3/27/06
W. Marciano

Muon Anomalous Magnetic Moment

Outline

1) Present Exp (821) vs Theory

$$\Delta a_\mu = 268\,(59)_{VP}\,(35)_{LBL}\,(2)_{EW}\,(63)_{Exp} \times 10^{-11}$$

2) New Physics Implications (SUSY...)

3) Standard Model Uncertainties
   (The $\tau \to \pi^- \pi^0 \nu$ Problem)

4) Future Theory + Exp. (E969 at BNL)

5) Conclusions

$$a_\mu = \frac{g_\mu - 2}{2} = \frac{\alpha}{2\pi} + \cdots$$

E821 $\longrightarrow$ $a_\mu^{exp} = 116\,592\,080\,(63) \times 10^{-11}$

Standard Model $\rightarrow$ $a_\mu^{SM} = 116\,591\,812\,(59)_{VP}(35)_{LBL}(2)_{EW} \times 10^{-11}$

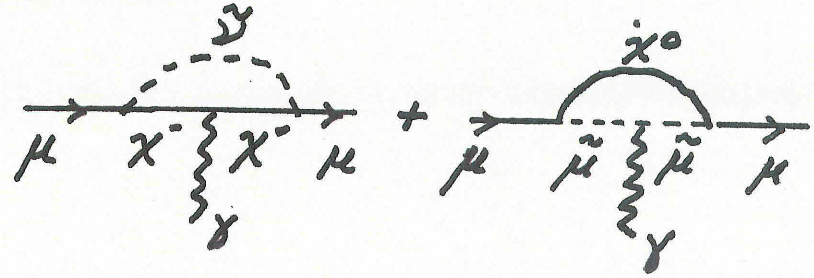
$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = \underline{268}\,\underbrace{\underbrace{(59)_{VP}(35)_{LBL}(2)_{EW}}_{(69)}(63)_{exp}}_{(93)} \times 10^{-11}$$

2.9 sigma Discrepency !

Future goal > 5 sigma

2) New Physics Implications:

i) SUSY Loops:



$$a_\mu^{SUSY} \simeq (\text{sign}\,\mu = +/-) \times 130 \times 10^{-11} \tan\beta \left(\frac{100\,\text{GeV}}{\bar{m}_{SUSY}}\right)^2$$

$$\Delta a_\mu = 268(93) \times 10^{-11}$$

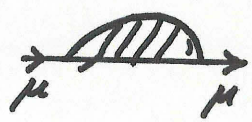
Natural Explanation!

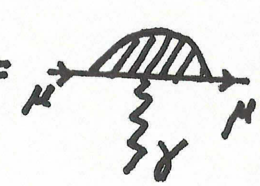
$\text{sign}\,\mu = +$ (Eliminates $\sim\frac{1}{2}$ Models)

$\bar{m}_{SUSY} \simeq 70\sqrt{\tan\beta}$ GeV $\simeq 100 \sim 500$ GeV

Someday $\tan\beta \simeq 2\left(\bar{m}_{SUSY}/100\,\text{GeV}\right)^2$ *Best Determination

ii) Muon Mass Generation via Loops (egs Dynamics, Extra Dim. ...)

$m_\mu^0 = 0$, $m_\mu =$  $\propto \Lambda$ scale of new physics

$\Delta a_\mu =$  $\sim \mathcal{O}(1) \frac{m_\mu^2}{\Lambda^2}$

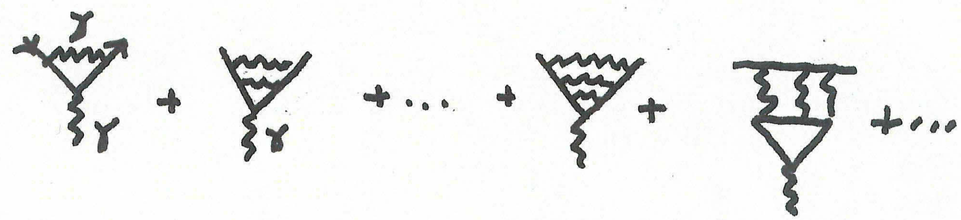
$\Delta a_\mu = 268 \times 10^{-11} \rightarrow \Lambda \simeq 2\,\text{TeV}$ (Scale Probed by LHC)

Many Examples!

E821 Results > 350 Citations

3.) Standard Model Uncertainties

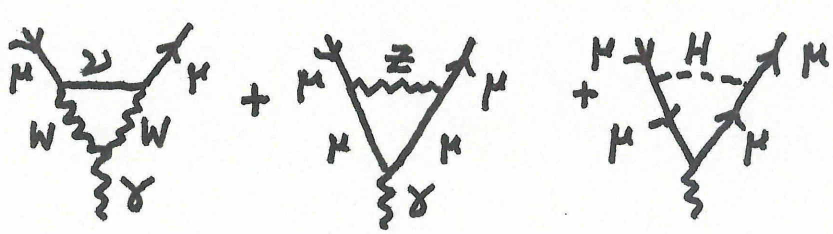
$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Hadronic}$$

i) $a_\mu^{QED} =$  5/loop

$$a_\mu^{QED} = \frac{\alpha}{2\pi} + 0.765857376\left(\frac{\alpha}{\pi}\right)^2 + 24.05050898\left(\frac{\alpha}{\pi}\right)^3 + 131.0\left(\frac{\alpha}{\pi}\right)^4 + 663(20)\left(\frac{\alpha}{\pi}\right)^5 \ldots$$

$\alpha^{-1} = 137.035999877(40)$ from a_e^{QED}

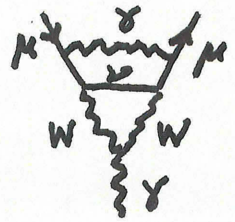
$$a_\mu^{QED} = 116\,584\,718.9\,(0.4)(0.1) \times 10^{-11}$$ Very Precise

ii) $a_\mu^{EW} =$  + 2loops ...

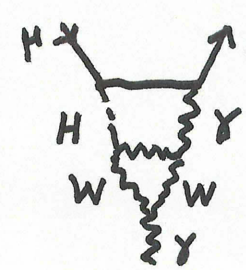
(tiny)

2loop order:

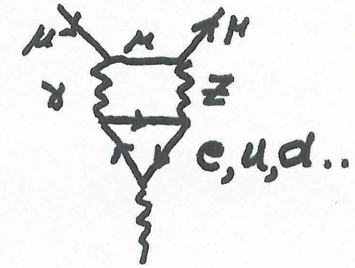
> 1600 diagrams



+ . . .



