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NECESSARY BANDWIDTH AND SPECTRAL PROPERTIES OF DIGITAL MODULATION

DAVID J. COHEN

U.S. DEPARTMENT OF COMMERCE
Malcolm Baldrige, Secretary

David J. Markey, Assistant Secretary
for Communications and Information

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TABLE OF CONTENTS

Subsection Page

ACKNOWLEDGMENT........ vii

ABSTRACT........ ix

SECTION 1

INTRODUCTION

BACKGROUND........ 1

OBJECTIVE........ 2

APPROACH........ 2

SECTION 2

CONCLUSIONS AND RECOMMENDATIONS

GENERAL CONCLUSIONS........ 3

SPECIFIC CONCLUSIONS........ 3

RECOMMENDATIONS........ 4

TABLE OF CONTENTS (Continued)

Subsection Page

SECTION 3

BANDWIDTH AND NECESSARY BANDWIDTH CONCEPTS

BANDWIDTH CONCEPTS.. 5

NONREGULATORY DEFINITIONS OF BANDWIDTH.................................... 6

REGULATORY DEFINITIONS OF BANDWIDTH....................................... 9

Necessary Bandwidth.. 11

Occupied Bandwidth... 12

Relationship of Occupied Bandwidth to Necessary Bandwidth................ 12

Authorized Bandwidth... 13

NTIA MANUAL FORMULAS FOR DIGITAL MODULATION.............................. 13

NECESSARY BANDWIDTH OF DIGITAL SIGNALS................................... 15

Definitions.. 15

Effects of Filtering and Tradeoffs of Bandwidth and E_b/N_o............. 16

SECTION 4

PHASE SHIFT KEYING (PSK) MODULATION

POWER SPECTRUM OF PSK MODULATION... 19

NECESSARY BANDWIDTH.. 22

SPECTRUM FILTERING AND THE K FACTOR...................................... 26

TABLE OF CONTENTS (Continued)

Subsection Page

SECTION 5

MINIMUM SHIFT KEYING (MSK) AND SINUSOIDAL FREQUENCY SHIFT KEYING (SFSK) 31

SECTION 6

FREQUENCY SHIFT KEYING (FSK)

POWER SPECTRAL DENSITY (PSD).. 37

NECESSARY BANDWIDTH OF F1 FREQUENCY SHIFT MODULATION.................... 46

NECESSARY BANDWIDTH OF BINARY CONTINUOUS PHASE FSK MODULATION.......... 47

SECTION 7

RESULTS 51

LIST OF REFERENCES R-1

LIST OF TABLES

Table Page

1 FORMULAS FOR DIGITAL MODULATION AS GIVEN IN THE NTIA MANUAL.. 14

2 BINARY FSK MODULATION WITH 99% ENERGY CONTAINMENT BANDWIDTH.... 49

3 NECESSARY BANDWIDTH.. 51

LIST OF FIGURES

Figure Page

1 A sample waveform of a binary digital random process........... 7

2 Nonregulatory bandwidths for the BPSK spectrum................ 10

3 Transmit and receive filters in a digital communications system.. 17

4 Power spectrum of a BPSK modulation........................... 20

5 Power spectrum of a QPSK modulation........................... 21

6 Block diagram of a digital simulation model.................. 23

7 Performance curves for a PSK receiver containing a boxcar filter with variable bandwidth................................ 25

8 Percent of power in the filter passband versus 2W/R (BPSK) where W = 1/2-sided bandwidth.................... 27

TABLE OF CONTENTS (Continued)

Figure		Page
9	Percent of power in the filter passband versus 2W/R (QPSK) where W = 1/2-sided bandwidth..........................	28
10	Percent of power in the filter passband versus K for both a BPSK and QPSK spectrum..............................	29
11	Power spectral density for MSK modulation....................	33
12	Power spectral density for SFSK modulation...................	34
13	The percent of power outside the bandwidth versus the two-sided bandwidth/bit rate..............................	35
14	Power spectral density for BFSK when $\|\cos 2\pi D/R\| = 1$............	39
15	Power spectral density for BFSK with a modulation index of M = 0.2..	40
16	Power spectral density for BFSK with a modulation index of M = 0.7..	41
17	Power spectral density for BFSK with a modulation index of M = 1.49...	42
18	Power spectral density for BFSK with a modulation index of M = 3.3..	43
19	Power spectral density for BFSK with a modulation index of M = 8.4..	44
20	Power spectral density for BFSK with a modulation index of M = 14.35..	45
21	Necessary bandwidth of FSK modulation as a function of the modulation index..	48

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Additional thanks go to Robert Mayher, Chief, Spectrum Engineering and Analysis Division, NTIA, for calling attention to an important relavent CCIR reference and for suggesting that a simulation of PSK system performance be accomplished to determine the necessary bandwidth of this digital modulation.

ABSTRACT

The National Telecommunications and Information Administration (NTIA) and the Interdepartment Radio Advisory Committee (IRAC) require, for their management of the spectrum, technical guidance concerning the necessary bandwidth and spectral characteristics of digital modulation. The necessary bandwidth is the minimum emission bandwidth required for an acceptable quality of service. The NTIA Manual includes all guidance and criteria relating to necessary bandwidth in Annex J entitled "Determination of Necessary Bandwidth." In this report, the concept of necessary bandwidth is applied to digital modulation. The necessary bandwidths for several digital modulations are considered in detail. These are phase shift keying (PSK), minimum shift keying (MSK), and frequency shift keying (FSK). The techniques used to determine necessary bandwidth are general and could be extended to other digital modulations. This report shows, for digital modulation systems, the necessary bandwidth, signal power, and bit error rate are all interrelated and tradeoffs among these factors are possible.

KEY WORDS

Bandwidth
Digital Modulation
Frequency Shift Keying (FSK)
Minimum Shift Keying (MSK)
Necessary Bandwidth
Phase Shift Keying (PSK)

SECTION 1

INTRODUCTION

BACKGROUND

The National Telecommunications and Information Administration (NTIA) is responsible for managing the radio spectrum allocated to the U.S. Federal Government. Part of NTIA's responsibility is to; "Establish policies concerning spectrum assignment, allocation and use, and provide the various Departments and agencies with guidance to ensure that their conduct of telecommunications activities is consistent with these policies" [Department of Commerce, 1981]. In support of these requirements, NTIA has undertaken a number of spectrum resource assessments. The objectives of these studies are to: assess spectrum utilization, identify existing and/or potential compatibility problems between systems of various departments and agencies, provide recommendations for resolving any compatibility conflicts, and recommend changes to promote efficient use of the radio spectrum and to improve spectrum management procedures.

The Technical Subcommittee (TSC) of the Interdepartment Radio Advisory Committee (IRAC) is a permanent subcommittee concerned with the technical aspects of the use of the electromagnetic spectrum. One purpose of the TSC is to develop and recommend new standards and existing standards pertinent to use of the radio spectrum. The TSC was tasked by the IRAC to prepare updated tables of necessary bandwidth formulas for the "NTIA Manual of Regulations and Procedures for Federal Radio Frequency Management" [NTIA Manual, 1981]. To accomplish this task, the TSC formed a special working group, TSC Working Group 9 (WG9). Initially, WG9 surveyed the range of modulation techniques employed by the Federal Government and identified which of these techniques lacked clear guidance to determine necessary bandwidth. It was determined that formulas for calculating necessary bandwidths are generally available for most analog modulation techniques and that a few formulas for calculating necessary bandwidths for some digital modulation techniques are available. These theoretical formulas include a design factor (K) that depends upon the implementation approach. The NTIA agreed to examine the spectral and performance characteristics of certain common digital modulation techniques and develop methodologies for calculating the necessary bandwidth for these modulation types. This report is the result of that NTIA examination. The discussions in this report relating to the bandwidth and spectral characteristics of these digital modulations may also be relevant to other spectrum management activities such as the determination of system and band technical spectrum efficiency factors (TSEF) [Shelton, et al., 1984].

OBJECTIVE

The objective of this report was to examine the power densities and necessary bandwidth properties of digital modulation.

APPROACH

To meet the objective of this study, the following approach was taken.

1. Nonregulatory and NTIA regulatory definitions of bandwidth were identified.

2. The most suitable definition for necessary bandwidth of digital signals was chosen.

3. For digital modulation, the interrelated effects of tradeoffs of bandwidth and E_b/N_o = (Energy/bit)/(Noise density) on system performance (bit error probability) were explored.

4. Formulas for the necessary bandwidths of the digital modulations, phase shift keying (PSK), minimum shift keying (MSK), and frequency shift keying (FSK), were developed.

SECTION 2

CONCLUSIONS AND RECOMMENDATIONS

GENERAL CONCLUSIONS

The necessary bandwidth is one of the measures of bandwidth in the NTIA Manual. It is included in the emission designator used for frequency management purposes and is used as a factor in spectrum standards and frequency assignments throughout the NTIA Manual. The necessary bandwidth, for each class of emission, is the minimum emission bandwidth that allows an acceptable quality of performance.

A variety of definitions of bandwidth are candidates for application in calculating the necessary bandwidth. These include the null-to-null, X dB, noise equivalent, Nyquist, and the fractional power containment bandwidth. All of these definitions of bandwidth have shortcomings, with the exception of fractional power containment bandwidths. The fractional power containment bandwidth is the definition of bandwidth that has been most used to define channel bandwidth for digital modulation.

SPECIFIC CONCLUSIONS

1. The fractional power containment bandwidth is the most appropriate measure of necessary bandwidth because it is a measure of the integrated power spectrum density and can be related to system performance.

2. The computer analysis of a digital PSK system described in Section 4 substantiates the conclusion that the necessary bandwidth of each type of digital modulation is not simply defined because it is dependent upon tradeoffs in a system's E_b/N_o, amount of filtering, and probability of error.

3. For certain modulations such as FSK and MSK, the necessary bandwidth is generally independent of tradeoffs in parameters and is just the 99% energy containment bandwidth. Often, the 99% power containment bandwidth is used as the necessary bandwidth for other digital signals.

4. Necessary bandwidth formulas for certain classes of digital modulation are contained in Annex J of the NTIA Manual. Additional necessary bandwidth formulas that should be included in Annex J for certain digital modulations are listed in Section 7.

RECOMMENDATIONS

The following are recommendations based on the technical findings in this report. Any action to implement these recommendations will be accomplished under separate action by modification of established rules, regulations, or procedures.

1. The necessary bandwidth formulas in Section 7 are recommended for inclusion in Annex J of the NTIA Manual.

2. Figure 10 should be included in Annex J of the NTIA Manual to relate, for spectrum management applications, the percentage of signal power to its corresponding K value for both BPSK and QPSK modulation. Necessary bandwidth values submitted to NTIA, which have K values greater than 1.5, should be explained and justified in the system review process.

3. It is commonly stated in the literature that certain digital modulations are more spectrally efficient than others. Further studies should be done to compute both the technical spectrum efficiency factor (TSEF) [Shelton, et al., 1984] and necessary bandwidth for various digital modulations that are utilized in systems that accomplish the same mission.

SECTION 3

BANDWIDTH AND NECESSARY BANDWIDTH CONCEPTS

This section examines the concept of bandwidth both from a regulatory and nonregulatory approach. The concept of necessary bandwidth is applied to digital modulation as a basis for use with specific modulation techniques.

BANDWIDTH CONCEPTS

There is no universally accepted method for computing the bandwidth of a signal; however, a variety of calculation methods are being utilized in telecommunications to specify the bandwidth for both analog and digital modulations. The commonality of all of these methods is that each is some measure of the effective width of the power spectral density (PSD) of the signal. The PSD for a random process is expressed in Equation 1 [Papoulis, 1965].

$$S(f) = F(Rxx(\tau)) \tag{1}$$

where $Rxx(\tau)$ is the autocorrelation function of the random process and $S(f)$ is its Fourier transform and f represents the frequency. $F(—)$ denotes the transform. The general method to determine the PSD of a random process, be it analog or digital, is first to either calculate, measure, or postulate the autocorrelation function of the random process and then take its Fourier transform. The autocorrelation of a process is defined as the statistical expected value as in Equation 2.

$$Rxx(\tau) = E\left[X(t)X(t+\tau)\right] \tag{2}$$

where

E = the expected value
X = a function dependent on time (t)

An example of a random process often encountered in digital communications is a sequence of statistically independent synchronous random pulses with equally likely amplitudes ±1 and symbol durations (T). A time waveform sample of this random process is shown in Figure 1. The autocorrelation function for this process is expressed by Equations 3 and 4.

$$Rxx(\tau) = 1 - |\tau|/T \text{ for } |\tau| \leq T \quad (3)$$

$$= 0 \text{ for } |\tau| > T \quad (4)$$

Taking the Fourier transform of the autocorrelation yields the PSD of the random process as in Equation 5.

$$S(f) = T\left[\frac{\sin(\pi fT)}{\pi fT}\right]^2 \quad (5)$$

The next two subsections are devoted to particular measures of bandwidth. The nonregulatory definitions (textbook definitions) of bandwidth often used in the literature are explained in the first subsection. The nonregulatory definitions are computational methods to calculate bandwidth directly from the PSD of the signal. The second subsection includes descriptions of regulatory terms included in the NTIA Manual that are used to measure bandwidth. These include the terms necessary bandwidth, occupied bandwidth, and authorized bandwidth.

