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# Subprime \& Prime Mortgages: Loss Distributions 

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# Abstract <br> Subprime \& Prime Mortgages: Loss Distributions 

This paper links the probabilities of default and prepayments to the distribution of losses associated with a synthetic portfolio of Fannie Mae and Freddie Mac mortgages randomly samples from 30 year fixed rate prime and subprime mortgages. The simulations exploit historical relationships found between mortgage characteristics and economic conditions in time and space as estimated in a competing risk conditional default and prepayment hazard model and a loss given default model.

Estimations of loss distributions indicate that subprime loans exhibit greater dispersion and higher loss rates than prime loans. For instance, expected or mean simulated losses for subprime loans are found to be 5 to 6 times higher than for prime loans. However, the use of simple risk sharing arrangements can greatly mitigate expected losses and reduce the variation of losses.

## Subprime \& Prime Mortgages: Loss Distributions

## Introduction

The growth of subprime lending in the mortgage market has helped to extend credit to borrowers who otherwise may not have had access to mortgage credit and homeownership in the past. Research has shown that borrowers of subprime mortgages often default more frequently than do borrowers of prime mortgages. In addition, borrowers of subprime mortgages also appear to react differently to changing interest rates and other economic conditions than do other borrowers (Pennington-Cross 2004 and Alexander et al. 2002). Few studies have explored the link between the likelihood of prepayment and default and the corresponding losses associated with defaults.

When mortgage borrowers default they generally impose losses on lenders, mortgage insurers, mortgage based security holders, and others. The losses differ by entity and each prices its services accordingly. In addition to setting prices these entities also use non-price credit rationing techniques (credit scores, loan to value ratios, income verification, etc.) to limit their exposure to risks. As a result, each institution bears an accompanying distribution of potential losses that varies by borrower and lender and is priced accordingly.

To accurately assess risk both the probability of loss and the expected magnitude of that loss should be known. Loss distributions can be characterized by conditional means and variances, which are contingent on the subsequent economic environment and type of loan (for example, prime and subprime). In the subprime portion of the mortgage market loans are differentiated at origination and typically classified as progressively more risky with letter grades such as prime, $\mathrm{A}-, \mathrm{B}, \mathrm{C}$, and D .

Typically worse credit history moves a borrower into a riskier and hence more costly (higher interest rate and fees) risk classification. To help reduce the loss on a given default lenders require that borrowers provide larger down payments to compensate for the perceived higher probability of default. While unlikely, it is possible for riskier
individual loan grades to experience lower expected losses while simultaneously experiencing higher defaults rates. Risk can be expressed as a function of the expected losses (the mean of the distribution) and the variance (a measure of dispersion). Due to larger variations in lending standards ("flexible lending") and the more precarious financial condition of subprime borrowers it is more likely that subprime lending has both higher loss rates and more variability in loss rates. If priced appropriately, however, no shortage in the supply subprime loans should result.

Probability distributions of loss rates are not directly observed, but they may be estimated. Using a sample of Fannie Mae and Freddie Mac 30 year fixed rate prime and subprime loans, this study estimates probability density functions (PDF) if loss rates for these two types of mortgages.

The results show that the mean of the subprime loss distribution is 5.15 times higher than the mean of the prime loss distribution when private mortgage insurance (PMI) is used and 6.00 times higher when PMI is not used. In addition, PMI insurance reduces expected default losses by more than 85 percent for both prime and subprime mortgages.

One additional feature available from the distributions of losses is that these distributions permit the determination of the economic capital requirements associated with various risk tolerances. Economic capital is defined as the amount of capital that is required to cover unexpected losses in a portfolio. For instance, the results of the simulations can be used to derive a 0.08 percent economic capital/asset offset for prime mortgages and a 1.14 percent economic capital/asset for subprime mortgages without mortgage insurance.

While these results are based on econometric relationships established during the 1990s and a simulation covering only from 1985 through the middle of 2002, estimated economic capital requirements contrasts with the current capital rules under the 1988 Basel Accord, which require a 2 percent capital ratio for most residential mortgages (assets have a .50 risk weight) and a 4 percent capital ratio for subprime mortgages (assets have a 1.00 risk weight). Consistent with Calem and LaCour-Little (2003), the
results of this paper indicate that the Basel standards greatly and inappropriately increase the cost of holding subprime and prime loans in portfolio relative to holding mortgage securities.

Assuming a stationary distribution, estimates of the distributions of the loss rates also permits estimates of the value at risk (VAR). Carey (1998) and Calem and LaCour-Little (2003) provide this method. While the VAR methodology has become a standard tool used to evaluate the credit risk of corporate debt ${ }^{1}$ it has received comparatively little attention in the mortgage market. ${ }^{2}$ This study addresses this lack of attention.

This paper follows a semi-parametric procedure to estimate and simulate the distributions of default and prepayment losses and estimate value at risk. It combines a previously published default and prepayment competing risk hazard model for fixed rate prime and nonprime mortgages by Pennington-Cross (2004) and a model that translates defaults into losses (loss given default) based primarily on the down payment, changes in house prices, and mortgage type to simulate losses from scenarios of metropolitan area house prices, metropolitan area unemployment rates, interest rates, and location drawn from history. Both models use loan history from two large secondary market institutions (Fannie Mae and Freddie Mac) over the period 1995-1999 and are likely to represent only the least risky portion of the prime and subprime mortgage market (Alt-A and A- risk categories).

Estimates of losses, VAR, and economic capital requirements depend on the composition of the synthetic portfolio and the time period used for the analysis. The simulations are restricted to those 198 metropolitan areas that have a repeat sales house price index reported by the Office of Federal Housing Enterprise and Oversight (OFHEO) from 1985 to present. While this time period may in general be viewed as a high growth and low risk environment, it does include two time periods when the real growth rate of the Gross Domestic Product declined for three consecutive quarters starting in the third quarter of

[^0]1990 and the first quarter of 2000. In addition, the economy also experienced some regionally based rolling recessions during this time period that substantially reduced the value of housing. For instance, in nominal terms, Dallas experienced a 14.6 decline in house prices starting in 1986 and did not recover to 1985 price levels until 1998. Another prominent example can be found in the New England area where Boston also lost over 10 percent of it's housing value and did not see prices return to their 1990 levels until the end of 1996 (See www.ofheo.gov for metropolitan area repeat sales price indices).

This paper does not conduct a stress test. Instead, history is used to parameterize distributions of losses that include interest rate, house price index and local unemployment variation in both stressful and buoyant economic scenarios. Consequently the results are more suited to baseline projections, although the methodology can be applied to a stressful time period. Since only the most simple mortgage type (30 year fixed rate home purchase owner occupied) is included in this paper, the results cannot be extended to more complicated mortgages such as adjustable rates, graduated payment programs, or other hybrid mortgages.

## Models of Default

The expected loss given all information $(x)$ available in period $\mathrm{t}, \mathrm{E}(L \mid x, z)_{\mathrm{t}}$, may be obtained by multiplying an estimate of the probability of default, $\pi_{d}(x)$, by the loss $\left(L(z)_{\mathrm{t}}\right)$ that would occur given default. The loss that would occur given default is measured as the fraction of the remaining or unpaid balance $\left(u p b_{\mathrm{t}}\right)$ of the loan that is lost after default. Expected loss is then defined as:

$$
\begin{equation*}
\mathrm{E}(L \mid x, z)_{\mathrm{t}}=\pi_{\mathrm{d}}(x)_{\mathrm{t}}^{*} L(z)_{\mathrm{t}} * u p b_{\mathrm{t}} \tag{1}
\end{equation*}
$$

The competing risk proportional hazard model of default and prepayment provides a useful way to estimate both the hazard functions for default $\pi_{d}(x)_{t}$ and prepayment $\pi_{p}(x)_{t}$. Deng et al. (2000) was the first to use an approach introduced by McCall (1996) to estimate the probability of default and prepayment for real estate loans. Unfortunately the sample excluded information on credit history.