Higgs



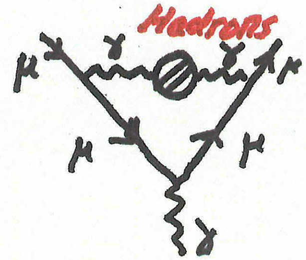
Anomaly

$$a_\mu^{EW}(2loop) = -40.7(1.0)(1.8) \times 10^{-11}$$

$$a_\mu^{EW}(3loop) \simeq O(10^{-12})$$ Negligible

$$\boxed{a_\mu^{EW} = 154(2) \times 10^{-11}}$$

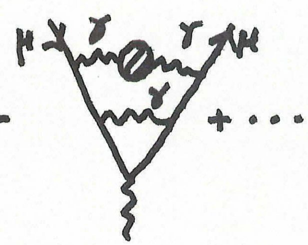
iii) $a_\mu^{Hadronic} =$



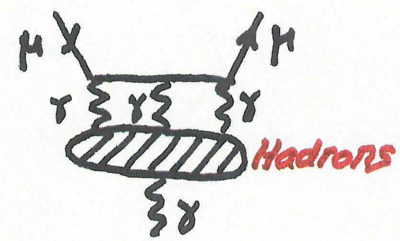
VP

Vacuum Pol.

+



+ . . .



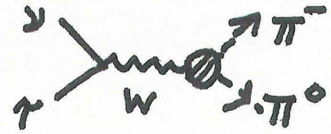
LBL

Light by Light

Use $e^+e^- \to$ Hadrons Cross Section + Dispersion Relation

$e^+e^- \to \pi^+\pi^-(\gamma)$ Dominates $\sim 72\%$ (ρ dominated)

Alternatives:

$\Gamma(\tau \to \pi^-\pi^0)$ + Isospin Corrections 

or Pure Lattice Calculation (T. Blum)

For LBL: Pion Pole + Short-Distance

or

Lattice (Some Day)

$$a_\mu^{LBL} = +120(35) \times 10^{-11}$$ Conservative Error

Interesting History

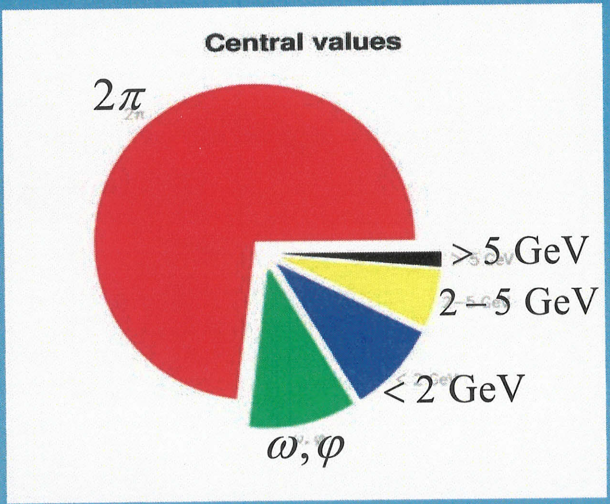
Now Stable

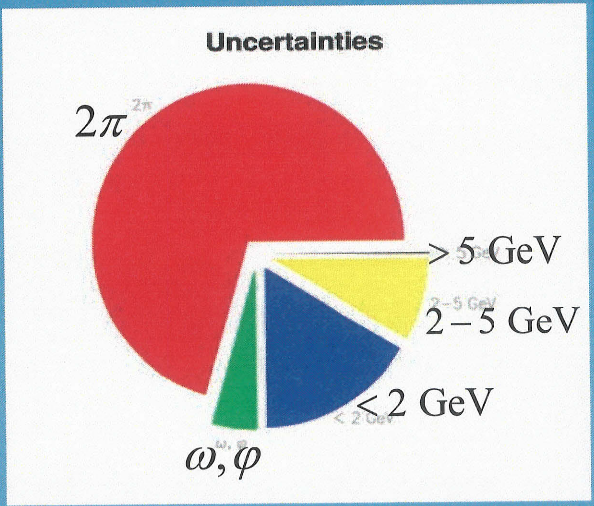
Role of e+e- → π+π- in evaluation of the hadronic contribution to muon (g-2)

Hadronic contribution to the muon (g-2) is calculated via dispersion integral:

$$a_\mu^{had}(l.o.) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R(s)$$

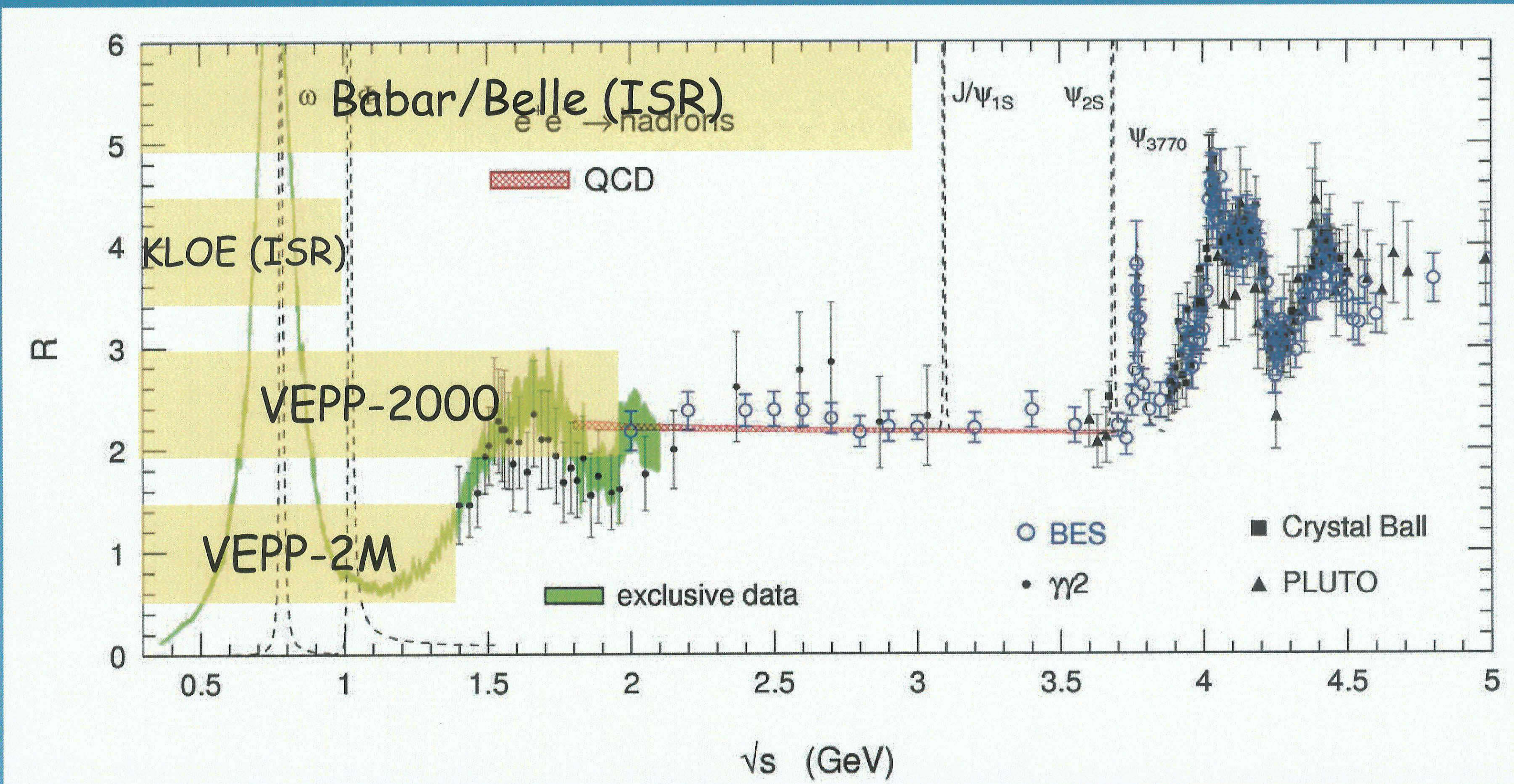
Contribution to the integral from different modes e+e-→hadrons:





e+e- → 2π gives dominant contribution both to the value and to the uncertainty of the hadronic contribution