NONREGULATORY DEFINITIONS OF BANDWIDTH

1. Null-to-Null Bandwidth - This bandwidth is the width of the main spectral lobe of the PSD bounded by spectra nulls if they exist. The majority of the baseband signal power for many digital modulation methods

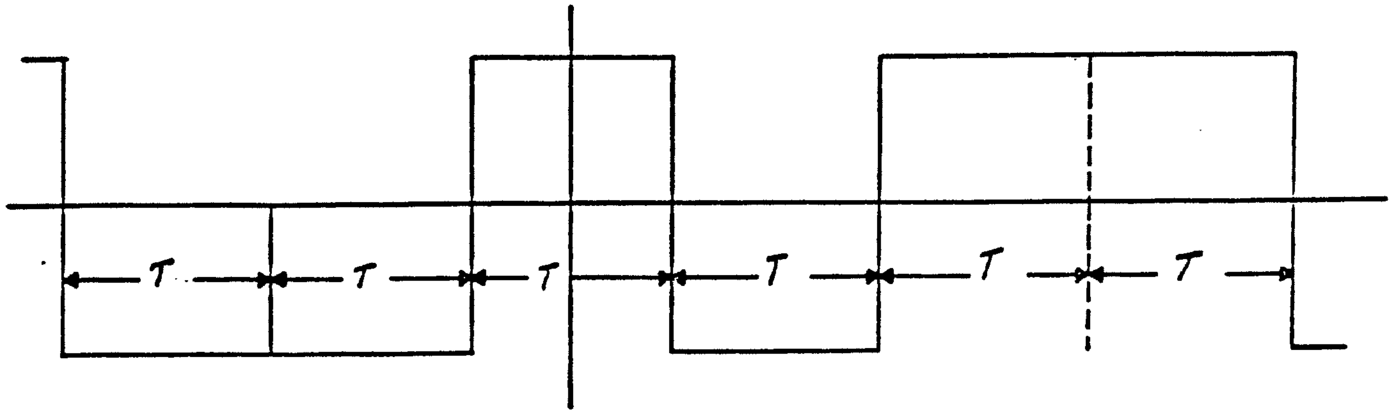

Figure 1. A sample waveform of a binary digital random process. This is an example of a nonreturn-to-zero (NRZ) waveform.

is contained between the first two nulls of the spectrum surrounding the origin ($f=0$).

2. Fractional Power Containment Bandwidth - A measure of bandwidth which is the amount of spectrum width needed to contain a certain percentage of the signal's power. It is defined by the relationship in Equation 6.

$$\int_{-B}^{B} S(f)df = \alpha \int_{-\infty}^{+\infty} S(f)df \tag{6}$$

where

$2B$ = power containment bandwidth (two-sided)

α = decimal equivalent of percent power (e.g., 0.95, 0.99, 0.999)

The most commonly utilized value for α is 0.99 (99 percent). Other values of fractional bandwidth are 0.95 (95 percent) and the more conservative 99.99 percent bandwidth. (See [Korn, 1980], [Amoroso, 1980] and [Shamugam, 1979]).

3. X dB Bandwidth (bounded PSD) - CCIR Recommendation 328-5 [CCIR, 1982a] defines the X dB bandwidth as the bandwidth where, beyond its lower and upper limits, any discrete spectrum component or continuous PSD is attenuated by at least X dB, relative to a given and predetermined zero dB reference level. A common example is the half power bandwidth that is the bandwidth where the PSD is 0.5 or 3 dB below the peak value.

4. Noise Equivalent Bandwidth - The noise equivalent bandwidth is defined as the bandwidth that satisfies Equation 7.

$$W_N S(f_c) = P \tag{7}$$

where

W_N = noise bandwidth

$S(f_c)$ = PSD at the carrier frequency (f_c)

P = total signal power for all frequencies

5. Nyquist Bandwidth - The Nyquist bandwidth results from a number of theorems by [Nyquist, 1928] relating to the passage of a signal through a band-limiting filter to a receiving device able to reconstruct the signal. For digital signals, it is that minimum bandwidth that passes a signal without intersymbol interference.

CCIR Report 378-3 [CCIR Report, 1978] lists the Nyquist bandwidth for a number of digital modulations. These include binary phase shift keying (BPSK) and quaternary phase shift keying (QPSK). The report states, however, that the Nyquist bandwidth is not a true indicator of the required channel bandwidth that, in practice, must be allotted to a digitally modulated signal. The CCIR report states that the channel bandwidth may be as low as 0.85 Nyquist for certain systems and may exceed 2-Nyquist bandwidth for other systems.

Figure 2 shows a plot of the PSD defined by Equation 5. Also shown in Figure 2 are some of the values of the nonregulatory bandwidths for this power spectrum. The figure shows the relative magnitudes of some of the bandwidths for the power spectrum of Equation 5.

REGULATORY DEFINITIONS OF BANDWIDTH

The NTIA Manual includes three regulatory definitions of bandwidths. These are necessary bandwidth, occupied bandwidth, and authorized bandwidth.

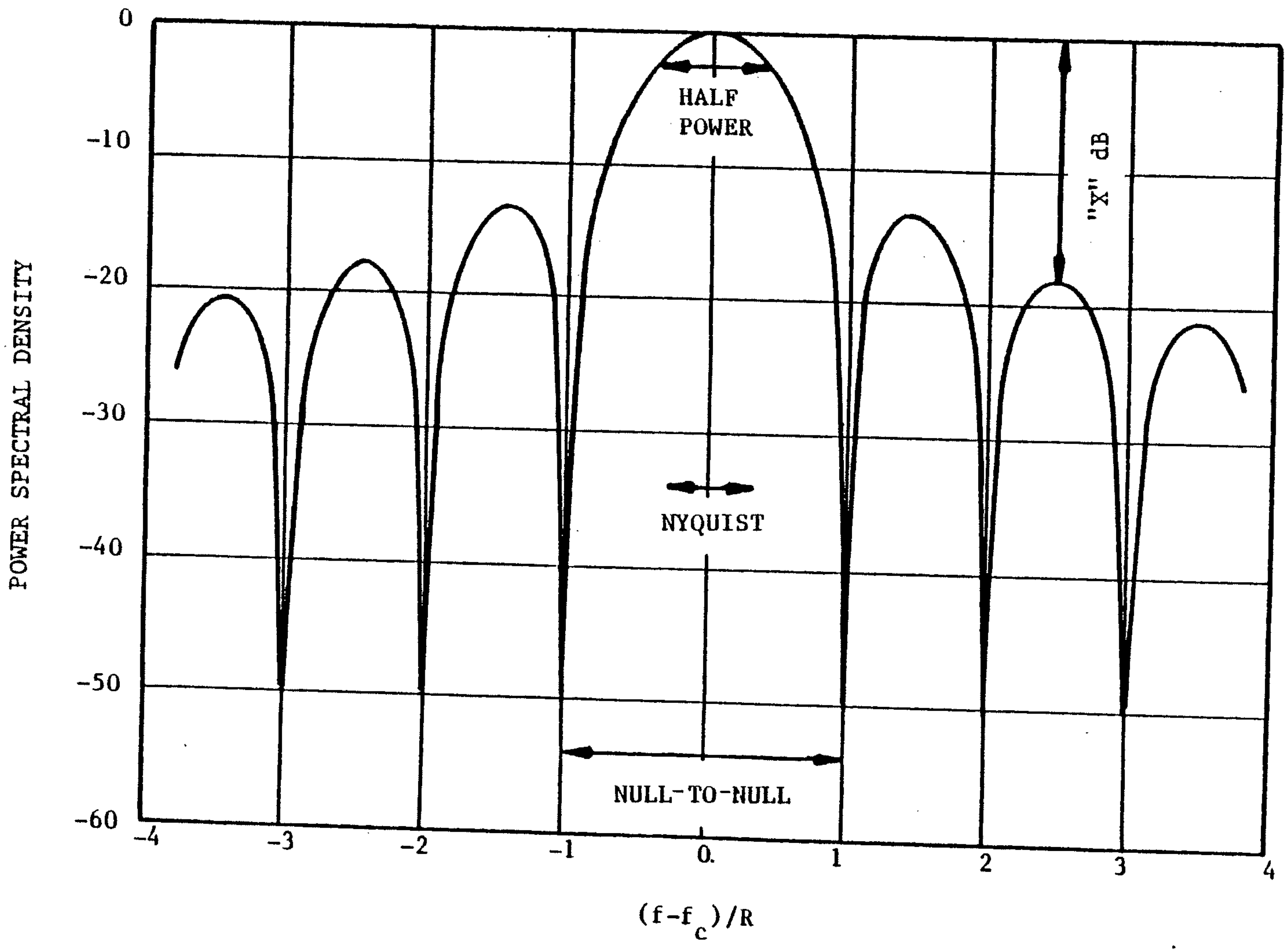

Figure 2. Nonregulatory bandwidths for the BPSK spectrum. (A) Half power bandwidth = .88R, (B) Nyquist bandwidth = R/2, (C) Null-to-Null = 2R, and (D) X dB bandwidth.

Necessary Bandwidth

The necessary bandwidth forms part of the emission designator and is used throughout the NTIA Manual as a factor in spectrum standards and frequency assignments.

Both the NTIA Manual and the International Telecommunication Union (ITU) Radio Regulations [ITU, 1982] define the necessary bandwidth as follows.

> "Necessary Bandwidth: For a given class of emission, the width of the frequency band which is just sufficient to ensure the transmission of information at the rate and with the quality required under specified conditions."

Although the definition for necessary bandwidth is specific, note that it does not include a formula, algorithm, or technique to determine the necessary bandwidth. CCIR Recommendation 328-5 [CCIR, 1982a] does give some additional application principles and states:

> "...(g.a) the necessary bandwidth should be established as the smallest value possible taking into account the modulation technique, while including the spectrum components useful to a good receiver to ensure communication with the quality required by the two correspondents (for example, maintaining telephone quality laid down, or the error rate admitted in telegraphy) under given technical conditions."

The NTIA Manual states that, except for radars, the necessary bandwidths may be determined by one of the following methods with order of preference shown.

1. Use of the appropriate formula from TABLE A (Annex J)

2. Computation in accordance with the latest Recommendations of the International Radio Consultative Committee (CCIR)

3. Direct measurements not covered by statements 1 or 2 above

4. Use of the best available technical information from other sources

5. See Part 5.5 of the NTIA Manual for the desired relationship of occupied bandwidth to necessary bandwidth.

Occupied Bandwidth

Another measure of bandwidth, closely related to necessary bandwidth, is the occupied bandwidth. This bandwidth, as defined by the ITU Radio Regulations and the NTIA Manual is as follows.

> "Occupied Bandwidth: The width of a frequency band such that below the lower, and above the upper, frequency limits, the mean powers emitted are each equal to a specified percentage of the total mean power of a given emission. Unless otherwise specified by the CCIR, for the appropriate class of emission, the value of β should be taken as 0.5 percent."

The CCIR Study Group 1, in 1981, reviewed this definition that is included in Recommendation 328-5 [CCIR, 1982a]. The CCIR noted that the β in this definition was not clearly specified. Study Group 1, to clarify this issue, added the following note to the occupied bandwidth definition in Recommendation 328-5 [CCIR, 1982a].

NOTE - The value of β could be determined by calculating the sum of the percentages of the total mean power above and below the necessary bandwidth. The occupied bandwidth is optimum when it equals the necessary bandwidth.

Relationship of Occupied Bandwidth to Necessary Bandwidth

The NTIA Manual, in Section 5.5, covers the desired relationship of occupied bandwidth to necessary bandwidth by stating: "All reasonable effort shall be made in equipment design and operation by Government agencies to maintain the occupied bandwidth of transmission of any authorized transmission as closely to the necessary bandwidth as is reasonably practicable within the state-of-the-art."

The CCIR in Recommendation 328-5 [CCIR, 1982a] states that "one way to determine the occupied bandwidth is to carry out measurement of the bandwidth actually occupied by a given emission and thus to ascertain, by comparison with the necessary bandwidth, that such an emission does not occupy an excessive bandwidth for the service to be provided."

Authorized Bandwidth

Also included in the NTIA Manual is the definition of authorized bandwidth.

> "Authorized Bandwidth: Authorized bandwidth is, for purposes of this Manual, the necessary bandwidth (bandwidth required for transmission and reception of intelligence) and does not include allowance for transmitter drift or Doppler shift."

NTIA MANUAL FORMULAS FOR DIGITAL MODULATION

Necessary bandwidth formulas are included in the ITU Radio Regulations in Appendix 6. The NTIA Manual was recently revised to include all guidance and criteria relating to necessary bandwidth in Annex J entitled: "Determination of Necessary Bandwidth." The tables of necessary bandwidth formulas are similar in both the national and international regulations and include primarily analog modulation entries, along with a few digital formulas. The formulas for digital modulation in the NTIA Manual are given in TABLE 1.

The modulation for the entries in TABLE 1 is multilevel pulse code modulation (PCM) where each signal is one of S possible signal states. When binary modulation is considered, S = 2.

The NTIA Manual, in addition, lists necessary bandwidth formulas for frequency and amplitude modulated signals with quantized or digital information.

TABLE 1

FORMULAS FOR DIGITAL MODULATION AS GIVEN IN THE NTIA MANUAL

Description of Emission	*Necessary Bandwidth*		*Designation of Emission*
	Formula	*Sample Calculation*	
Microwave radio relay system	$B_n = R/\text{Log}_2 S + 2DK$	**Digital modulation used to send 10 megabits per second by use of frequency shift keying with 4 signaling states and 2 MHz peak deviation of the main carrier** $R = 10 \times 10^6$ **bits per second;** $D = 2$ **MHz;** $K = 1$; $S = 4$; $B_n = 9$ **MHz**	**9M00F7DDT**
Microwave radio relay system	$B_n = 2RK/\text{Log}_2 S$	**Digital modulation used to send 10 megabits per second by use of phase shift keying with 4 signaling states.** $R = 10 \times 10^6$ **bits per second;** $K = 1$; $S = 4$; $B_n = 10$ **MHZ**	**10M00G7DDT**
Composite transmission digital modulation using DSB-AM (Microwave radio relay system)	$B_n = 2RK/\text{Log}_2 S$	**Digital modulation used to send 5 megabits per second by use of amplitude modulation of the main carrier with 4 signaling states** $R = 5 \times 10^6$ **bits per second** $K = 1$ $S = 4$ $B_n = 5$ **MHz**	**5M00K7DD**

NECESSARY BANDWIDTH OF DIGITAL SIGNALS

Definitions

The bandwidth properties of digital modulation have been examined previously by [Skylar, 1983] and [Amoroso, 1980]. They investigated many of the definitions of bandwidth included in this report and concluded that no single definition of bandwidth is universally applicable to digital modulation.

