NOTES AND CORRESPONDENCE

Comments on ''Moist Convective Velocity and Buoyancy Scales''

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Recently, both Rennó and Ingersoll (1996, hereafter RI) and Emanuel and Bister (1996, hereafter EB) derive theoretical formulas for the convective available potential energy (CAPE), the fractional area covered by active updrafts, and the vertical velocity in the active updrafts for atmospheres in radiative–convective equilibrium. The theories of RI and EB differ primarily in the means of closure for the mass flux of cumulus convection. Emanuel and Bister state that ''the predictions of our theory differ considerably from those of Rennó and Ingersoll; for example, the buoyancy and velocity scales are independent of the magnitude of the radiative forcing.'' The numerical experiments EB present support this notion that CAPE is relatively insensitive to the magnitude of the radiative forcing of the atmosphere. The quote above and the inference of EB's paper is that the RI theory for CAPE will not satisfy these numerical experiments and therefore is invalid. In this comment, we present an alternative formula for CAPE using the same closure assumptions as RI. Furthermore, we will show that the predictions of CAPE by RI *are not inconsistent* with the numerical experiments presented in EB. Thus we will conclude it is inappropriate for EB to dismiss the theory of RI based upon the evidence presented by EB *to date.*

Rennó and Ingersoll derive the following expression for the CAPE of an atmosphere in radiative–convective equilibrium:

$$\mathrm{CAPE}_{\mathrm{RI}} = \frac{\eta F_{\mathrm{in}}}{2M}, \qquad (1)$$

where M is the mean cumulus mass flux, η is the efficiency of the heat engine, and F_{in} is the heat input at the surface [RI Eq. (20)]. The factor of 2 arises because RI state that to first order negatively buoyant saturated downdrafts contribute an approximately equal quantity to the buoyancy flux that positively buoyant saturated updrafts contribute to the buoyancy flux. The heat input at the surface, F_{in}, is the sum of the latent, sensible, and net longwave radiative heat flux at the surface [RI Eq. (19)]. The efficiency of the heat engine η can be approximated as the difference between the surface temperature T_{sfc} and the temperature at the level the atmosphere is effectively radiatively cooled to space, $\overline{T}$, divided by the surface temperature:

$$\eta \sim \frac{T_{\mathrm{sfc}} - \overline{T}}{T_{\mathrm{sfc}}}. \qquad (2)$$

As their closure for the cumulus mass flux, RI approximate M by the radiatively determined subsidence mass flux M_{rad}. Rennó and Ingersoll use graybody radiative transfer theory to deduce M_{rad}. Here we present an alternative derivation following the concepts of Betts and Ridgway (1988) and Sarachik (1978). Our closure assumption is the same as RI, namely that $M = M_{\mathrm{rad}}$. We differ from RI only in that we do not express M_{rad} in terms of graybody radiative transfer theory. For an atmosphere in radiative equilibrium, the subsidence warming must balance the radiative cooling of the atmosphere (Betts and Ridgway 1988):

$$M_{\mathrm{rad}}(s_t - s_b) = -\overline{Q}_A, \qquad (3)$$

where $-\overline{Q}_A$ (>0) is the radiative cooling of the atmosphere (in W m^{-2}), s_t is the dry static energy at the tropopause, and s_b is the dry static energy in the subcloud layer (see Fig. 1 for an explanation of some of the thermodynamic symbols using a mean tropical sounding from the TOGA COARE field experiment). For an atmosphere in radiative convective equilibrium, the radiative cooling of the atmosphere balances the *nonradiative* heat input from the surface, F^*_{in}:

$$-\overline{Q}_A = F^*_{\mathrm{in}}. \qquad (4)$$

Strictly speaking, the heating due to the dissipation of kinetic energy should be included in (4). However, this dissipation term scales as ηF_{in}, which is small in comparison to F^*_{in}, provided η is much less than unity. Fur-

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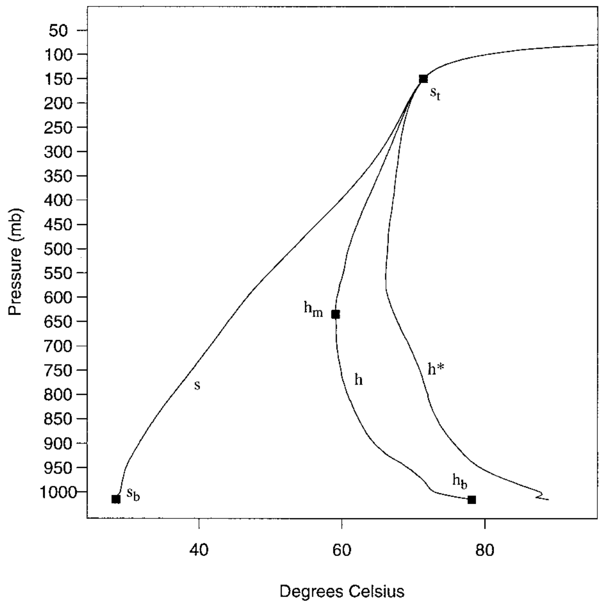