R(s) measurements at low s



At low s the cross-section is measured independently for each final state

Hadronic Vacuum Polarization (L.O.) (lowest order)

New e^+e^- results CMD2, SMD, KLOE

CMD2, SMD: $e^+e^- \to \pi^+\pi^-$; KLOE: $e^+e^- \to \gamma\pi^+\pi^-$ (Radiative Return)

*All Consistent Now

$$a_\mu^{\text{Hadronic}}(\text{L.O.}) = 6917(59) \times 10^{-11}$$

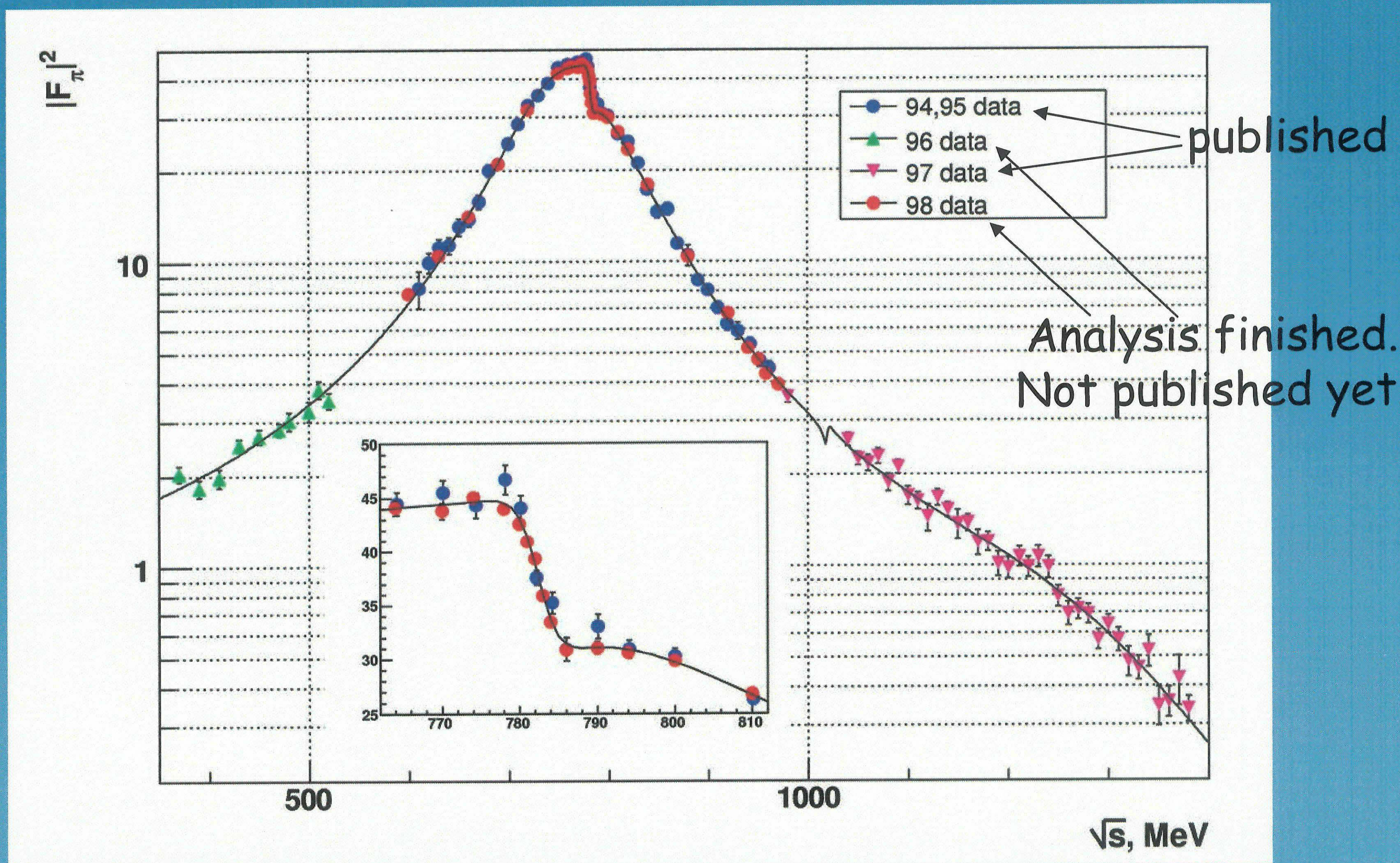
$$+ \text{Higher Order } (-98 \times 10^{-11}) + \text{LBL } (+120(35) \times 10^{-11})$$