The inability to specify a unique bandwidth measure for digital modulation makes it difficult to specify a computational method for determining the necessary bandwidth, since the necessary bandwidth is the particular bandwidth sufficient to achieve an acceptable rate of transmission and quality of service. There is, for the necessary bandwidth, the dual problem of specifying what quality of service and which of the definitions of bandwidth described earlier is appropriate.

Another difficulty in determining the necessary bandwidth of digital modulation, as pointed out in CCIR SPM Document P/17 [CCIR, 1978] is the fact that unlike analog signals, there is no need for digital signals to preserve the signal waveform during transmission through the system. The only requirement is to transmit the digital information contained in the signal and therefore, the bandwidth required is less than that required for transmission of the signal undistorted.

Each of the six nonregulatory definitions of bandwidth described earlier is a candidate for calculating the necessary bandwidth. The null-to-null and X dB bandwidths are most useful for defining bandwidths when the power spectrum has a significant rolloff that can be associated with a well-defined bandwidth. The filters used in digital systems have gradual rolloffs in PSD to minimize intersymbol interference, which eliminates these bandwidth definitions from consideration. The noise-equivalent bandwidth is an equivalent measure of bandwidth useful for the calculation of noise power and is not relatable to the required channel bandwidth for acceptable performance. The Nyquist bandwidth, as previously stated, is not a true indicator of the required channel bandwidth that must be allotted to a digitally modulated signal. This leaves the fractional power containment bandwidth. This is the definition of bandwidth that has been used the most to define channel bandwidth for digital modulation. The fractional power containment bandwidth is an excellent measure of necessary bandwidth because it is a measure of the integrated PSD (i.e., the percentage of total power contained within a frequency range) and can be related to system performance.

Effects of Filtering and Tradeoffs of Bandwidth and E_b/N_o

The necessary bandwidth of a digital modulation is often affected by the characteristics of transmit and receive filters. The effects of filtering can be understood by an analysis of the digital communication system as illustrated in Figure 3.

In this system, the signal at the output of the modulator is unfiltered and, for PSK modulation, the power spectrum is of the form $(\sin X/X)^2$ where X is a frequency variable. Spectrum efficiency requires that this waveform be filtered to transmit only those frequency components that are needed. In the transmitter, a filter is added to limit the bandwidth of the signal. The signal then passes through the transmission channel. In the receiver, additional filtering is included to limit interference from signals in adjacent channels.

The transmitter and receiver filters distort the characteristics of the signal and may only pass a certain percentage of the signal energy. The filters stretch the individual bits beyond the symbol period into a previous, or subsequent, symbol period. This distortion results in intersymbol interference. The more narrow the filters, the smaller the channel spacings (bandwidth) and the greater the spectrum efficiency. However, along with this, there is greater intersymbol interference and less signal power for digital detection. This results in an increase in the probability of bit error.

A tradeoff factor to bandwidth in a digital system is the signal-to-noise ratio parameter E_b/N_o. The dimensionless E_b/N_o is commonly used to characterize digital systems. Its formula is E_b/N_o=(Energy/bit)/(Noise density). The bit error probability for a digital system can be reduced by either increasing the bandwidth or the E_b/N_o. The E_b/N_o is increased by simply raising the signal power. The system designer for a digital system

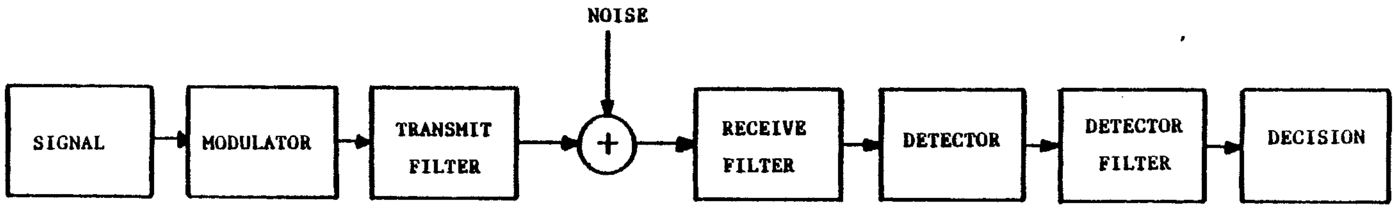

Figure 3. Transmit and receive filters in a digital communications system.

must make tradeoffs among the three parameters; bandwidth (filtering), E_b/N_o, and probability of bit error. The specification of the necessary bandwidth for a particular modulation may not be unique because of this inherent interdependence of bandwidth, the bit error probability and E_b/N_o.

The next three sections of this report, using computerized models and measured data, will develop formulas for the necessary bandwidth of certain digital modulations. Account will be taken, where necessary, of the interdependence of bandwidth, bit error probability, and E_b/N_o. Modulation types considered are PSK, MSK, and FSK. The spectral characteristics of these modulations are computed and shown, along with the derivation of formulas for necessary bandwidth.

SECTION 4

PHASE SHIFT KEYING (PSK) MODULATION

POWER SPECTRUM OF PSK MODULATION

PSK is a digital phase modulation signal that shifts the carrier phase at the symbol rate between signal states (S) representing distinct equally spaced angular positions. BPSK is characterized by two signal states and the waveform is described by Equation 8.

$$X(t) = Y(t)(A\cos \omega_c t) \tag{8}$$

where

$X(t)$ = PSK waveform
$Y(t)$ = random binary waveform with period (T) and equiprobable levels -1 and 1.
A = a constant proportional to average power
ω_c = angular carrier frequency

The PSD for M-ary unfiltered PSK modulation is independent of the number of signal states (S) and is a function of the symbol rate ($1/T_s$). It is of the form expressed in Equation 9.

$$S(f) = A^2 T_s \left[\frac{\sin \pi T_s (f - f_c)}{\pi T_s (f - f_c)} \right]^2 \tag{9}$$

where

T_s = symbol rate ($T_s = T_b (\log_2 S)$)

and

$T_b = (T_s)/(\log_2 S)$

where

T_b = bit length

The PSDs for BPSK (S=2) and QPSK (S=4) are shown in Figures 4 and 5.

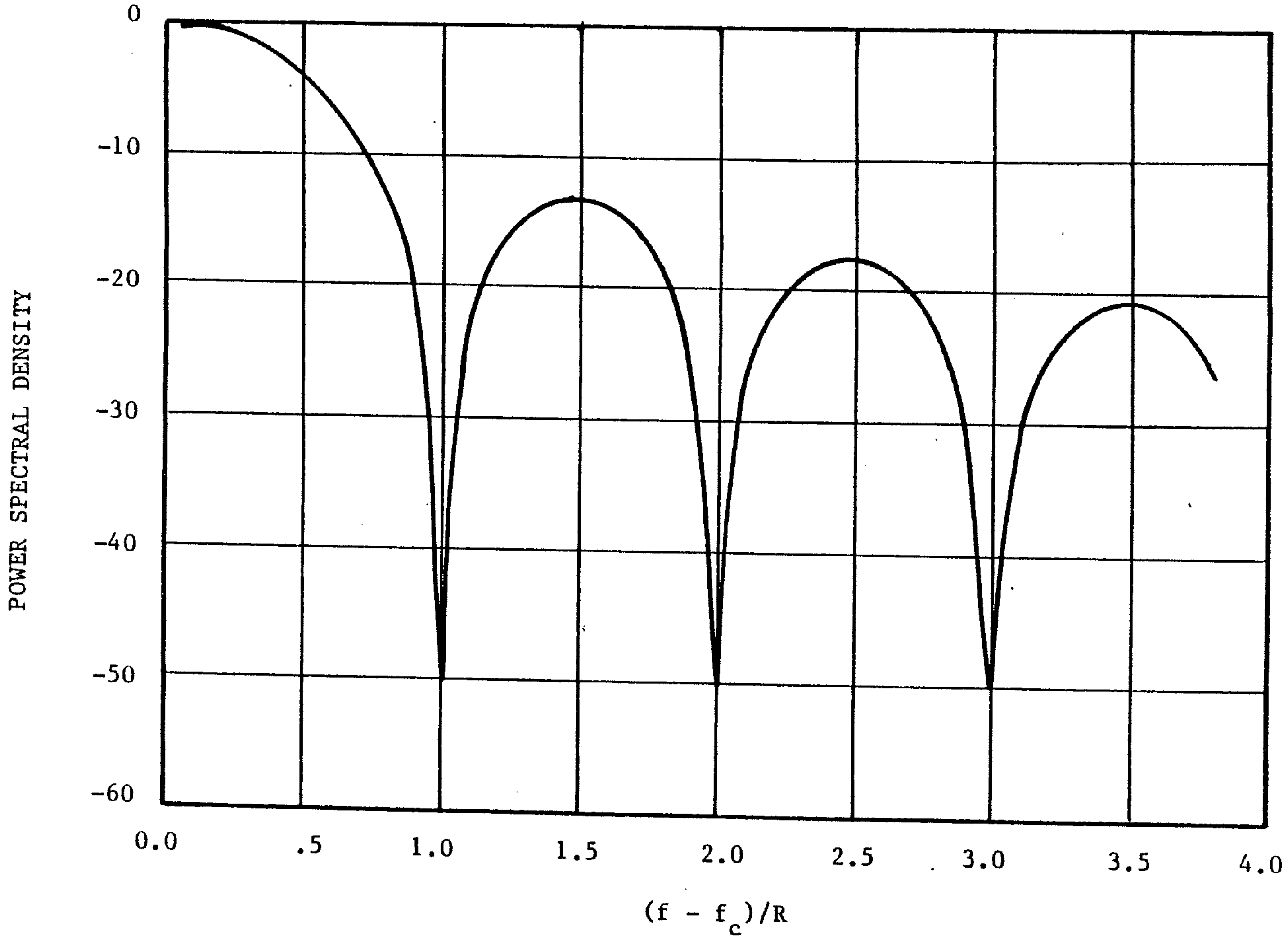

Figure 4. Power spectrum of a BPSK modulation.

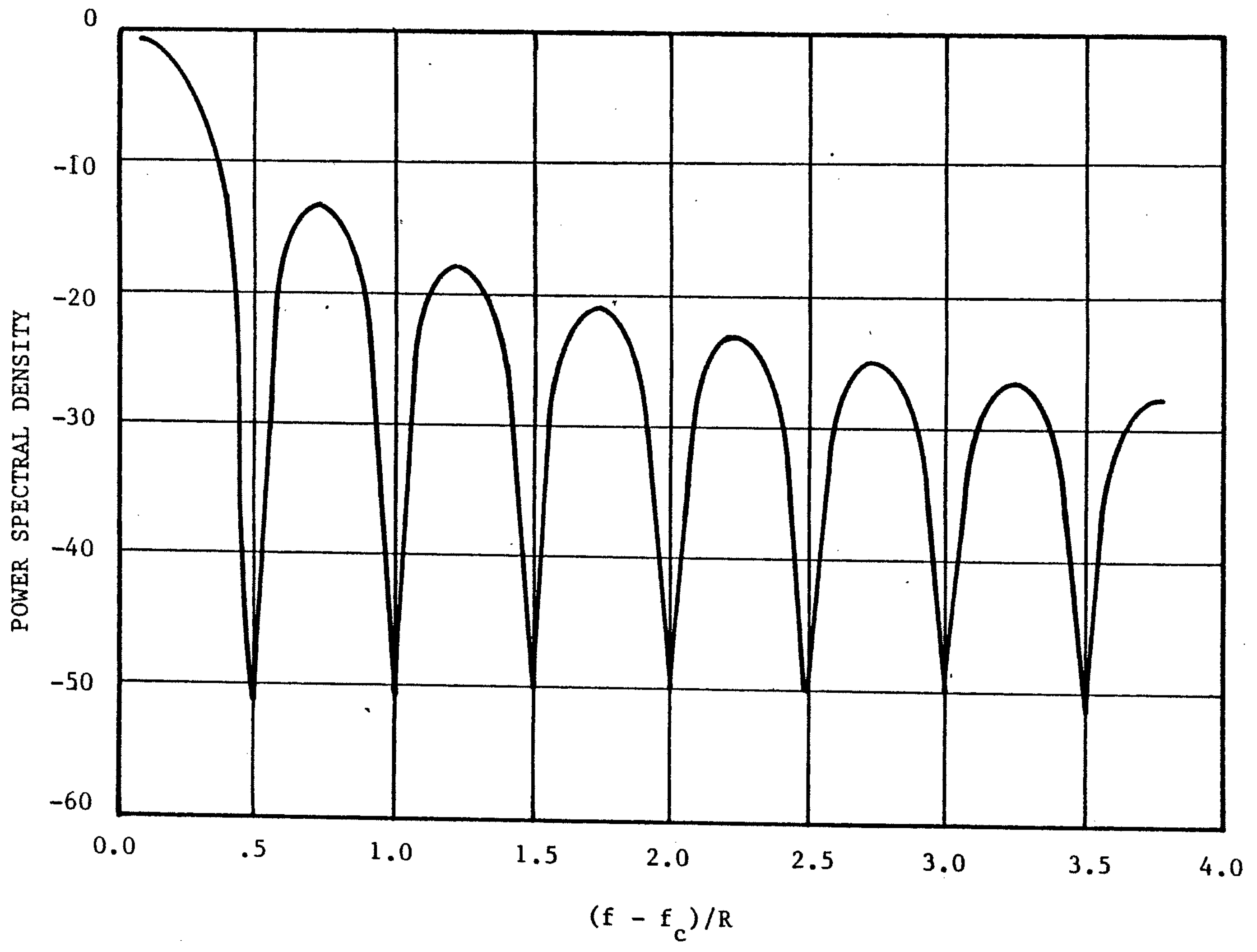

Figure 5. Power spectrum of a QPSK modulation.

NECESSARY BANDWIDTH

The formula for the necessary bandwidth of M-ary PSK modulation in Annex J of the NTIA Manual is shown in Equation 10.

$$B_n = \frac{2RK}{\log_2 S} \tag{10}$$

where

B_n = necessary bandwidth

K = design factor

R = bit rate

The conceptual basis for this formula is that, to a first approximation, the necessary bandwidth for PSK is proportional to $2/T_s$. K is added to this approximation to complete the bandwidth formula. K accounts for the characteristics of the signal's PSD and other factors of the digital system.