Pennington-Cross (2004) demonstrated the importance of including borrower credit history using FICO scores in the estimation of likelihood of defaults and prepayments. In addition, Pennington-Cross (2004) showed that subprime and prime loans prepay and default at different rates (subprime has a higher baseline probability of default and prepayment) and react differently to the financial incentives to default or refinance a mortgage. Specifically, subprime loans always defaulted at significantly higher rates than prime mortgages.

$$
\begin{equation*}
\pi_{d s}(x)_{\mathrm{t}}>\pi_{d p}(x)_{\mathrm{t}} \tag{2}
\end{equation*}
$$

Where s indexes subprime loan types and pindexes prime loan types. Subprime loans also prepay more often when interest rates $(i)$ are steady or decrease and prepay less often when interest rates decline substantially.

$$
\begin{align*}
& \text { if } \Delta i \geq 0 \text { then } \pi_{p s}(x)_{\mathrm{t}}>\pi_{p p}(x)_{\mathrm{t}}  \tag{3}\\
& \text { if } \Delta i \ll 0 \text { then } \pi_{p s}(x)_{\mathrm{t}}<\pi_{p p}(x)_{\mathrm{t}} \tag{4}
\end{align*}
$$

Subprime loans also react differently to local economic conditions $\left(u_{\mathrm{t}}\right)$ and borrower credit scores $\left(c_{\mathrm{t}}\right)$, where $u_{\mathrm{t}}$ and $c_{\mathrm{t}}$ are elements of x .

$$
\begin{align*}
& \delta\left(\pi_{p s}(x)_{t}\right) \delta(x) \neq \delta\left(\pi_{p p}(x)_{\mathrm{t}} / \delta(x)\right.  \tag{5}\\
& \delta\left(\pi_{d s}(x)_{\mathrm{t}}\right) \delta(x) \neq \delta\left(\pi_{d p}(x)_{\mathrm{t}} / \delta(x)\right. \tag{6}
\end{align*}
$$

Loans in which the contract rate on the mortgage was at least 100 basis points higher than the average contract rate reported in the Freddie Mac Primary Mortgage Market Survey (PMMS) at origination were identified as subprime in Pennington-Cross (2004). This study added to the empirical literature by analyzing loan defaults and prepayments during the period 1995 through 1999. It included prime and subprime loans in the sample obtained from Fannie Mae and Freddie Mac. The availability of credit scores limited the maximum age of the loan to 5 years. This analysis does not reflect the entire subprime market, but rather those loans purchased by the Fannie Mae or Freddie Mac.

Hazard functions allow the user to solve for the time to prepayment and default. Define the time to prepayment as $T_{p}$ and the time to default as $T_{d}$, which are random variables that have a continuous probability distribution, $\mathrm{f}\left(t_{w}\right)$, where $t_{w}$ is a realization of
$T_{w}(w=p, d)$. The joint survivor function for loan j is then $S_{j}\left(t_{p}, t_{d}\right)=\operatorname{pr}\left(T_{p}>t_{p}, T_{d}>t_{d} \mid x_{t j}\right)$. The joint survivor function has the following form:

$$
\begin{equation*}
S_{j}\left(t_{p}, t_{d}\right)=\exp \left(-\theta_{p} \sum_{t=0}^{t_{p}} \exp \left(\beta_{p}^{\prime} x_{j t}\right)-\theta_{d} \sum_{t=0}^{t_{d}} \exp \left(\beta_{d}^{\prime} x_{j t}\right)\right) \tag{7}
\end{equation*}
$$

Note t indexes time in months for outcome $\mathrm{p}, \mathrm{d}$, or c , which indicates whether the loan is prepaid, defaulted, or continued, and j indexes the N individual loans. The baseline hazard function is one element of the matrix $x_{j t}=\left\{x_{l j t}, x_{2 j t}, x_{3 j t}, \ldots, x_{k j t}\right\}$ for up to k regressors of j loans for t periods. Thus $x_{j t}$ is a $(\mathrm{j} \times \mathrm{t}) \times \mathrm{k}$ matrix. The coefficient vector $\left(\beta_{w}\right)(\mathrm{k} \times 1)$ can be used to approximate the underlying continuous time baseline hazard for the default and prepayment probabilities. The vector of parameters $\left(\beta_{w}\right)$ also represents other time varying and time constant effects of regressors on the probability of terminating. When $x_{j t}$ is mean centered, $\theta_{p}$ and $\theta_{d}$ represent the mean propensity to default or prepay (the constant). Only the shortest mortgage duration is observed, $T_{j}=\min \left(T_{p}, T_{d}, T_{c}\right)$. The hazard probabilities of mortgage prepayment, $\pi_{p j}(t)$, default, $\pi_{d j}(t)$, or continuing, $\pi_{c j}(t)$, in time period $t$ are defined as a function of survival rates:

$$
\begin{align*}
& \pi_{p j}(t)=S_{j}(t, t)-S_{j}(t+1, t)-.5\left(S_{j}(t, t)+S_{j}(t+1, t+1)-S_{j}(t, t+1)-S_{j}(t+1, t)\right)  \tag{8}\\
& \pi_{d j}(t)=S_{j}(t, t)-S_{j}(t, t+1)-.5\left(S_{j}(t, t)+S_{j}(t+1, t+1)-S_{j}(t, t+1)-S_{j}(t+1, t)\right) \\
& \pi_{c j}(t)=S_{j}(t, t)
\end{align*}
$$

The term multiplied by $1 / 2$ is the adjustment made because actual duration is measured in months instead of continuously. Using equation 8 and taking logs the log likelihood of the proportional competing risks model is summed across all N loans.

$$
\begin{equation*}
\sum_{j=1}^{N} \delta_{p j} \log \left(\pi_{p j}\left(T_{j}\right)\right)+\delta_{d j} \log \left(\pi_{d j}\left(T_{j}\right)\right)+\delta_{c j} \log \left(\pi_{c j}\left(T_{j}\right)\right) \tag{9}
\end{equation*}
$$

$\delta_{o j}, o=p, d, c$ indicate if the $\mathrm{j}^{\text {th }}$ loan is terminated by prepayment, default, or censoring.

Table 1 provides the estimated coefficients obtained by maximizing the likelihood function defined in equation 9. The results show that prime and subprime loans respond similarly to the incentives to prepay and default and although the responses are often in the same direction they are of different magnitude. Consequently the "speed" of default or prepayment may be very different for identical loans if one is classified as subprime
and the other prime. For instance, defaults are more likely and prepayments are less likely for loans with borrowers with lower (worse) credit scores (FICO) for both prime and subprime loans. This result is intuitively appealing because it indicates that households with a poor record of managing their finances in the past are likely to continue in this manner and, as a result, are less likely to default on their mortgage. In addition, the higher the probability that the value of the home is less than the outstanding mortgage (the probability of negative equity or $p n e q_{j t}$ ) the more likely it is the loan will default and less likely it will prepay. ${ }^{3}$ Again, this result is consistent with the financial incentives to default on a mortgage when the loan amount is larger than the value of the property. The coefficient on the variable $r e f i_{j t}$ indicates that the more interest rates have dropped relative to the rate on the mortgage, the more likely it is the loan will be prepaid. ${ }^{4}$ Therefore, as it makes more and more financial sense to refinance a loan more

[^1]and more borrowers do refinance and reduce their monthly payments. The loss of a job is a financial hardship that must be strongly associated with the ability to pay a mortgage. Therefore, as expected the results of the hazard model estimation show that metropolitan areas with higher unemployment rates are associated with higher rates of default and lower rates of prepayment. Parameters are used to provide a baseline hazard in a quadratic formulation of the age and age squared of the loan in months. Lastly, $\theta_{p}$ and $\theta_{d}$ indicate that the central tendency of defaults and prepayments are much higher for subprime loans than prime loans. For more details on these results see Pennington-Cross (2004).