$$a_\mu^{\text{Hadronic}} = 6939(59)_{VP}(35)_{LBL} \times 10^{-11}$$

$$a_\mu^{SM} = 116\,591\,812\,(59)_{VP}(35)_{LBL}(2)_{EW} \times 10^{-11}$$

$$\Delta a_\mu = 268(59)_{VP}(35)_{LBL}(2)_{EW}(63)_{Exp} \times 10^{-11}$$

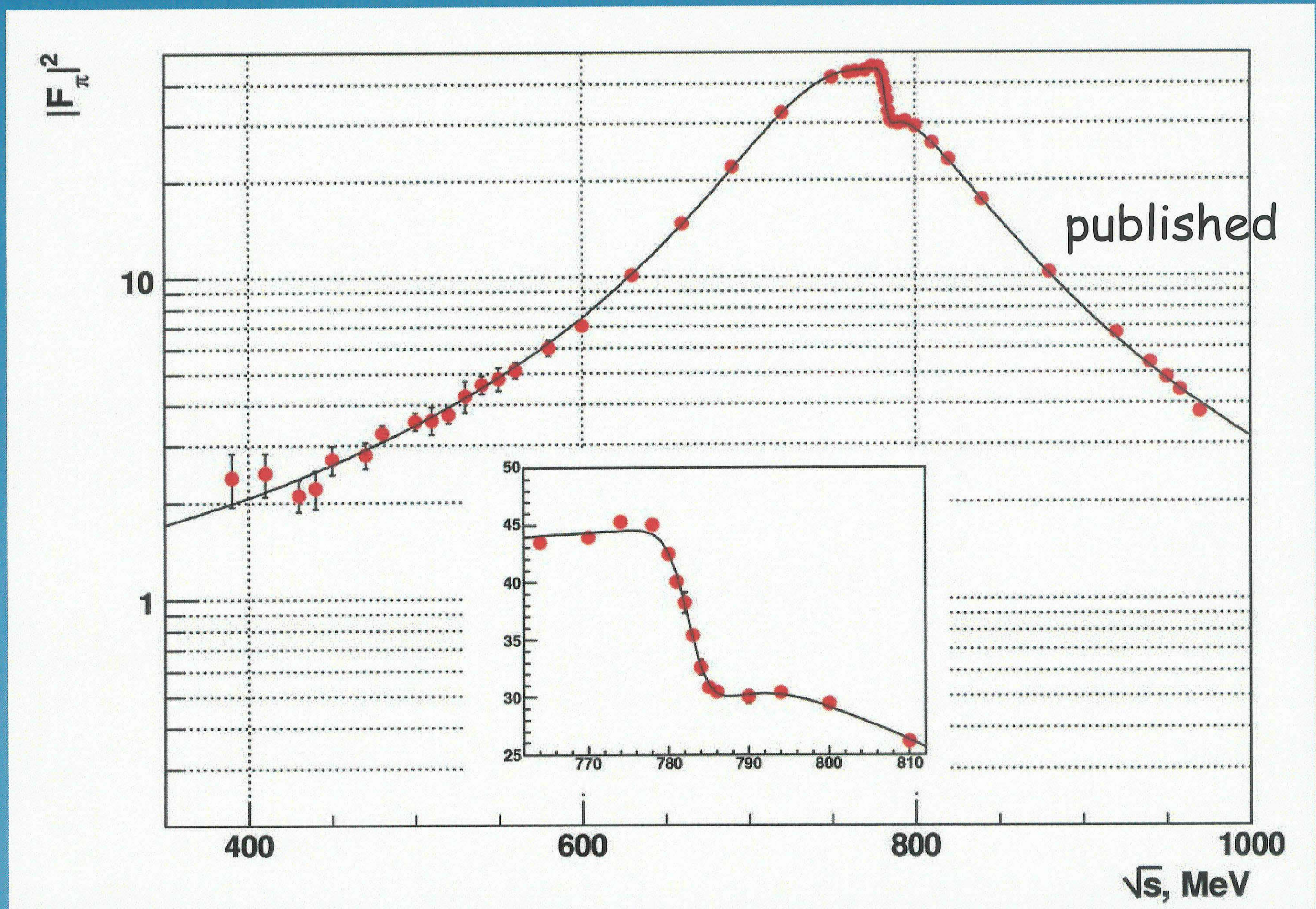
Pion formfactor (CMD-2)



Systematic error

0.7% | 0.6% (95)/ 0.8% (98) | 1.2-4.2%

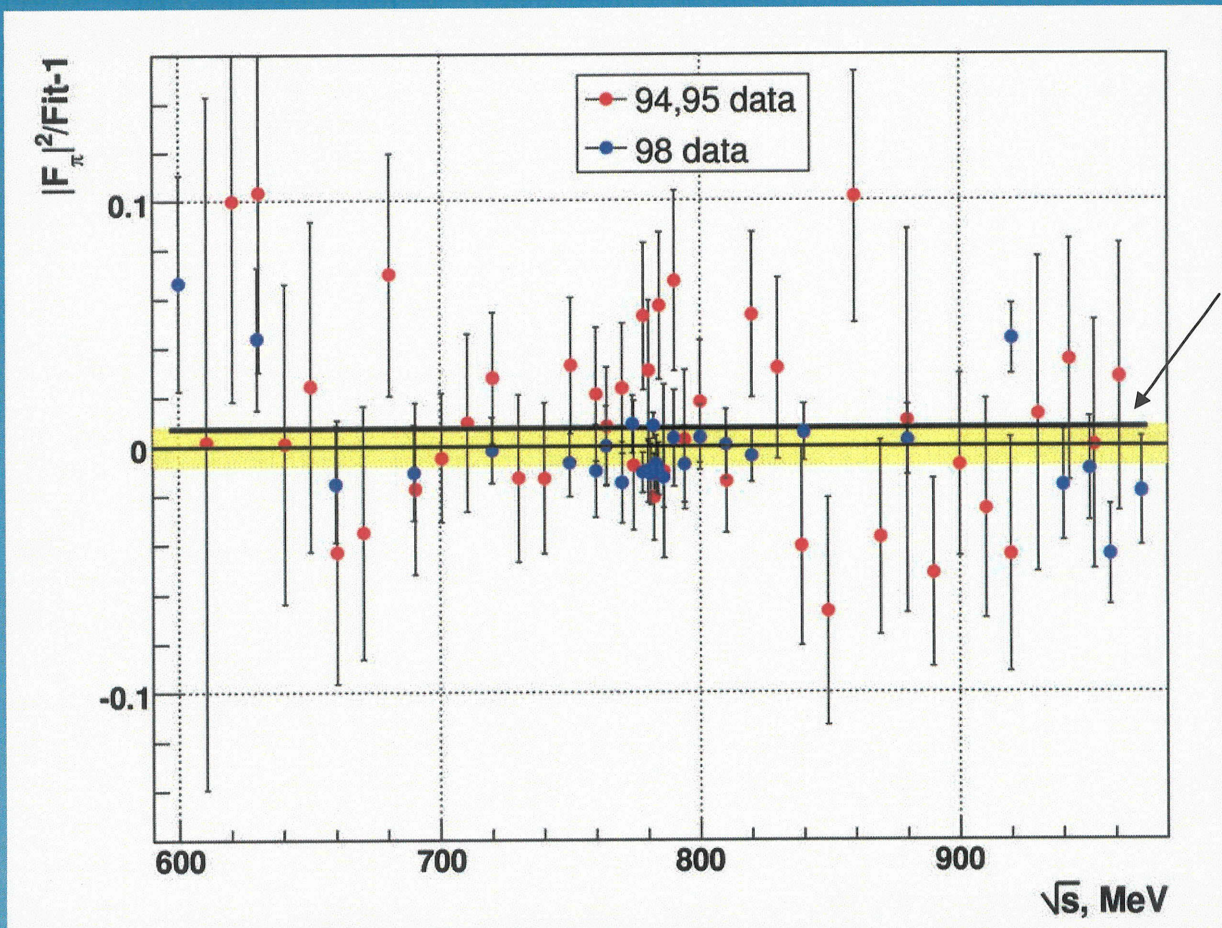
Pion formfactor (SND)



