



Angles of the CKM unitarity triangle measured at Belle

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- Introduction
- Determining $\sin(2\phi_1)$ (β)
- Determining $\sin(2\phi_2)$ (α)
- Determining ϕ_3 (γ)
- Summary



Introduction I

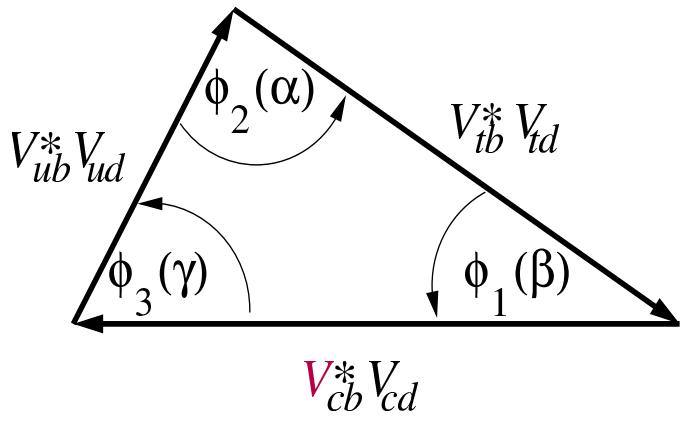
CKM weak mixing matrix:

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$U^\dagger U = 1 \Rightarrow$$

$$\begin{aligned} V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} &= 0 \\ V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} &= 0 \\ V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} &= 0 \end{aligned}$$

$$\begin{aligned} V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\ V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} &= 0 \end{aligned}$$



$$\begin{aligned} \phi_1 (\beta) &= \arg \left(\frac{V_{cb}^* V_{cd}}{-V_{tb}^* V_{td}} \right) \\ \phi_2 (\alpha) &= \arg \left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}} \right) \\ \phi_3 (\gamma) &= \arg \left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}} \right) \end{aligned}$$

Does the triangle close? i.e., are there >3 generations?

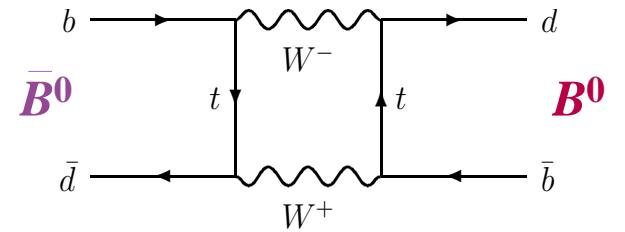


Introduction II

(some formalism)

$$\begin{aligned} |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle \\ |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \end{aligned}$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}}} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} = e^{i2\phi_1} \quad (\text{phase of } V_{td}^* V_{tb})$$



$$\begin{aligned} |B^0(t)\rangle &= e^{-(\Gamma/2+i\bar{m})t} \left[\cos\left(\frac{\Delta m}{2}t\right)|B^0\rangle + \left(\frac{q}{p}\right)i \sin\left(\frac{\Delta m}{2}t\right)|\bar{B}^0\rangle \right] \\ |\bar{B}^0(t)\rangle &= e^{-(\Gamma/2+i\bar{m})t} \left[\left(\frac{p}{q}\right)i \sin\left(\frac{\Delta m}{2}t\right)|B^0\rangle + \cos\left(\frac{\Delta m}{2}t\right)|\bar{B}^0\rangle \right], \end{aligned}$$

$$|\langle f | H | B^0(t) \rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2} [1 + |\lambda|^2 + (1 - |\lambda|^2) \cos(\Delta m t) - 2 \operatorname{Im} \lambda \sin(\Delta m t)]$$

$$|\langle f | H | \bar{B}^0(t) \rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2} [1 + |\lambda|^2 - (1 - |\lambda|^2) \cos(\Delta m t) + 2 \operatorname{Im} \lambda \sin(\Delta m t)]$$

$$\lambda = \left(\frac{q}{p}\right) \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} = e^{i2\phi_1} e^{i2\phi} \quad (\text{one weak phase})$$



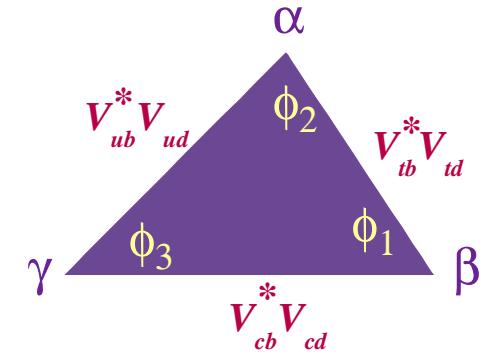
Introduction III

$$\frac{N_{\bar{B}^0 \rightarrow f} - N_{B^0 \rightarrow f}}{N_{\bar{B}^0 \rightarrow f} + N_{B^0 \rightarrow f}} = \mathcal{A}_f \cos(\Delta m \Delta t) + \mathcal{S}_f \sin(\Delta m \Delta t)$$

$$\mathcal{A}_f = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad \mathcal{S}_f = \frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2}$$

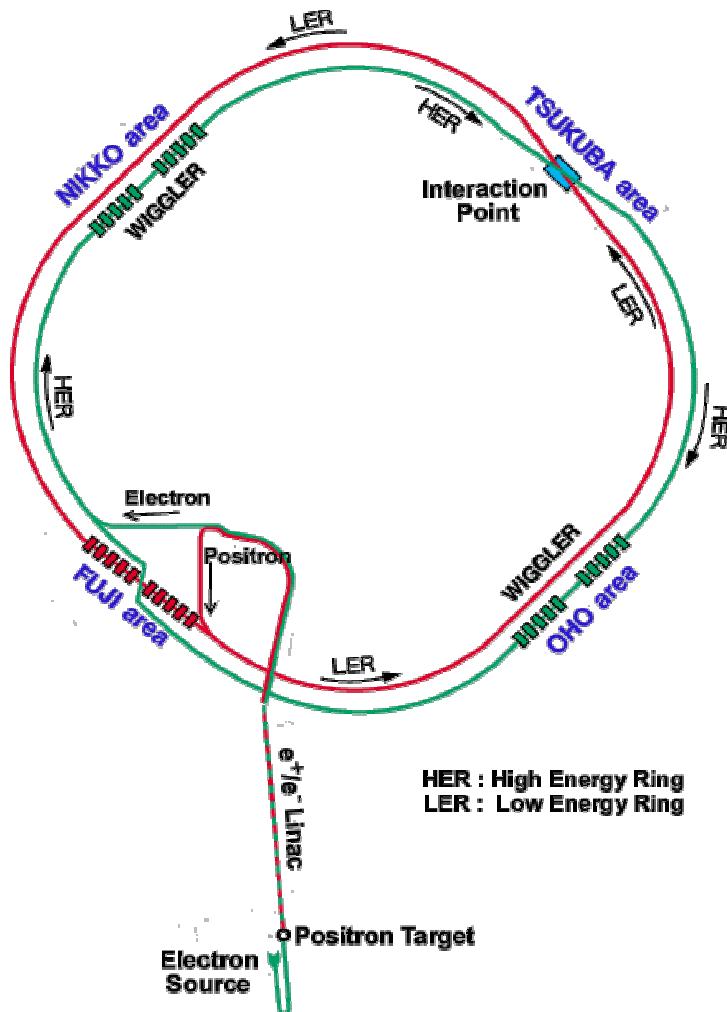
$$\lambda = \left(\frac{q}{p} \right) \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} = e^{i2\phi_1} e^{i2\phi} \quad (\text{one weak phase})$$

$$\Rightarrow \mathcal{A}_f \approx 0, \quad \mathcal{S}_f \approx \sin 2(\phi_1 + \phi) = -\sin 2\phi'$$

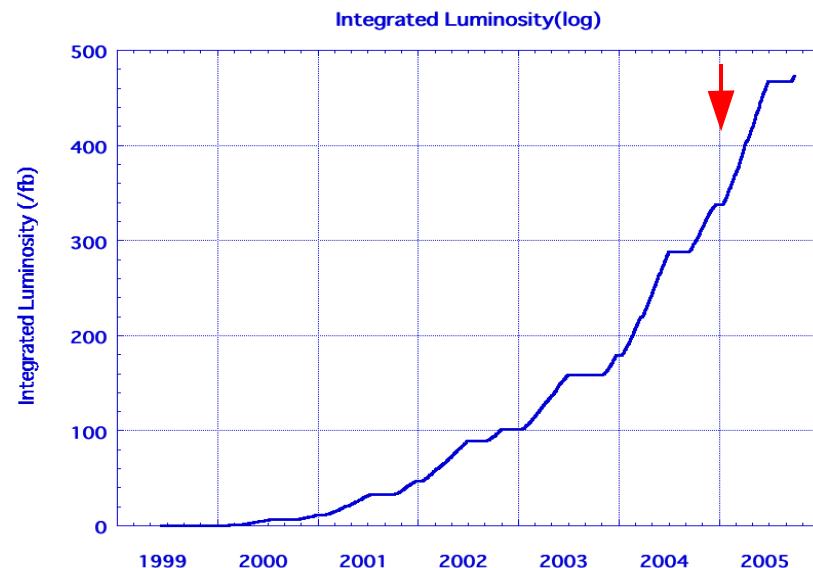




Belle at KEKB



$e^+e^- \rightarrow Y(4S) \rightarrow \bar{B}B$
3.5 GeV on 8.0 GeV



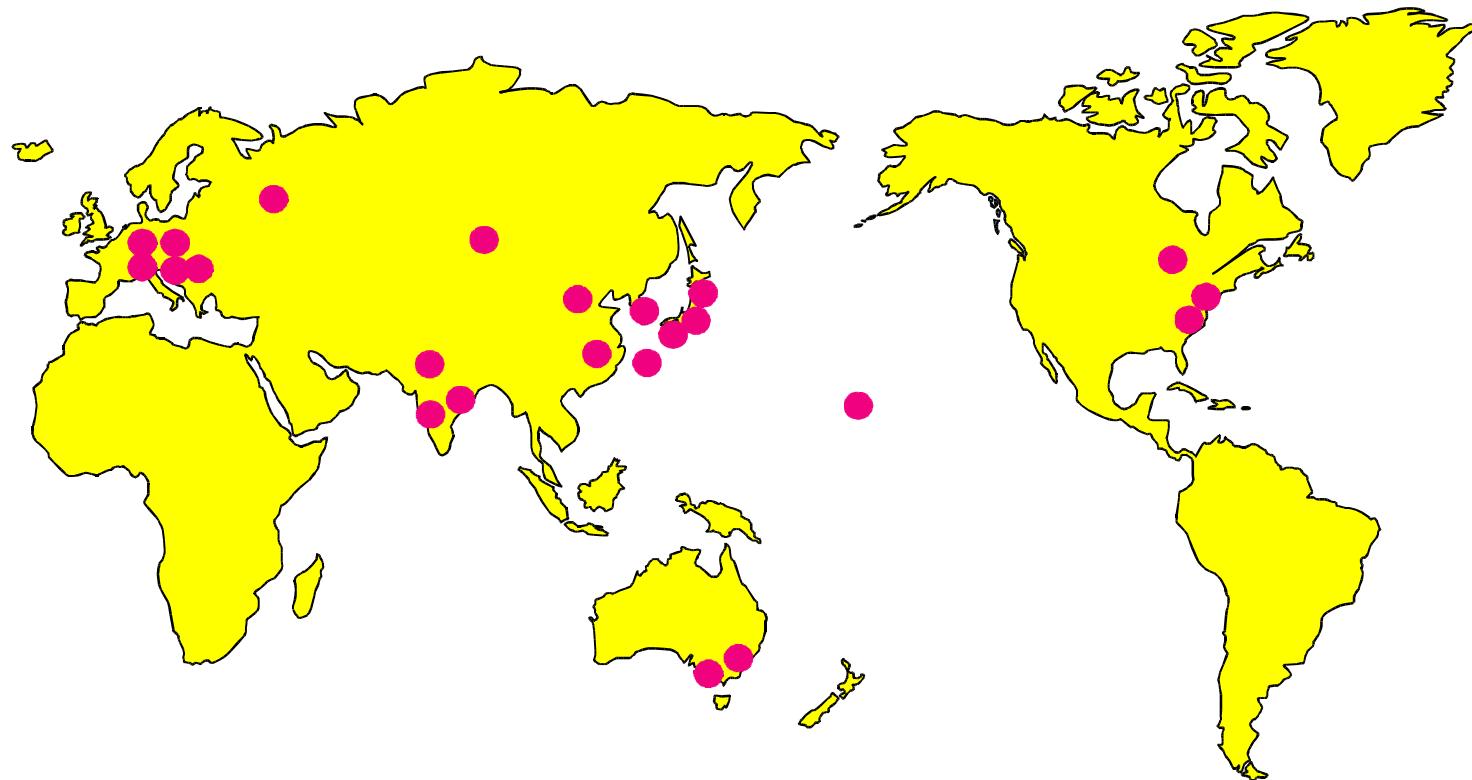
$$\int L dt = 476 \text{ fb}^{-1} \text{ on 4 Oct 2005}$$

$$L_{peak}(\text{max}) = 1.6 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

