave action-wave energy divided by Doppler-shifted frequency.) Therefore,

$$\mathbf{G}P = 2k\mathbf{k}\psi^2 \quad (15)$$

(cf. Young and Rhines, 1980, or Hoskins et al., 1983). The generalization of this result to the case of stationary external Rossby waves in vertical shear is

$$\mathbf{G}P = 2k\mathbf{k}\{\psi_e{}^2\}, \quad (16)$$

with P given by (6). This relation can be obtained from Eqs. (2.27–2.29) in HPP by noting that

$$\mathbf{G} = 2k\mathbf{k}\partial c/\partial K^2, \quad (17)$$

and that the choice of normalization in HPP is such that $\{\psi_e{}^2\} = 1$. Combining (16) and (17) one sees that P is of the same sign as $\partial c/\partial K^2$. In all continuous models that we have examined in which $\bar{q}_y > 0$ for $z > 0$, $\partial c/\partial K^2$ is positive for the external mode, just as it is for the simplest Rossby wave in a uniform flow, and therefore P is also positive. We can now write

$$k_I = -\omega_I/G_x = D_p/(PG_x) = D_p/(2k^2\{\psi_e{}^2\}). \quad (18)$$

Calculations of k_I/γ in Charney's model for the cases of uniform thermal damping, $\gamma_T = \gamma$, and decaying potential vorticity damping $\gamma_Q = \gamma \exp(-z)$, are shown in Fig. 2b as a function of nondimensional phase speed. One can think of the negative of the phase speed as the surface wind $\bar{u}(0)$ required for the wave to be stationary.

In the limit $c \to 0-$, a simple expression exists for the external mode's vertical structure: $\psi_e = z\exp(-rz/2)$. One can then evaluate k_I explicitly from (18). One finds that as $c \to 0-$, $k_I/\gamma \to -(1 + r)/4$ for uniform thermal damping. For potential vorticity damping of the form $\gamma_Q = \gamma \exp(-\alpha_Q z)$,

$$k_I/\gamma \to -\frac{(1 + r)^3\alpha_Q}{r(2 + r)(1 + r + \alpha_Q)}. \quad (19)$$

These expressions give some indication of how the spatial amplification depends on the shear and static stability of the basic state (i.e., on the value of r) and on the vertical structure of the damping coefficient. From (19) one finds spatial amplification as $c \to 0-$ for all positive α_Q, but confinement of the damping to low levels (large α_Q) yields larger amplification rates.

In HPP, we define a quantity γ_e which is the appropriate Rayleigh friction coefficient for use in an equivalent barotropic model that captures the external mode's contribution to the stationary wave field when the atmosphere is weakly dissipative. Following the reasoning in HPP, one can show that

$$\gamma_e = 2u_e k_I = u_e D_p/(K^2\{\psi_e^2\}), \tag{20}$$

where u_e is the zonal wind at the appropriately determined equivalent barotropic level. (The zonal group velocity of a stationary wave in the equivalent barotropic model is then $2u_e$.) Equation (20) reduces to Eq. (2.31) in HPP for the special case in which Ekman pumping is the only dissipative mechanism present. If mechanisms are present which tend to damp eddy temperatures or low level potential vorticities, then the appropriate choice of γ_e may be negative.

3. Large damping

In this section we treat the spatial and temporal instabilities of the Charney problem in the presence of finite damping. Finite damping effects are essential in the computation of ω_I when the westward phase speed of the wave with respect to the mean surface wind is small; the perturbation theory breaks down in this limit because of the vanishing of the denominator in (9). These calculations also allow one to determine if destabilization occurs for physically interesting values of the damping coefficients. Additionally, we are interested in the extent to which the destabilization by thermal or potential vorticity damping is altered in the presence of strong Ekman pumping. All results discussed below were obtained by numerically solving the eigenvalue problem for ω or k using the technique described in section 4 of HPP. Only results for $r = 0.5$ will be presented. We restrict attention to the effects of damping on the external mode and its continuation, the Charney mode instability; the influence of dissipation on the Green modes will not be considered (but see Wang et al., 1985).