Systematic error

3.2% 1.3%

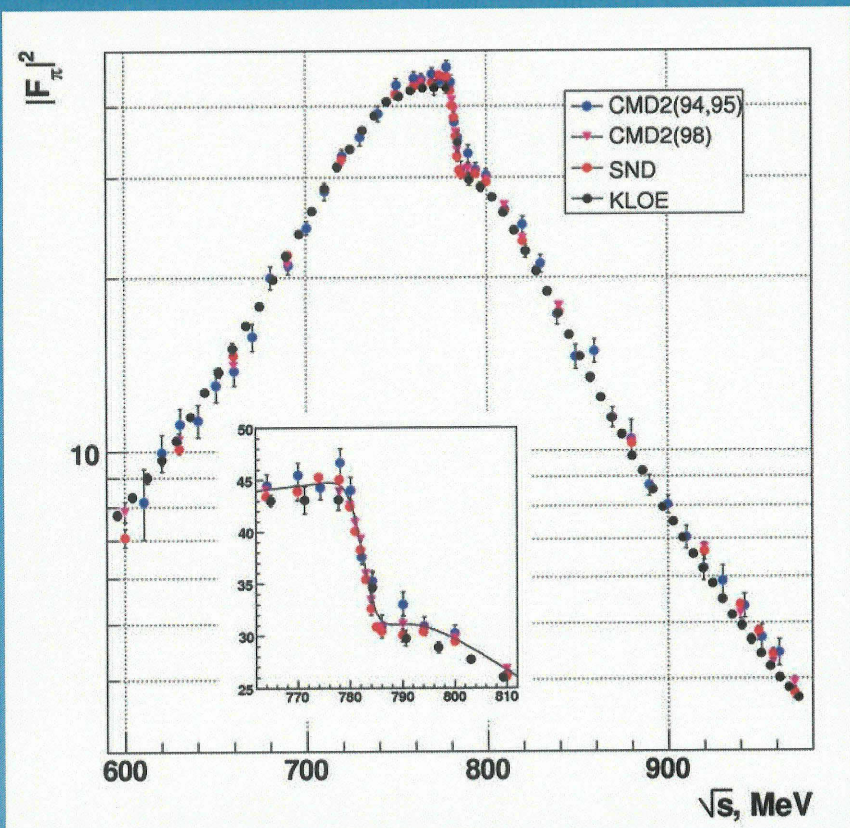
Comparison of CMD2(95) and CMD2(98)

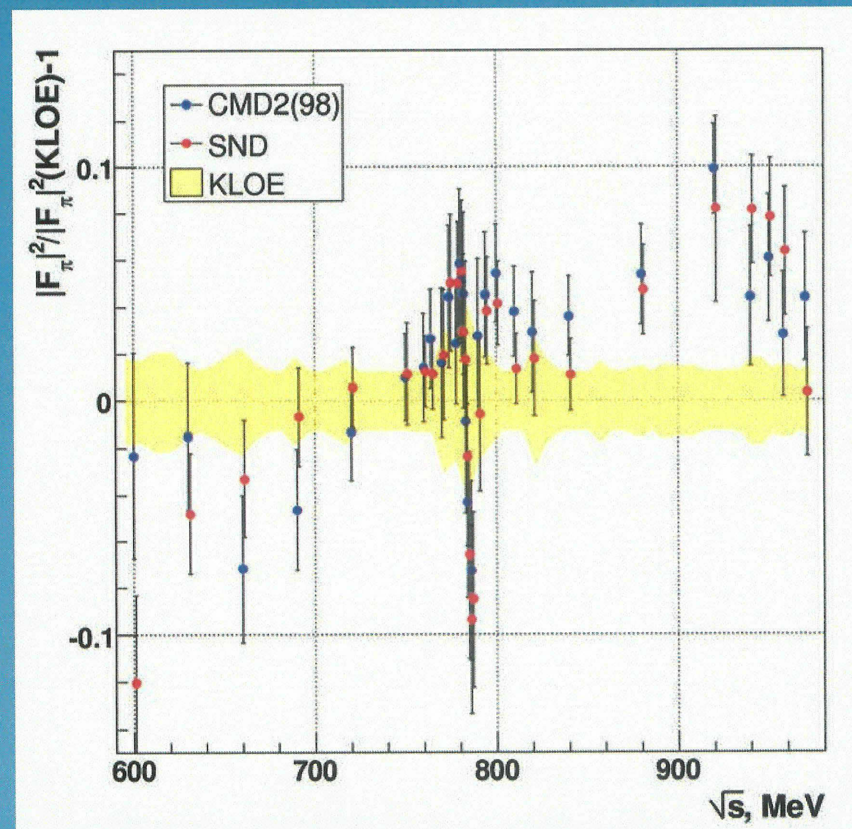


Δ(95-98)≈0.7%±0.5%

Plotted is $\frac{\Delta F}{F} = \frac{|F_\pi|^2(\text{exp})}{|F_\pi|^2(\text{CMD-2 fit})} - 1$

Comparison with KLOE





Implication to a_μ (very unofficial)

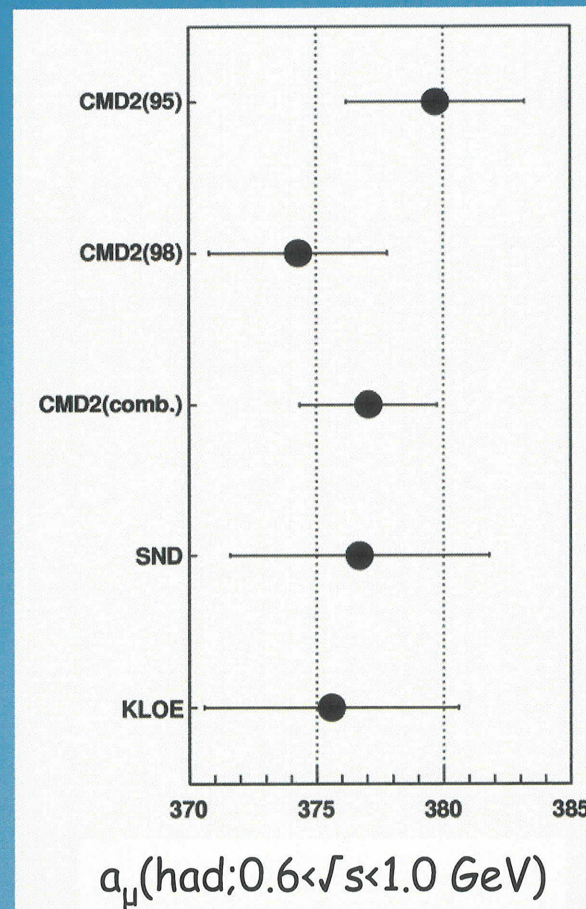
Davier, Marciano-2004:

$\Delta a_\mu = (23.9 \pm 7.2_{had,LO} \pm 3.5_{other} \pm 5.8_{exp}) \cdot 10^{-10}$

- $0.610 < \sqrt{s} < 0.960$ GeV

CMD-2 (95):	379.7 ± 2.6 ± 2.3 (3.5)
CMD-2 (98):	374.3 ± 1.8 ± 3.0 (3.5)
CMD-2 (comb):	377.05 ± 2.2 ± 1.5 (2.7)
SND:	376.7 ± 1.3 ± 4.9 (5.1)
KLOE:	375.6 ± 0.8 ± 4.9 (5.0)

S.Eidelman, private communication



The $\tau \to \pi^- \pi^0 \nu$ Problem

$\Gamma(\tau \to \pi^- \pi^0 \nu)$ + Isospin (QED) Corrections seem to disagree with $e^+e^- \to \pi^+\pi^-$

τ Data From Aleph, Cleo, Opal

Alleviates $\sim \frac{1}{2}$ Discrepency

But $\sigma(e^+e^- \to \pi^+\pi^-)$ data from CMD2, SND + KLOE also consistent

Main Difference $\sqrt{s} \gtrsim 1\,\text{GeV}$ Spectrum

Recent Belle: $\tau \to \nu_\tau \pi^- \pi^0$ data statistically powerful!