357 fb $^{-1}$ on resonance (386M BB)
analyzed thus far



The Belle Collaboration

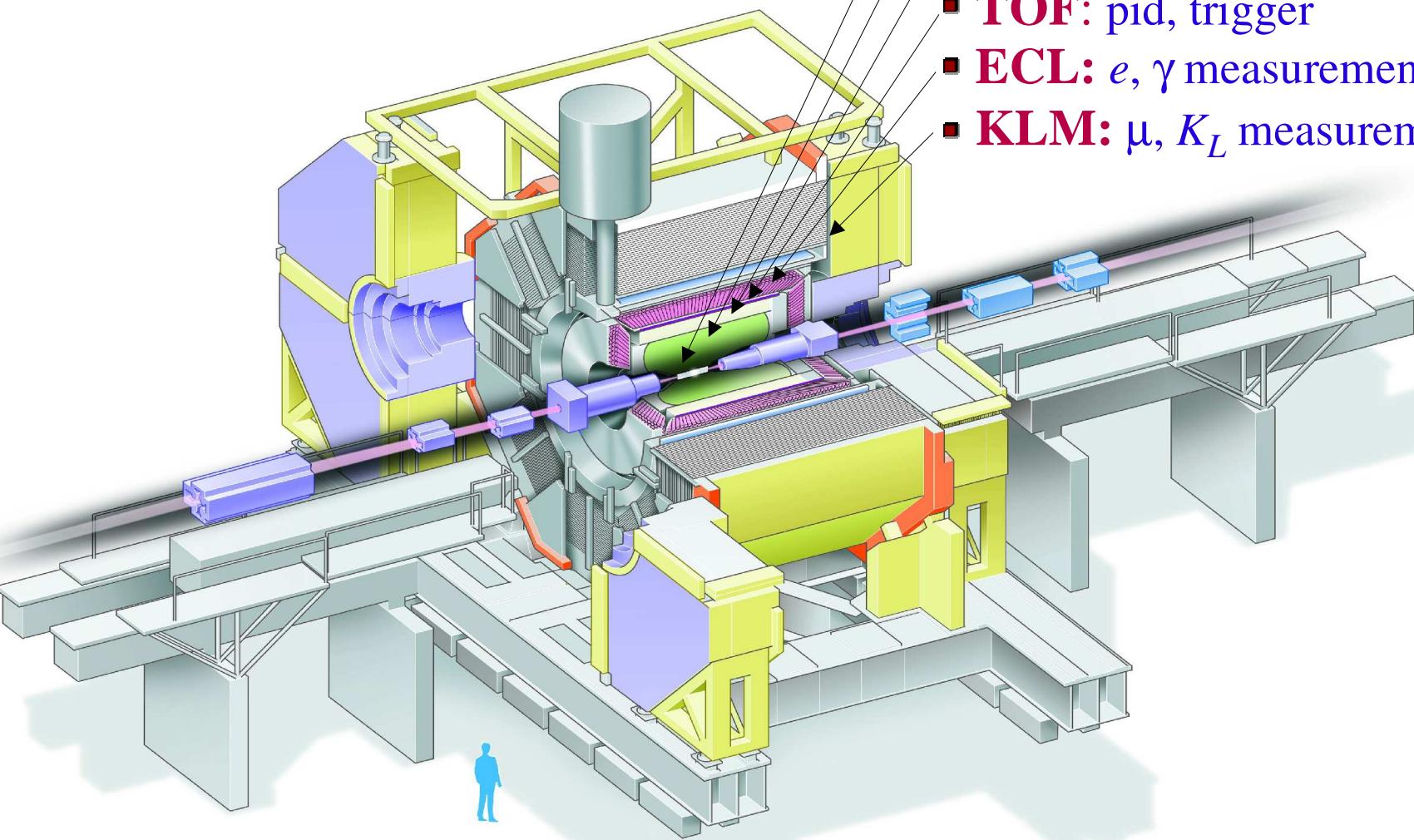


About 400 people from 59 institutions, many nations

US: Cincinnati
Hawaii
Princeton
Virginia Tech



The Belle detector



- **SVD:** vertexing (lifetime)
- **CDC:** tracking, dE/dx for pid
- **ACC:** aerogel Cerenk. Counter
- **TOF:** pid, trigger
- **ECL:** e, γ measurement
- **KLM:** μ, K_L measurement



Analysis Overview

1) $B \rightarrow f$ selection:

$$m_{bc} = \sqrt{(E_{beam}^*)^2 - (p_B^*)^2}$$
$$\Delta E = E_B^* - E_{beam}^*$$

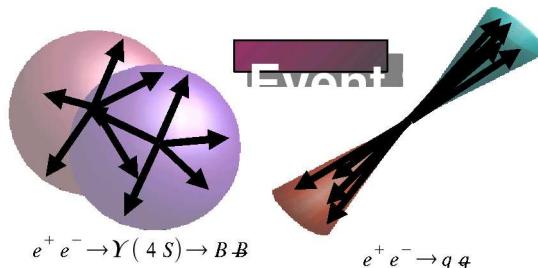
(e.g., for $B \rightarrow \pi^+ \pi^-$:
 $5.271 < m_{bc} < 5.287 \text{ GeV}/c^2$
 $|\Delta E| < 0.064 \text{ GeV}$)

2) Flavor tagging:

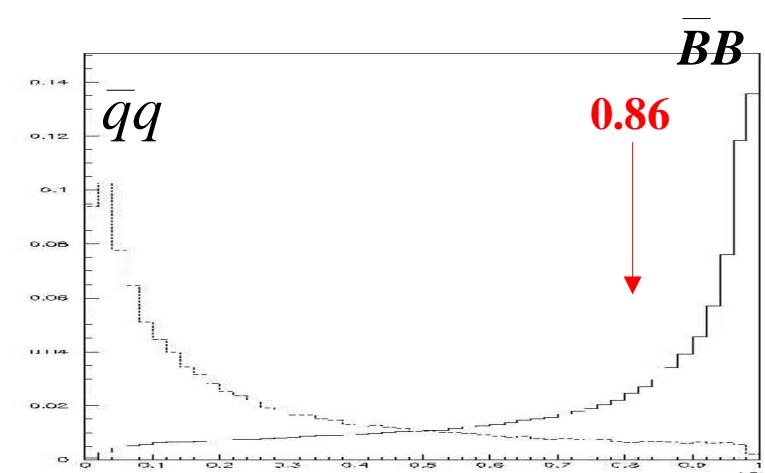
mainly K^\pm, μ^\pm, e^\pm

output: $q = \pm 1, \text{ quality } r = 0-1$

3) Continuum suppression:



$$KLR \equiv \frac{\mathcal{L}_{B\bar{B}}}{(\mathcal{L}_{B\bar{B}} + \mathcal{L}_{q\bar{q}})}$$



4) Vertexing and Δt fit

KLR

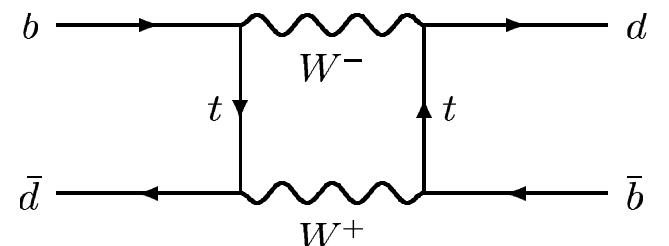


Measurement of $\sin(2\phi_1)$ with $B^0 \rightarrow J/\psi K^0$

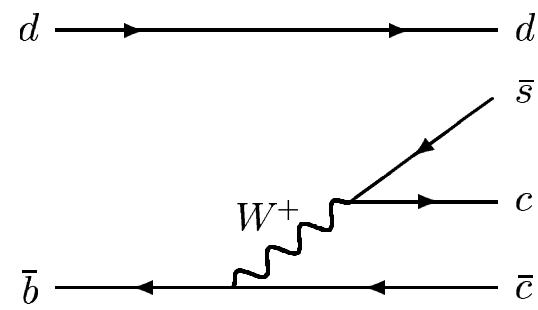
$$\begin{aligned}
 \lambda &= \sqrt{\frac{M_{12}^*}{M_{12}}} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} = - \left(\frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right) \\
 &= - \frac{V_{td} V_{tb}^* V_{cb} V_{cd}^*}{V_{td}^* V_{tb} V_{cb}^* V_{cd}} \\
 &= - \frac{-V_{cb} V_{cd}^* / (V_{td}^* V_{tb})}{-V_{cb}^* V_{cd} / (V_{td} V_{tb}^*)} \\
 &= - \frac{|\mathcal{M}| e^{-i\phi_1}}{|\mathcal{M}| e^{i\phi_1}} \\
 &= -e^{-2i\phi_1}
 \end{aligned}$$

$$\Rightarrow \mathcal{A}_{(J/\psi K^0)} = 0 \quad \mathcal{S}_{(J/\psi K^0)} = \sin(2\phi_1)$$

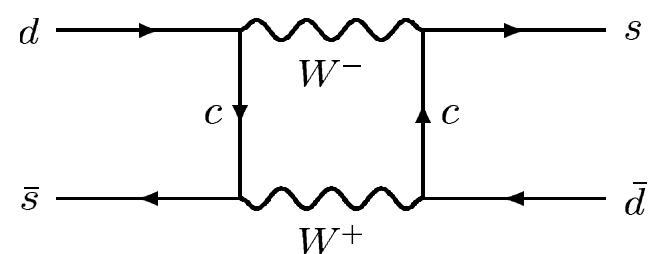
\bar{B}^0 - B^0 oscillation:



Tree:



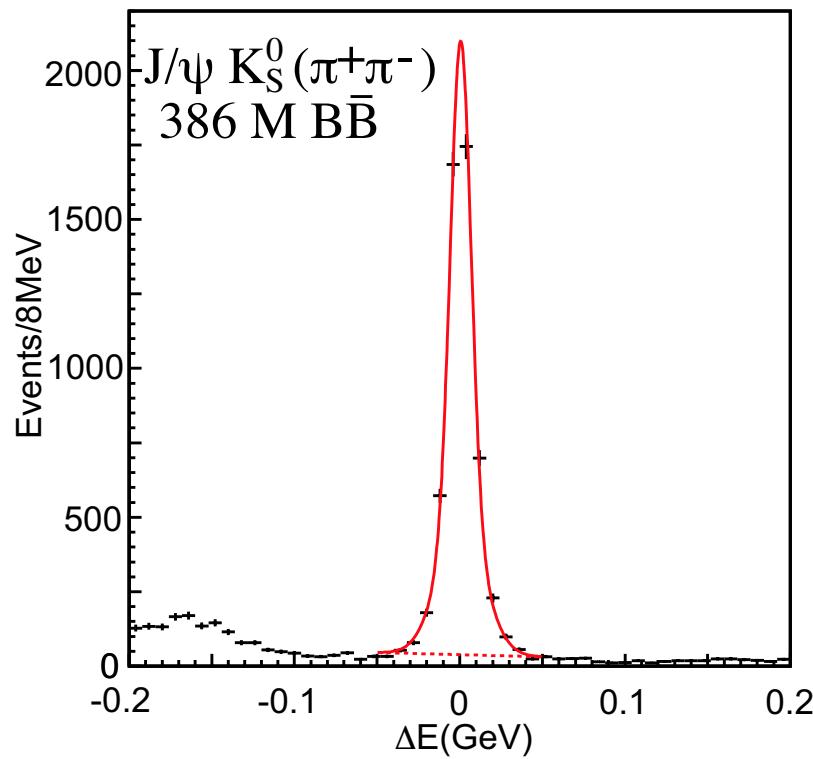
\bar{K}^0 - K^0 oscillation:



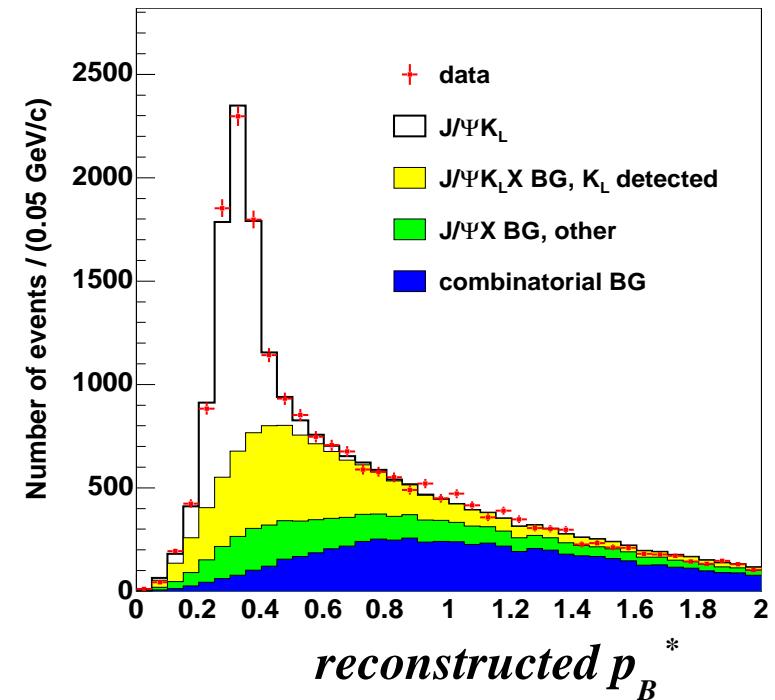


Measurement of $\sin(2\phi_1)$ with $b \rightarrow ccs$ (hep-ex/0507037)

357 fb⁻¹:



$B^0 \rightarrow J/\psi K_S$ ($CP = -1$)
5264 events
98% pure

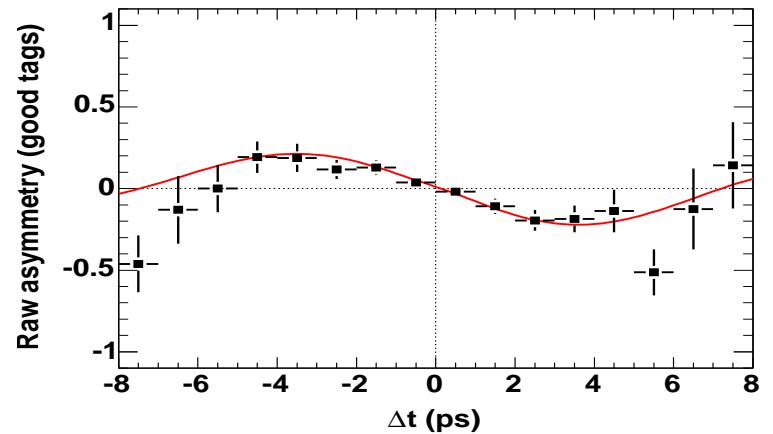
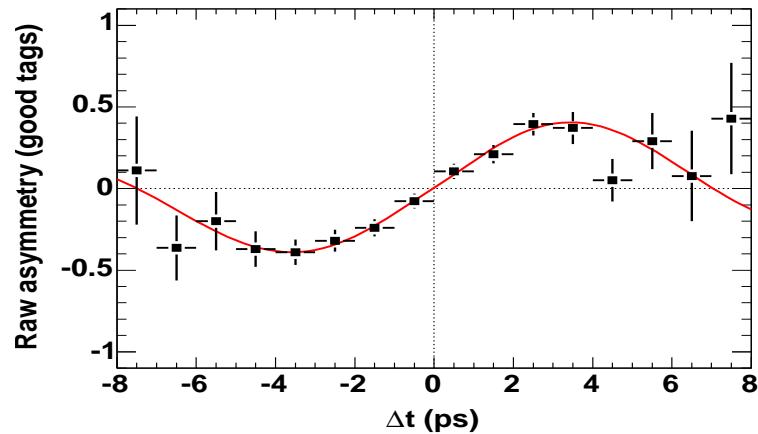
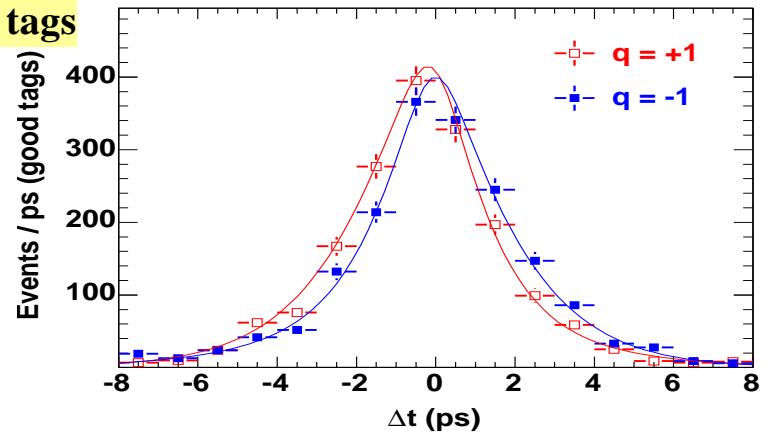
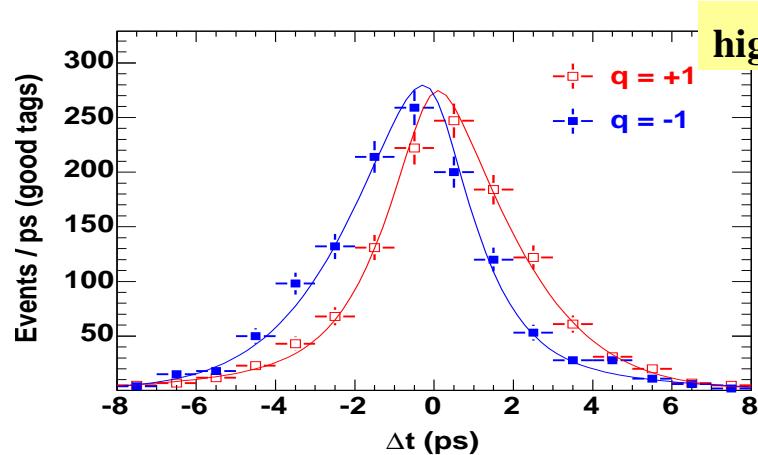