Results for the temporal problem with constant thermal damping are given in Fig. 3a. In this figure, we show the imaginary part of ω in the domain $0.1 \leqslant k \leqslant 1.5$ and $0 \leqslant \gamma \leqslant 1$, using the same nondimensionalization as in section 2. We set $l = 0$ for simplicity. The dividing line between positive and negative ω_R is shown by the heavy line. In the undamped problem with $r = 0.5$, the Charney mode instability exists for $k > k_c \equiv (r(2 + r)/4)^{1/2} \approx 0.56$; the unstable modes are eastward propagating, while the neutral external mode which exists when $k < k_c$ is westward propagating (see Fig. 1). In the presence of small damping, it is noteworthy that the external mode continues smoothly into the Charney mode as the phase speed becomes eastward and the mode develops a steering level within the flow.

From Fig. 3a it is apparent that the most unstable eastward propagating Charney mode is markedly stabilized by damping, while the westward propagating external mode is destabilized and remains unstable even at large values of the damping coefficient. Although the most unstable mode remains eastward propagating for all values of damping considered, the introduction of instabilities propagating westward with respect to the surface wind is potentially relevant to the maintenance of quasi-stationary long waves, as discussed by Haltiner (1967) and White and Clark (1975).

Figure 3b shows the spatial amplification rate k_I of a stationary external mode subjected to constant thermal damping, for surface wind in the range $0 \leqslant \bar{u}(0) \leqslant 1$. [The perturbation theory of section 2 is a good approximation for a given $\bar{u}(0)$ as long as γ is sufficiently small that the contours in Fig. 3 are equally spaced in the vertical.] The k_I are everywhere negative,

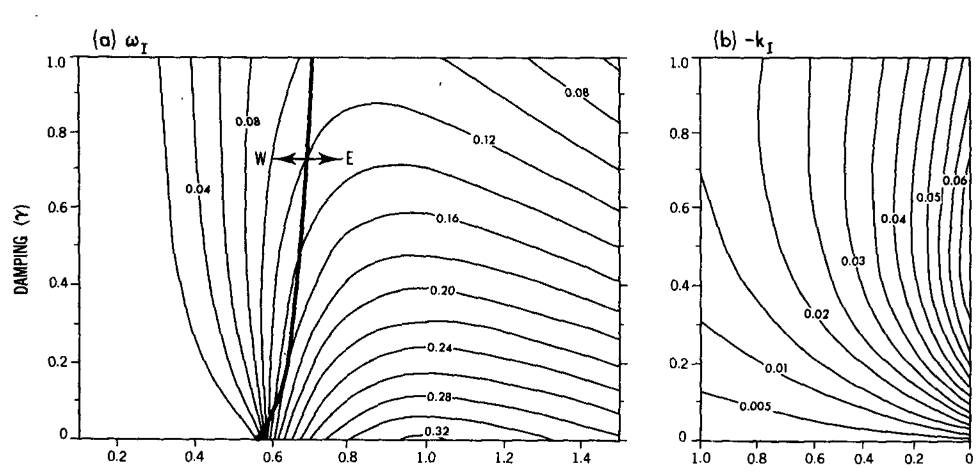

FIG. 3. (a) Growth rate ω_I for modes of Charney's model ($r = 0.5$) with uniform thermal damping, as a function of horizontal wavenumber and strength of the damping γ; (b) k_I for stationary modes as a function of the surface wind $\bar{u}_0$ required to make the mode stationary and the strength of the damping. (See text for nondimensionalization.)

corresponding to growth in the downstream direction. The greatest destabilization is found for profiles with small surface wind. When $\bar{u}(0) = 0$, the growth rate attains a maximum value $k_I = -0.072$ at $\gamma = 0.44$. Since the nondimensional wavelength, $\lambda_e \equiv 2\pi/k_e$, of this stationary mode is ≈ 11, the maximum value of $|k_I\lambda_e|$ is ≈ 0.8, which corresponds to very significant growth over one wavelength.

Next we consider the effects of potential vorticity damping with profile $\gamma_Q = \gamma \exp(-z)$. The temporal growth rates as a function of γ and of the real horizontal wavenumber are given in Fig. 4a. As with thermal damping, the Charney modes are stabilized and the westward propagating modes destabilized. As discussed in section 4, our interest in these results is primarily confined to the quasi-stationary waves ($\omega_R \approx 0$). From the figure we see that the shallow potential vorticity damping creates quasi-stationary instability as long as γ is less than ≈ 0.6 (for this value of r). For larger γ the damping is stabilizing. As γ increases from zero, the phase speed of the most unstable wave decreases, passing through zero at $\gamma \approx 0.25$.