Spectrum Shape Disagrees with Aleph!

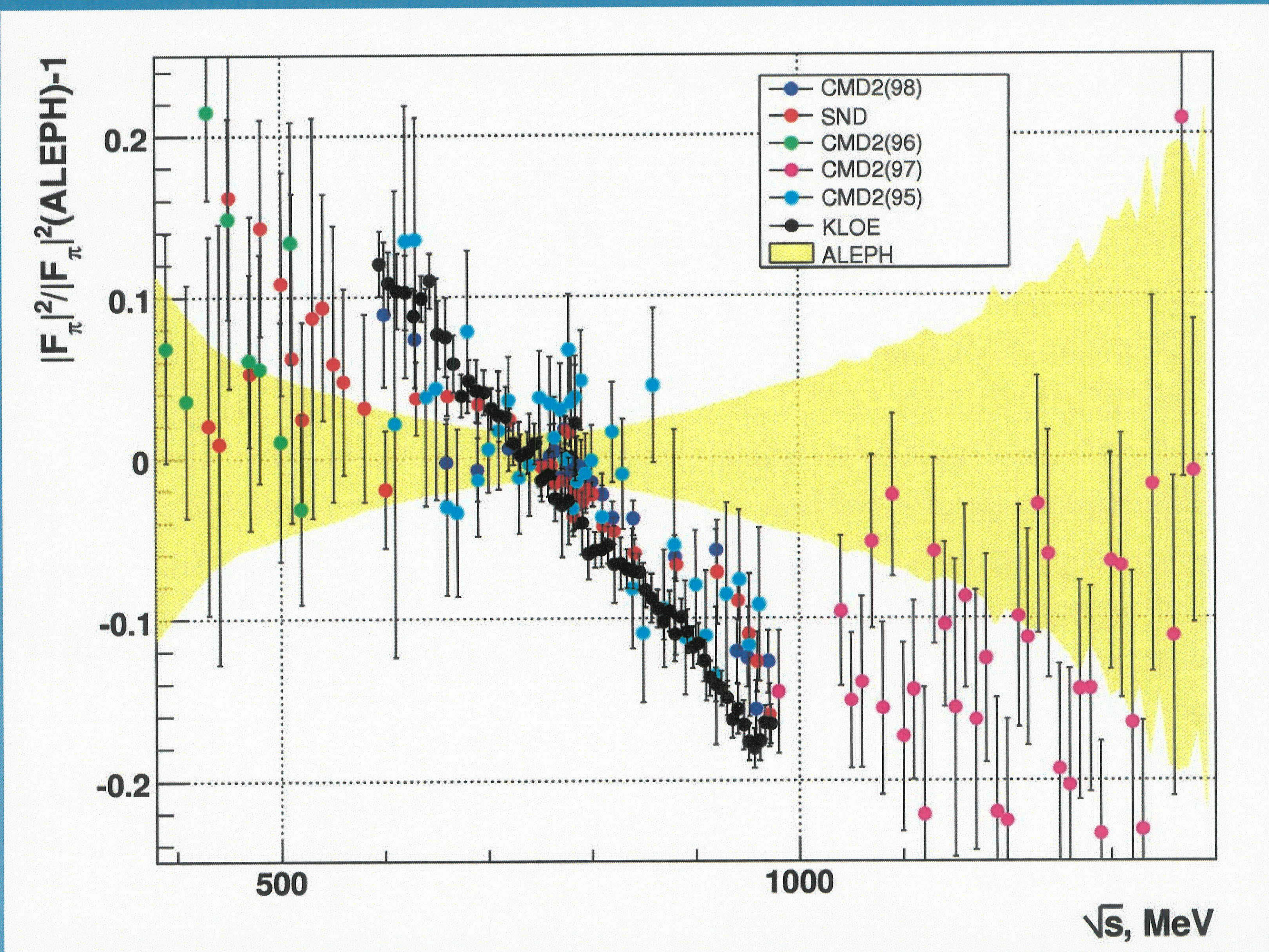
$m_\rho = 773.9 \pm 0.1$ MeV vs 775.5 ± 0.7 MeV Aleph

+ other differences Γ_ρ, $m_{\rho'}$, $m_{\rho''}$

$BR(\tau \to \nu \pi^- \pi^0)$ somewhat smaller (25.15%)

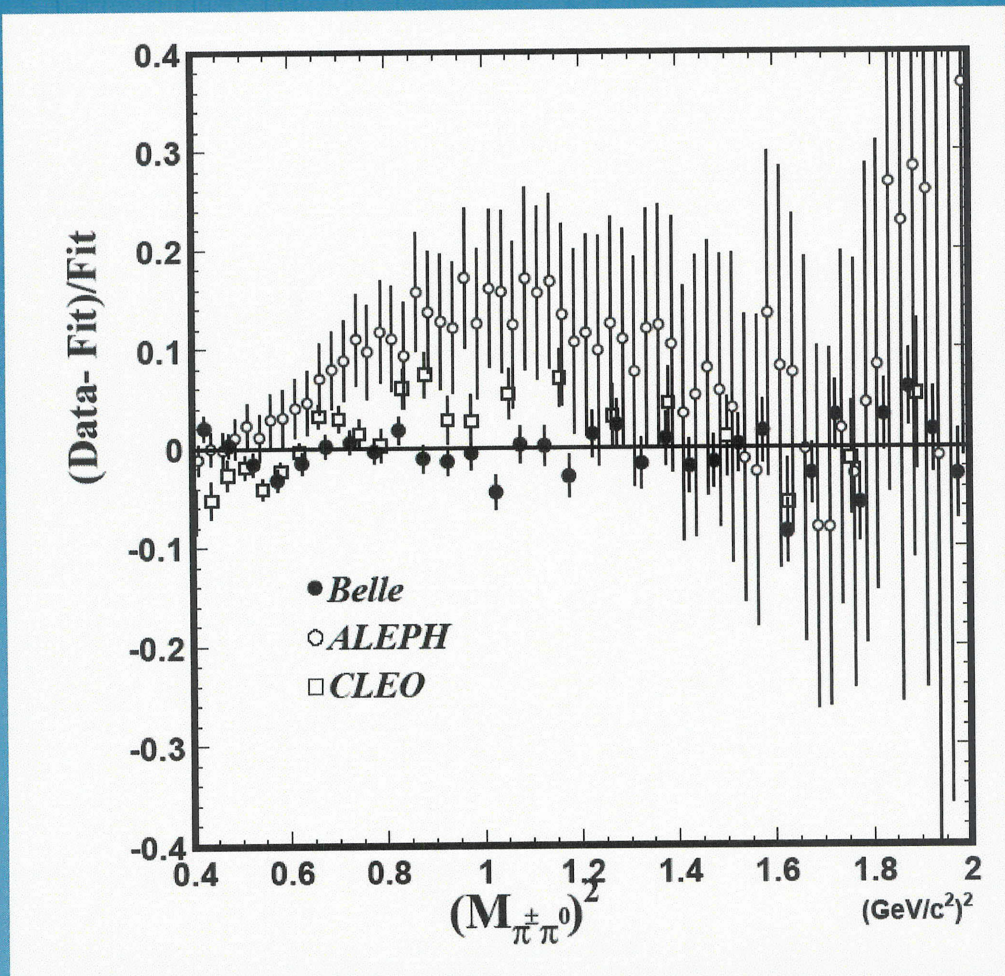
But claim to agree with Aleph on a_μ^{Hadronic}? Isospin Corr.!