The value of B_n can be obtained from either system measurements or computer simulations of the performance of a PSK system. For purposes here, it was decided to obtain the value of B_n via computer simulation of a digital receiver.

The computer model chosen for the simulation was the Digital Time Waveform Simulator (DTWS) developed by [Shalvi, 1980]. This computer model uses Monte Carlo techniques to compute the bit error probability for any one of a variety of different digital receivers. Here, the simulation modeled a BPSK receiver and noted the effects of changes in the system channel bandwidth. A block diagram of the receiver structure simulated in the computer model is shown in Figure 6.

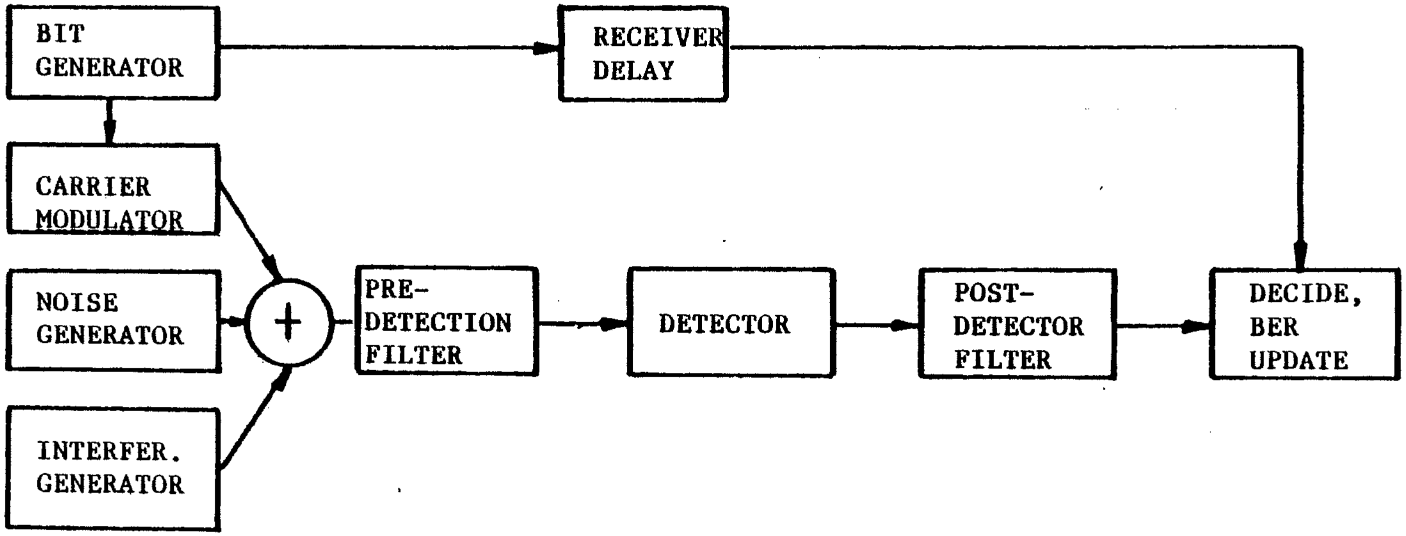

Figure 6. Block diagram of a digital simulation model. This is a block diagram of the receiver modeled on the computer. The computer model used Monte Carlo techniques.

The effects on system performance of changes in the system channel bandwidth were modeled by varying the passband bandwidth of the receiver predetection filter and noting how this change affected bit error probability. The filter represents the composite effects of both transmitter and receiver filtering. The predetection filter was made into a ideal bandpass boxcar filter (B_F) with transfer function $H(f) = 1$ for $-W < f < W$. In the simulation, the boxcar filter was modeled by a 20-pole Butterworth filter with a selectable break frequency (3 dB). The desired signal was unfiltered BPSK modulated with a PSD described by Equation 9. The computer model, which included the effects of intersymbol interference, was used to determine the probability of bit error for three different predetection filter bandwidths. These were $B_F = 3/2R$, $B_F = 2R$, and $B_F = 4R$. The probability of bit error performance curves (A, B, C,) for each of these bandwidths are shown in Figure 7. Curve D shows the performance when the predetection bandwidth is $B_F = B$. In this case, the integrate-and-dump circuit of the receiver is a matched filter and the receiver is optimum. The performance curves illustrate that a particular system probability of error is achievable by making a tradeoff between bandwidth and signal power as discussed earlier. For example, a 10^{-3} bit error rate (BER) can be achieved with either $B_F = 2R$ and $E_b/N_o = 7.5$ dB, or $B_F = 1.5R$ and $E_b/N_o = 9.3$ dB.

The necessary bandwidth of digital PSK modulation can be estimated from the data in Figure 7. When the system bandwidth changes from $B_F = 1.5R$ (curve A) to $B_F = 2R$ (curve B), it is accompanied by a large change in bit error probability. However, beyond $B_F = 2R$ (curve B), increases in bandwidth have little effect on decreasing the bit error probability. Moreover, since the performance curve B is close to the limiting performance curve D ($B_F = B$) it can be concluded that $B_F = 2R$ is a sufficient bandwidth for a BPSK system under these performance conditions. $B_F = 2R$ is judged sufficient to ensure transmission of information with the quality required and is considered an appropriate measure of the BPSK necessary bandwidth.

The results of this computer simulation can be compared with a similar study by [Parks, 1969] of the effects of bandlimiting on the detection of binary signals. Parks used analytical methods to model the performance of a pulse-code modulation (PCM) receiver that includes an integrate-and-dump detector. Inherent in the receiver was an ideal band-limiting filter with a

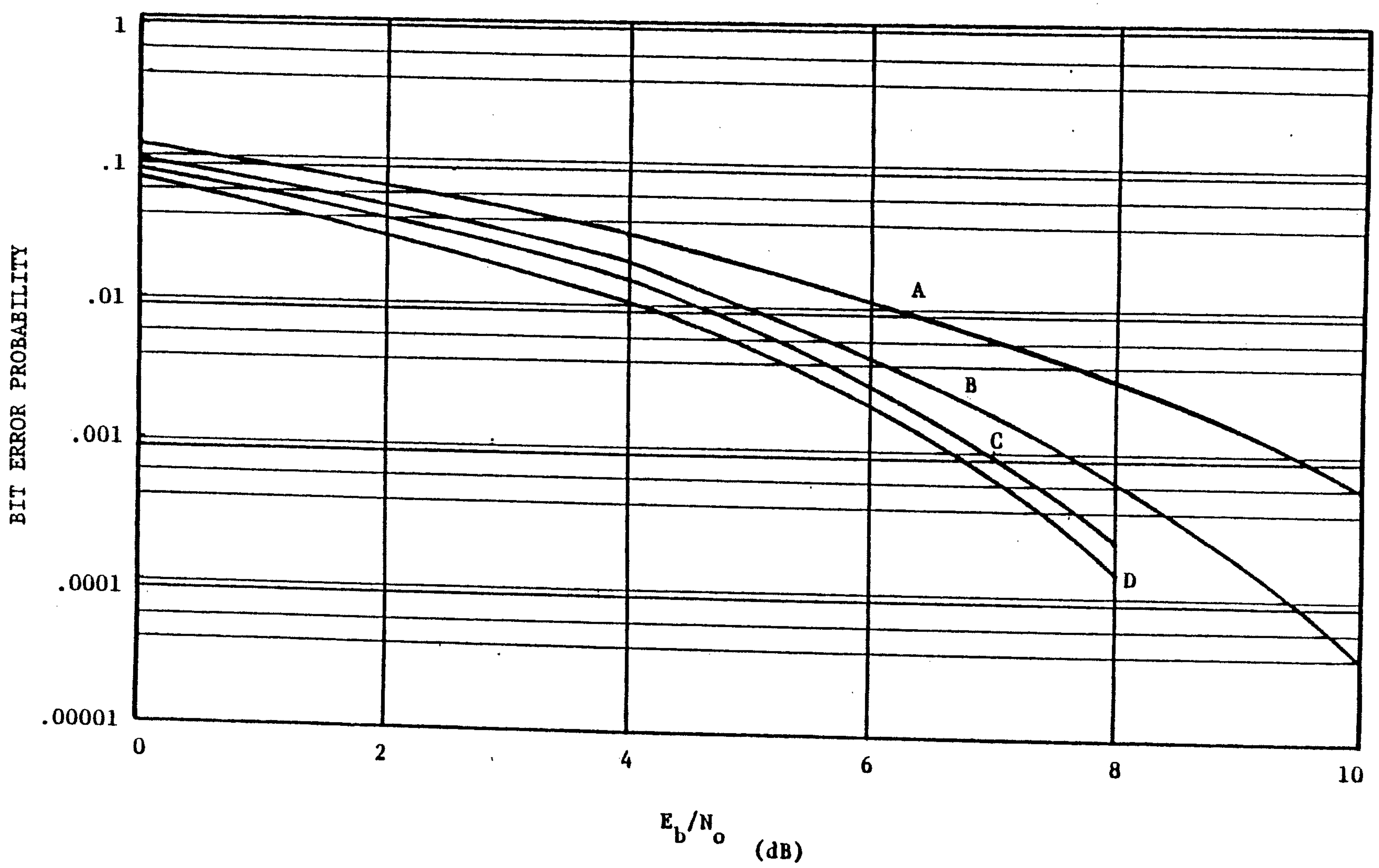

Figure 7. Performance curves for a PSK receiver containing a boxcar filter with variable bandwidth. The bit error probability is plotted vs. E_b/N_o using the filter bandwidth, B_F, as a parameter. Curve A has $B_F = 1.5R$, Curve B has $B_F = 2R$, Curve C has $B_F = 4R$, and Curve D has $B_F = \infty$.

sharp cutoff. Analytical methods were used to compute the additional power required with varying amounts of filtering to give the same performance as the optimum receiver (no filters). Parks showed that when the receiver had bandwidths greater than about 1.4R (88% power containment), the performance of the receiver approached optimum. Conversely, when the bandwidth of the filter is decreased below 1.4R, increasing amounts of power are needed to give the same performance as the optimum receiver. The United Kingdom, in an input to the CCIR SPM Document P/17 [CCIR, 1978] for WARC-79, gave examples of necessary bandwidth calculations for terrestrial digital systems. From experience, they found the necessary bandwidth for PSK digital radio-relay systems to be about 1.5 to 2.0 times the symbol rate of the system.

SPECTRUM FILTERING AND THE K FACTOR

The predetection filter is a window function of the signal's PSD. However, as the filter widens to contain the tails of the power spectrum, the additional portions of the signal's power spectrum passed by the filter contain less and less additional power. The effects of the windowing are illustrated in Figures 8 and 9 that plot the percent of contained energy in the filters passband ($-W < f < W$) versus $2W/R$ passband bandwidth normalized to the bit rate. The percentage of powers passed by individual bandwidths for BPSK modulation are $2W = 3/2R$(88%), $2W = 2R$(90%), and $2W = 4R$(95%).

A useful plot that should be included in the NTIA Manual is the percent of contained energy versus the design factor (K) shown in Figure 10. The design factor is defined in Equation 10. In this figure, the abscissa is K where $K = WT$. This figure relates the percentage of energy to its corresponding K values for both BPSK and QPSK modulations. (NOTE: One curve is sufficient for both modulation types.) An upper bound to K, suitable for most spectrum management applications, is $K = 1.5$. Values of K reported to NTIA as being greater than $K = 1.5$ should be explained and justified.

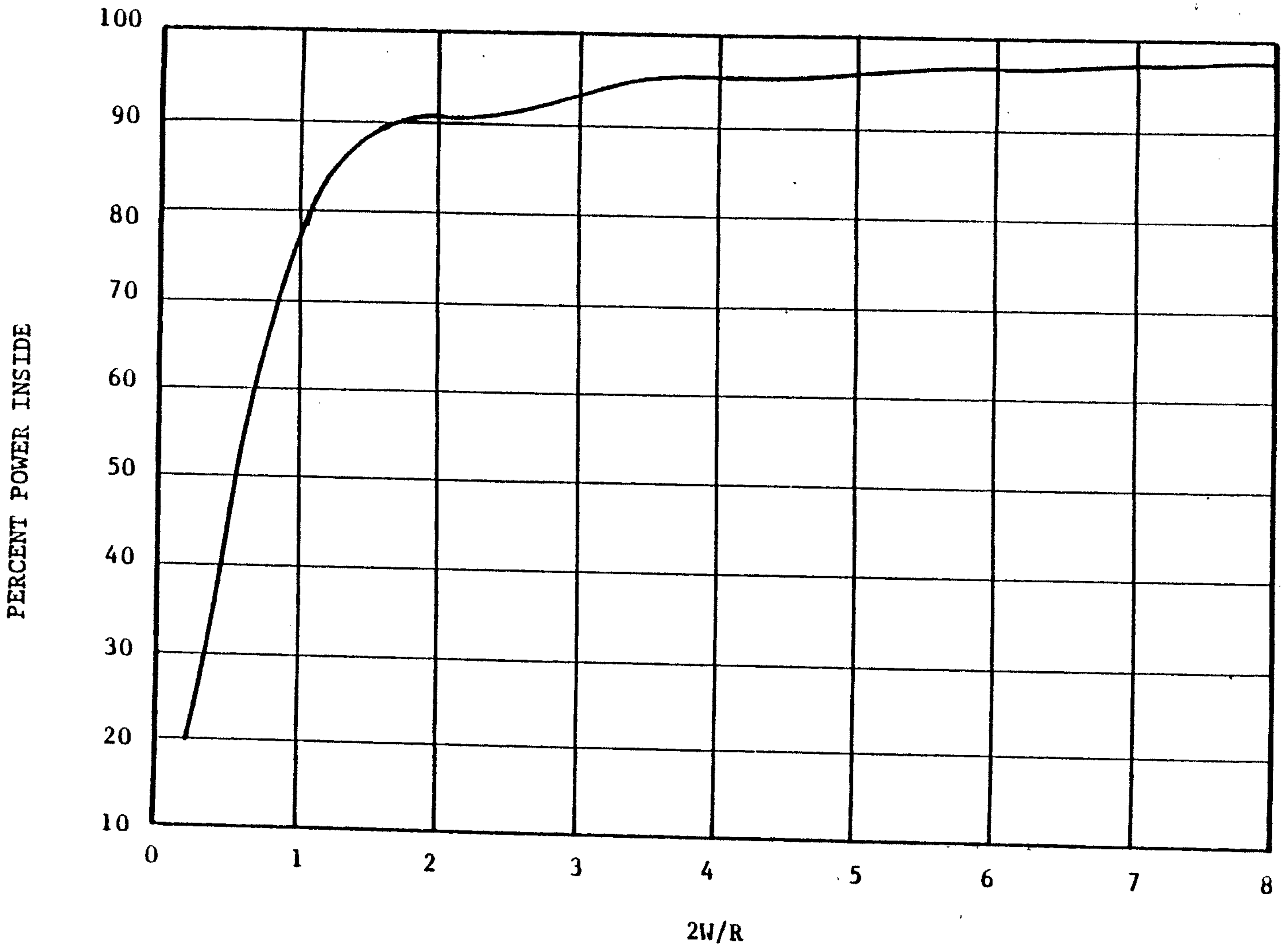

Figure 8. Percent of power in the filter passband versus 2W/R (BPSK) where W = 1/2-sided bandwidth.