$B^0 \rightarrow J/\psi K_L$ ($CP = +1$)
4792 events
60% pure

Note: cannot use ΔE because we don't have E_{KL}



Measurement of $\sin(2\phi_1)$ with $b \rightarrow ccs$ (hep-ex/0507037)



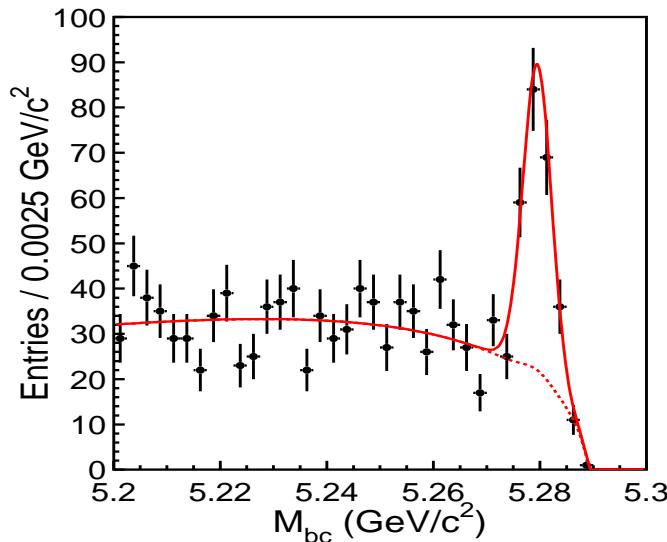
$$\begin{aligned} \sin(2\phi_1) &= 0.652 \pm 0.039 \pm 0.020 \\ \Rightarrow \quad \phi_1 &= (20.3^{+1.7}_{-1.6})^\circ \end{aligned}$$

close to BaBar 210 fb^{-1} :

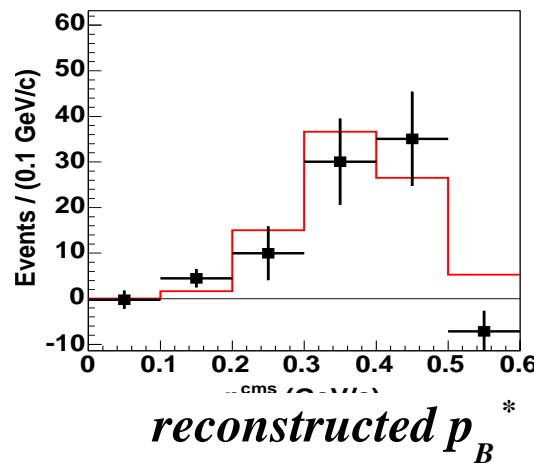
$$\begin{aligned} \sin(2\phi_1) &= 0.722 \pm 0.040 \pm 0.023 \\ |\lambda| &= 0.950 \pm 0.031 \pm 0.013 \end{aligned}$$



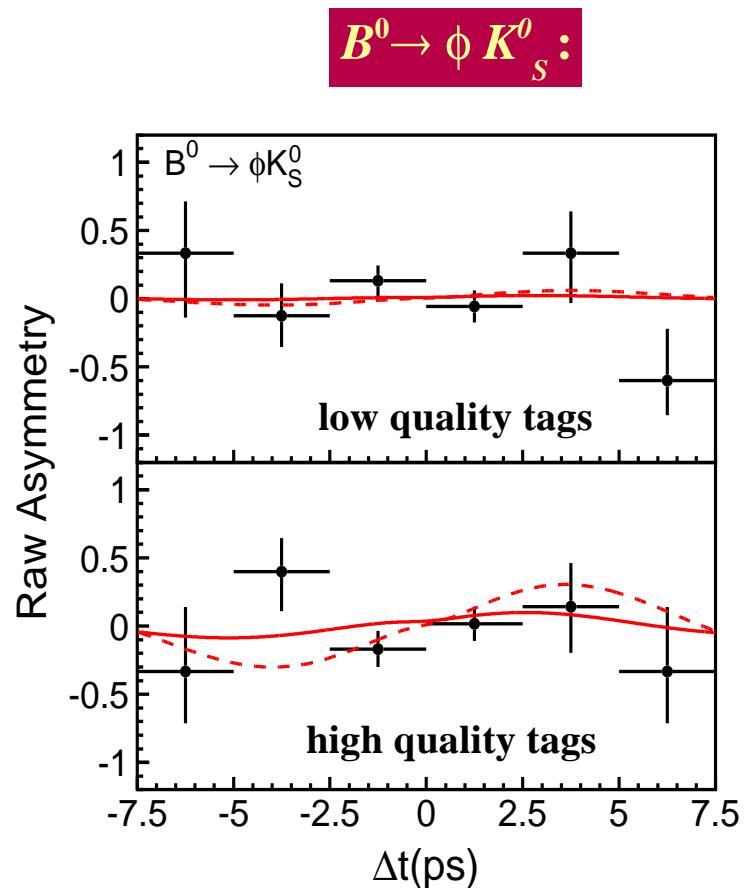
Measurement of $\sin(2\phi_1)$ with $b \rightarrow sss$ (hep-ex/0507037)



$B^0 \rightarrow \phi K_s$ ($CP = -1$)
 $N = 180 \pm 16$
 57% pure



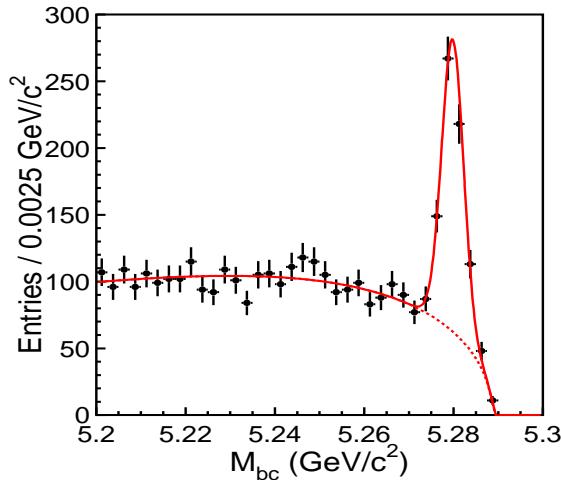
$B^0 \rightarrow \phi K_L$ ($CP = +1$)
 $N = 78 \pm 13$
 12% pure



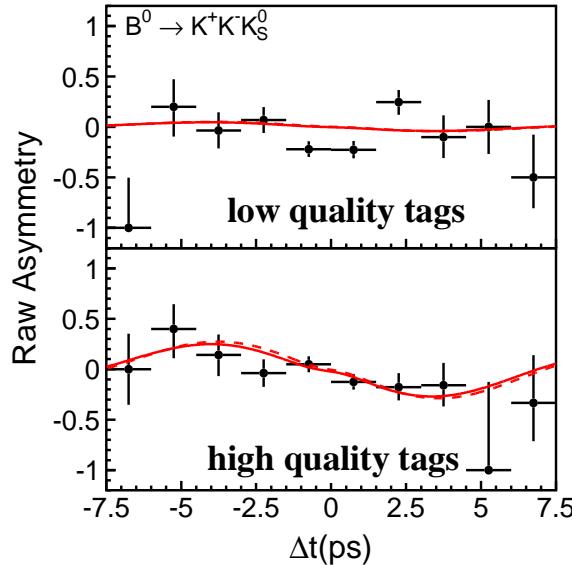
$\sin(2\phi_1) = + 0.44 \pm 0.27 \pm 0.05$
 $A = + 0.14 \pm 0.17 \pm 0.07$



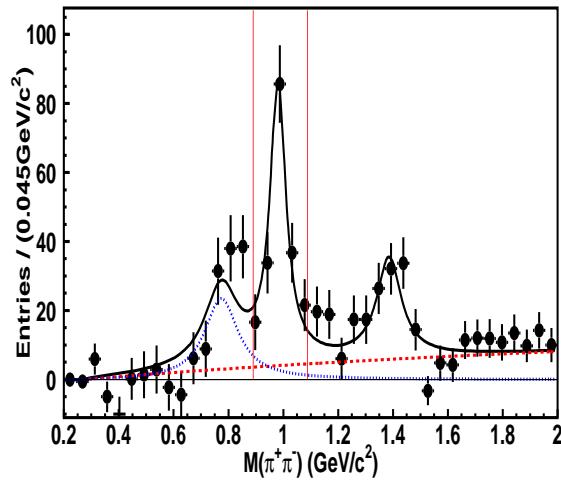
Measurement of $\sin(2\phi_1)$ with $b \rightarrow qqs$ (hep-ex/0507037)



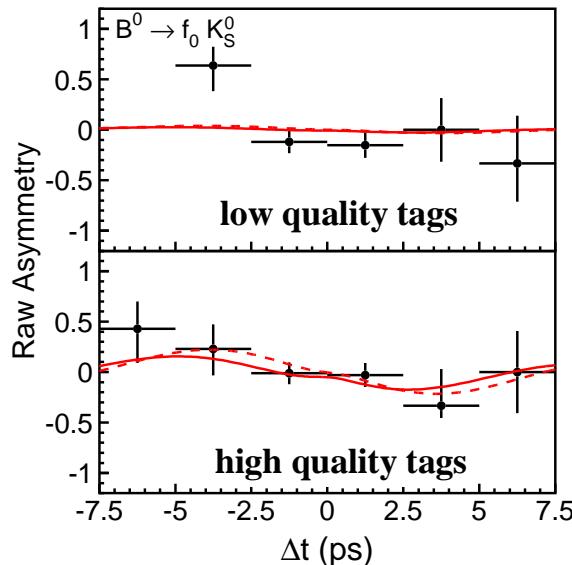
$B^0 \rightarrow K^+ K^- K_S$
 $(CP = +1 \text{ mostly})$
 $N = 536 \pm 29$
 55% pure



$\sin(2\phi_1) =$
 $-0.52 \pm 0.16 \pm 0.03$
 $A =$
 $-0.06 \pm 0.11 \pm 0.07$



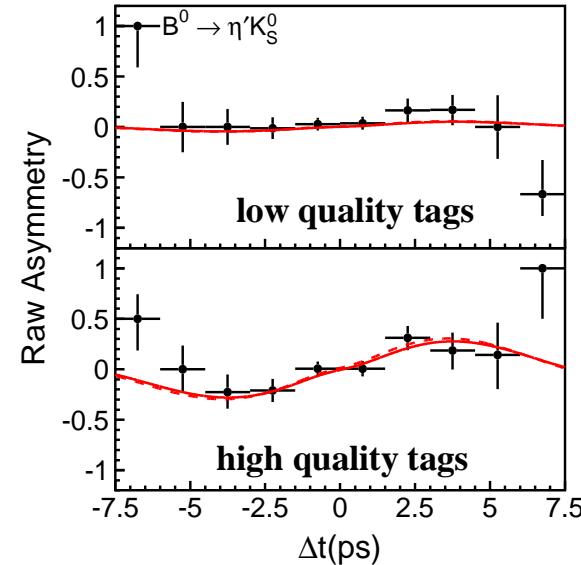
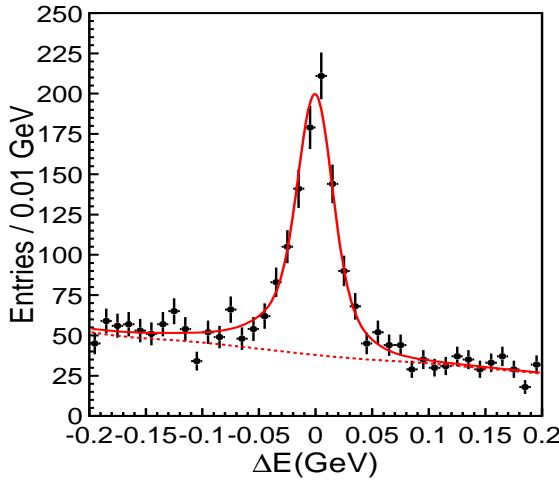
$B^0 \rightarrow f_0(980) K_S$
 $(CP = +1)$
 $N = 145 \pm 16$
 47% pure



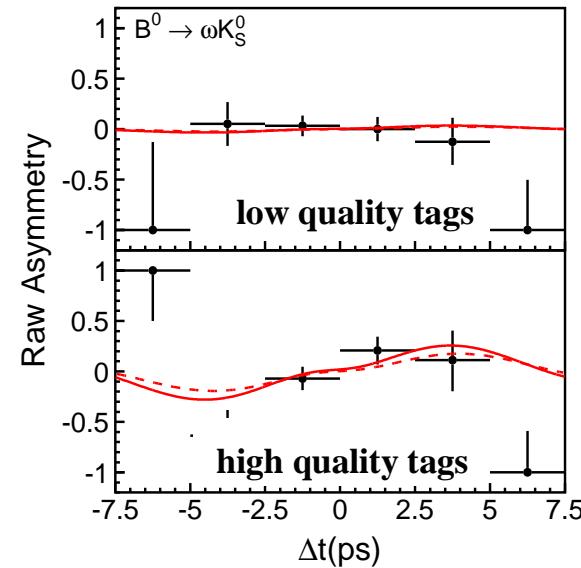
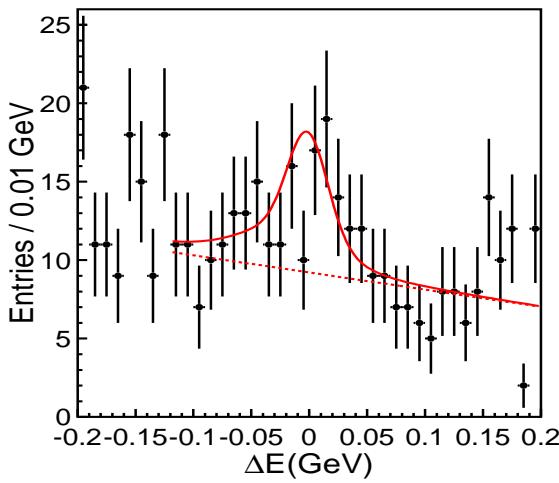
$\sin(2\phi_1) =$
 $-0.47 \pm 0.36 \pm 0.08$
 $A =$
 $-0.23 \pm 0.23 \pm 0.13$