## Models of Loss Given Default

The first step in quantifying the loss to the lender, a secondary mortgage holder, or a mortgage backed security holder, is to estimate how much of the outstanding balance of the mortgage can be recovered from the sale of the property (the inverse of the loss rate) once a loan has defaulted.

For several reasons it is likely that homes for sale due to foreclosure sell at discounts relative to other houses on the market. Lenders who are trying to dispose of a defaulted mortgage property are earning no income from the mortgage since no payments are being made by the borrower. Therefore, they may wish to sell the property faster than the typical homeowner because they may experience higher opportunity costs of holding on to a non-performing asset. This desire should lower the lenders' reservation price and lead to a lower transaction price on average for foreclosed property.

[^2]In addition properties associated with a mortgage default typically are in greater need for repair. A homeowner who is evicted for failure to repay the mortgage is more likely to be under financial stress and therefore fail to adequately maintain the property. When a homeowner decides that the mortgage has more value than the home itself, the homeowner has little incentive to maintain the home in order to keep the value of the property up. As a result of these and other factors, it seems likely that foreclosed properties suffer from a lack of maintenance.

Thus even loans that have fairly large current loan to value ratios (lots of home owner equity) are unlikely to recover all of the outstanding balance from the sale of the house, unless property values have appreciated considerably in the area as a whole.

In addition to these maintenance issues, an individual unable to initially obtain a prime loan may possess a lower propensity to behave responsibly with respect to other obligations, including a willingness to maintain the property values. Without empirical evidence to the contrary, indicators of whether a loan is prime or subprime may well affect the ultimate recovery rate and the potential loss to the lender.

Previous empirical examinations of the loss on sale of a foreclosed property have consistently found that the amount of the outstanding principal recovered is strongly related to the loan to value ratio at foreclosure (the current loan to value ratio, CLTV) or the LTV at origination (Calem and LaCour-Little 2003, Lekkas, Quigley, and Van Order 1993, Clauretie 1990, Crawford and Rosenblatt 1995, Clauretie 1989). In addition, although the mechanism is unclear, there is also some evidence linking the age and size of the loan to the recovery rate (Calem and LaCour-Little 2002, Lekkas, Quigley, and Van Order 1993).

These results however differ across studies and samples, in part because there is substantial variation across the country in how states treat the rights of the borrowers and lenders during the foreclosure process. Pence (2003) provides a comprehensive summary of the variations in foreclosure state laws. Following Pence's (2003) definitions three
foreclosure classifications are used in this paper: 1) twenty-one states require a judicial foreclosure process so that the lender must proceed through court to foreclose, while all other states allow a nonjudicial procedure called power of sale which is typically simpler, cheaper and quicker; 2) nine states allow a statutory right of redemption so that up to a year after sale of the property the homeowner can redeem the property by paying the foreclosure price plus any foreclosure expenses; and 3) nine states allow a deficiency judgment to be used by the lender to collect any losses on a foreclosure from the borrower's other assets.

Foreclosure laws plainly affect the cost of foreclosing and the time it takes for a foreclosure process to be completed, but the effects on the selling price of the home are less transparent. However, in locations where the borrower has the right to buy back the home even after foreclosure sale of the property, one would expect the selling price of the property to be depressed, because the new owner cannot immediately obtain clear title on the property. ${ }^{5}$

In addition, the requirement of a judicial foreclosure process may lower the resale price of the property because it is vacant or rented for a longer period of time. However, this effect is probably smaller than the effects of the right of redemption, because the new owner of the property at least has a clean title. In addition, deficiency judgment provides more power to the lender and therefore may lead to a quicker resolution of the foreclosure process, providing less time for the property to deteriorate. Again, this effect should be much less important than the right of redemption in terms of house prices and the recovery from sale.

While these alternative hypotheses make intuitive sense they are not consistently nor uniformly supported by empirical evidence. For example, Wood (1997) finds evidence that Fannie Mae recovery rates are higher in right of redemption states and lower in deficiency judgment states, a counter-intuitive result. Overall, the econometric evidence

[^3]of the relationship between foreclosure laws and recovery on sales is mixed (for example, Crawford and Rosenblatt (1995), Clauretie (1989), Ciochetti (1997), and Clauretie and Herzog (1990)).

Table 2 provides the univariate summary statistics of the recovery rate or loss severity estimation data set, which includes a random sample of Fannie Mae and Freddie Mac foreclosed properties from 1995 through 1999. The mean recovery rate (sale price/outstanding unpaid balance) is 97.88 percent. Therefore, from the sale of the property on average losses on foreclosed loans amounted roughly to 2.12 percent of the outstanding balance. The current loan to value (CLTV) variable dummies indicate that almost three quarters of the foreclosed loans have CLTVs between 70 and 90 percent. In addition, the subprime variable identifies loans that paid interest rates on their mortgages greater than 100 basis points higher than the prevailing contract interest rate. This definition is consistent with Pennington-Cross (2004). Lastly, indicators of the state laws governing foreclosures are included. Just over 40 percent of the loans are in states that require judicial foreclosure, 6.0 percent are in statutory right of redemption states, and 31.0 percent are in deficiency judgment states.

## Recovery Rate Results

The general form of the recovery rate model appears a follows:

$$
\begin{equation*}
\mathrm{R}_{\mathrm{jt}}=\alpha_{\mathrm{o}}+\Sigma_{\mathrm{m}} \alpha_{\mathrm{m}} * \mathrm{Y}_{\mathrm{mjt}}+\mu_{\mathrm{jt}} \tag{10}
\end{equation*}
$$

Where $R$ is the recovery rate, $Y$ represents $m$ explanatory variables, and $j$ and $t$ index time and each foreclosed property. Three specifications are presented. Taking each in turn, specification I is a series of dummy variables that form a piecewise linear decompositon of CLTV (the loan to value ratio at closure of the loan). In this specification, loans with large amounts of equity have much better recovery rates than loans with small amounts of equity. For instance, loans with CLTV less than or equal to 40 percent recover 112.64 percent of the unpaid balance of the loan and loans with CLTV greater than 100 percent only recover 73.32 percent. The CLTVs are then interacted with indicators of whether the loan is a subprime loan. The recovery rate for subprime loans with CLTVs less than or equal to 80 percent is 7.68 percentage points lower than for
prime loans. As the equity position of the borrower deteriorates the difference between the prime and subprime recovery rates shrinks down to 4.36 percentage points for high CLTV categories. These results for specification I have a very high adjusted $\mathrm{R}^{2}$ of 0.9531 . Specifications II and III provide only marginal improvement in explanatory power with adjusted $\mathrm{R}^{2}$ of 0.9539 and 0.9563 .

Specification II uses Specification I as a more restrictive case but it also includes indicators of the types of state level foreclosure laws. To the extent that these indicators affect recovery rates, the estimator of Specification I will be biased. As expected, the requirement of judicial foreclosure proceedings decreases recovery rates by 3.39 percent and the right of deficiency judgment has a 3.73 percent positive impact on recovery rates. In short, state foreclosure laws can have a significant impact on the ability of a lender to recoup losses by selling a foreclosed home. However, a loan in a state that permits the borrower the right of statutory redemption (the right to buy the house back after foreclosure for up to a year) has no relationship to recovery rates.