FIG. 1. Mean atmospheric sounding from the TOGA COARE IOP. The sounding shows the vertical profile of dry static energy (s), moist static energy (h), and saturation moist static energy (h^*) divided by the specific heat at constant pressure (c_p). Data courteously provided by R. Brown of the University of Washington.

thermore, for a temperature profile that roughly follows the moist adiabat of near-surface air,

$$s_t - s_b = Lq_b, \tag{5}$$

where q_b is the specific humidity of subcloud layer air and L is the latent heat of vaporization (Sarachik 1978). Substitution of (3), (4), and (5) into (1) leads to

$$\mathrm{CAPE_{RI}} \approx \frac{\eta L q_b}{2} \cdot \left(\frac{F_{\mathrm{in}}}{F^*_{\mathrm{in}}}\right). \tag{6}$$

The net longwave flux at the ocean surface is about 50 W m^{-2}, whereas the sensible and latent heat fluxes are typically 10 W m^{-2} and 130 W m^{-2}, respectively. Thus F_{in} is about 190 W m^{-2}, whereas F^*_{in} is about 140 W m^{-2}. Although the difference between F_{in} and F^*_{in} is about 25%, RI ignore this difference approximating F_{in} as the latent heat flux alone, using a value of 155 W m^{-2} (RI, p. 581). Because of this, we also neglect the difference between F_{in} and F^*_{in}, and thus we arrive at our formula for the RI theory of CAPE:

$$\mathrm{CAPE_{RI}} \approx \frac{\eta L q_b}{2}. \tag{7}$$

Emanuel and Bister derive an expression for CAPE using the moist static energy budget of subcloud-layer air as closure for the cumulus mass flux. One can derive the following expression for CAPE from Eq. (25) of EB:

$$\mathrm{CAPE_{EB}} \approx \eta(h_b - h_m) \cdot \left(\frac{T_{\mathrm{irr}}}{\overline{\overline{T}}}\right), \tag{8}$$

where h_b is the moist static energy of subcloud layer air and h_m is the moist static energy of midtropospheric air (again see Fig. 1), and where T_{irr} is the temperature at which mechanical energy is dissipated. Furthermore, EB make the approximation $T_{\mathrm{irr}} \approx \overline{\overline{T}}$, and with this approximation the EB prediction for CAPE is

$$\mathrm{CAPE_{EB}} \approx \eta(h_b - h_m). \tag{9}$$

Comparing (9) to (7), one important difference is that EB's CAPE depends on the properties of midtropospheric air as well as near-surface air.

Quantitatively, we can use the mean sounding from the TOGA COARE experiment in the western Pacific to evaluate the predictions of RI and EB. From the sounding in Fig. 1, the CAPE of a surface parcel raised pseudoadiabatically is 2920 J kg^{-1}. The h_b and h_m are 78 600 J kg^{-1} and 59 400 J kg^{-1} respectively, while the q_b and q_m are 20.0 g kg^{-1} and 5.7 g kg^{-1} respectively. The efficiency of the heat engine, $\eta \sim (T_b - \overline{\overline{T}})/T_b$, is approximated as 0.1. From (7) RI predicts a CAPE of 2500 J kg^{-1}, while from (9) EB predicts 1920 J kg^{-1}. Given the extreme sensitivities of the calculated CAPE to what level the raised parcel is selected from and to whether or not the condensed water is carried with the parcel and contributes to a lower density for the parcel (Xu and Emanuel 1989), the predictions of RI and EB both give reasonable estimates for the CAPE of tropical atmospheres. Of course, it may not be fair to compare the predictions of RI and EB with the CAPE calculated from this sounding since it is likely that a mean upward vertical velocity exists in the TOGA COARE region and the predictions of RI and EB are for atmosphere in radiative convective equilibrium with a zero mean vertical velocity.