Measurement of $\sin(2\phi_1)$ with $b \rightarrow qqs$ (hep-ex/0507037)



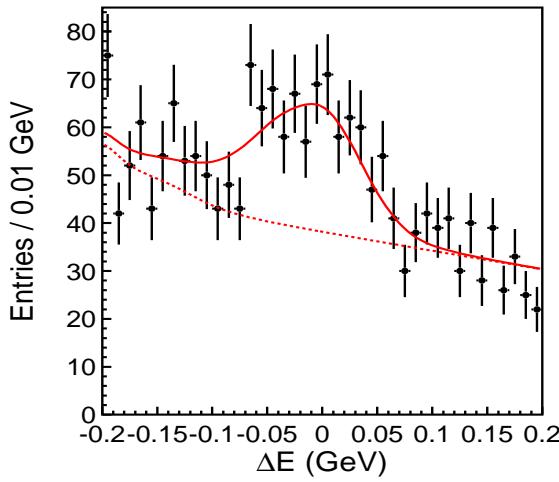
$\sin(2\phi_1) =$
+ 0.65 ± 0.18 ± 0.04
 $A =$
- 0.19 ± 0.11 ± 0.05



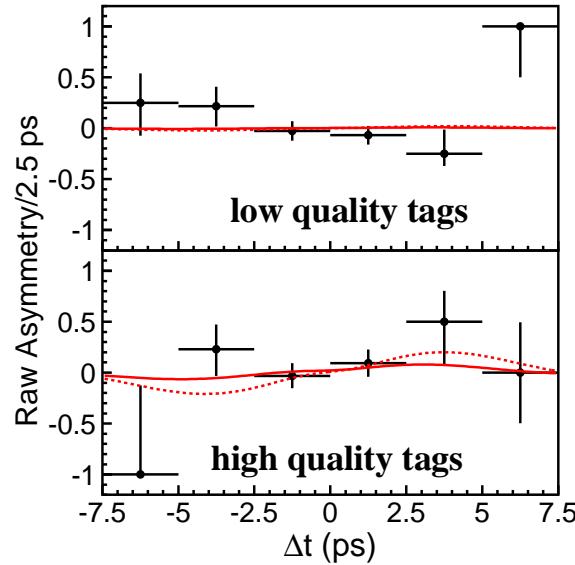
$\sin(2\phi_1) =$
+ 0.75 ± 0.64^{+0.13}_{+0.16}
 $A =$
+ 0.26 ± 0.48 ± 0.15



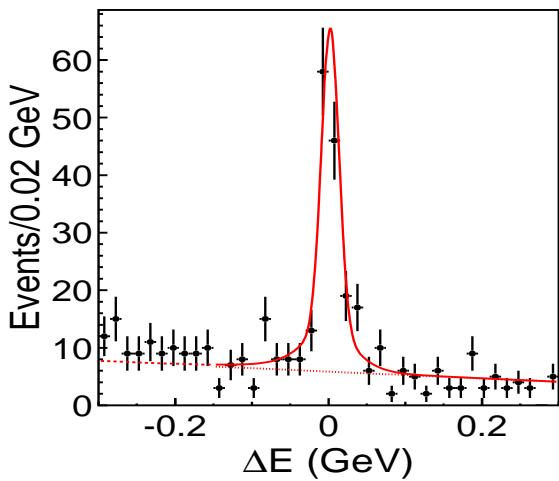
Measurement of $\sin(2\phi_1)$ with $b \rightarrow qqs$ (hep-ex/0507037)



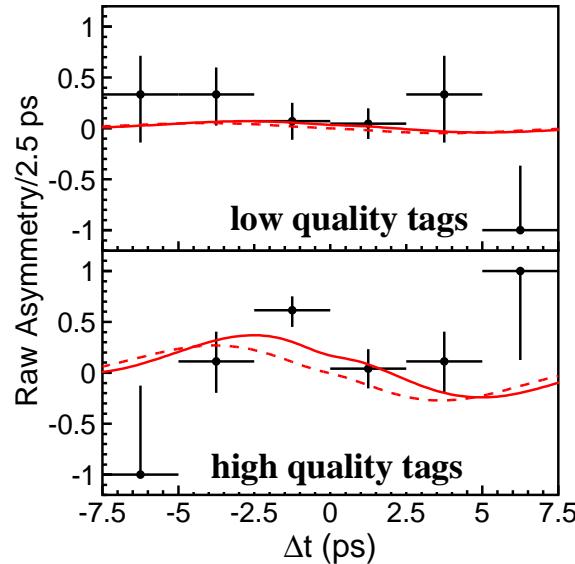
$B^0 \rightarrow \pi^0 K_s$
($CP = -1$)
 $N = 344 \pm 30$
25% pure



$\sin(2\phi_1) =$
 $+ 0.22 \pm 0.47 \pm 0.08$
 $A =$
 $+ 0.11 \pm 0.18 \pm 0.08$



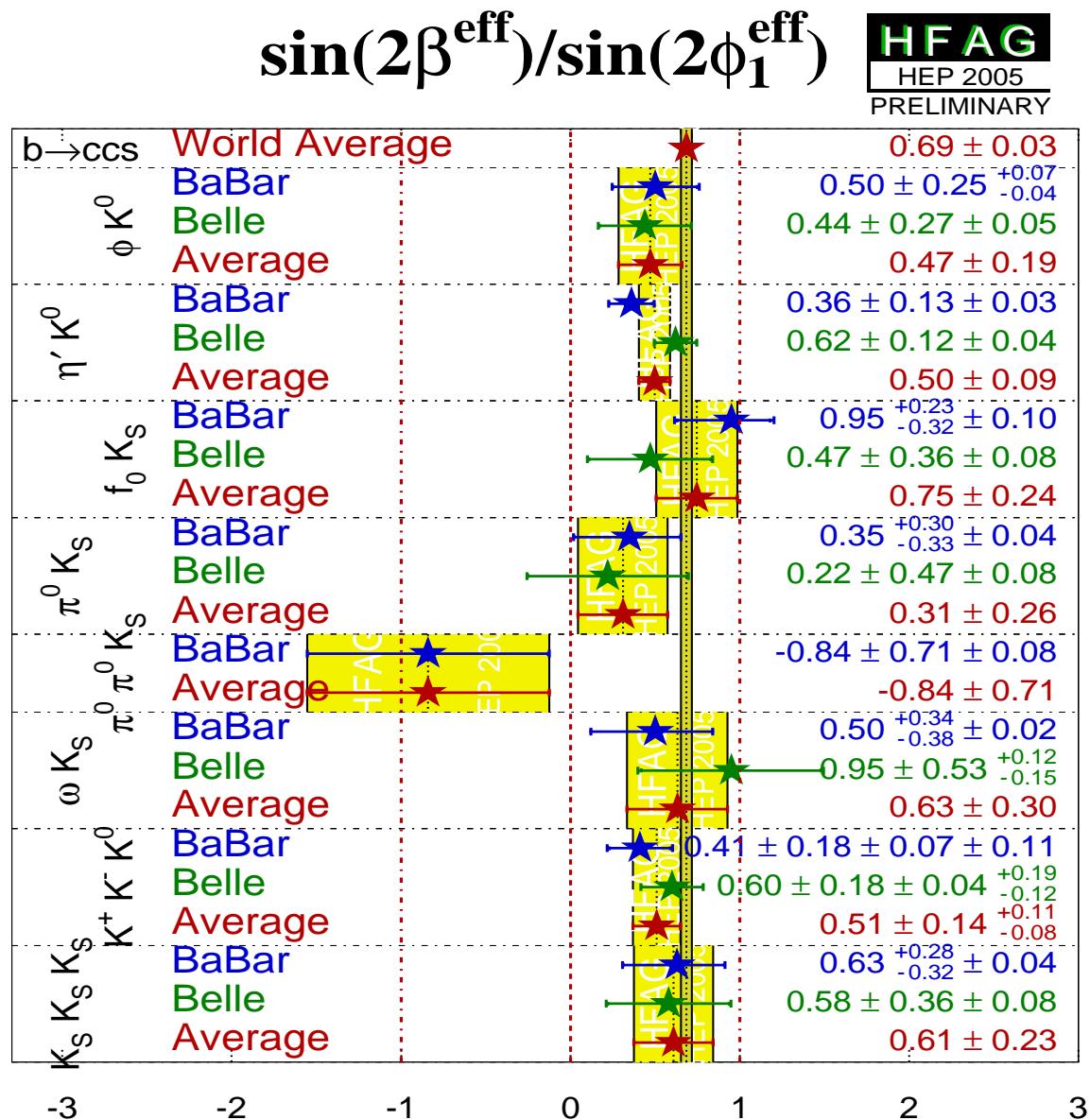
$B^0 \rightarrow K_s K_s K_s$
($CP = +1$)
 $N = 105 \pm 12$
64% pure



$\sin(2\phi_1) =$
 $- 0.58 \pm 0.36 \pm 0.08$
 $A =$
 $+ 0.50 \pm 0.23 \pm 0.06$



Measurement of $\sin(2\phi_1)$ summary



⇒ modulu final-state interactions
(see Soni, hep-ph/0502235),
results appear consistent
with SM

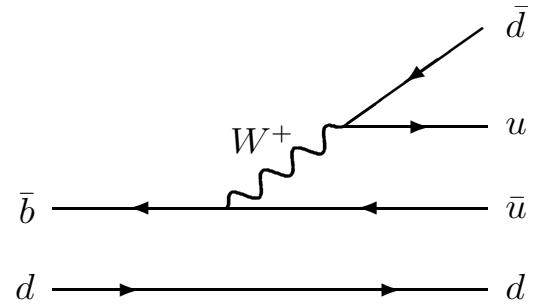


Measurement of $\sin(2\phi_2)$ with $B^0 \rightarrow \pi^+\pi^-$

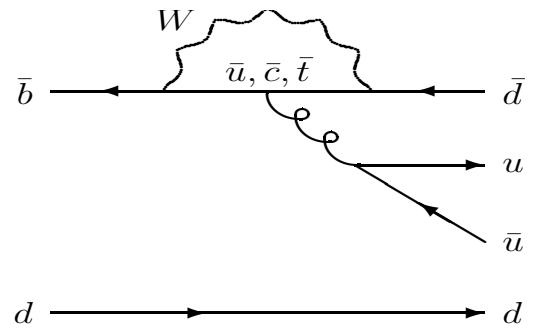
$$\begin{aligned}
 \lambda &= \sqrt{\frac{M_{12}^*}{M_{12}}} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} = + \left(\frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \right) \left(\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \right) \\
 &= \frac{-V_{tb}^* V_{td} / (V_{ub}^* V_{ud})}{-V_{tb} V_{td}^* / (V_{ub} V_{ud}^*)} \\
 &= \frac{|\mathcal{M}'| e^{i\phi_2}}{|\mathcal{M}'| e^{-i\phi_2}} \\
 &= e^{2i\phi_2}
 \end{aligned}$$

$$\Rightarrow \mathcal{A}_{\pi\pi} = 0 \quad \mathcal{S}_{\pi\pi} = \sin(2\phi_2)$$

Tree:



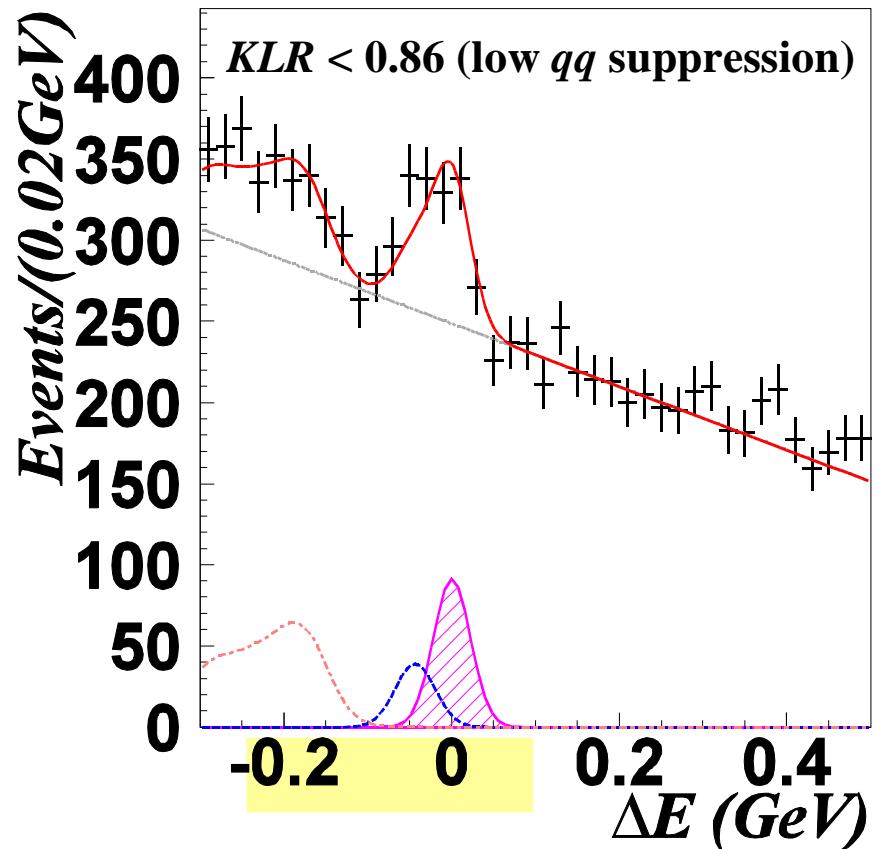
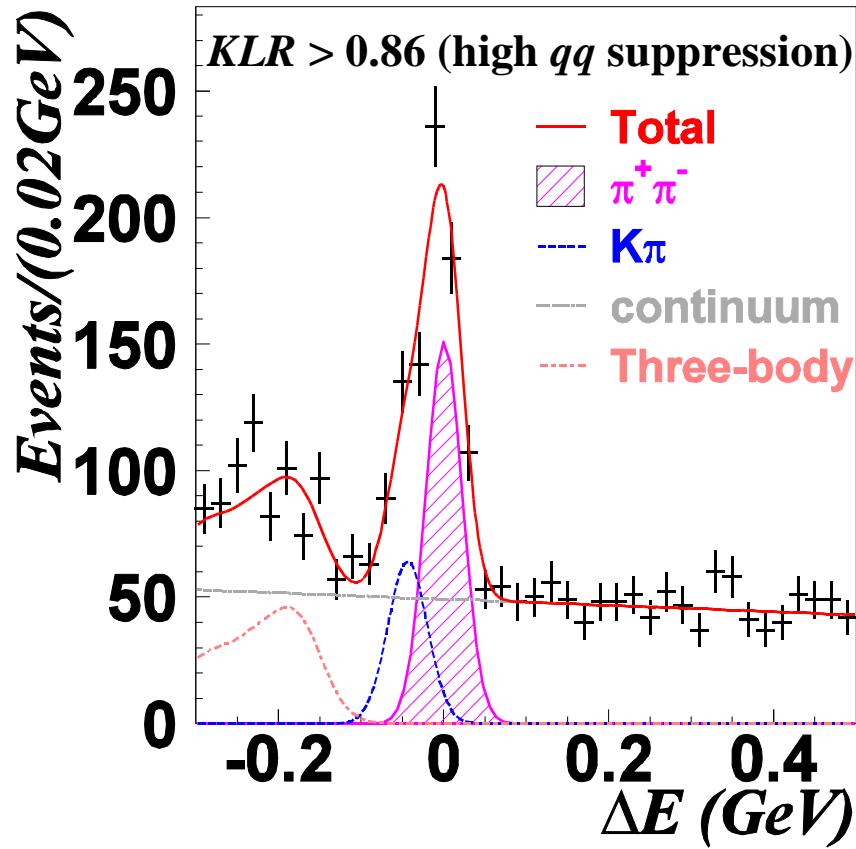
Penguin:



...if no penguin. But there is a penguin contribution, which “breaks” these equalities



$B^0 \rightarrow \pi^+ \pi^-$ sample (253 fb^{-1}) (hep-ex/0502035)



$$N_{\pi\pi} = 415 \pm 13$$

$$N_{\pi\pi} = 251 \pm 8$$

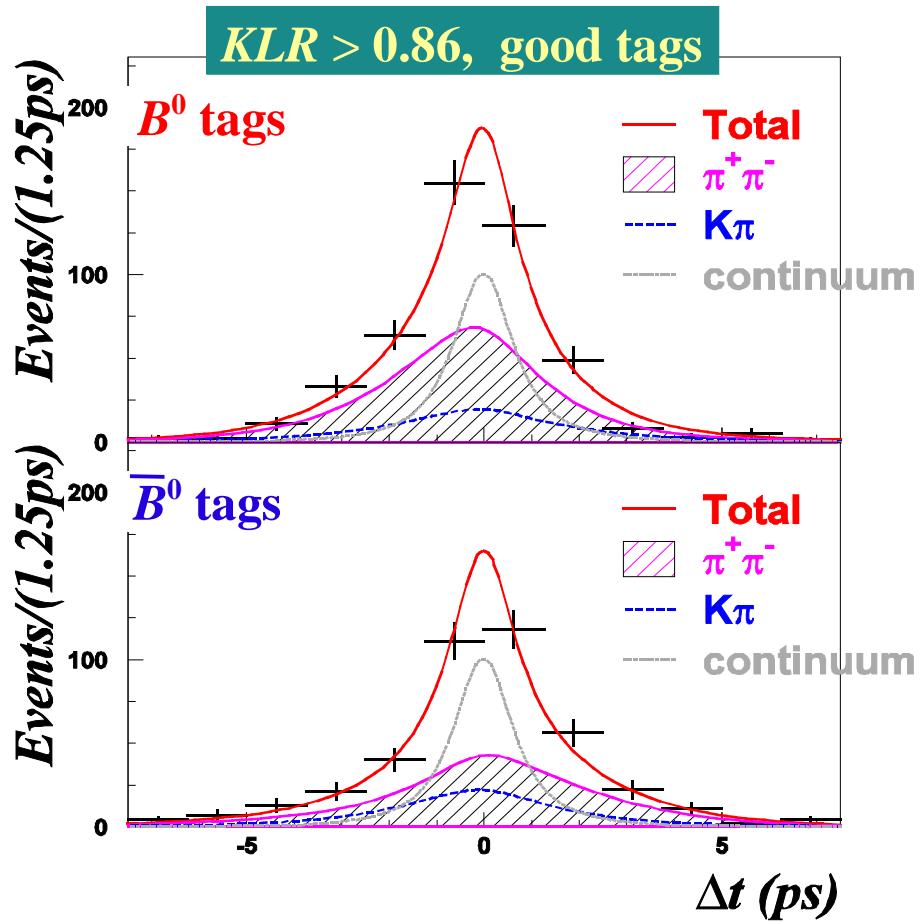


Maximum likelihood fit to Δt ($B^0 \rightarrow \pi^+ \pi^-$)

$$\begin{aligned} \mathcal{L}_i = & \int [f_{\pi\pi} P_{\pi\pi}(\Delta t') + f_{K\pi} P_{K\pi}(\Delta t')] \cdot R_{hh}(\Delta t_i - \Delta t') \\ & + f_{q\bar{q}} P_{q\bar{q}}(\Delta t') \cdot R_{q\bar{q}}(\Delta t_i - \Delta t') dt' \end{aligned}$$

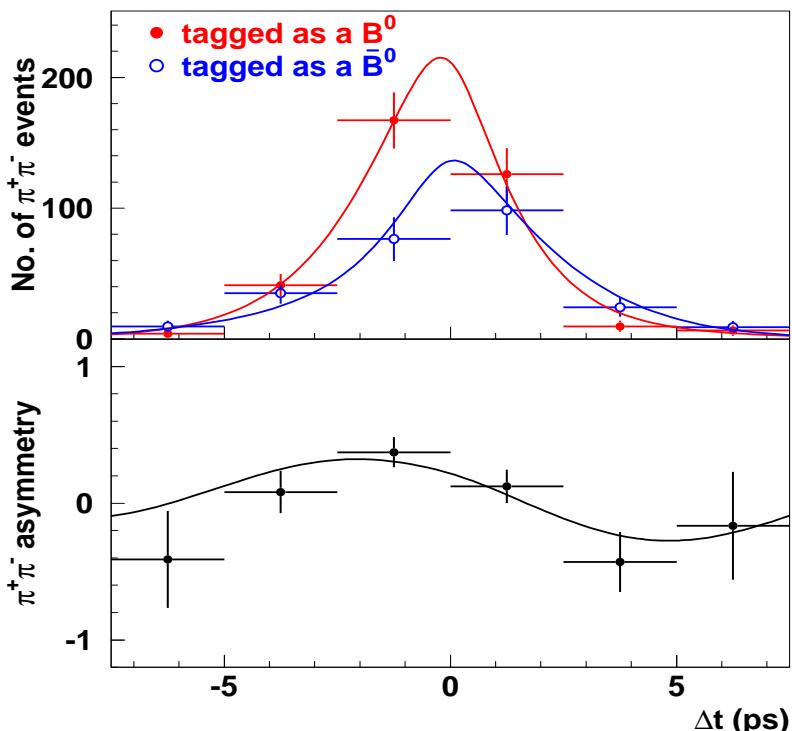
$$\begin{aligned} P_{B^0 \rightarrow \pi\pi}^{(\ell)} &= \frac{e^{-|\Delta t|/\tau_B}}{\mathcal{N}} \left\{ 1 + q(1 - 2\omega_\ell) [\mathcal{A}_{\pi\pi} \cos(\Delta m \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m \Delta t)] \right\} \\ P_{K\pi} &= \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} \left\{ 1 + q(1 - 2\omega_\ell) \mathcal{A}_{K\pi}^{\text{eff}} \cos(\Delta m \Delta t) \right\} \quad (\mathcal{A}_{K\pi} = -0.109 \pm 0.019) \\ P_{q\bar{q}} &= f \frac{e^{-|\Delta t|/\tau_{q\bar{q}}}}{2\tau_{q\bar{q}}} + (1 - f) \delta(\Delta t), \end{aligned}$$

$$f_{\pi\pi} = \frac{F_{\pi\pi}(\Delta E, M_{bc}) \cdot f_\ell(\pi\pi)}{[F_{\pi\pi}(\Delta E, M_{bc}) + F_{K\pi}(\Delta E, M_{bc})] \cdot f_\ell(\pi\pi) + F_{q\bar{q}}(\Delta E, M_{bc}) \cdot f_\ell(q\bar{q})}$$



$$A_{\pi\pi} = 0.56^{+0.11}_{-0.12} \quad S_{\pi\pi} = -0.67 \pm 0.16$$

m_{bc} - ΔE 2D fit for
event yields in bins
of Δt :





Systematic Uncertainties

Uncertainty	$A_{\pi\pi}$	$S_{\pi\pi}$
Wrong tag fraction	± 0.01	± 0.01
τ_B , Δm , $A_{K\pi}$	± 0.01	< 0.01
Resolution function	± 0.01	± 0.04
Background Δt shape	< 0.01	< 0.01
Background fractions	± 0.04	± 0.02
Fit bias	± 0.01	± 0.01
Vertexing	$+0.03$ -0.01	± 0.04
Tag side interference	$+0.02$ -0.04	± 0.01
Total	± 0.06	± 0.06

← includes uncertainty
in final state radiation

← O. Long *et al.*,
PRD 68, 034010 (2003)



Constraints upon ϕ_2 (α) and $|P/T|$

Gronau and Rosner,
PRD 65, 093012, 2002:

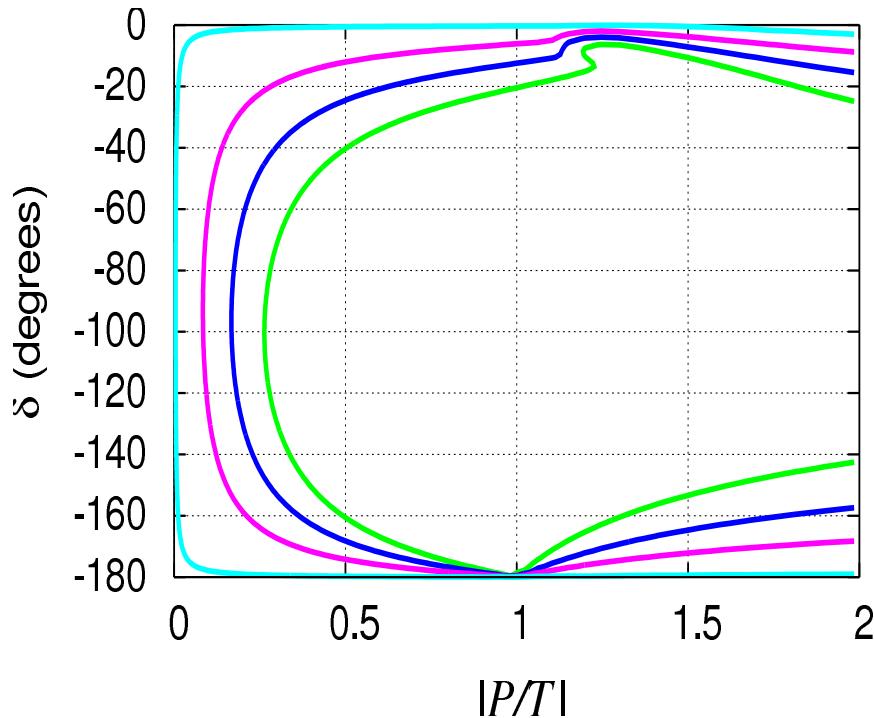
$$\begin{aligned}
 A(B^0 \rightarrow \pi^+ \pi^-) &= -(|T| e^{i\delta_T} e^{i\phi_3} + |P| e^{i\delta_P}) \\
 A(\bar{B}^0 \rightarrow \pi^+ \pi^-) &= -(|T| e^{i\delta_T} e^{-i\phi_3} + |P| e^{i\delta_P}) \\
 \Rightarrow \lambda_{\pi\pi} \equiv \frac{q}{p} \frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} &= e^{i\phi_2} \frac{1 + |P/T| e^{i(\delta + \phi_3)}}{1 + |P/T| e^{i(\delta - \phi_3)}} \\
 &\quad (\delta \equiv \delta_P - \delta_T)
 \end{aligned}$$

Take $\phi_1 = 0.725 \pm 0.037$
 $\Rightarrow 2$ constraints &
 3 unknowns
 $(\phi_2, \delta, |P/T|)$

$$\begin{aligned}
 A_{\pi\pi} &\equiv \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} = \frac{-2|P/T| \sin(\phi_1 + \phi_2) \sin \delta}{1 - 2|P/T| \cos(\phi_1 + \phi_2) \cos \delta + |P/T|^2} \\
 S_{\pi\pi} &\equiv \frac{2Im\lambda}{|\lambda|^2 + 1} \\
 &= \frac{2|P/T| \sin(\phi_1 - \phi_2) \cos \delta + \sin 2\phi_2 - |P/T|^2 \sin 2\phi_1}{1 - 2|P/T| \cos(\phi_1 + \phi_2) \cos \delta + |P/T|^2}
 \end{aligned}$$



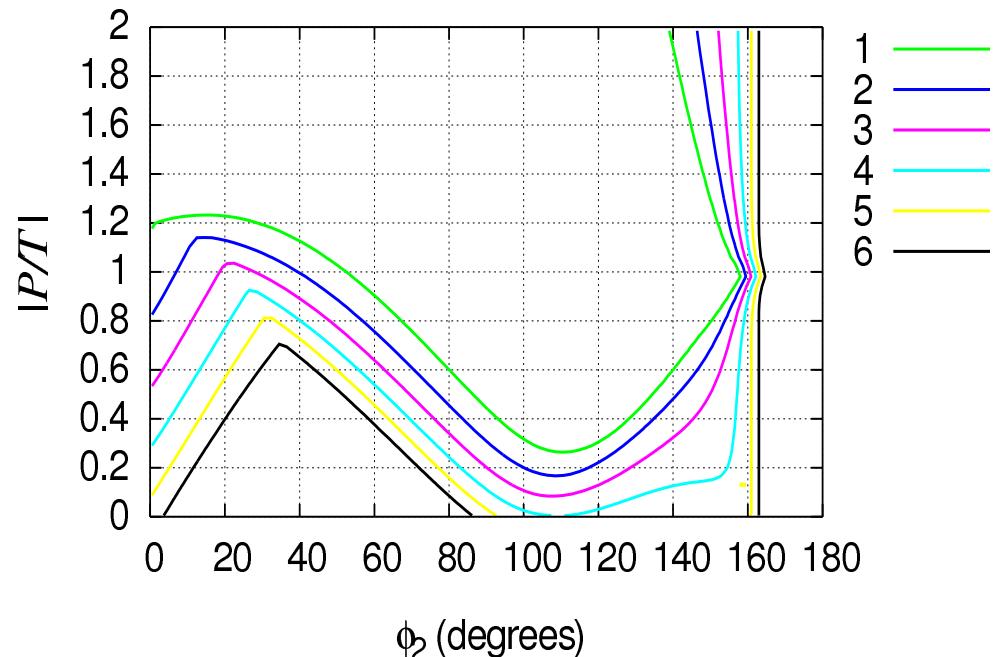
Constraints upon ϕ_2 (α) and $|P/T|$ cont'd