Specification III introduces the age of the loan in years and years squared as well as the size of the loan and size squared at origination in 10,000 of dollars. Recovery rates reasonably increase at a decreasing rate as the loan lasts longer before foreclosure and peak at the loan age at foreclosure of 5.75 years. The recovery rate also increases at a decreasing rate as loan amount increases and peaks at a loan amount of $\$ 147,083$. While the increase in explanatory power is negligible for specification II and III they do provide intuitive appealing results consist with prior expectations.

## The Simulation Procedure

This section describes the procedure used to simulate credit loss distributions. The procedure applies to newly originated loans and assumes that coefficient estimates, as discussed in the previous section, are known and constant through time.

The simulation starts by randomly drawing dynamic scenarios for metropolitan area house prices and unemployment rates, and interest rates for 5-year time horizons from the
historical distribution of these variables. For each draw, the paths of these variables are interacted with loan specific information such as the loan to value ratio and credit score of the borrower to estimate the probability of default and prepayment in each month and the dollars lost given default.

The first task is to define synthetic portfolio using representative loans derived from a random sample of 30 year fixed rate mortgages. The primary defining loan level information used in the default model is credit score and LTV at origination. Seven categories of LTVs and 12 categories of FICO scores are defined, creating 84 loans to represent the simulated synthetic portfolio. ${ }^{6}$ To determine the weighting or importance of each category the distribution found in the data set of 24,018 prime loans and 65,992 subprime loans, randomly sampled, as defined in Pennington-Cross (2004) from Fannie Mae and Freddie Mac is used. Therefore, the synthetic portfolios will represent the distribution of loan types that these two institutions used when the loans were originated (1995 through 1998) and any change in this pattern or institutions will confound the results.

Once the representative loans are identified the five-year scenario needs to be defined. Each loan is randomly assigned a metropolitan area and an origination date (year and month). The characteristics of this location and time are used to define what conditions the loan will experience for the next five years. These risk factors, as defined by location and time, are generated by Monte-Carlo re-sampling with replacement, for the historical period of 1985 through June of 1997. Therefore, the latest origination date available to the simulation will complete the 5 -year time period in June 2002. As a result, the simulation runs from 1985 through June of 2002. As articulated in Carey (1999) and Calem and LaCour-Little (2003) the re-sampling "historical" approach allows the simulation to avoid the need to estimate in a separate model the variance and correlation structure between all exogenous variables, because these relationships are embedded in

[^4]the experiences of the randomly selected locations and time periods. Further, since location is such an important component to real estate any parametric simulation must also identify and estimate spatial correlations of key drivers of mortgage defaults. Again, these correlations are embedded into the randomly selected locations and time periods used in this paper.

Metropolitan area house prices and their standard errors are collected from the OFHEO repeat sales house price index (HPI). With the LTV at origination, changes in house prices, and the HPI standard errors the probability of the loan being in negative equity can be simulated for each of the 60 months. The unemployment rate is also needed at the metropolitan area level and is collected from the Bureau of Labor and Statistics web site (www.bls.gov). Mortgage interest rates for 30 year fixed rate loans are collected from the Freddie Mac Primary Mortgage Market Survey to create the refi variable to indicate the potential savings from refinancing the loan.

Given the random selection of a time period and location all the information needed to calculate the probability of default is available for the next 60 months. The conditional probability of borrower j prepaying or defaulting on the mortgage in period $t$ is modeled by:

$$
\begin{equation*}
\pi\left(T_{w}=t \mid x_{j t}, \theta_{w}, T>t-1\right)=1-\exp \left(-\exp \left(x_{j t} \beta_{w}\right) \theta_{w}\right) \tag{11}
\end{equation*}
$$

where $x_{j t}$ represents the matrix of regressors, $\theta_{w}$ is the location parameters which represent the central tendency of the likelihood function, and $\beta$ is a vector of parameters measuring the effects of the $x_{j t}$ on the probability $(\pi)$ of $w(w=$ default, prepay, or continue) occurring.

To transform conditional probabilities into cumulative probabilities and to calculate the remaining unpaid balance at risk in each month as well as the dollars defaulted, a simple and mechanical process is used. All loan amounts are normalized as 100-dollar loans. The original loan amount to borrower j is $u p b_{j 0}$ and over the next month $p_{j 0}\left(u p b_{j 0}\right)$ dollars of principal is paid, the probability of default is $\pi_{j 0}$, and the probability of prepay is $\pi_{j p 0}$.

The unpaid principal balance at the end of the month is defined as $u p b_{j 1}=\left(1-\pi_{j d 0^{-}}\right.$ $\left.\pi_{j p o}\right) u p b_{j 0}-p_{j 0}\left(u p b_{j 0}\right)$ and the defaulted amount is $d e f_{j 0}=\pi_{j d 0} * u p b_{j 0}$. In the next period the same process is repeat but with everything pushed forward one period, so that $u p b_{j 2}=(1-$ $\left.\pi_{j d 1}-\pi_{j p 1}\right) u p b_{j 1}-p_{j 1}\left(u p b_{j 1}\right)$ and the defaulted amount is $d e f_{j 1}=\pi_{j d 1} * u p b_{j 1}$ and, in general, in time period $\mathrm{t} u p b_{j t}=\left(1-\pi_{j d t-1}-\pi_{j p t-1}\right) u p b_{j t-1}-p_{j t-1}\left(u p b_{j t-1}\right)$ and $d e f_{j t}=\pi_{j d t} * u p b_{j t}$. Following this process a vector of 60 defaulted dollar amounts is created for each of the 82 representative loan/location/time combinations.

Once $d e f_{j t}$ is determined these dollars need to be translated into losses. Using the loss given default model specification I the loss severity rate is estimated from 1 minus the recovery rate or $\operatorname{serv}_{j t}$ and is defined as the percent of the outstanding balance $\left(u p b_{j t}\right)$ that is lost after the home is sold by the lender. The dollar amount lost from the sale of the property is, therefore, defined as Loss $\operatorname{Sale}_{j t}=\left(\operatorname{serv}_{j t} / 100\right)^{*} u p b_{j t}$. Following Calem and LaCour-Little (2003) and OFHEO (1999), and consistent with Wilson (1995), it is assumed that it costs 10 percent of $d e f_{j t}$ to dispose of the property and 5 percent of $d e f_{j t}$ for foreclosure transaction costs. The property disposal costs are assumed to be paid when the property is sold (disposed) and the foreclosure transaction costs are assumed to occur when the loan is foreclosed and defaulted. Lastly, the opportunity cost of funding the loan while it is delinquent (lost interest payments from the last payment date until foreclosure) is also included. The simulation assumes that the loan is 3 months delinquent at the date of default (no payments in the last 3 months until default) and an additional 2 months is needed to dispose of the property. The Freddie Mac PMMS is used to generate lost interest payments or funding cost for this 5 month time period. For most of the simulations, results are presented with and without the use of PMI. PMI is a form of risk sharing for loans with original LTVs greater than 80 percent. Specifically, losses are fully reimbursed for all losses up to a maximum of 20 percent of the unpaid principal balance for original LTVs up to 90 percent and greater than 80 percent. If the original LTV is greater than 90 percent full reimbursement is capped at 25 percent of the original unpaid balance. All losses are discounted by the one-year constant maturity Treasury Bill rate as reported by the St. Louis Federal Reserve is used.