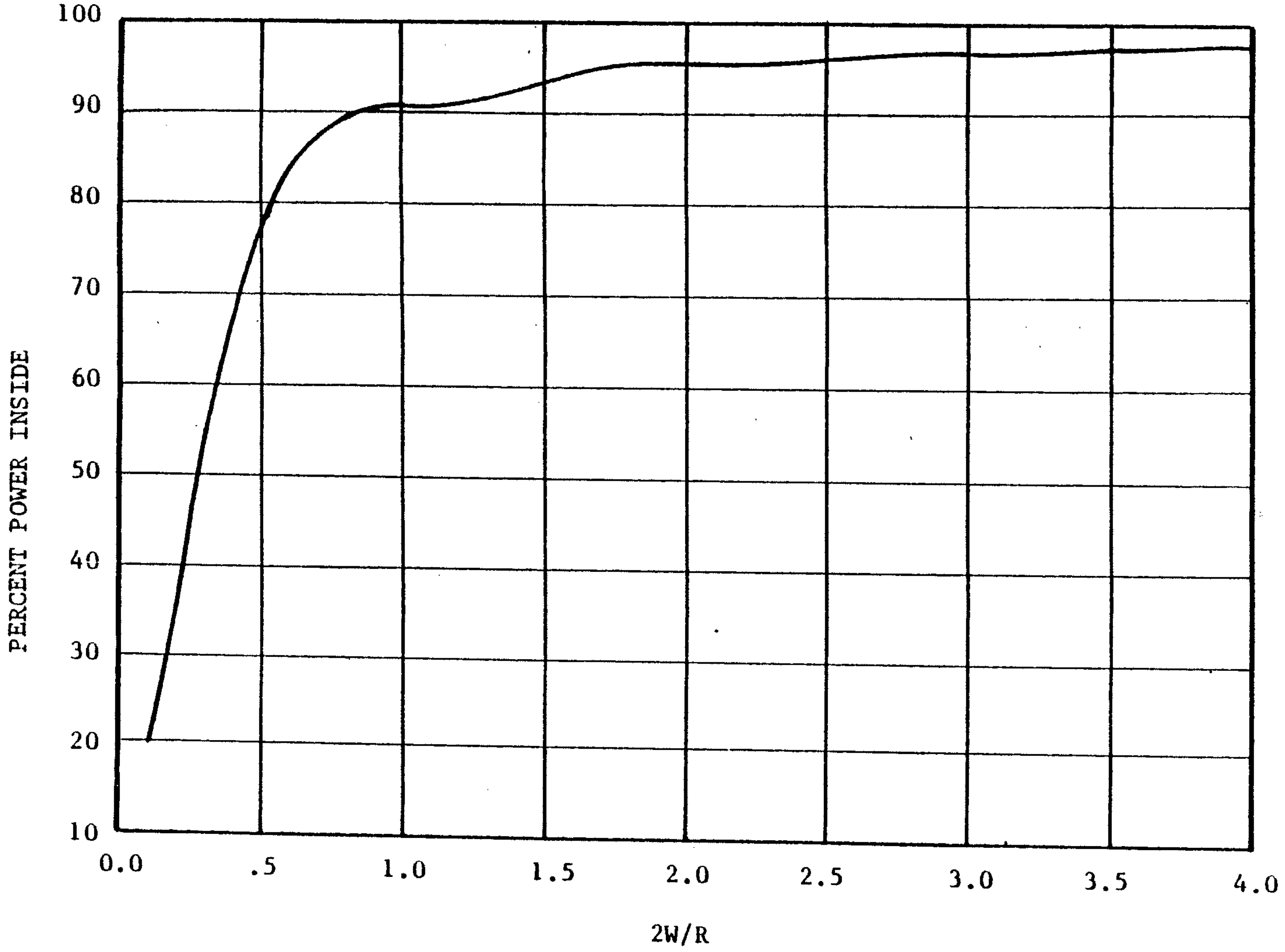

Figure 9. Percent of power in the filter passband versus 2W/R (QPSK) where W = 1/2-sided bandwidth.

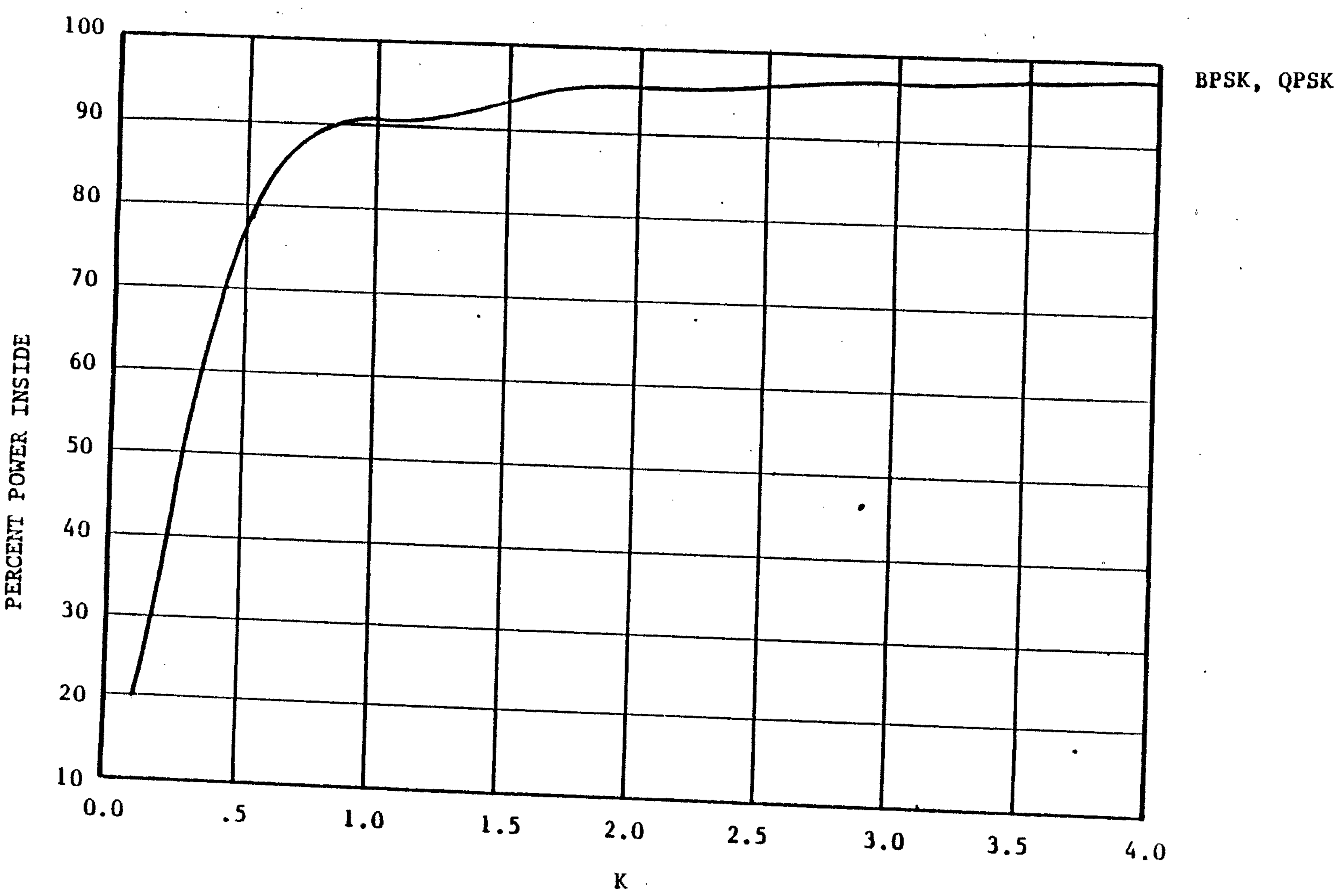

Figure 10. Percent of power in the filter passband versus K for both a BPSK and QPSK spectrum. K = W/R.

SECTION 5

MINIMUM SHIFT KEYING (MSK) AND SINUSOIDAL FREQUENCY SHIFT KEYING (SFSK)

MSK modulation is a constant envelope signal with the important additional characteristic of phase continuity during bit transitions. The phase continuity makes the PSD of MSK compact, falling off rapidly with increasing frequency. This makes MSK an attractive digital modulation method when spectral efficiency is considered.

The most popular form of MSK modulation is really a rectangular pulse FSK signal (on the I and Q channels) with a frequency shift between mark and space of 1/2T. The spectrum for this form of MSK is expressed by Equation 11.

$$G(f - f_c) = \frac{16T}{\pi^2}\left[\frac{\cos[2\pi(f - f_c)T]}{1 - 16(f - f_c)^2 T^2}\right]^2 \qquad (11)$$

The MSK signal format can be generalized as described by [Amoroso, 1976] to a larger class of continuous envelope signaling formats. The general class of MSK signals is best understood as two antipodal pulse streams modulating the inphase and quadrature components. This constant envelope signal can be represented [LaRosa, et al., 1982] as the horizontal projection of the complex vector expressed by Equation 12.

$$\exp J(\phi(0) + 2\pi f_c t)\left\{\cos\left(\frac{\pi}{2}\,\frac{t}{T} - U\sin\frac{2\pi t}{T}\right) \pm j\sin\left(\frac{\pi t}{2T} - U\sin\frac{2\pi t}{T}\right)\right\} \qquad (12)$$

The factor U is a pulse shaping factor that accounts for parametric variations of the pulse. When U = 0, the signaling is the ordinary MSK. When U = 0.25, the modulation is SFSK. The term sinusoidal is used because, for SFSK modulation, the rate of change of frequency during the pulse period is sinusoidally varying.

Figures 11 and 12 show the PSD for this class of signals where U = 0 (MSK) and U = 0.25 (SFSK). These power spectrums are for unfiltered signals. A plot of power containment versus two-sided bandwidth/bit rate for MSK modulations is shown in Figure 13. The curve falls off quickly which demonstrates the small bandwidth required for MSK signals. The 99% energy containment bandwidth for these types of modulation is 1.18 bit rate for MSK and 2.20 bit rate for SFSK. (These are two-sided bandwidths).

[Mathwich, et al., 1974] have studied the effects of filtering and limiting on the performance of MSK. They found that when narrow transmit filters are used, the use of limiters after the filter expands the spectrum to almost the same bandwidth required without filtering or limiters. Their measurements demonstrated that the 99% energy containment bandwidth, with limiting and filtering, is in the range of 1.08 to 1.15 bit rate. This value is almost identical to the unfiltered bandwidth of 1.18 bit rate.

It is concluded that the necessary bandwidths for MSK and SFSK signals are the unfiltered 99% energy containment bandwidths of 1.18/bit rate and 2.20/bit rate, respectively. These values are upper bounds since, as discussed in Section 3, the necessary bandwidths for each digital system depend upon filtering, bit error probability, and E_b/N_o. A smaller bandwidth could be utilized if either the bit error probability is allowed to increase, or if greater power is available.

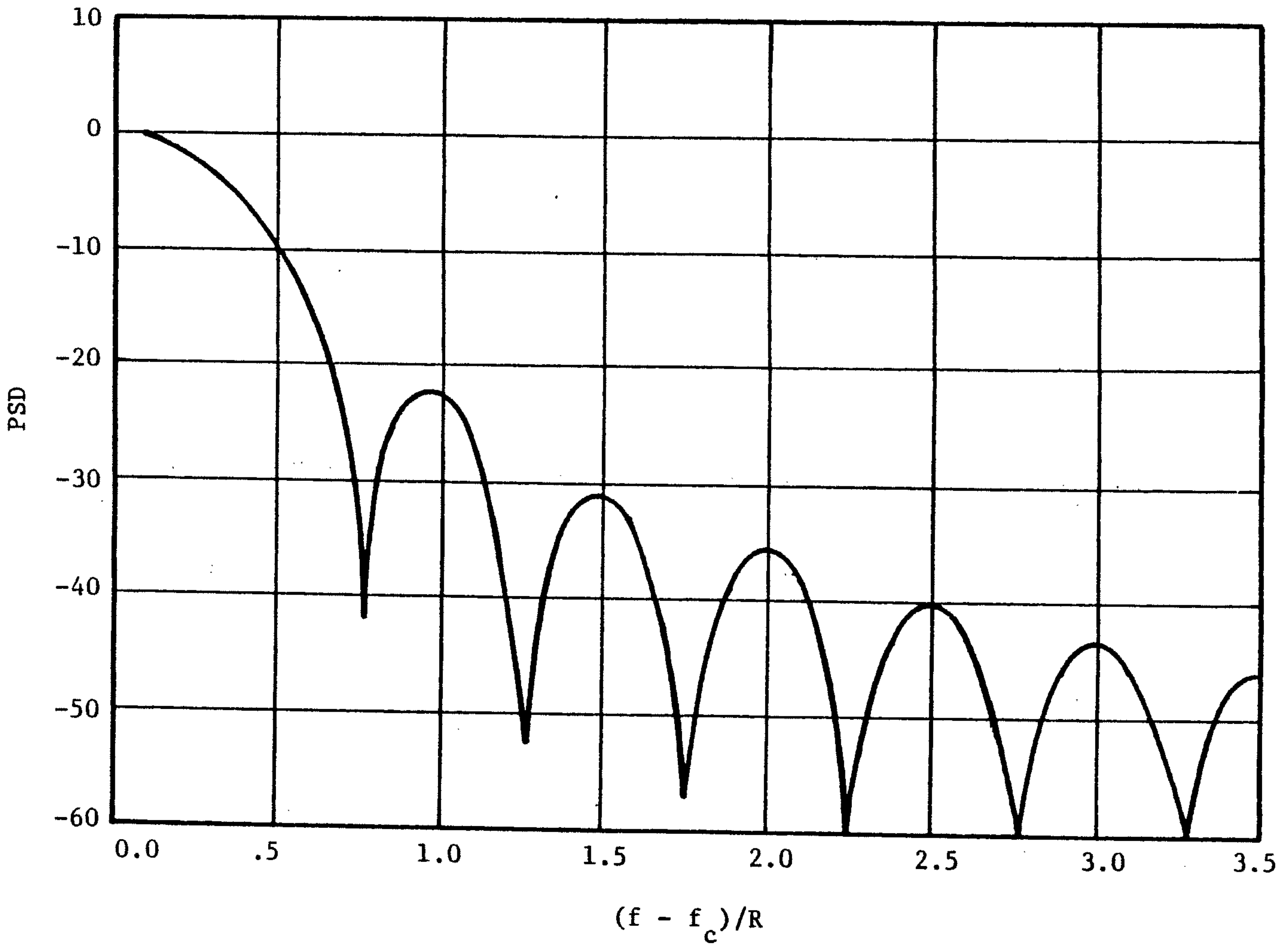

Figure 11. Power spectral density for MSK modulation.