For $|P/T|=0.6$ (for example)
 $72^\circ < \phi_2 < 146^\circ$ (95% CL)

1
2
3
4

For any $|P/T|$
 $\delta < -4^\circ$ (95% CL)
For any δ
 $|P/T| > 0.17$ (95% CL)





Isospin analysis for ϕ_2

$SU(2)$ isospin analysis:

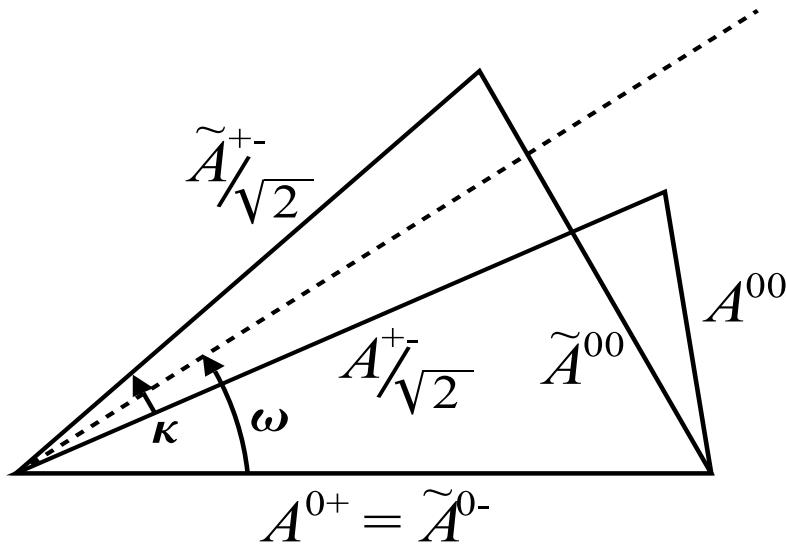
Gronau and London,
PRL 65, 3381 (1990)

$$\frac{A(B^0 \rightarrow \pi^+ \pi^-)}{\sqrt{2}} + A(B^0 \rightarrow \pi^0 \pi^0) = A(B^+ \rightarrow \pi^+ \pi^0)$$

$$\frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{\sqrt{2}} + A(\bar{B}^0 \rightarrow \pi^0 \pi^0) = A(B^- \rightarrow \pi^- \pi^0)$$

6 param. + 6 observables \Rightarrow all determined

Recent measurements (253 fb^{-1}) of
 $\bar{B}^0(B^0) \rightarrow \pi^0 \pi^0$ now make this possible



$$|A_{\text{th}}^{+-}| = \sqrt{a^{+-}(1 - \mathcal{A}_{\pi\pi})}$$

$$|\bar{A}_{\text{th}}^{+-}| = \sqrt{a^{+-}(1 + \mathcal{A}_{\pi\pi})}$$

$$|A_{\text{th}}^{0+}| = |A_{\text{th}}^{0+}| = \sqrt{a^{0+}}$$

$$|A_{\text{th}}^{00}|^2 = \frac{|A_{\text{th}}^{+-}|^2}{2} + |A_{\text{th}}^{0+}|^2 - \sqrt{2} |A_{\text{th}}^{+-}| |A_{\text{th}}^{0+}| \cos(\omega - \kappa/2)$$

$$|\bar{A}_{\text{th}}^{00}|^2 = \frac{|\bar{A}_{\text{th}}^{+-}|^2}{2} + |A_{\text{th}}^{0+}|^2 - \sqrt{2} |\bar{A}_{\text{th}}^{+-}| |A_{\text{th}}^{0+}| \cos(\omega + \kappa/2)$$

$$B_{\text{th}}^{\pi^+ \pi^-} = \left(|A_{\text{th}}^{+-}|^2 + |\bar{A}_{\text{th}}^{+-}|^2 \right) / 2 = a^{+-}$$

$$B_{\text{th}}^{\pi^0 \pi^0} = \left(|A_{\text{th}}^{00}|^2 + |\bar{A}_{\text{th}}^{00}|^2 \right) / 2$$

$$B_{\text{th}}^{\pi^0 \pi^+} = |A_{\text{th}}^{0+}|^2 (\tau_{B^\pm} / \tau_{B^0}) = a^{+0} \cdot (\tau_{B^\pm} / \tau_{B^0})$$

$$\mathcal{A}_{\text{th}}^{\pi^0 \pi^0} = \frac{|\bar{A}_{\text{th}}^{00}|^2 - |A_{\text{th}}^{00}|^2}{|\bar{A}_{\text{th}}^{00}|^2 + |A_{\text{th}}^{00}|^2}$$

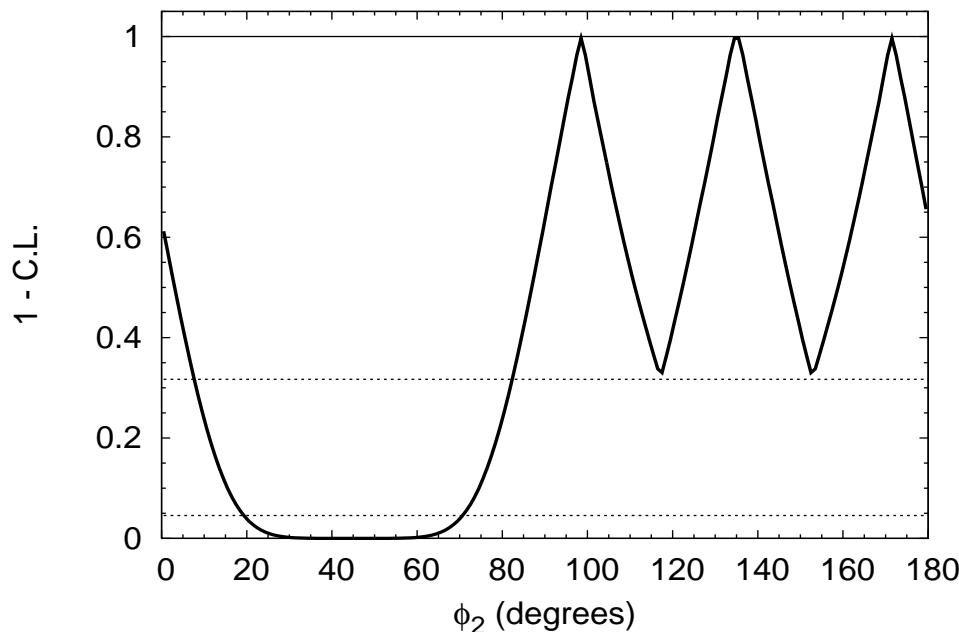
$$\mathcal{A}_{\text{th}}^{\pi^+ \pi^-} = \mathcal{A}_{\pi\pi}$$

$$\mathcal{S}_{\text{th}}^{\pi^+ \pi^-} = \sqrt{1 - \mathcal{A}_{\pi\pi}^2} \sin(2\phi_2 + \kappa)$$

Use HFAG values for $B(\pi^+\pi^-)$, $B(\pi^+\pi^0)$, $B(\pi^0\pi^0)$, $\mathcal{A}(\pi^0\pi^0)$

Calculate χ^2 :

$$\chi^2(\vec{y}) = \sum \frac{(x_{\text{exp}} - x_{\text{th}})^2}{\sigma_{\text{exp}}^2} + \chi^2_{FC}(\mathcal{A}_{\text{th}}^{\pi^+\pi^-}, \mathcal{S}_{\text{th}}^{\pi^+\pi^-})$$



$0^\circ < \phi_2 < 19^\circ$ and $71^\circ < \phi_2 < 180^\circ$
(95% CL)



$\sin(2\phi_2)$ with $B^0 \rightarrow \rho^+ \rho^-$

First results: 253 fb⁻¹
(hep-ex/0507039)

Advantages:

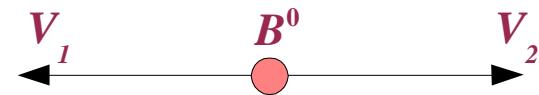
- $CP = +1$ like $\pi^+ \pi^-$
- “large” branching fraction ($> 20 \times 10^{-6}$)
- branching fraction for $B^0 \rightarrow \rho^0 \rho^0 < 1.1 \times 10^{-6}$
 - ⇒ small penguin amplitude
 - ⇒ $A_{\rho\rho} = 0$ and $S_{\rho\rho} = \sin(2\phi_2)$

But complications:

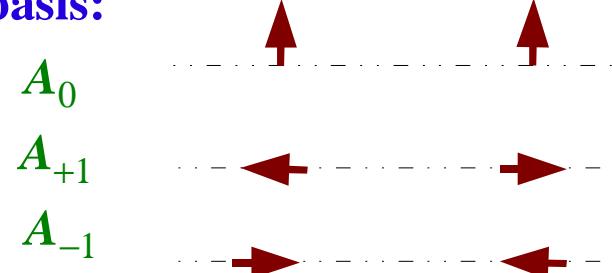
- different polarization ⇒ different CP states
- nonresonant contribution
- possible isospin $I=1$ contribution



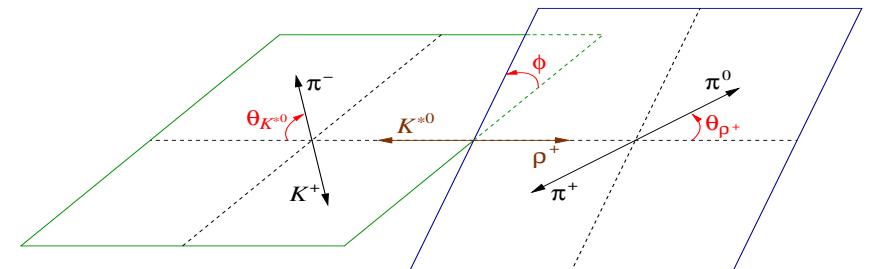
$B^0 \rightarrow VV$ polarization



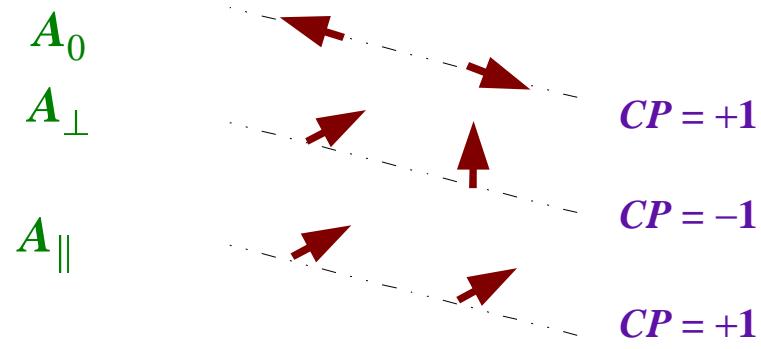
Helicity basis:



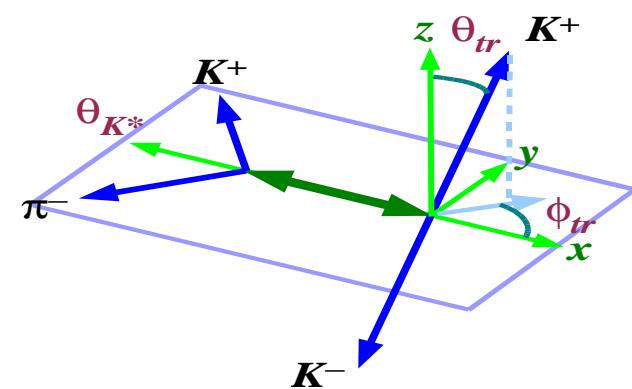
Variables θ_1, θ_2, ϕ :



Transversity basis:



Variables $\theta_K, \theta_{tr}, \phi_{tr}$:



Factorization: $f_L \equiv \frac{|A_0|^2}{|A_0|^2 + |A_{+1}|^2 + |A_{-1}|^2} = \frac{|A_0|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2} \approx 1 - O\left(\frac{m_V^2}{m_B^2}\right) \approx 1$

Kagan, PLB 601, 151 (2004)



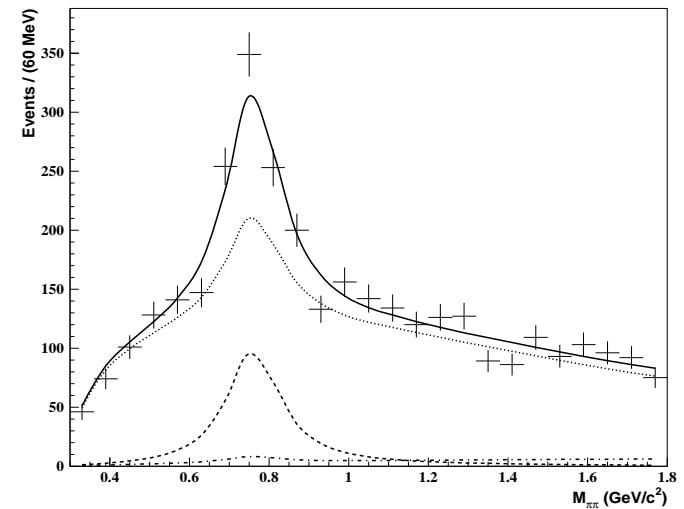
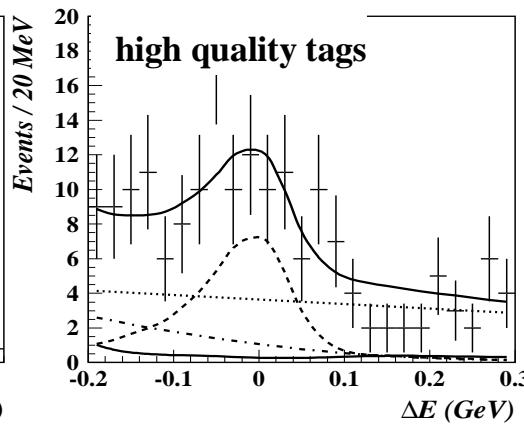
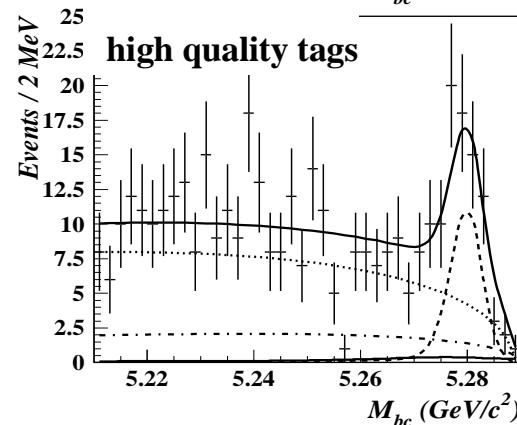
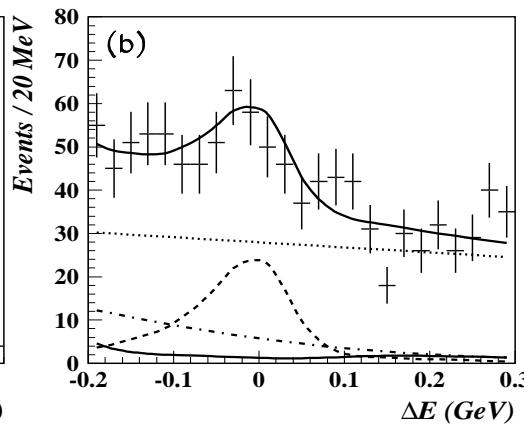
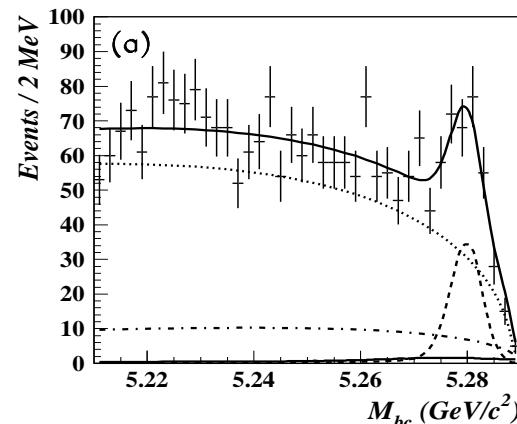
$\sin(2\phi_2)$ with $B^0 \rightarrow \rho^+\rho^-$ (cont'd)