This process is repeated for all 82 loans in their random location and 5 year time period. Total discounted loan losses are then calculated by summing across time individually for each loan and then across each loan type, so that for the synthetic portfolio as a whole Losses $=\Sigma_{\mathrm{j}} w_{j} \Sigma_{\mathrm{t}} \delta_{t}$ Loss $_{j t}$, where $w_{j}$ is the weighted importance of representative loan type j and the sum of all weights $=1, \delta$ is the discounting term for each time period t and $\operatorname{Loss}_{j t}$ is defined as the total losses associated with the $\operatorname{de} f_{j t}$ amount, where j indexes the 82 loan types, and t indexes the 60 time periods for the full 5 year period. The cumulative 5 year loss rate is then defined as discounted losses (Losses) divided by total loan amount or synthetic portfolio size at origination (Loss Rate $=$ Losses $\left./ \Sigma_{j} u p b_{j 0}\right)$. This provides the estimated losses for one draw. The simulation conducts 5,000 draws in order to represent the shape of the distribution of credit losses (probability density function) for the prime and subprime synthetic portfolios.

## The Simulation Results

## Loss Rates

Figure 1 presents the simulated distribution of loss rates for the prime and subprime synthetic portfolios. The dashed lines represent the distribution without the use of PMI, the solid lines represent the distributions of loss rates when PMI was used, and the lines with x's are the subprime distributions. The distributions are probability density functions, so that the integral over the whole distribution equals 100 and the $y$-axis can be interpreted as the percent of all draws from the simulation that had that loss rate. The xaxis represents the loss rates as discussed in the previous section.

There are at least three striking results: (1) the synthetic subprime portfolio experiences much higher losses than the synthetic prime portfolio on average; (2) all distributions have fat tails to the right; (3) the introduction of PMI reduces average losses and the variability of losses for both the synthetic subprime and prime portfolio. In fact, as shown in table 4, the mean expected loss rate for the synthetic prime portfolio drops from 0.26 percent to 0.04 percent when PMI is introduced and for the synthetic subprime portfolio the mean expected loss rate drops from 1.57 percent to 0.26 percent. Table 5 provides estimates of the expected loss rates at different points on the distributions. The
expected loss rates reported at each point on the distribution $(5,25,50, \ldots 100$ percent) indicate the maximum loss rate that can be expected 5 percent of the time, 25 percent of the time, 50 percent of the time, $\ldots$, and 100 percent of the time. For instance, for the synthetic prime portfolio without PMI 5 percent of the distribution has loss rates up to 0.17 percent or conversely 95 percent of the distribution has loss rates greater than 0.17 percent. In contrast, for the synthetic subprime portfolio without PMI the loss rate at the 5 percent point on the distribution is 1.16 percent. The maximum loss rates are 0.88 percent and 3.48 percent for the synthetic prime and subprime portfolios without PMI. In contrast, the maximum simulated synthetic portfolio loss rate with PMI is 0.18 percent for prime and 1.03 percent for subprime. For a given portfolio size it is then possible to calculate how much capital is required to meet expected losses. For instance, from the 5000 draws in the simulation a 100 dollar portfolio has an average loss rate for the synthetic prime portfolio with PMI of 0.04 percent so 4 cents is the average expected loss. In contrast, for 95 percent of the draws 6 cents will cover expected losses and for 99 percent of the draws 11 cents will cover expected losses.

## Economic Capital

To estimate how much economic capital is required it is necessary to know the risk tolerance of the firm and the average expected loss. Table 4 provides the estimated average expected loss as discussed above. It is possible to use default rates on bonds issued by firms to identify acceptable insolvency rates for various types of firms. For example, Calem and LaCour-Little (2003) calculate that for bonds rated BBB by Standard and Poor's (S\&P) the five year cumulative default rate is 1.65 percent and for the S\&P A- rated bonds the five year cumulative default rate is 0.70 percent. These default rates can be interpreted as the probability standard for an A- or a BBB rated firm to remain solvent and can be used to calculate the risk and loss rate tolerances of A- and BBB rated firms. For example, using the simulation results for the synthetic prime portfolio with PMI the expected loss rate is up to 0.09 percent for 98.35 of the simulated draws and up to 0.12 percent for 99.3 percent of the simulated draws. As a result for a 100 dollar portfolio 9 cents of capital and 12 cents of capital will cover losses at the BBB and A- risk tolerances. For the synthetic subprime portfolio with PMI 50 cents for BBB
and 65 cents for A- tolerances will cover losses. Since the subprime market typically does not use private mortgage insurance as a risk sharing mechanism, it is also important to note that the requirements rise to 92 cents for BBB risk tolerances and one dollar and 14 cents for A- risk tolerances.

The firm is assumed to set yields on the performing assets to cover all mean expected losses. To calculate economic capital mean expected losses are subtracted from losses associated with a particular probability threshold. For instance, for a synthetic portfolio of subprime loans without PMI the mean expected loss rate is 1.57 percent and the loss rate tolerance for a BBB rated subprime lender is 2.49 percent. Given the assumption of uniform loan sizes in the portfolio, a 100 dollar portfolio for a BBB rated firm implies an economic capital requirement of 0.92 percent. For an A- rated firm the economic capital requirement increases to 1.14 percent. For the synthetic prime portfolio the economic capital requirements are always lower. For instance, the synthetic prime portfolio with PMI has the required economic capital of 0.05 and 0.08 percent for the BBB and A- rated firms respectively. All of these requirements are substantially below the typical capital requirements of 2 percent for prime loans and 4 percent for subprime loans.

These results must be viewed with caution because they represent the portfolio of two large and unique financial institutions (Fannie Mae and Freddie Mac) created from a random sample of 30 year fixed rate loans. Therefore, the results do not represent the mortgage market as a whole, but instead these two institutions' involvement in it. In particular, it is expected that other institutions are involved in riskier segments of both the prime and subprime market. ${ }^{7}$ In addition, the time period that the coefficients that are used to simulate the expected default probabilities and expected losses come from a prosperous time period during the end of the 1990s. Lastly, the period of the simulation does not cover any severe national recessions.

[^5]
## Subprime Risk Categories

The subprime mortgage market is segmented into separate risk classifications, which are typically referred to by a letter such as A-, B, C, or D. The IndyMac bank web site provides definitions of risk classifications along with the interest rate premium borrowers will pay for each loan type. In general, the worse a borrower's credit score the higher the interest rate. For instance, table 6 shows that the B+ loan type allows credit scores as low as 600 with an interest rate premium of 1.875 over the prime rate ( 5.875 on $11 / 19 / 02$ ), while the B loan type allows credit scores down to 575, but the interest rate premium increases to 2.25 points. This pattern continues until the D loan type allows credit scores down to 500 and the interest rate premium is 5.125 points. Borrowers with worse credit history rate are required to provide a larger down payment. For instance, B+ loans require at least a 5 percent down payment, B loans require at least a 10 percent down payment, and D loans require at least a 30 percent down payment.

The simulation procedure is run to create expected losses and the distribution of these losses for each loan type. Information is not publicly available on the distribution of LTV and FICO for each category so all loans have identical down payments and identical credit scores in the simulation. This will artificially reduce the variation in losses in a synthetic portfolio because some of the variation in losses is derived from the heterogeneous nature of the loans and their underwriting characteristics. Therefore, estimates of capital requirements should only be used for comparative purposes across loan types. In addition, a prime loan type is added for better comparison of prime loans. In this section prime loans are defined as having 720 credit scores and a 20 percent down payment, which is approximately the mean for the synthetic prime portfolio discussed in the previous section.