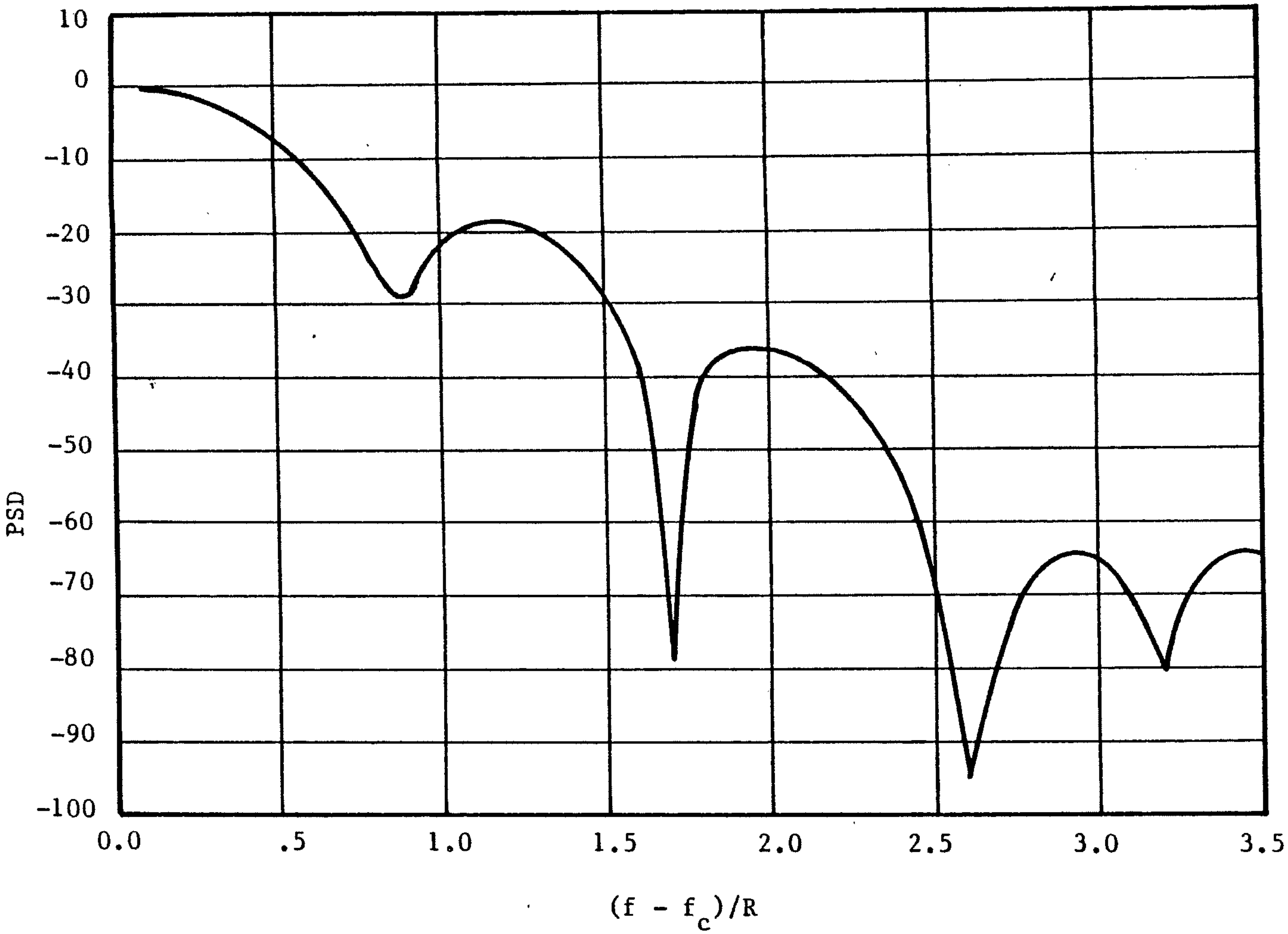

Figure 12. Power spectral density for SFSK modulation.

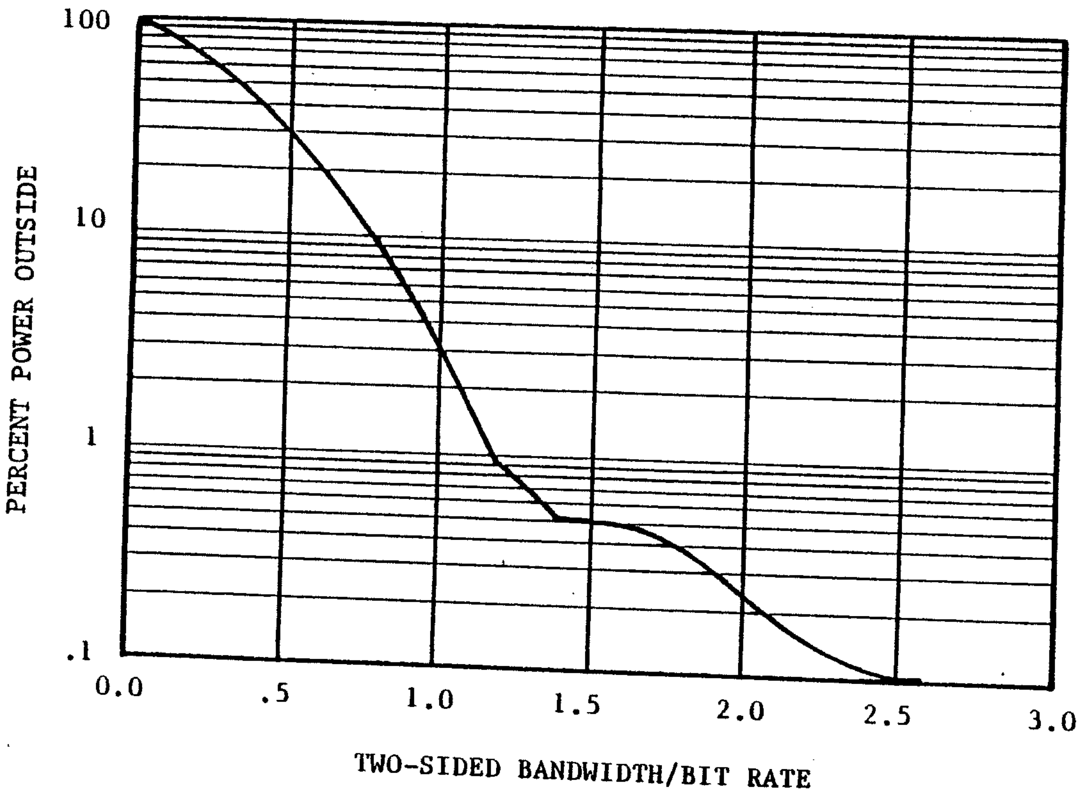

Figure 13. The percent of power outside the bandwidth versus the two-sided bandwidth/bit rate. (MSK power containment.)

SECTION 6

FREQUENCY SHIFT KEYING (FSK)

POWER SPECTRAL DENSITY (PSD)

Binary FSK is a common digital modulation characterized by the fact that the carrier frequency is switched between two frequencies to represent either the mark or space in sequences of binary signals. The case considered for analysis in this report will be the more common continuous-phase constant-envelope BFSK modulation. This modulation results from frequency modulating an oscillator rather than from switching between the outputs of two continuously running oscillators. The signal has the form expressed by Equation 13.

$$e(t) = A \cos \left[\omega_c t + \omega_d \int_{-\infty}^{t} f(t') \, dt' + \theta\right] \tag{13}$$

and

$e(t)$ = FSK signal

ω_d = angular deviating frequency

$f(t')$ = random binary waveform

θ = initial phase angle of the carrier.

The power spectrum for this digital random process is given by [Pelchat, 1964] in Equations 14 and 15.

$$S(X) = 4/R \left[\frac{2(D/R)}{\pi((2D/R)^2 - X^2)}\right]^2 \frac{(\cos(2\pi D/R) - \cos\pi X)^2}{(1 - 2\cos(2\pi D/R)\cos\pi X + \cos^2(2\pi D/R))} \quad ; |\cos(2\pi D/R)| < 1 \tag{14}$$

where

$X = 2(f-f_c)/R$ = normalized frequency variable

R = bit rate

D = peak deviation that equals half the difference between the maximum and minimum values of the instantaneous frequency

and

$$S(X) = \tfrac{1}{4}\,\delta(X + 2\pi D/R) + \tfrac{1}{4}\,\delta(X - 2\pi D/R) + 2/R\left[\frac{2D/R}{\pi\left[(2D/R)^2 - X^2\right]}\right]^2 (1 - \cos(2\pi D/R)\cos\pi X) \qquad ;\left|\cos(2\pi D/R)\right| = 1 \tag{15}$$

A lengthy procedure in random signal processing was used by Pelchat to arrive at this PSD for a random bit stream frequency modulating a carrier. As discussed previously, using the statistical characteristics of the signal first, the autocorrelation function for the process was calculated. Then, its Fourier transform was taken to yield the PSD.

When $\cos(2\pi D/R) = 1$, this PSD consists of a continuous spectrum and two discrete spectral lines. The power discrete spectral density for this special case is shown in Figure 14. Note the line component at $f = f_c + (1/2)R$. A corresponding line is also located symmetrically on the other side of the carrier. The discrete lines of the spectrum correspond to the periodic portions of the modulating process. Each line contains 1/4 of the power and the continuous portion of the spectrum contains 1/2 of the power.

Examples of PSDs for other cases are shown in Figures 15 through 20. All PSDs are normalized in frequency to $2(f - f_c)/R$. Figure 14 shows that for small values of the 2D/R modulation index (M), the spectral power is concentrated close to the origin in a frequency range much less than the bit rate. The PSD for M = 0.7 (Figure 15) has a unique shape showing "ears" and a steep drop in PSD for $2(f - f_c)/R$ greater than 0.5. When the modulation index is larger, Figure 19 shows that the PSD is closely centered around the

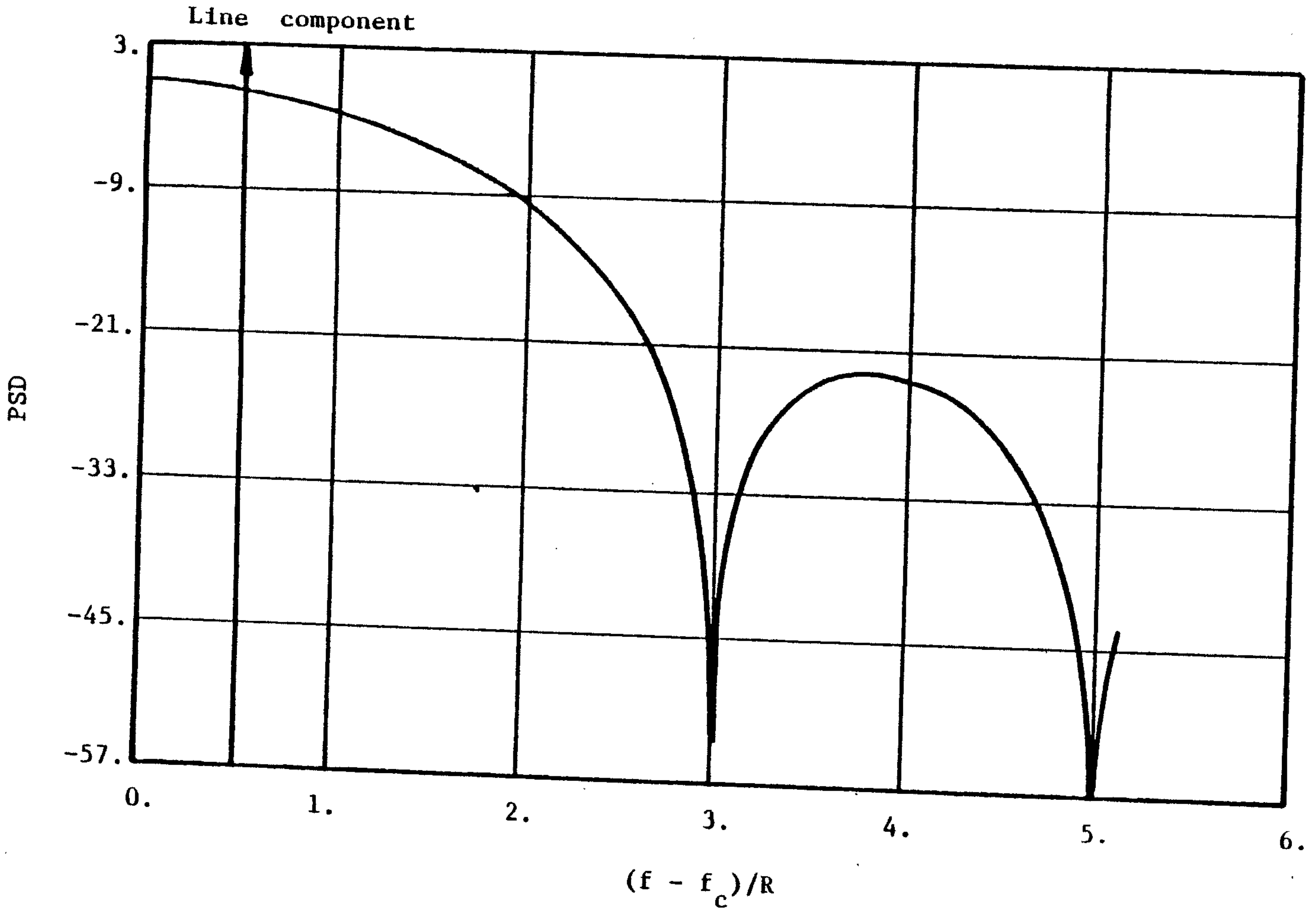

Figure 14. Power spectral density for BFSK when $|\cos 2\pi D/R| = 1$. The spectrum contains a continuous portion and line components at $f = f_c \pm (1/2)R$.