253 fb^{-1} :

$$0.62 < m_{\pi\pi} < 0.92 \text{ GeV}/c^2$$

$$-0.10 < \Delta E < 0.06 \text{ GeV}$$

$$5.273 < m_{bc} < 5.290 \text{ GeV}/c^2$$



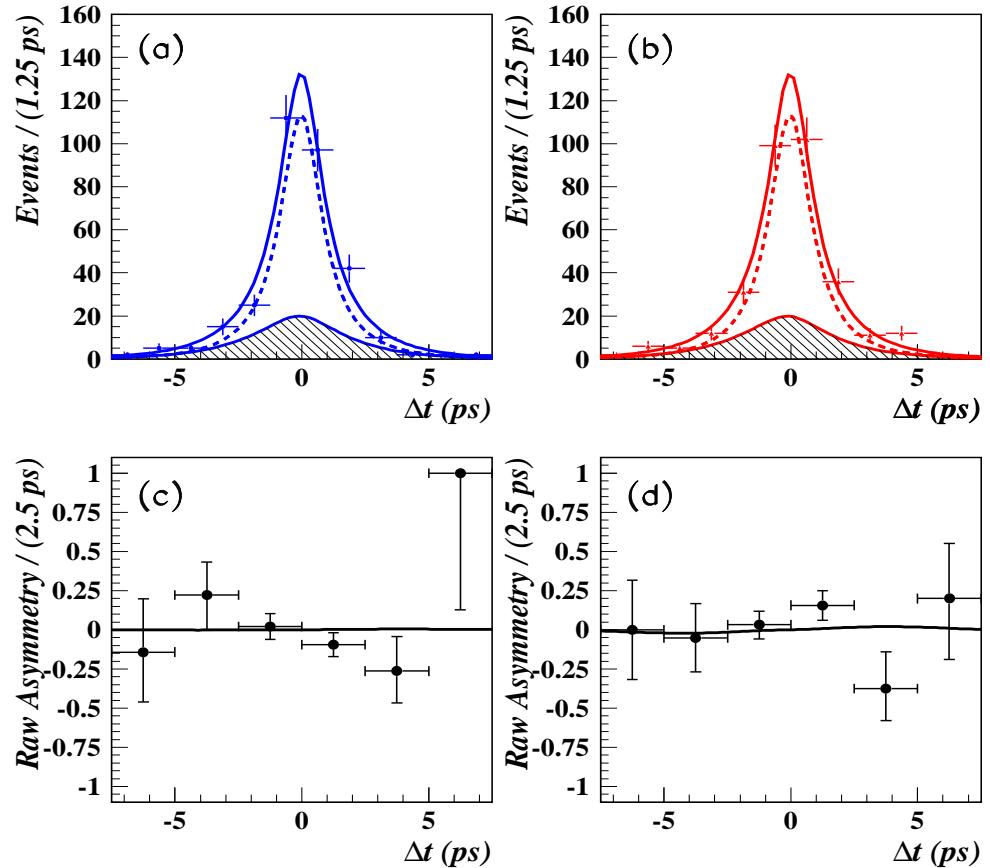
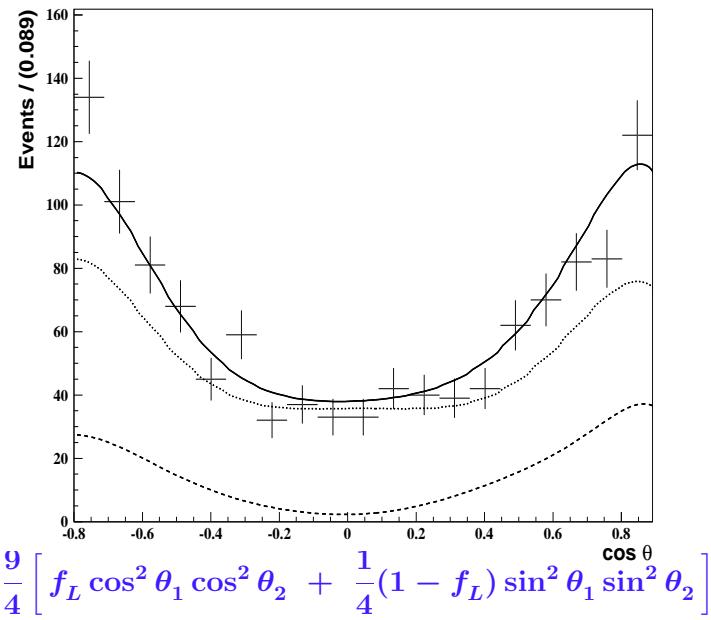
$$f_{\rho\pi\pi} = (8.9 \pm 6.2) \%$$

$$N_{\rho\rho} = 142 \pm 13$$



$\sin(2\phi_2)$ with $B^0 \rightarrow \rho^+\rho^-$ (cont'd)

253 fb^{-1} :



$$f_L = (95.1 \pm 3.3 - 3.9 \pm 3.0) \%$$

$$A_{\rho\rho} = 0.00 \pm 0.30^{+0.10}_{-0.09}$$

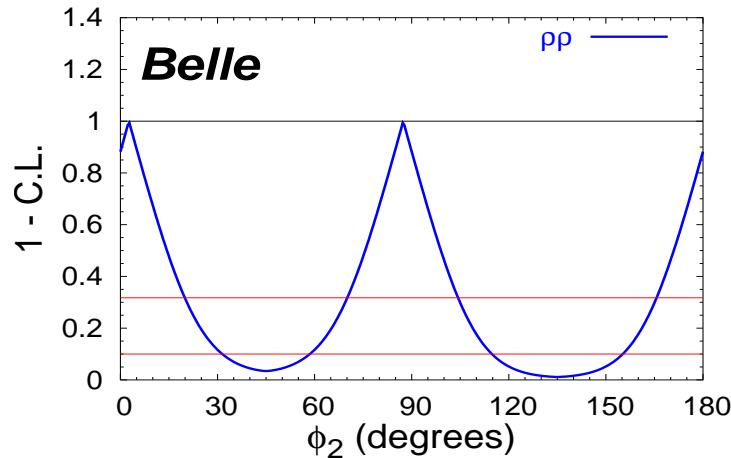
$$S_{\rho\rho} = 0.09 \pm 0.42 \pm 0.08$$

no CPV



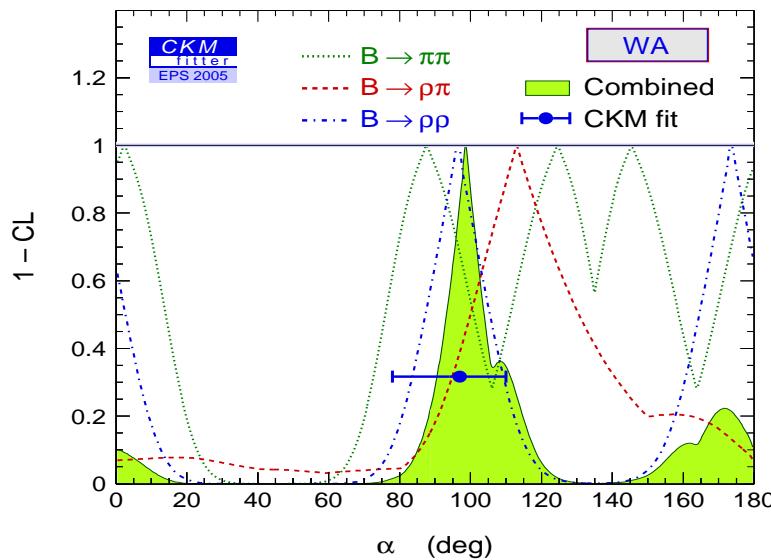
$\sin(2\phi_2)$ with $B^0 \rightarrow \rho^+\rho^-$ (cont'd)

253 fb^{-1} :



$$\phi_2 = (87 \pm 17)^\circ$$

$$59^\circ < \phi_2 < 115^\circ \quad (90\% \text{ CL})$$



(<http://ckmfitter.in2p3.fr>)

CKM fitter

Belle + BaBar:

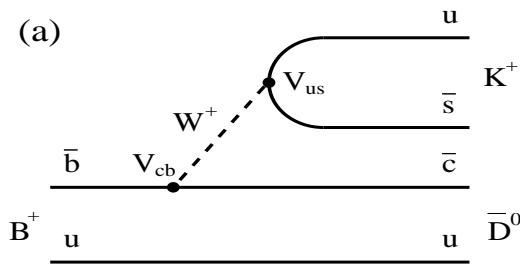
$$\phi_2 = (96 \pm 13)^\circ$$



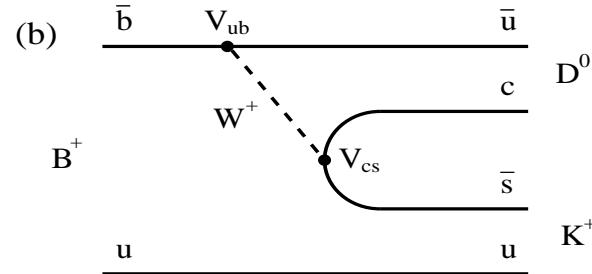
Measurement of ϕ_3

A. Bondar *et al.*, 2002 (unpublished);
 Giri *et al.*, PRD 68, 054018, 2003

$$B^+ \rightarrow \bar{D}^{0(*)} K^+$$



$$B^+ \rightarrow D^{0(*)} K^+$$



if $\bar{D}^0/D^0 \rightarrow K_S \pi^+\pi^-$, amplitudes interfere

$$m_+ = m(K_s^0, \pi^+)$$

$$m_- = m(K_s^0, \pi^-)$$

$$r = \left| \frac{A(B^+ \rightarrow D^0 K^+)}{A(B^+ \rightarrow \bar{D}^0 K^+)} \right| \sim 0.1 - 0.2$$

$$M_+ = A(m_+^2, m_-^2) + r e^{i(\delta + \phi_3)} A(m_-^2, m_+^2)$$

$$M_- = A(m_-^2, m_+^2) + r e^{i(\delta - \phi_3)} A(m_+^2, m_-^2)$$

$$\begin{aligned} |M_{\pm}|^2 &= (r^2)_- |A(m_+^2, m_-^2)|^2 + (r^2)_+ |A(m_-^2, m_+^2)|^2 + \\ &\quad 2 |A(m_+^2, m_-^2)| |A(m_-^2, m_+^2)| r \cos(\delta + \theta_{(m_+^2, m_-^2)} \pm \phi_3) \end{aligned}$$

amplitude A determined from $D^0 \rightarrow K_S \pi^+\pi^-$ Dalitz plot (from continuum)

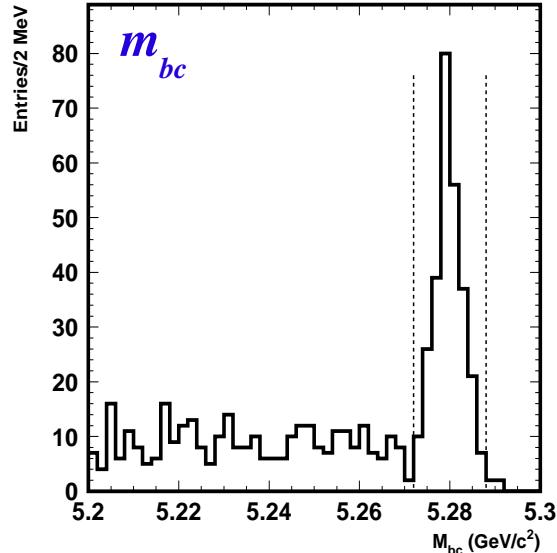
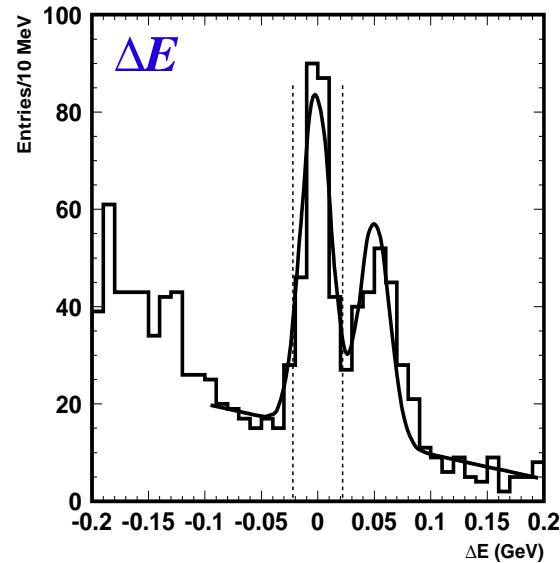