Table 7 provides the simulated mean expected loss rates for each of the loan types without PMI. The mean expected loss rate is much lower for prime than it was for the synthetic portfolio of prime loans, indicating that the average looking loan does not represent the average experience of a true portfolio of loans. Among subprime loans, the D type loans have the highest mean expected loan loss rate of 4.86 percent while the $B$
rated loans have the lowest with a mean expected loss rate of 3.37 percent. In general, the results are consistent with the premium paid by borrower. Interestingly, B rated loans have a slightly lower mean expected loss rate than $\mathrm{B}+$ rated loans. This may be an indication that the increase in the required down payment reduced losses more than the decrease in credit history quality increased losses.

Figure 2 plots the 5-year cumulative loss rate distribution for each loan type. Again caution must be used when interpreting these results because they do not reflect a portfolio but rather a single representative loan. The dramatically higher expected loss rates of the riskiest portions of the subprime market is reinforced as well as the separation of loss rates between prime type loans and subprime type loans.

## Conclusion

Following the nonparametric simulation procedure of Carey (1998 and 2002) and Calem and LaCour-Little (2003) this paper estimates distributions of credit losses associated with a representative synthetic portfolio of prime and subprime loans from Fannie Mae and Freddie Mac. These distributions are derived from a model of the competing risks of default and prepayment in a proportional hazards framework by Pennington-Cross (2004) and a model of loss severity. The estimated loss severity model shows that the amount recovered from the sale of the foreclosed property is sensitive to the loan to value ratio at foreclosure, state level foreclosure laws, and whether the loan is prime or subprime.

If no PMI is used to mitigate loss exposure, the typical (median) loss on the synthetic subprime portfolio is approximately 5.23 times higher than the synthetic prime portfolio. The results also show that risk-sharing agreements based on down payments requirements can substantially reduce expected losses. For instance, the simulations show that the introduction of PMI reduces median losses for synthetic prime and subprime portfolios by over 85 percent. Estimates of economic capital requirements are much lower than current regulatory requirements, which may help to explain the high rate of securitization of prime and subprime mortgages. Lastly, an analysis using a representative loan type simulation revealed that expected mean loan losses generally
follows pricing patterns in the subprime market, despite the fact that the highest risk classification loans require larger down payments to mitigate the higher default rates.

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Table 1: Competing Risk Proportional Hazard Coefficient Estimates

|  | Variable | Prime |  | Subprime |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Coeff | T stat | Coeff | T stat |
| Default |  |  |  |  |  |
|  | $\mathrm{FICO}_{j}$ | -1.806 | -15.86 | -1.476 | -36.30 |
|  | $\mathrm{pneq}_{\mathrm{jt}}$ | 0.447 | 5.53 | 0.288 | 10.99 |
|  | $\mathrm{refi}_{\mathrm{j} t}$ | 0.018 | 0.90 | 0.017 | 2.59 |
|  | refi ${ }^{*}(\mathrm{refi}<0)_{\text {jt }}$ | 0.038 | 0.66 | -0.021 | -1.32 |
|  | $u^{\text {rate }}{ }_{j t}$ | 0.108 | 3.82 | 0.070 | 6.82 |
|  | $\mathrm{age}_{\mathrm{jt}}$ | -0.080 | -3.38 | -0.005 | -0.68 |
|  | age ${ }_{\text {jt }}$ | 0.111 | 2.11 | 0.003 | 0.17 |
|  | $\theta_{\text {dj }}$ | 0.00016 | 4.90 | 0.0006 | 12.34 |
| Prepay |  |  |  |  |  |
|  | $\mathrm{FICO}_{\mathrm{j}}$ | 0.090 | 4.92 | 0.316 | 25.96 |
|  | $\mathrm{pneq}_{\mathrm{j} t}$ | -0.039 | -4.28 | -0.090 | -13.86 |
|  | $\mathrm{refi}_{\mathrm{j} t}$ | 0.138 | 55.77 | 0.075 | 40.46 |
|  | refi* ${ }^{*}$ (refi $\left.<0\right)_{\text {jt }}$ | -0.081 | -7.83 | 0.025 | 3.91 |
|  | urate $_{\text {jt }}$ | -0.079 | -13.61 | -0.098 | -23.64 |
|  | $\mathrm{age}_{\mathrm{jt}}$ | 0.082 | 22.36 | 0.067 | 28.46 |
|  | age ${ }_{\text {j }}$ t | -0.136 | -18.41 | -0.103 | -22.26 |
|  | $\theta_{\text {pi }}$ | 0.00353 | 24.62 | 0.0085 | 40.28 |

Source Pennington-Cross (2004) tables 2 and 3 with FICO column.

Table 2: Recovery On Sale Summary Statistics

| Variable | Mean | Std Dev | Minimum | Maximum |
| :--- | ---: | :---: | ---: | ---: |
| Recover Rate | 97.884 | 22.755 | 0.001 | 278.004 |
| Age $_{\text {Age }^{2}}$ Loan Amount | 2.519 | 0.886 | 0.333 | 4.750 |
| Loan Amount $^{2}$ | 7.129 | 4.578 | 0.111 | 22.563 |
| CLTV<=40 | 9.768 | 4.354 | 1.610 | 30.473 |
| 40<CLTV <=60 | 114.378 | 97.783 | 2.592 | 928.573 |
| 60<CLTV <=70 | 0.001 | 0.025 | 0 | 1 |
| $70<$ CLTV <=80 | 0.017 | 0.128 | 0 | 1 |
| $80<$ CLTV <=85 | 0.108 | 0.310 | 0 | 1 |
| $85<$ CLTV <=90 | 0.255 | 0.436 | 0 | 1 |
| $90<$ CLTV <=95 | 0.241 | 0.428 | 0 | 1 |
| $95<$ CLTV <=100 | 0.241 | 0.428 | 0 | 1 |
| CLTV >100 | 0.113 | 0.317 | 0 | 1 |
| CLTV *subprime<=80 | 0.022 | 0.147 | 0 | 1 |
| $80<C L T V ~ * s u b p r i m e ~<=90 ~$ | 0.003 | 0.054 | 0 | 1 |
| CLTV *subprime >90 | 0.1061 | 0.239 | 0 | 1 |
| Judicial | 0.037 | 0.308 | 0.188 | 0 |
| SRR | 0.401 | 0.490 | 0 | 1 |
| DJ | 0.060 | 0.238 | 0 | 1 |
| Number of Observations | 16,272 |  | 0 | 1 |
| Source OFHEO Sam | 0.310 | 0.462 | 0 | 1 |

Source OFHEO. Sample of 1995 through 1999 foreclosures. Recovery rate is defined as the percent of the outstanding balance recovered from the sale of the property, Age is defined in years and represented the years that the loan lasted before it foreclosed, Loan Amount is in 10,000 of dollars and represents the original loan amount, CLTV is the current loan to value ratio of the loan at foreclosure, subprime indicates when the difference between the contract interest rate at origination is greater than 100 basis points, Judicial indicates when a loan is located in a state that requires a judicial foreclosure process, SRR indicates when a loan exists in a state that allows a statutory right of redemption for the borrower for up to a year, and DJ indicates when the loan exists in a state that allows creditors to collect foreclosure losses through a deficiency judgment against other borrower assets.