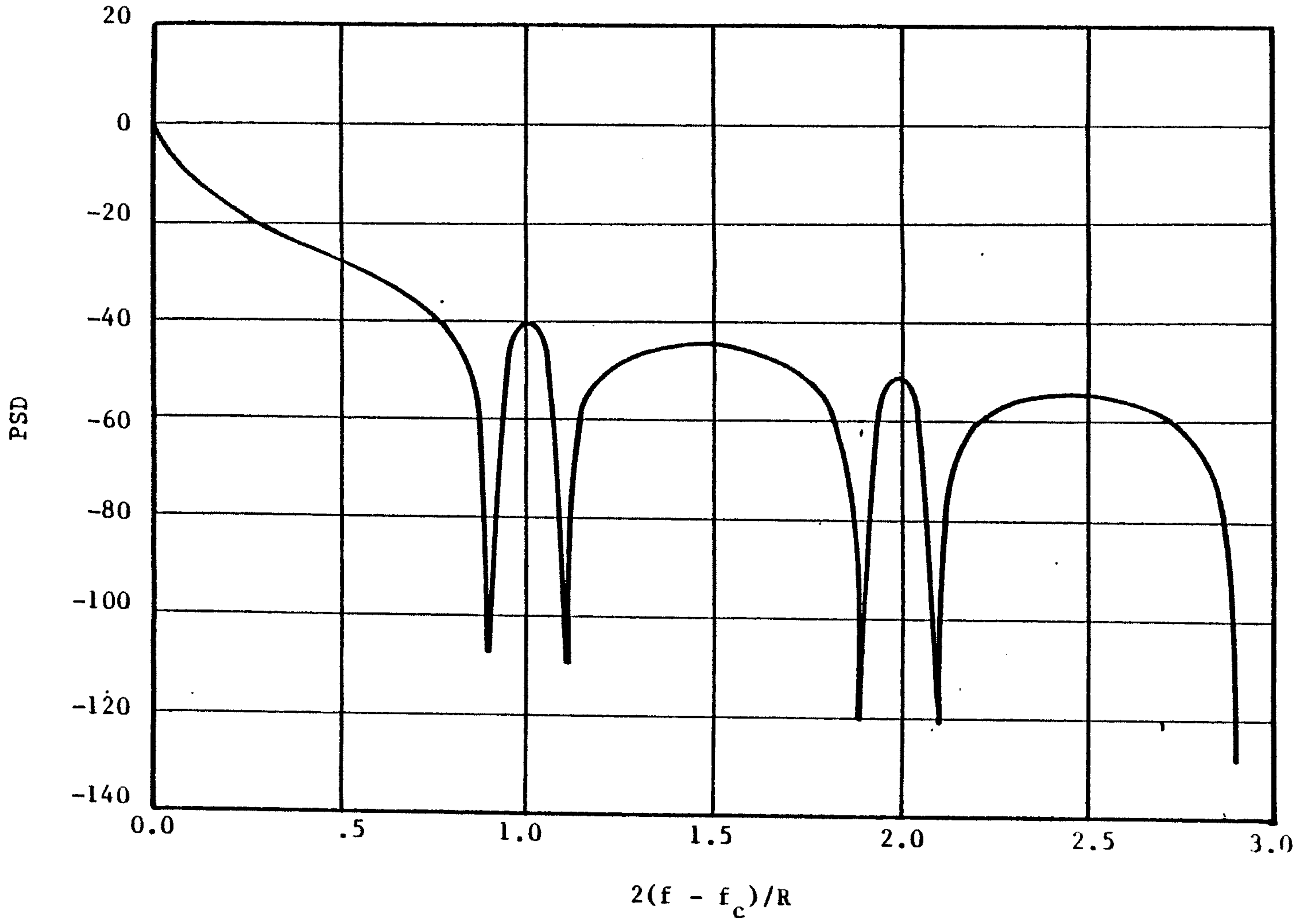

Figure 15. Power spectral density for BFSK with a modulation index of $M = 0.2$.

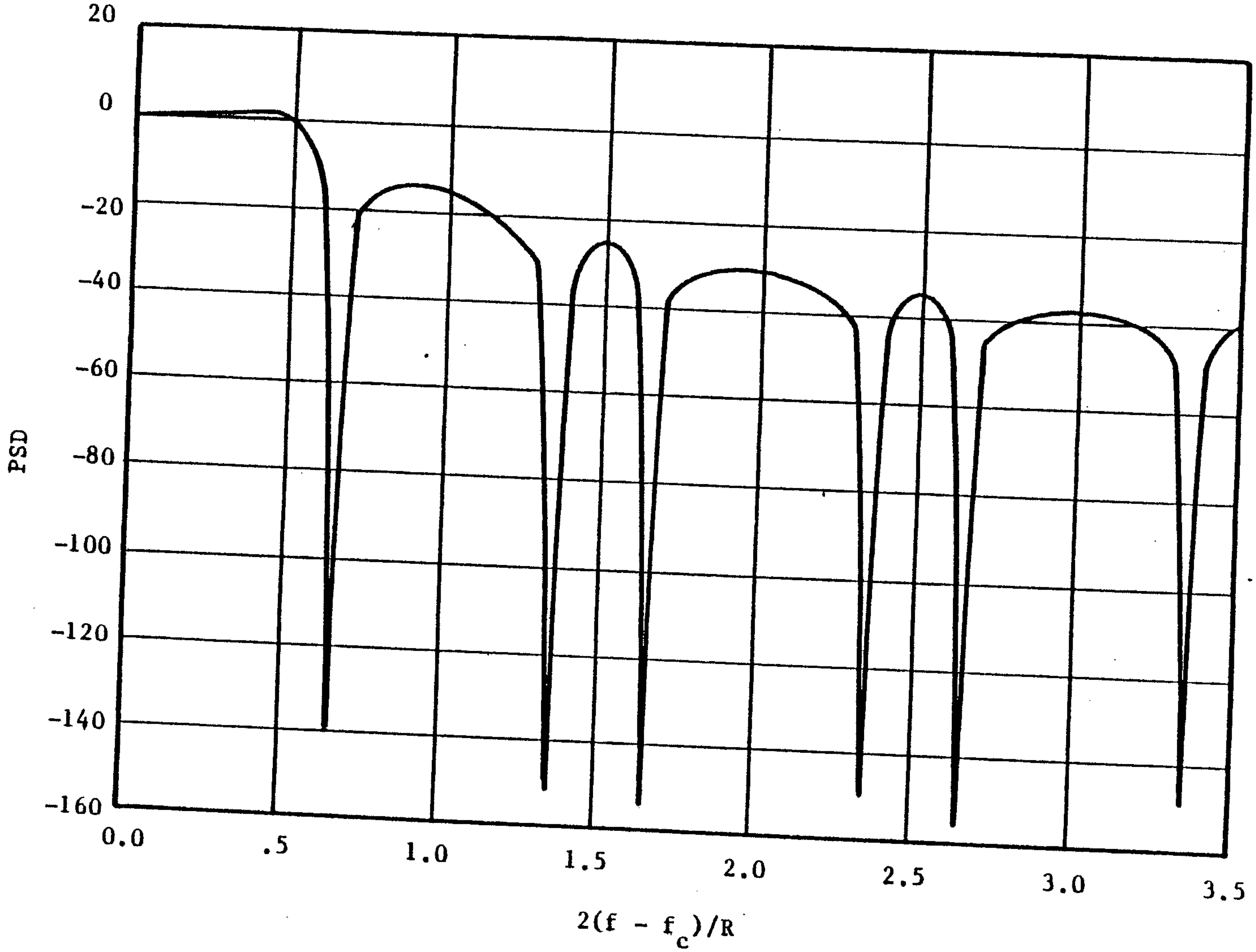

Figure 16. Power spectral density for BFSK with a modulation index of M = 0.7.

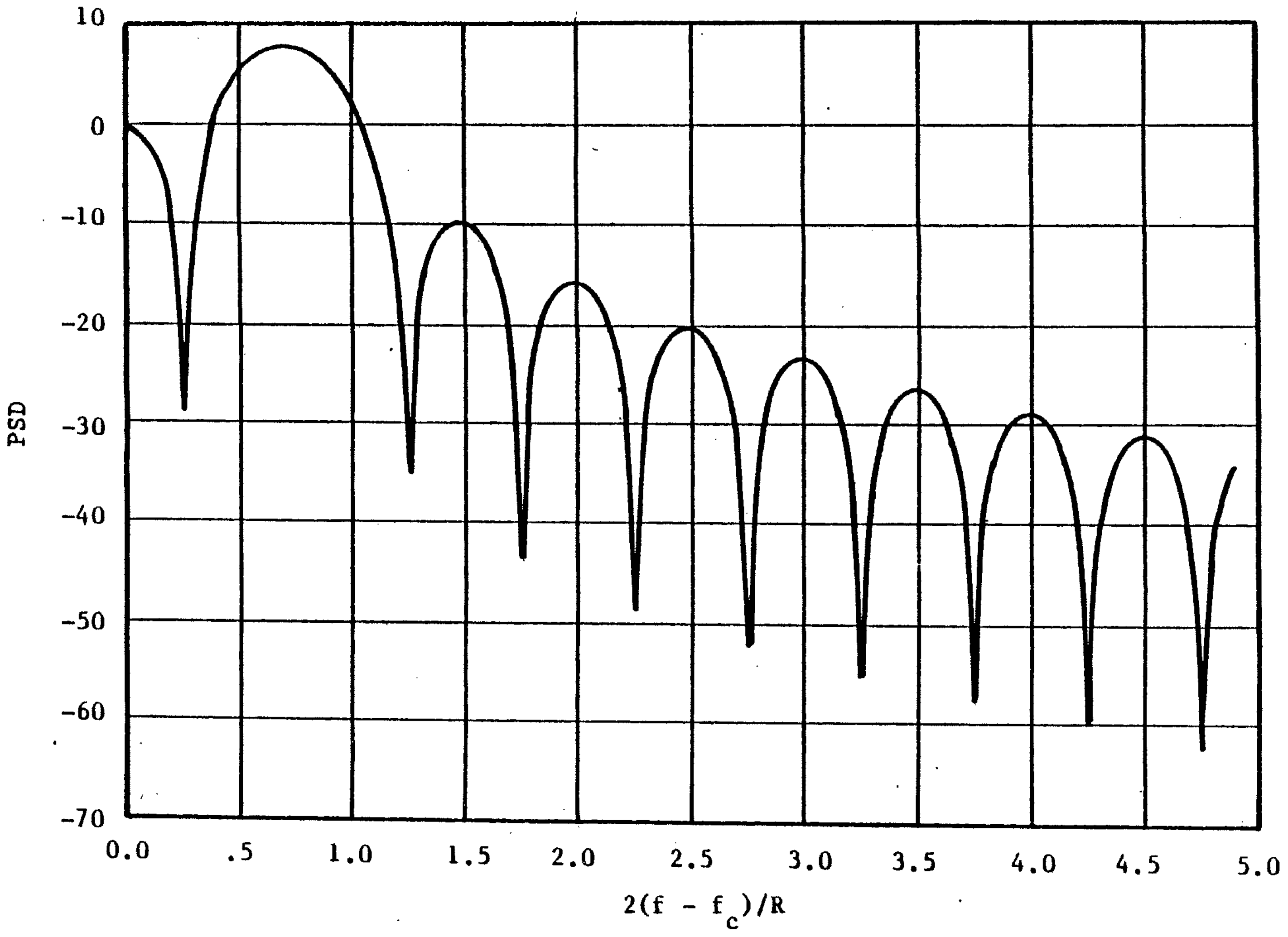

Figure 17. Power spectral density for BFSK with a modulation index of $M = 1.49$.