Comparison with ALEPH ($\tau \to \pi\pi^0\nu$)



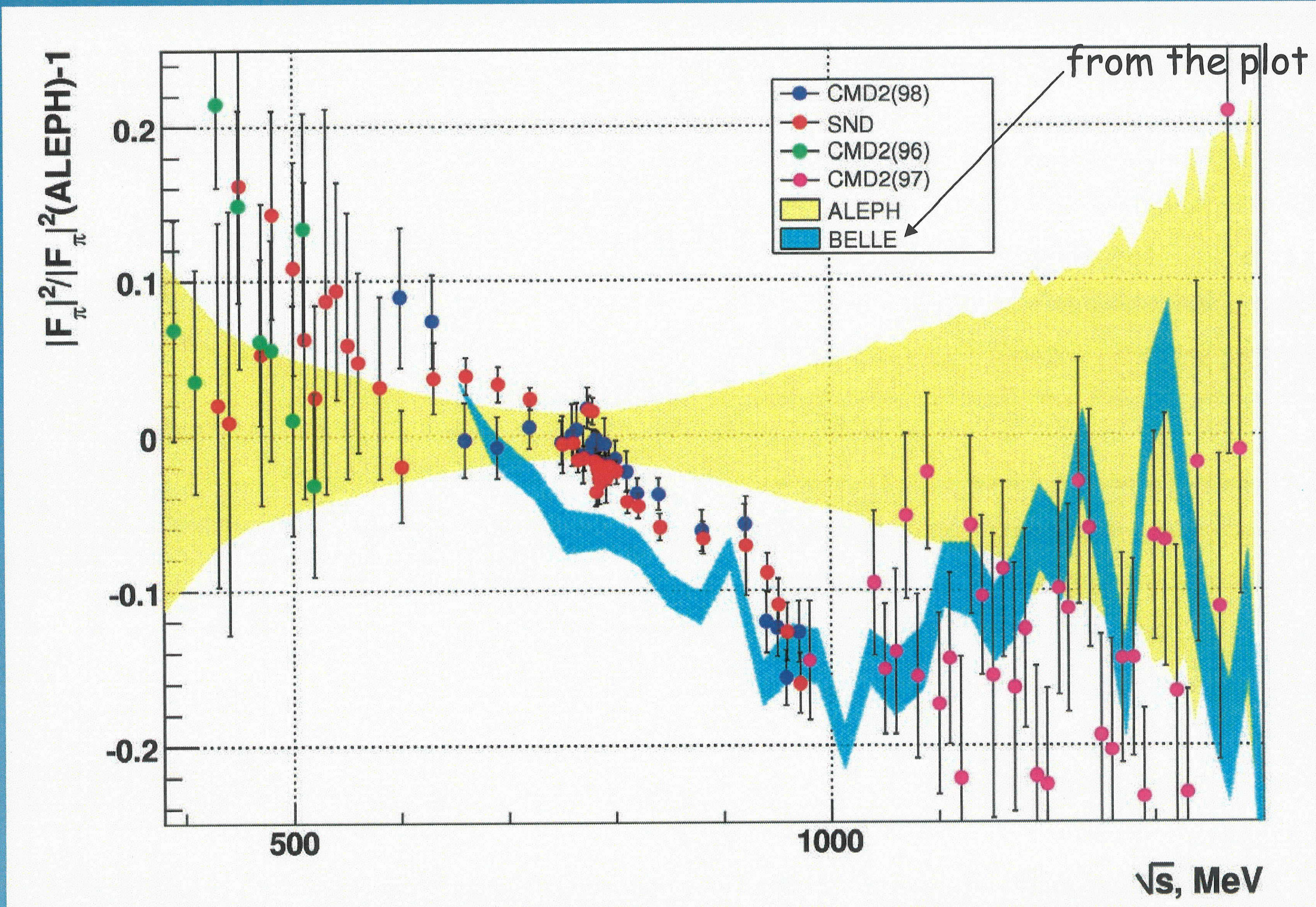
Recent Belle analysis of tau decays

Pion formfactor, calculated from the spectral function of $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay



EPS2005 Proceedings - hep-ex/0512071

Comparison with ALEPH and BELLE tau decay data



4) Future Theory + Exp (E969 at BNL)