Emanuel and Bister imply that the theory of RI would fail to satisfy the numerical experiments with cumulus ensemble models presented in EB. In the first experiment, the height-independent radiative cooling rate of the atmosphere is varied and the CAPE remains roughly constant. Emanuel and Bister suggest that RI would predict that the CAPE would rise as the radiative cooling rate of the atmosphere is raised. This is not what RI would predict. As the radiative cooling rate rises, the heat input from the ocean, F_{in}, increases, but according to (3), if the mean vertical temperature profile remains near moist adiabatic, the radiative determined subsidence rate M_{rad} would also rise. Thus (1) suggests that RI would also predict the CAPE to be relatively constant as the radiative cooling rate of the atmosphere rises. Our formula (7) for the RI prediction of CAPE predicts that the CAPE would rise with the surface specific humidity of air and the heat engine efficiency and these quantities are not shown in the figures of EB. Given that the sea surface temperature is fixed in these experiments and the strong dependence of specific humidity on temperature (i.e., Clausius–Clapeyron), it is entirely possible that the near-surface specific humidity q_b remains fixed as the radiative cooling rate is varied. The second experiment of EB (called Case I and Case II) varies the vertical profile of the radiative cooling rate, concentrating the radiative cooling in the upper troposphere (Case I) and in the lower troposphere (Case II). CAPE increases as the effective level of radiative cooling rises. As the effective level of radiative cooling rises, $\overline{T}$ decreases, and through (2) η increases. Because both EB and RI predict that CAPE rises with η, both theories are consistent with this second experiment. In the third experiment, the fraction of rain that falls outside of the cloud is varied. As this fraction increases, more of the rain evaporates, and the troposphere is moister. Emanuel and Bister's column model shows that CAPE decreases by less than 10% as this fraction increases from 0.07 to 0.16. Because the atmosphere is more moist, (7) might suggest that RI would predict CAPE to increase. However, it is not clear that surface air moisture will decrease as the moisture of the whole troposphere decreases. Furthermore, a drier troposphere will raise the effective temperature at which the atmosphere is radiatively cooled $\overline{T}$, suggesting a decrease in η and hence a decrease in CAPE. Without more data, it is unclear what RI would predict the variation of CAPE to be for this third experiment. Perhaps EB could provide an analysis of how well (7) would predict the variations of CAPE simulated by their numerical models. In summary, it is entirely possible that the predictions of RI as expressed in (7) are entirely consistent with the numerical experiments presented in EB.

Part of the confusion about what RI predicts arises from a figure in RI that shows the CAPE increases with the heat input from the surface (RI, Fig. 5). We suspect these curves are misleading since we suspect that these curves were calculated for the case where the radiatively determined subsidence rate remains constant. As argued above, if the nonradiative heat input from the surface rises, the radiative cooling rate of the atmosphere must rise in radiative–convective equilibrium. Through (3) the radiatively determined subsidence rate must also rise if the mean vertical temperature profile remains relatively constant. Thus, it is misleading for RI to show curves where the heat input from the ocean surface rises but the radiatively determined subsidence velocity does not vary.

Emanuel and Bister suggest that an advantage of their theory is that it predicts CAPE to rise as the humidity of the middle troposphere decreases in rough accord with observations. However, CAPE may rise when middle tropospheric humidity decreases because the observations sample states of different mean vertical velocity. If the mean velocity of the domain falls (i.e., mean subsidence increases), the middle troposphere will dry out and convection will be less frequent. As convection decreases, the venting of the boundary layer decreases and the specific humidity and moist static energy of the surface air will increase. For a negligible change in upper tropospheric temperatures, CAPE will increase when the moist static energy of surface air increases. In this way, the correlation of high CAPE with small middle tropospheric humidity can occur. Because EB's (and RI's) theory applies so far only to the case of zero mean vertical velocity, the observation that high CAPE accompanies small middle tropospheric humidity may not be support of EB's theory.

In summary, we have presented an expression for CAPE that uses the same closure assumptions as RI. The primary advantage of our expression for CAPE over the expressions given in RI [RI Eqs. (39) or (40)] is that in our expression the CAPE of tropical atmospheres in radiative–convective equilibrium is not explicitly dependent on the rate of heat input or the radiative timescale. This is because to a first order, the heat input from the surface and the radiative timescale are strongly coupled through our Eq. (4). It is this understanding that tropical CAPE is not explicitly dependent on the rate of heat input, or the radiative timescale, that is absent from RI but made clear by EB. However, because the predictions of RI as expressed in our Eq. (7) are similar to the predictions of EB, it will take further analysis of the numerical experiments performed or new experiments to demonstrate that the predictions of EB and RI do differ and how they differ from numerical experiments performed with cumulus ensemble models. In the particular, the predicted dependence of CAPE on midtropospheric humidity by EB remains to be verified.

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