Table 3: Recovery On Sale Estimates

| Variable | Spec. I |  | Spec. II |  | Spec. III |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff | T-Stat | Coeff | T-Stat | Coeff | T-Stat |
| CLTV<=40 | 112.64 | 16.35 | 112.96 | 16.51 | 88.72 | 13.04 |
| $40<$ CLTV < $=60$ | 117.43 | 88.89 | 117.47 | 87.26 | 93.11 | 47.88 |
| $60<C L T V$ <=70 | 107.45 | 202.53 | 107.45 | 175.88 | 80.56 | 50.49 |
| $70<C L T V$ <=80 | 103.04 | 285.17 | 102.93 | 224.33 | 75.16 | 48.73 |
| $80<C L T V$ <=85 | 99.91 | 270.09 | 100.39 | 217.34 | 72.44 | 46.01 |
| $85<$ CLTV <=90 | 95.50 | 255.80 | 96.17 | 208.15 | 69.20 | 45.23 |
| 90<CLTV <=95 | 89.02 | 153.20 | 88.60 | 139.95 | 62.87 | 42.53 |
| $95<C L T V$ <=100 | 86.62 | 73.64 | 83.99 | 68.77 | 58.41 | 33.50 |
| CLTV >100 | 73.32 | 23.27 | 75.86 | 24.09 | 48.31 | 14.33 |
| CLTV * subprime <=80 | -7.68 | -10.17 | -7.21 | -9.62 | -5.92 | -8.08 |
| $80<$ CLTV * subprime <=90 | -6.07 | -10.24 | -5.70 | -9.70 | -4.18 | -7.27 |
| CLTV * subprime >90 | -4.36 | -4.19 | -3.84 | -3.72 | -2.69 | -2.67 |
| Judicial |  |  | -3.39 | -8.03 | -3.97 | -9.58 |
| SRR |  |  | -0.56 | -0.77 | 0.63 | 0.88 |
| DJ |  |  | 3.73 | 8.33 | 0.80 | 1.78 |
| Age |  |  |  |  | 3.68 | 3.69 |
| Age ${ }^{2}$ |  |  |  |  | -0.32 | -1.72 |
| Loan Amount |  |  |  |  | 3.53 | 20.63 |
| Loan Amount ${ }^{2}$ |  |  |  |  | -0.12 | -15.52 |
| Adjusted R ${ }^{2}$ | 0.9531 |  | 0.9539 |  | 0.9563 |  |

Table 4: Mean Loss Rates

|  | With PMI | Without PMI |
| :---: | :---: | :---: |
| Prime | $0.04 \%$ | $0.26 \%$ |
| Subprime | $0.19 \%$ | $1.57 \%$ |

Table 5: Portfolio Losses \& Economic Capital

|  | Loss Rate |  | Economic Capital |  |
| :---: | :---: | :---: | :---: | :---: |
| Distribution Percentile | Prime | Subprime | Prime | Subprime |
| Without PMI |  |  |  |  |
| 5 | $0.17 \%$ | $1.16 \%$ |  |  |
| 25 | $0.21 \%$ | $1.34 \%$ |  |  |
| 50 | $0.24 \%$ | $1.50 \%$ |  |  |
| 75 | $0.29 \%$ | $1.73 \%$ | $0.03 \%$ | $0.16 \%$ |
| 95 | $0.41 \%$ | $2.21 \%$ | $0.15 \%$ | $0.64 \%$ |
| 99 | $0.55 \%$ | $2.61 \%$ | $0.29 \%$ | $1.04 \%$ |
| 100 | $0.88 \%$ | $3.48 \%$ | $0.62 \%$ | $1.91 \%$ |
| BBB (98.35) | $0.51 \%$ | $2.49 \%$ | $0.24 \%$ | $0.92 \%$ |
| A- (99.3) | $0.57 \%$ | $2.71 \%$ | $0.31 \%$ | $1.14 \%$ |
| With PMI |  |  |  |  |
| 5 | $0.03 \%$ | $0.11 \%$ |  |  |
| 25 | $0.03 \%$ | $0.13 \%$ |  |  |
| 50 | $0.03 \%$ | $0.15 \%$ |  |  |
| 75 | $0.04 \%$ | $0.21 \%$ | $0.00 \%$ | $0.02 \%$ |
| 95 | $0.06 \%$ | $0.37 \%$ | $0.02 \%$ | $0.19 \%$ |
| 99 | $0.11 \%$ | $0.62 \%$ | $0.07 \%$ | $0.43 \%$ |
| 100 | $0.18 \%$ | $1.03 \%$ | $0.15 \%$ | $0.84 \%$ |
| BBB (98.35) | $0.09 \%$ | $0.50 \%$ | $0.05 \%$ | $0.32 \%$ |
| A- (99.3) | $0.12 \%$ | $0.65 \%$ | $0.08 \%$ | $0.47 \%$ |

Table 6: Subprime Loan Type Definitions

| Loan Type | LTV | FICO | Interest Rate <br> Premium |
| :---: | :---: | :---: | :---: |
| B+ | $95 \%$ | 600 | 1.875 |
| B | $90 \%$ | 575 | 2.25 |
| C+ | $85 \%$ | 550 | 2.75 |
| C | $75 \%$ | 525 | 3.875 |
| D | $70 \%$ | 500 | 5.125 |
| Prime | $80 \%$ | 720 | 0 |

$\overline{\text { Prime Mortgage Rate }=5.875 \text {, Source: IndyMac Bank 11/19/02 posted rates }}$

Table 7: Representative Loan Type Mean Loss Rates Without PMI

|  |  |
| :---: | :---: |
| Loan Type | Mean Loss Rate |
| B+ | $3.55 \%$ |
| B | $3.37 \%$ |
| C+ | $3.70 \%$ |
| C | $4.08 \%$ |
| D | $4.86 \%$ |
| Prime | $0.05 \%$ |

Figure 1


Figure 2


## Appendix: Results Using Alternative Recovery On Sale Specifications

Specification II
Table A1: Mean Loss Rates Specification II

|  | With PMI | Without PMI |
| :---: | :---: | :---: |
| Prime | $0.04 \%$ | $0.26 \%$ |
| Subprime | $0.18 \%$ | $1.58 \%$ |

Table A2: Portfolio Losses \& Economic Capital Specification II

|  | Loss Rate |  | Economic Capital |  |
| :---: | :---: | :---: | :---: | :---: |
| Distribution Percentile | Prime | Subprime | Prime | Subprime |
| Without PMI |  |  |  |  |
| 5 | $0.18 \%$ | $1.19 \%$ |  |  |
| 25 | $0.21 \%$ | $1.37 \%$ |  |  |
| 50 | $0.25 \%$ | $1.52 \%$ |  |  |
| 75 | $0.30 \%$ | $1.73 \%$ | $0.03 \%$ | $0.15 \%$ |
| 95 | $0.41 \%$ | $2.15 \%$ | $0.14 \%$ | $0.57 \%$ |
| 99 | $0.51 \%$ | $2.52 \%$ | $0.24 \%$ | $0.94 \%$ |
| 100 | $0.71 \%$ | $3.14 \%$ | $0.45 \%$ | $1.57 \%$ |
| BBB (98.35) | $0.48 \%$ | $2.40 \%$ | $0.22 \%$ | $0.82 \%$ |
| A- (99.3) | $0.53 \%$ | $2.59 \%$ | $0.27 \%$ | $1.01 \%$ |
| With PMI |  |  |  |  |
| 5 | $0.03 \%$ | $0.11 \%$ |  |  |
| 25 | $0.03 \%$ | $0.13 \%$ |  |  |
| 50 | $0.03 \%$ | $0.15 \%$ |  |  |
| 75 | $0.04 \%$ | $0.20 \%$ | $0.00 \%$ | $0.02 \%$ |
| 95 | $0.06 \%$ | $0.33 \%$ | $0.02 \%$ | $0.15 \%$ |
| 99 | $0.10 \%$ | $0.51 \%$ | $0.06 \%$ | $0.33 \%$ |
| 100 | $0.16 \%$ | $0.80 \%$ | $0.12 \%$ | $0.62 \%$ |
| BBB (98.35) | $0.08 \%$ | $0.43 \%$ | $0.05 \%$ | $0.26 \%$ |
| A- (99.3) | $0.11 \%$ | $0.56 \%$ | $0.08 \%$ | $0.38 \%$ |