Results for the spatial growth rate of a stationary wave in the presence of this low level potential vorticity damping are shown in Fig. 4b, as a function of γ and the surface wind $\bar{u}(0)$. The greatest spatial amplification, $k_I \approx -0.12$, occurs at zero surface wind and $\gamma \approx 0.2$. As compared with the constant thermal damping case, the spatial growth rate is larger for the same γ. However, for sufficiently large potential vorticity damping the mode becomes evanescent.

In reality, any downstream amplification due to thermal damping or potential vorticity mixing will be opposed by decay due to drag in the surface boundary layer. If the perturbation theory of section 2 is relevant, one can simply add the contributions to k_I from the individual damping mechanisms. However, surface drag of realistic strength distorts the modal structure, particularly near the surface, to the extent that simple addition of the separate k_I's is not adequate. To illustrate this point, we choose $\bar{u}(0) = 0.2$ and $r = 0.5$ and compute k_I for the stationary wave, with and without Ekman pumping, as a function of the strength of the thermal or potential vorticity damping. Figure 5a summarizes the results for $\gamma_{EK} = 0$, and Fig. 5b for $\gamma_{EK} = 0.25$. [If our unit of time is $N(f\partial\bar{u}/\partial z)^{-1} = 5 \times 10^4$ s, as estimated in section 2, the value $\gamma_{EK} = 0.25$ corresponds to an e-folding time of 2×10^5 s ≈ 2.5 days for the spindown of a barotropic eddy.] The dashed line in each figure represents the case of uniform thermal damping and the dashed–dotted line the case of shallow thermal damping with $\gamma_T = \gamma \times \exp(-z/2)$. The solid line represents potential vorticity damping with $\gamma_Q = \gamma \exp(-z)$.

It is clear from the figure that the thermal and potential vorticity damping have less of an effect on k_I when strong Ekman pumping is present. One can still see the tendency of the uniform thermal damping to increase the downstream amplitude when $\gamma < 0.2$, but it is incapable of overcoming the decay due to the boundary layer damping. For the case of shallow thermal damping, the effects of the Ekman pumping are greater, and, except at very small γ, the thermal damping simply increases the downstream decay. The difference between uniform and shallow thermal damping in Fig. 5b may provide one way of interpreting the results of Hendon and Hartmann (1982) mentioned in the Introduction.

Potential vorticity damping is still capable of overcoming the decay due to surface drag; indeed, $\gamma > 0.05$ (corresponding to an e-folding time of ≈ 10 days using the basic state parameters listed above) is sufficient to produce downstream amplification. This is in part due to the fact that the downstream decay induced by the Ekman pumping is itself rather weak for this set of parameters. As discussed in HPP, the decay due to Ek-

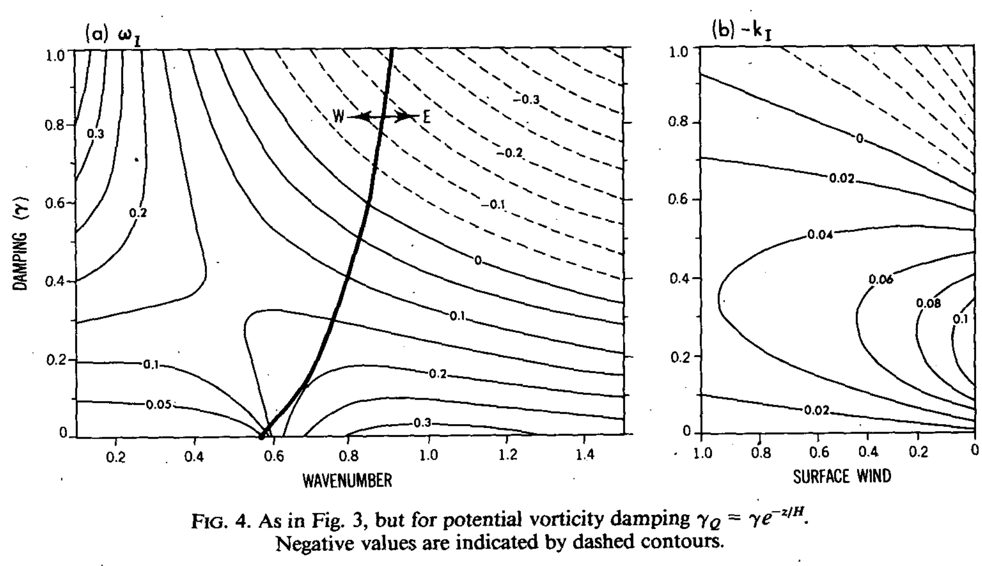

FIG. 4. As in Fig. 3, but for potential vorticity damping $\gamma_Q = \gamma e^{-z/H}$. Negative values are indicated by dashed contours.