$$\Delta a_\mu = 268\,(59)_{VP}(35)_{LBL}(2)_{EW}(63)_{Exp} \times 10^{-11}$$

$(59)_{VP} \rightarrow (30)_{VP} \rightarrow (25)_{VP}$

$(35)_{LBL} \rightarrow (20)_{LBL}$

$(63)_{Exp} \rightarrow (30)_{Exp}$ (E969) or $(27)_{Exp}$ if ave with E821

BaBar, Belle, Cleo, Nooosibirsk $\rightarrow (30)_{VP}$

Tau Data? $\longrightarrow (25)_{VP}$

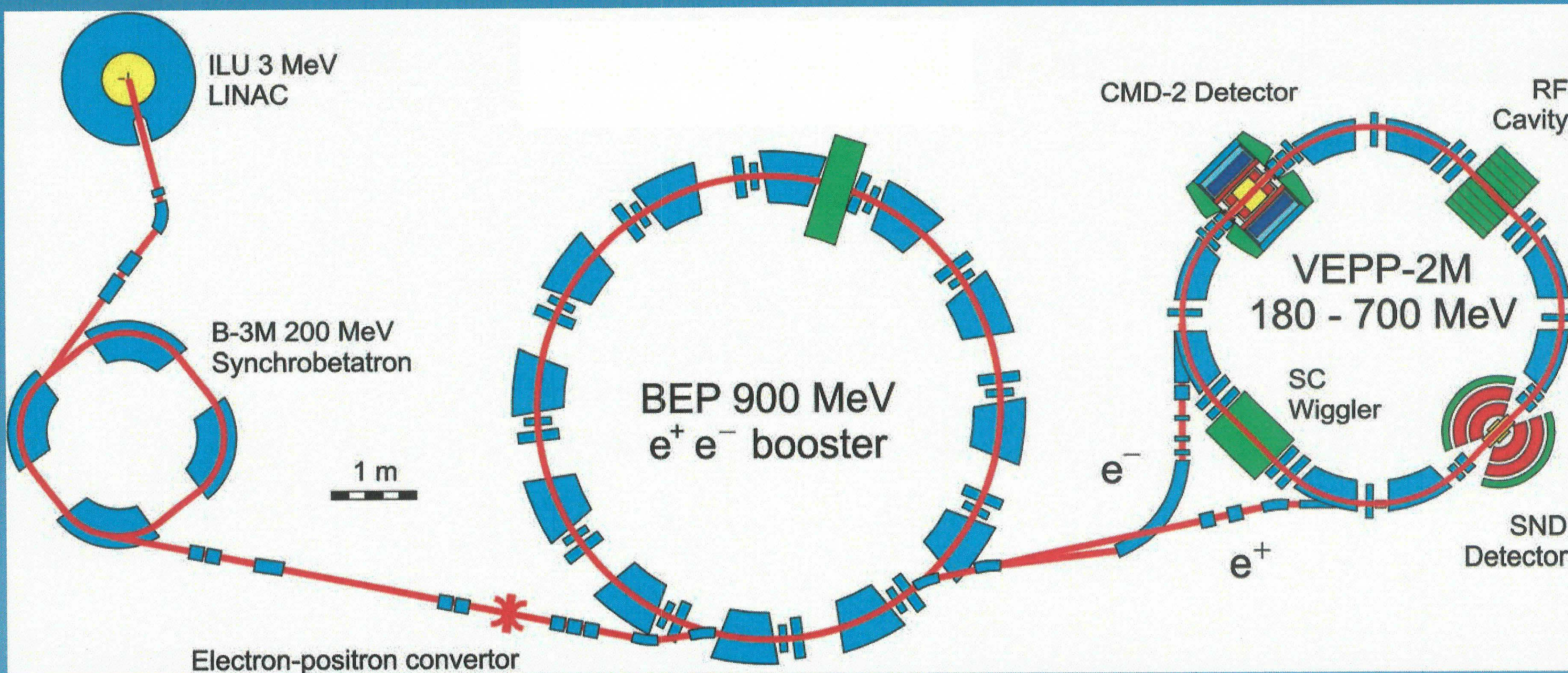
$$\Delta a_\mu = 268\,(36)(27)_{Exp} \times 10^{-11} \quad \text{or} \quad 268(32)(27)_{Exp} \times 10^{-11}$$

6 sigma   6.4 sigma

Remains 5σ down to $\longrightarrow 225 \times 10^{-11}$ 209×10^{-11}

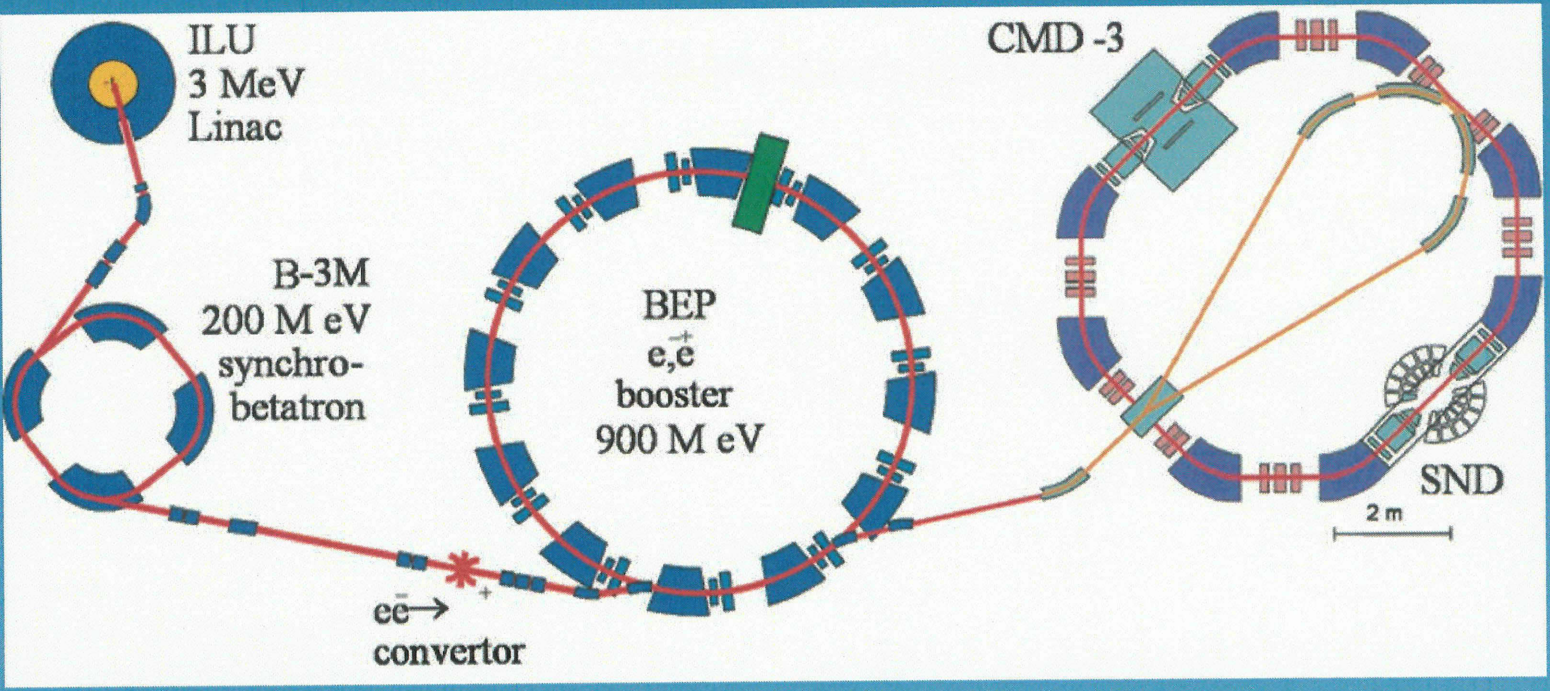
$(167 \times 10^{-11} \rightarrow 4$ sigma$)$

VEPP-2M collider



- VEPP-2M collider: 0.36-1.4 GeV in c.m., $L \approx 10^{30}\ 1/cm^2 s$ at 1 GeV
- Detectors CMD-2 and SND: 50 pb^{-1} collected in 1993-2000

Future measurements at VEPP-2000



- Factor >10 in luminosity
- Up to 2 GeV c.m. energy
- CMD-3: major upgrade of CMD-2 (new drift chamber, LXe calorimeter)
- measure 2π mode to 0.2-0.3%
- measure 4π mode to 1-2%
- overall improvement in R precision by factor 2-3

Conclusions: E969 Goal $\pm 30 \times 10^{-11}$ Well Motivated

$> 5\sigma$ Discovery Potential

(Even if τ-data moves Δa_μ down somewhat)

Complements LHC Discoveries eg $\tan\beta_{SUSY}$

Exp. + Theory Unc. Well Matched
Over Next Few Years

E969 - Must Do Experiment