Figure A1
Cumulative 5-Year Loss Rate Distribution Specification II


Specification III
Table A3: Mean Loss Rates Specification III

|  | With PMI | Without PMI |
| :---: | :---: | :---: |
| Prime | $0.03 \%$ | $0.25 \%$ |
| Subprime | $0.17 \%$ | $1.49 \%$ |

Table A4: Portfolio Losses \& Economic Capital Specification III

|  | Loss Rate |  | Economic Capital |  |
| :---: | :---: | :---: | :---: | :---: |
| Distribution Percentile | Prime | Subprime | Prime | Subprime |
| Without PMI |  |  |  |  |
| 5 | $0.17 \%$ | $1.12 \%$ |  |  |
| 25 | $0.20 \%$ | $1.28 \%$ |  |  |
| 50 | $0.23 \%$ | $1.44 \%$ |  |  |
| 75 | $0.28 \%$ | $1.64 \%$ | $0.03 \%$ | $0.15 \%$ |
| 95 | $0.38 \%$ | $2.02 \%$ | $0.13 \%$ | $0.53 \%$ |
| 99 | $0.50 \%$ | $2.40 \%$ | $0.25 \%$ | $0.92 \%$ |
| 100 | $0.75 \%$ | $3.19 \%$ | $0.50 \%$ | $1.70 \%$ |
| BBB (98.35) | $0.47 \%$ | $2.30 \%$ | $0.22 \%$ | $0.81 \%$ |
| A- (99.3) | $0.52 \%$ | $2.47 \%$ | $0.27 \%$ | $0.98 \%$ |
| With PMI |  |  |  |  |
| 5 | $0.02 \%$ | $0.11 \%$ |  |  |
| 25 | $0.03 \%$ | $0.13 \%$ |  |  |
| 50 | $0.03 \%$ | $0.15 \%$ |  |  |
| 75 | $0.03 \%$ | $0.19 \%$ | $0.00 \%$ | $0.02 \%$ |
| 95 | $0.05 \%$ | $0.31 \%$ | $0.02 \%$ | $0.14 \%$ |
| 99 | $0.09 \%$ | $0.44 \%$ | $0.05 \%$ | $0.27 \%$ |
| 100 | $0.17 \%$ | $0.89 \%$ | $0.14 \%$ | $0.72 \%$ |
| BBB (98.35) | $0.07 \%$ | $0.40 \%$ | $0.04 \%$ | $0.23 \%$ |
| A- (99.3) | $0.10 \%$ | $0.52 \%$ | $0.07 \%$ | $0.35 \%$ |

Figure $\mathbf{A} 2$
Cumulative 5-Year Loss Rate Distribution Specification III



[^0]:    ${ }^{1}$ Crouhy, Galai, and Mark (2000) and Gordy (2000) discuss thes types of models.
    ${ }^{2}$ Loan Performance Inc. has developed a proprietary VAR approach. See The Market Pulse 2000, volume 8, number 3 for an introduction (downloaded from www.loanperformance.com). In addition, Quigley and Van Order (1991) and Calem and LaCour-Little (2001) conduct a study in the same spirit as this paper on mortgage risk.

[^1]:    3 To determine how loan performance is affected by the equity in the home, the loan to value ratio $\left(l t v_{j}\right)$ is updated in each month to estimate the current loan to value $\left(l t v c_{j t} t\right)$. To calculate $l t v c_{j t}$ the outstanding balance of the mortgage and the value or current price of the house must be updated through time. The unpaid balance of the mortgage is calculated assuming that payments are received on time, and the house price is updated using the Office of Federal Housing and Enterprise Oversight (OFHEO) repeat sales price index at the metropolitan area level. But since the actual value of the home is estimated, not observed, it may be more accurate to estimate the probability that the household is in negative equity. Following Deng, Quigley, and Van Order (2000) the standard error (se) estimates reported by OFHEO, which are derived from the repeat sales house price index estimation procedure and the cumulative normal density function $(\Phi)$, can be used to calculate the probability that the house has more debt than value -- the probability of negative equity $\left(p n e q_{j t}=\Phi\left(l t v c_{j t} / s e_{s-t}\right)\right.$ ). The standard error estimates depend on how long ago the home was purchased. Let $s$ index the current date and $t$ the date of the transaction so that $s$ - $t$ is the time since the transaction. In general the larger $s-t$ the higher the estimated standard error from the house price index estimation procedure. Therefore, pneq $_{j t}$ is sensitive to changes in house prices, mortgage payments, and the standard errors.
    ${ }^{4}$ To determine if it makes sense for the borrower to refinance a mortgage, the present discounted cost $(P D C)$ of all future payments for the current mortgage is compared to the PDC of all future payments if the borrower refinances. Ignoring transaction costs, if the cost of refinancing is lower than the cost of continuing to pay then the option to refinance or prepay is 'in the money.' To address the refinance option assume that the borrower can obtain a loan for the remaining term of the original loan, but does so at current market interest rates. The discounted term is assumed to be the 10 -year constant maturity Treasury bill reported for each month. For fixed rate mortgages given the original balance $(O)$, the term of the mortgage $(T M)$, and the interest rate on the mortgage $(i)$ can be use to calculate the monthly payments, $P_{j}=i_{j} * O\left[\frac{\left(1+i_{j}\right)^{T M}}{\left(1+i_{j}\right)^{T M}-1}\right]$. The monthly payments $\left(\mathrm{P}_{\mathrm{j}}\right)$ are constant through the life of the loan and are discounted by d in each month (m) until the mortgage is fully paid in TM months, $P D C_{j c}=\sum_{m=0}^{T M} \frac{P_{j}}{\left(1+d_{j}\right)^{m}}$. The $P D C_{j c}$ is then recalculated for each month for each borrower for as long as the loan exists. This process is then repeated for the refinanced mortgage to calculate $P D C_{j r}$ except that

[^2]:    the unpaid balance of the current mortgage becomes the original balance in equation 5 , the term of the loan ( $T M$ ) is the remaining term of the original loan, and the interest rate on the refinanced mortgage is the market rate as defined by the Freddie Mac Primary Mortgage Market Survey in that month. The call option is defined as $r e f i_{j t}=\left[\frac{\left(P D C_{j c}-P D C_{j r}\right)}{P D C_{j c}}\right]$. The variable $r e f f_{j t}$ is defined as the percentage reduction in the present value of future payments the borrower, $\mathfrak{j}$, will gain in time period t if the mortgage is refinanced. This specification of the call option is likely to be a good representation of how much the option to prepay is in the money for prime loans.

[^3]:    ${ }^{5}$ Note that while the redemption option exists in nine states, it is rarely exercised by the household (Capone 1996).

[^4]:    ${ }^{6}$ Mortgages tend to cluster near underwriting requirements thus making it unnecessary to have a large number of representative loans. In addition, the law of large numbers allows for the diversification of idiosyncratic risk in sufficiently large portfolios. See Gordy (1999) for a discussion as applied to conditional loss rates and portfolio loss distributions.

[^5]:    ${ }^{7}$ Alternative specifications of the loss severity model revealed very similar results. In particular, using coefficient estimates from Quigley and Van Order (1995), which rely on LTV at origination and the age of the mortgage, estimated mean loss rates for prime loans were 0.10 and 0.48 percent with and without PMI. In addition, see the appendix for tables and figures using specifications II and II of the loss given default estimation.