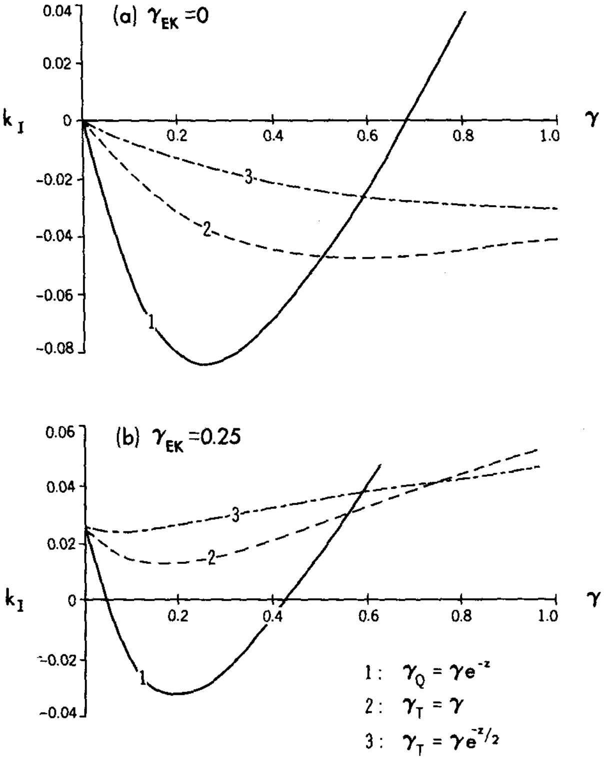

FIG. 5. Spatial amplification rates k_I as a function of the strength of the damping for 1) potential vorticity damping, 2) uniform thermal damping, and 3) shallow thermal damping, for the stationary wave in Charney's model [r = 0.5, $\bar{u}(0)$ = 0.2]. (a) Results obtained with no Ekman pumping; (b) results with large Ekman pumping, γ_{EK} = 0.25.

man pumping is small when the zonal mean surface wind is small.

4. Discussion and conclusions

In the absence of damping, Charney's model does not possess any instabilities that propagate westward with respect to the surface wind. However, when one adds thermal damping the westward propagating external Rossby waves are destabilized. Alternatively, one can show that a stationary wave in a flow $\bar{u}(z) > 0$ with realistic vertical shear will amplify eastward, in the direction of its group velocity, when thermal damping is present. A perturbation theory valid for sufficiently small damping provides one way of understanding this destabilization; the "damping" in fact acts as a source of wave activity for the mode. One cannot conclude from the fact that the thermal damping tends to destroy the available potential energy of the wave that it also tends to reduce its amplitude.

For reasonable midlatitude values of the basic state parameters, this destabilization persists even when the strength of the thermal damping, γ_T, is as large as (1 day)$^{-1}$. However, the presence of Ekman pumping of realistic strength reduces the range of γ_T for which destabilization occurs. This is particularly true if the thermal damping is confined to the low levels where the effects of surface friction are most pronounced. Calculations such as those summarized by Fig. 5, as well as those of Hendon and Hartmann (1982) make it seem unlikely that shallow sensible heat fluxes induced by the presence of a quasi-stationary wave would, as suggested by Haltiner (1967) and White and Clark (1975), result in significant amplification of the wave. If these heat fluxes are distributed through a greater depth, perhaps by moist convection associated with extratropical cyclones, the potential for amplication increases.

External waves that propagate westward with respect to the surface wind are also destabilized by processes that damp low-level potential vorticity and surface temperature perturbations. Calculations with Charney's model suggest that this result is fairly robust, with substantial destabilization persisting in the face of realistic Ekman pumping, and at fairly modest levels of the potential vorticity damping.