Measurement of ϕ_3

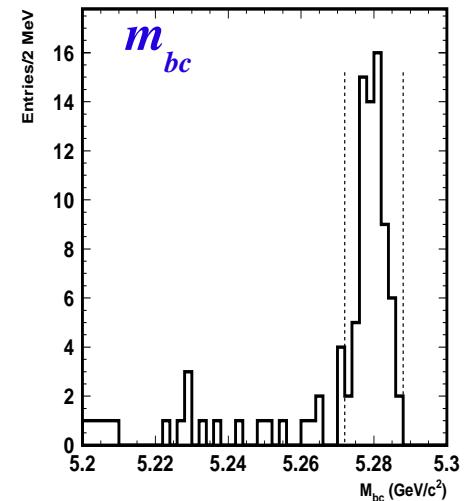
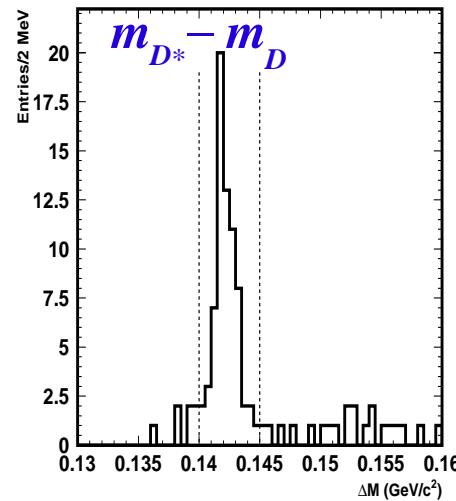
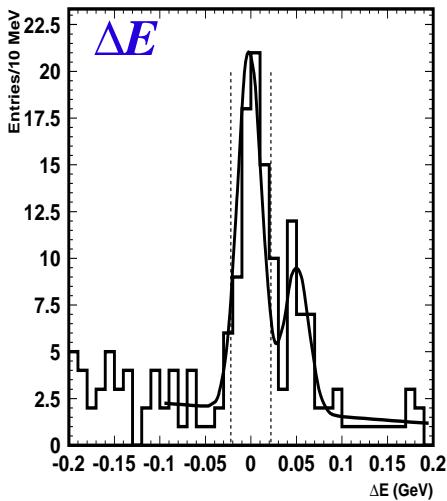
(hep-ex/0411049)

253 fb⁻¹

$B^\pm \rightarrow D^0 K^\pm$:
 $N = 209 \pm 16$
75% pure



$B^\pm \rightarrow D^{0*} K^\pm$:
 $N = 58 \pm 8$
87% pure

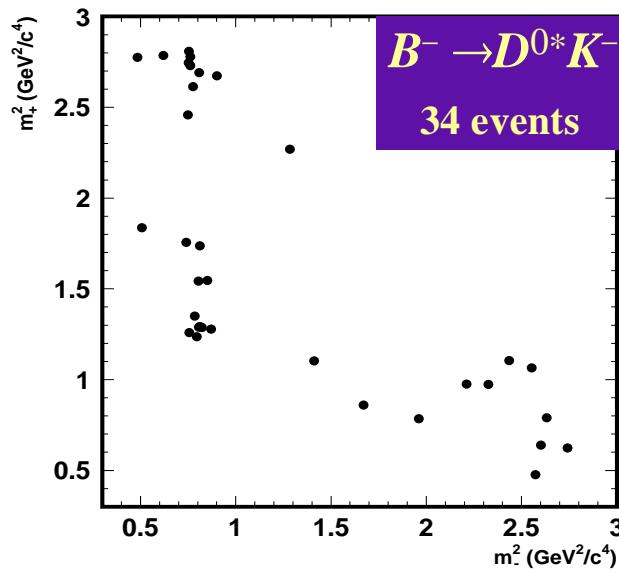
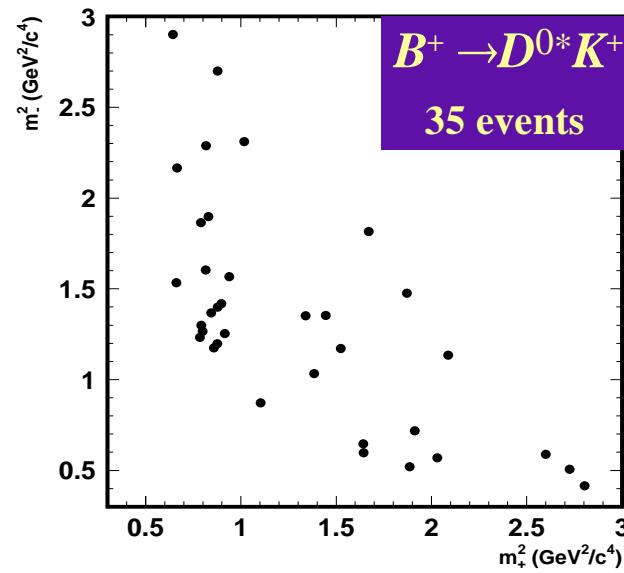
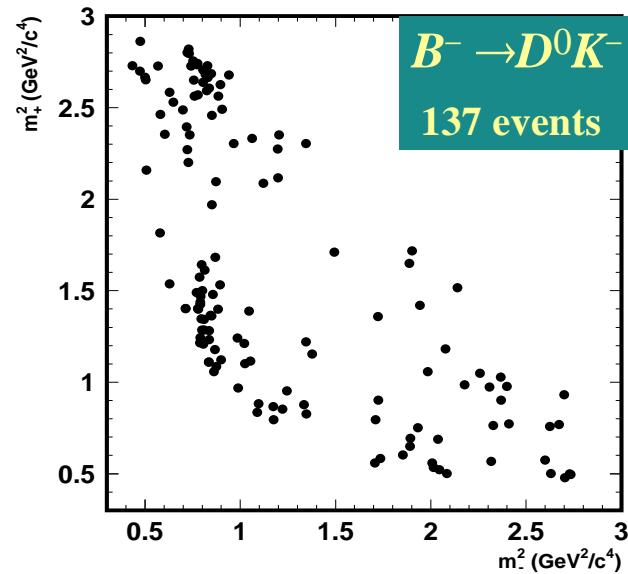
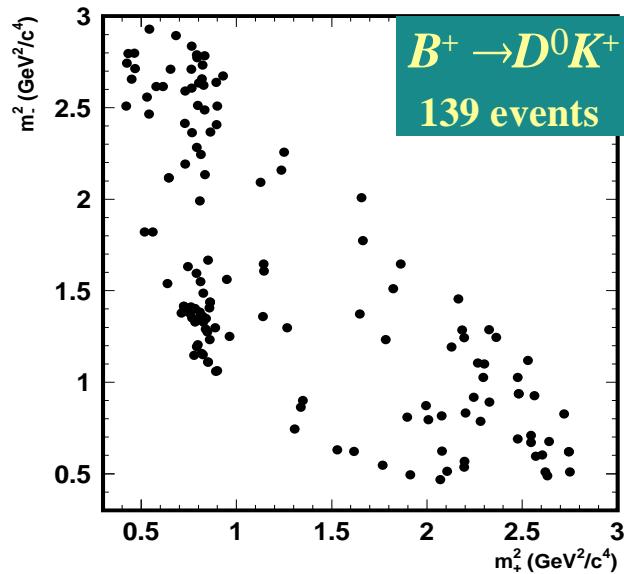




Measurement of ϕ_3

(hep-ex/0411049)

253 fb⁻¹





Measurement of ϕ_3

(hep-ex/0411049)

Do unbinned ML fit for ϕ_3, δ, r

Use toy MC with Feldman-Cousins ordering to calculate frequentist confidence regions for parameters ϕ_2, δ, r

Projecting 2σ regions for $(B^\pm \rightarrow D^0 K^\pm) + (B^\pm \rightarrow D^{0*} K^\pm)$:

$$\phi_3 = (68^{+14} \pm 13 \pm 11)^\circ$$

$$22^\circ < \phi_3 < 113^\circ \quad (95\% \text{ CL})$$

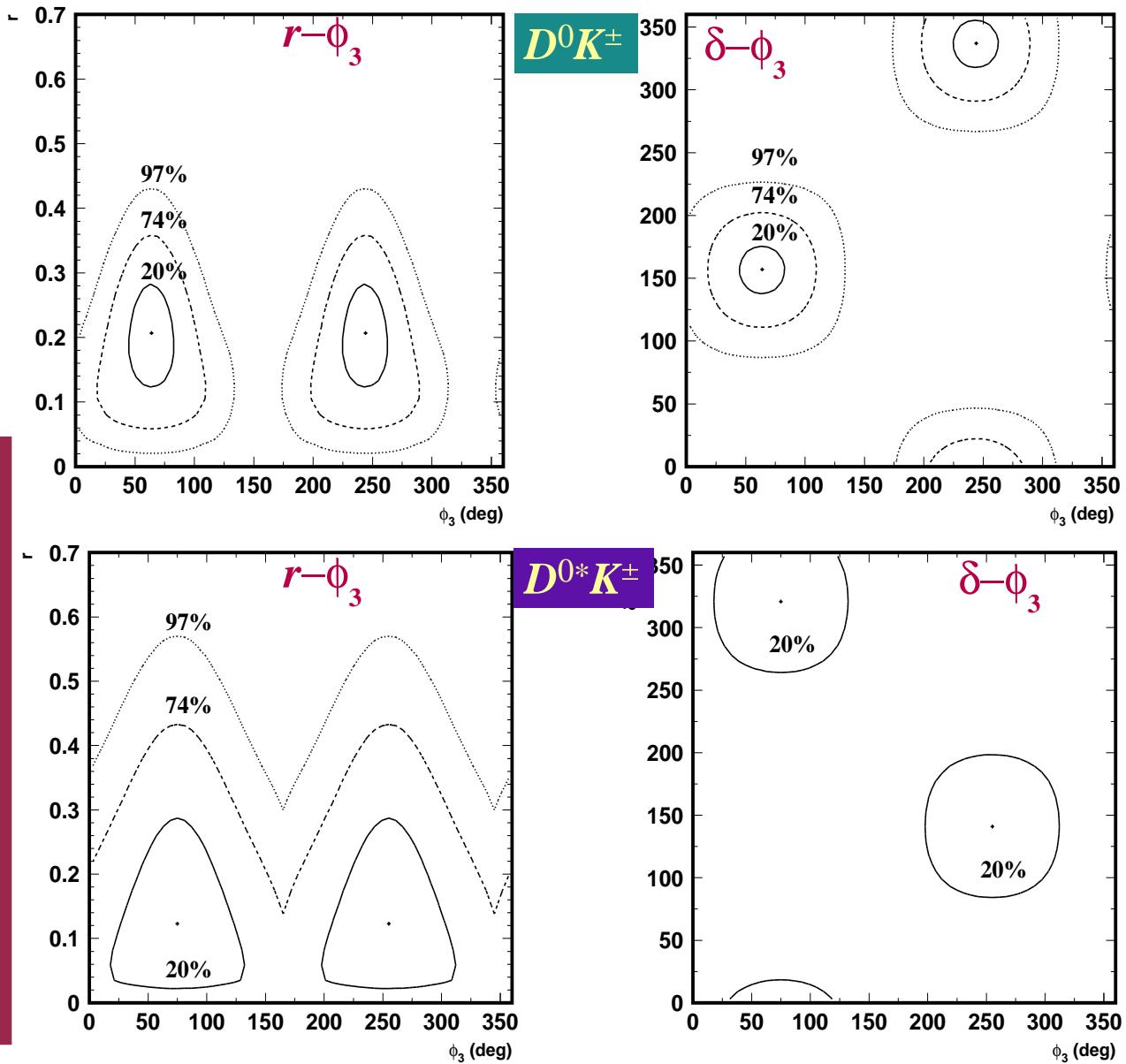
$$\delta_D = (157 \pm 19 \pm 11 \pm 21)^\circ$$

$$\delta_{D^*} = (321 \pm 57 \pm 11 \pm 21)^\circ$$

$$r_D = 0.21 \pm 0.08 \pm 0.03 \pm 0.04$$

$$r_{D^*} = 0.12^{+0.16}_{-0.11} \pm 0.02 \pm 0.04$$

CL of CPV = 98%





Summary I

357 fb⁻¹ :

$$\sin(2\phi_1): 0.652 \pm 0.039 \pm 0.020 \Rightarrow \phi_1 = (20.3^{+1.7}_{-1.6})^\circ$$

$b \rightarrow qqs$ penguin: consistent with SM, largest difference is 2σ (before: 2.3σ for avg.)

253 fb⁻¹ :

$\sin(2\phi_2)$: we have observed large CP violation in $B \rightarrow \pi^+\pi^-$:

$$A_{\pi\pi} = +0.56 \pm 0.12 \text{ (stat)} \pm 0.06 \text{ (syst)}$$

$$S_{\pi\pi} = -0.67 \pm 0.16 \text{ (stat)} \pm 0.06 \text{ (syst)}$$

($A_{\pi\pi}$ indicates direct CPV at 4σ significance)

$$|P/T| > 0.17 \text{ (95% CL)} \quad \delta < -4^\circ \text{ (95% CL)}$$

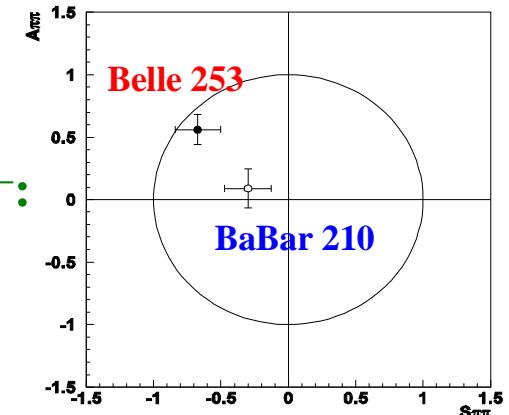
isospin analysis of $B \rightarrow \pi\pi$: $0^\circ < \phi_2 < 19^\circ$, $71^\circ < \phi_2 < 180^\circ$ (95% CL)

isospin analysis of $B \rightarrow \rho\rho$: $59^\circ < \phi_2 < 115^\circ$ (90% CL)

253 fb⁻¹ :

$$\phi_3: (68^{+14}_{-15} \pm 13 \pm 11)^\circ \quad 22^\circ < \phi_3 < 113^\circ \text{ (95% CL)}$$

CL of CP violation = 98%



- CKM fitter**
- $\phi_1 = (22 \pm 1.2)^\circ$
- Belle + BaBar:**
- $\phi_2 = (99 \pm 13 - 8)^\circ$
 - $\phi_3 = (63 \pm 15 - 12)^\circ$

Does the triangle close?

$$\begin{aligned} \phi_1 + \phi_2 + \phi_3 - 180^\circ \\ = (4 + 20 - 15)^\circ \end{aligned}$$

Yes! ...and...

**outstanding agreement
with other measurements
(ε , V_{ub} , B^0 - B^0 mixing)**