We would like to think of this potential vorticity damping as mimicking the effects of mixing by cyclone-scale transients. For this approach to have even qualitative validity, there must be some scale separation between the cyclone-scale transients and the larger-scale waves. From this point of view, the effects of potential vorticity damping on the Charney modes is of little interest as these modes are responsible for the cyclone-scale transients themselves. In addition, if the mixing by cyclone-scale transients is to damp the large-scale wave, the latter must be more or less stationary on the time scale of the development of the cyclone-scale eddies. Therefore, potential vorticity damping is not a very plausible way of approximating the effects of these smaller scale transients on very long waves with rapid westward phase propagation. However, we do feel that this may be a useful idealization for the effects of smaller-scale transients on quasi-stationary long waves. Indeed, White and Green (1982) have constructed a model of the long waves based on this premise (but it is difficult to relate the effects of potential vorticity mixing in their highly truncated model in a channel to the present analysis of its effects on wavetrains).

As mentioned in the Introduction, estimates by Holopainen et al. (1982) indicate that the average rate of damping of the stationary eddy potential enstrophy by transient eddies is of order (5 days)$^{-1}$ averaged over the extratropical troposphere in Northern Hemisphere winter. Figure 5 of Holopainen et al. indicates that the potential enstrophy destruction is larger near the surface and the tropopause, and smaller at middle tropospheric levels. Whether there are corresponding maxima in the local rate of potential enstrophy destruction per unit of potential enstrophy ($\equiv 2\gamma_Q$) is less clear. In any case, it is reasonable to assume that the horizontal mixing at low levels is primarily due to the initial growth stage of baroclinically unstable disturbances, while the maximum in the potential enstrophy destruction near the tropopause results at least partly from mixing during the decay of mature disturbances (e.g., Hoskins, 1983). Other processes, such as the barotropic instability of zonally asymmetric flow discussed by Simmons et al., (1983), may also play a role in the upper tropospheric maximum, as Holopainen et al. point out that much of the upper tropospheric mixing is itself associated with low frequency eddies.

Our calculations indicate that *the low-level mixing alone is of the right magnitude to produce significant downstream amplification of a stationary wave.* For the basic state parameters on which Fig. 5 is based, the amplitude of the stationary wave would *decrease* by $\approx$30% over one wavelength if Ekman pumping were the only relevant dissipative mechanism ($2\pi k_e^{-1}k_I \approx 0.3$). The amplitude would *increase* by $\approx$20% over one wavelength if, in addition to this Ekman pumping, the nondimensional potential vorticity damping were set equal to $\gamma_Q = 0.1 \exp(-z)$. If we choose $N(f\partial\bar{u}/\partial z)^{-1} = 5 \times 10^4$ s, this value of γ_Q corresponds to a damping rate of $\approx$(3 days)$^{-1}$ for potential enstrophy at the surface and $\approx$(5 days)$^{-1}$ averaged over the lowest scale height. Note however that *upper level potential vorticity damping is stabilizing.* The net effect of the mixing by cyclone-scale transients on the wave activity of the stationary waves therefore results from the competition between two rather distinct processes: the low-level mixing accompanying the growth of cyclone-scale eddies acts as a source of stationary wave activity; the upper level mixing associated with decaying eddies acts as a sink.

Despite the complications introduced by the upper tropospheric mixing, we feel that this analysis helps one understand how a stationary or quasi-stationary external Rossby wave can avoid being dissipated by the strong horizontal mixing generated by baroclinic instabilities. Contrary to what one might intuitively suspect, the low-level down-gradient potential vorticity fluxes (and heat fluxes near the surface) generated by these instabilities can actually increase the amplitude of the external wave. This is a type of "wave–wave" interaction that would not show up as such in standard energy or potential enstrophy analyses. The results of Hayashi and Golder (1983) on the space–time spectral energetics of a general circulation model with a uniform lower boundary (and therefore a zonally symmetric climate) are consistent with this picture. They find that low frequency waves (periods greater than 20 days) are maintained by conversion from zonal available potential energy, while wave–wave interactions are draining energy from these waves on average (see their Table 3). Our analysis suggests that cyclone-scale eddies are acting as a catalyst, in that the mixing on these scales is distorting the longer wave in such a way that this long wave can then tap the available potential energy on planetary scales.

Acknowledgments. The authors would like to acknowledge valuable conversations with S. Fels and Y. Hayashi on this topic. One of us (RLP) was supported by NOAA Grant 04-7-022-44017.

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