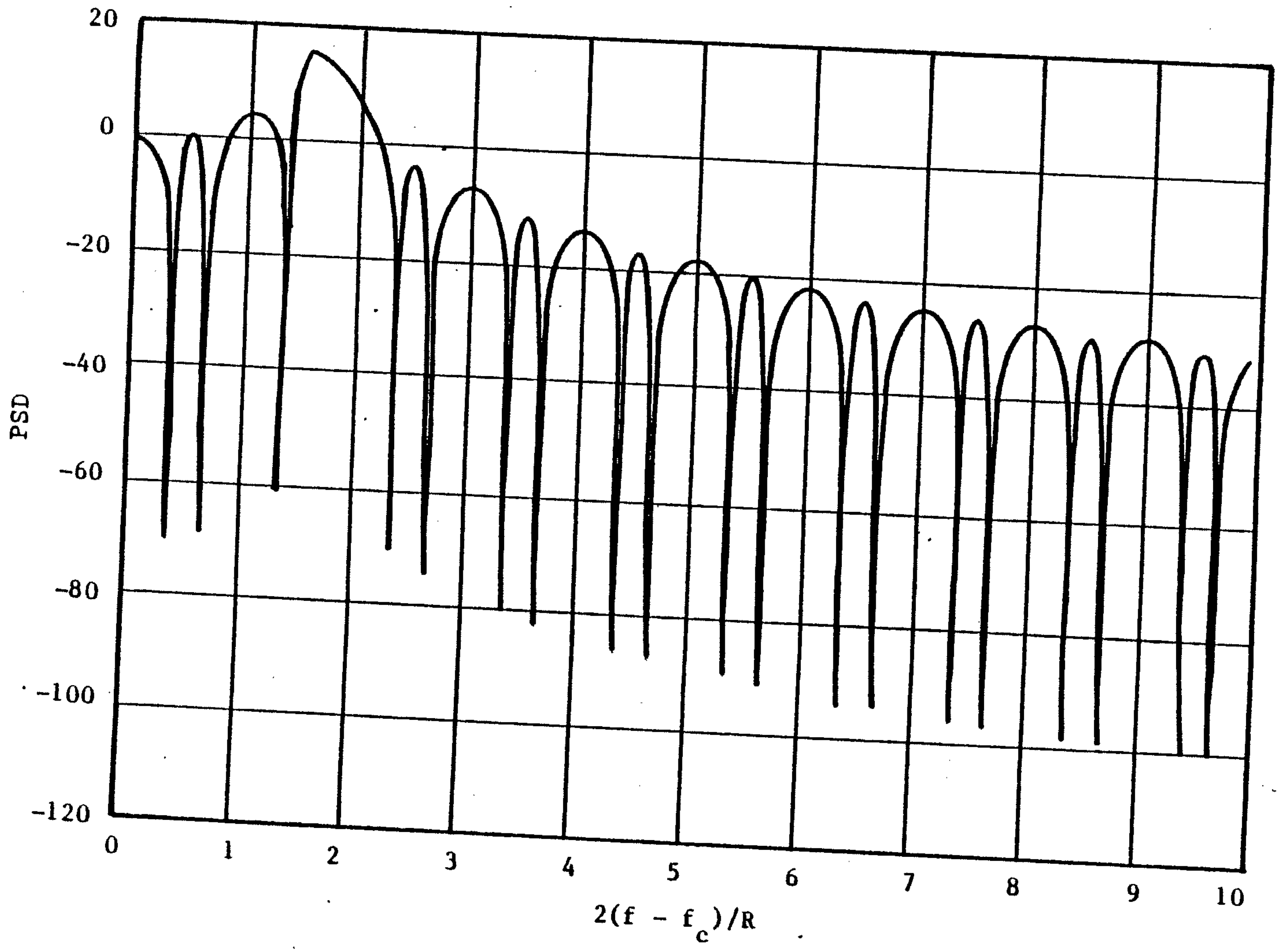

Figure 18. Power spectral density for BFSK with a modulation index of M = 3.3.

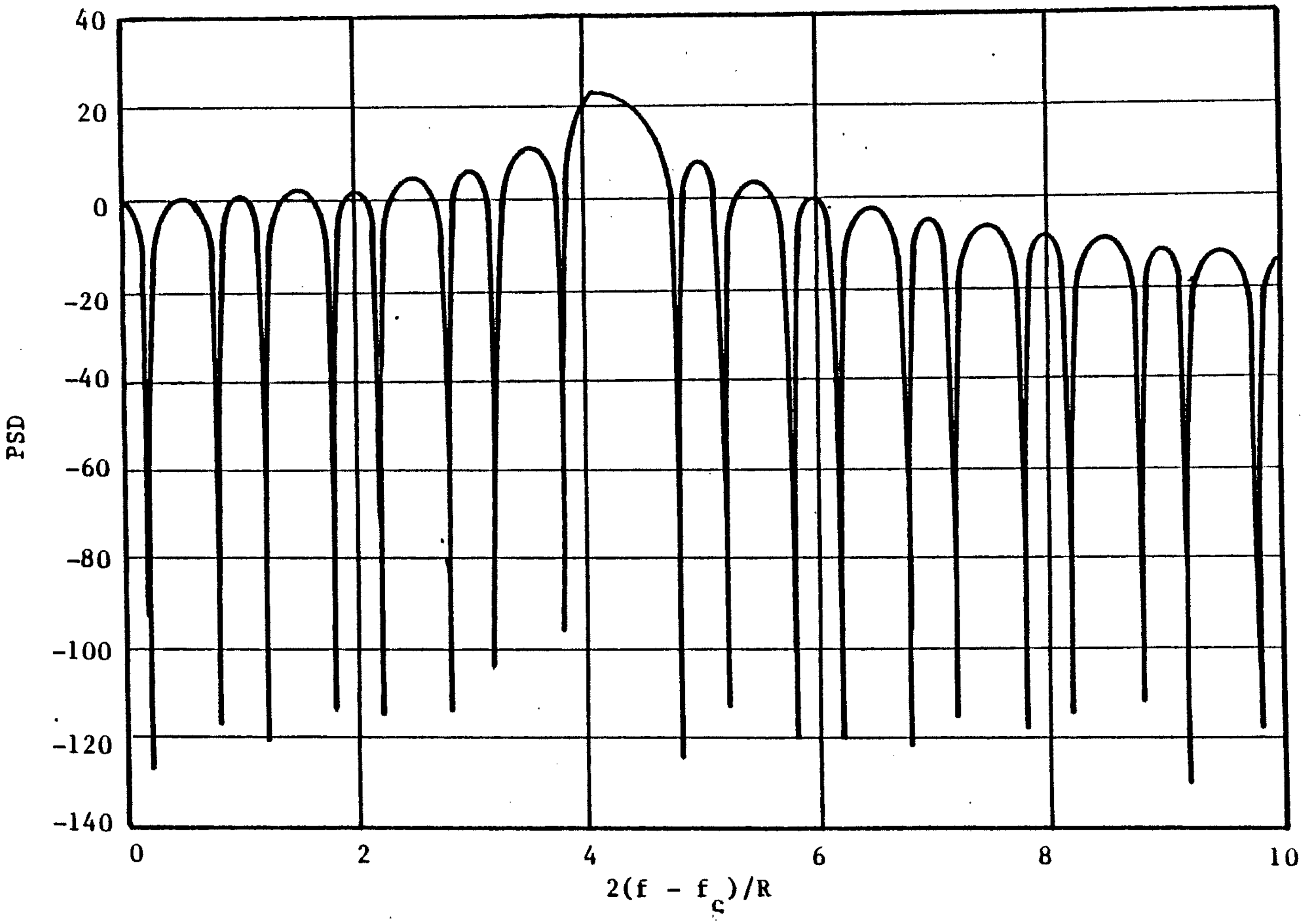

Figure 19. Power spectral density for BFSK with a modulation index of $M = 8.4$.

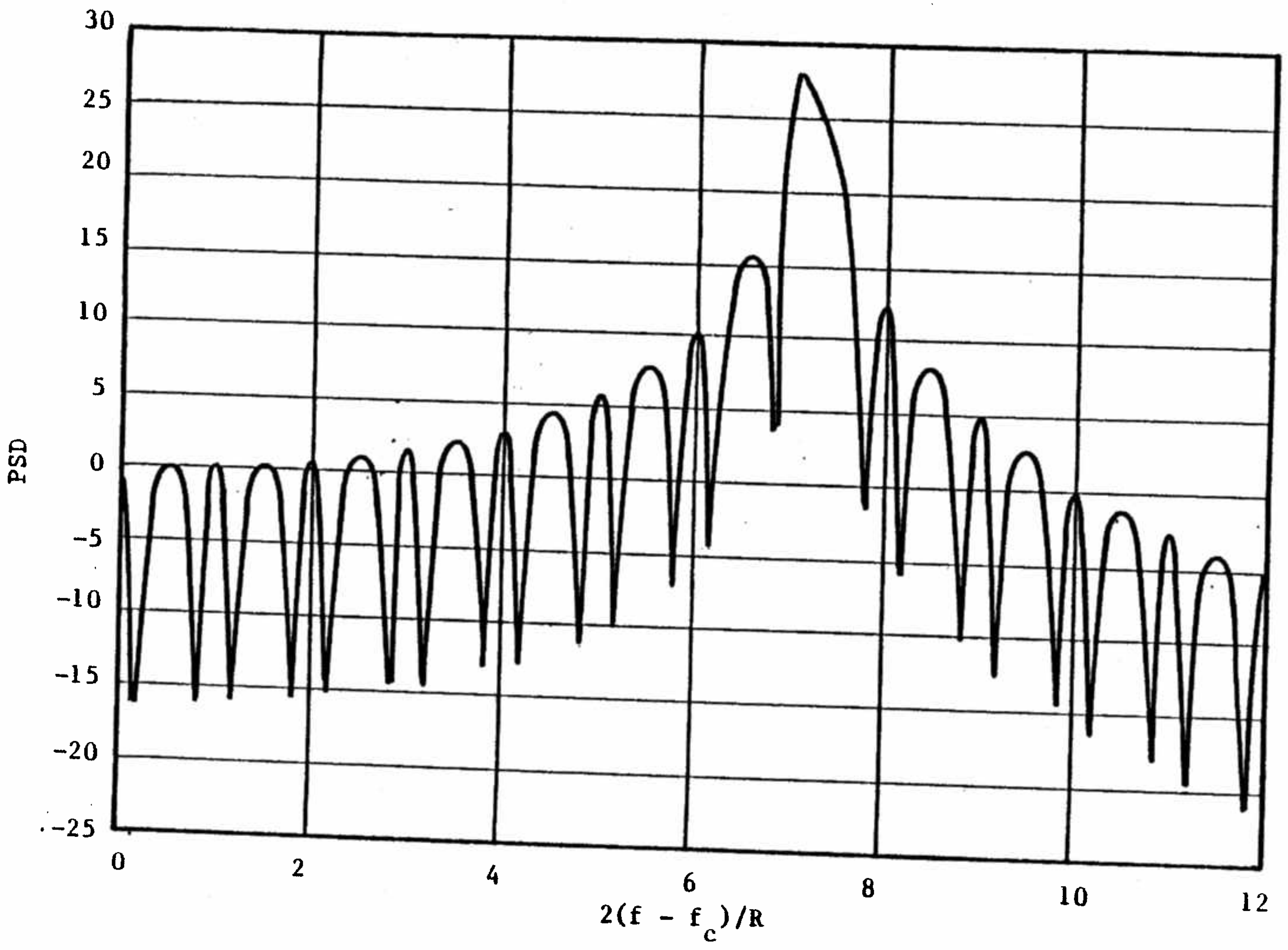

Figure 20. Power spectral density for BFSK with a modulation index of M = 14.35.

normalized deviating frequency (D/R). The central lobes of the spectrum in Figures 18 through 20 are asymmetrical. This is realistic since from Equation 14, the power spectrum is made up of a product of an envelope term and a term that is periodic in X. These figures can also be used to determine when the envelope of the spectrum is below a particular X dB level.

NECESSARY BANDWIDTH OF F1 FREQUENCY SHIFT MODULATION

F1 modulation is a telegraphic radiotelephone FSK emission using binary or quantized digital information. CCIR Recommendation 328-5 [CCIR, 1982a] and Report 179-1 [CCIR, 1982b] specifies the necessary bandwidth and spectral characteristics of F1B modulation. These CCIR texts do not specify the method of modulation (continuous phase or two continuous running oscillators), and only the envelope of the spectrum was considered. The permissible amounts of out-of-band power, above and below the frequency limits of the necessary bandwidth in CCIR Recommendation 328-5, are specified to be approximately 0.5% of the total mean power radiated. This condition is equivalent to making the necessary bandwidth the 99% containment bandwidth.

The formulas listed in CCIR Recommendation 328-5 for the necessary bandwidth of F1B modulation are:

Necessary Bandwidth	Modulation Index
$2.6D + 0.55B$	$1.5 < M < 5.5$
$2.1D + 1.9B$	$5.5 < M < 20.0$

In these equations, B is the modulation rate, in bauds, and D is the peak deviation (i.e., half the difference between the maximum and minimum values of the instantaneous frequency). The modulation index is $M = 2D/B$.

Recently, Annex J in the NTIA Manual included a new formula for the necessary bandwidth of F1 telegraphy without error-correction (signal channel) modulation that is given in Equation 16.

$$B = 2M + 2DK \qquad (16)$$

where

B = Modulation, rate in bauds (digital symbol rate)
$M = B/2$
$K = 1.2$ (typically)

This formula was added to the NTIA Manual to make it consistent with changes in the ITU Radio Regulations which were a result of WARC-79. At WARC-79, the necessary bandwidth formula for F1 was changed to that in Equation 16. This change was based on a CCIR special preparatory paper by the United Kingdom [CCIR, 1978], which used a fitting procedure to obtain a single necessary bandwidth formula for all frequency shift emissions (F1, F4, and F6).

NECESSARY BANDWIDTH OF BINARY CONTINUOUS PHASE FSK MODULATION

The two modulations, binary continuous phase FSK and F1, have similar characteristics. Both of these modulations are FSK emissions using binary digital information. CCIR Recommendations 328-5 and Report 179-1 based the necessary bandwidth of F1B modulation on the 99% energy containment bandwidth. Therefore, it follows to also base the necessary bandwidths for binary continuous phase FSK modulation on the 99% energy containment bandwidth.

Curve A in Figure 21 shows the 99% power contained bandwidths for the FSK power spectrum as a function of the modulation index. The abscissa is the modulation index ($M = 2D/B$) and the ordinate is the bandwidth/bit rate. Also plotted on Figure 21 are curves of the NTIA Annex J necessary bandwidth (curve B) and the CCIR Recommendation 328-5 necessary bandwidth for F1 modulation (curve C). Above the modulation index ($M = 1$), all of the representations of necessary bandwidth are virtually identical.

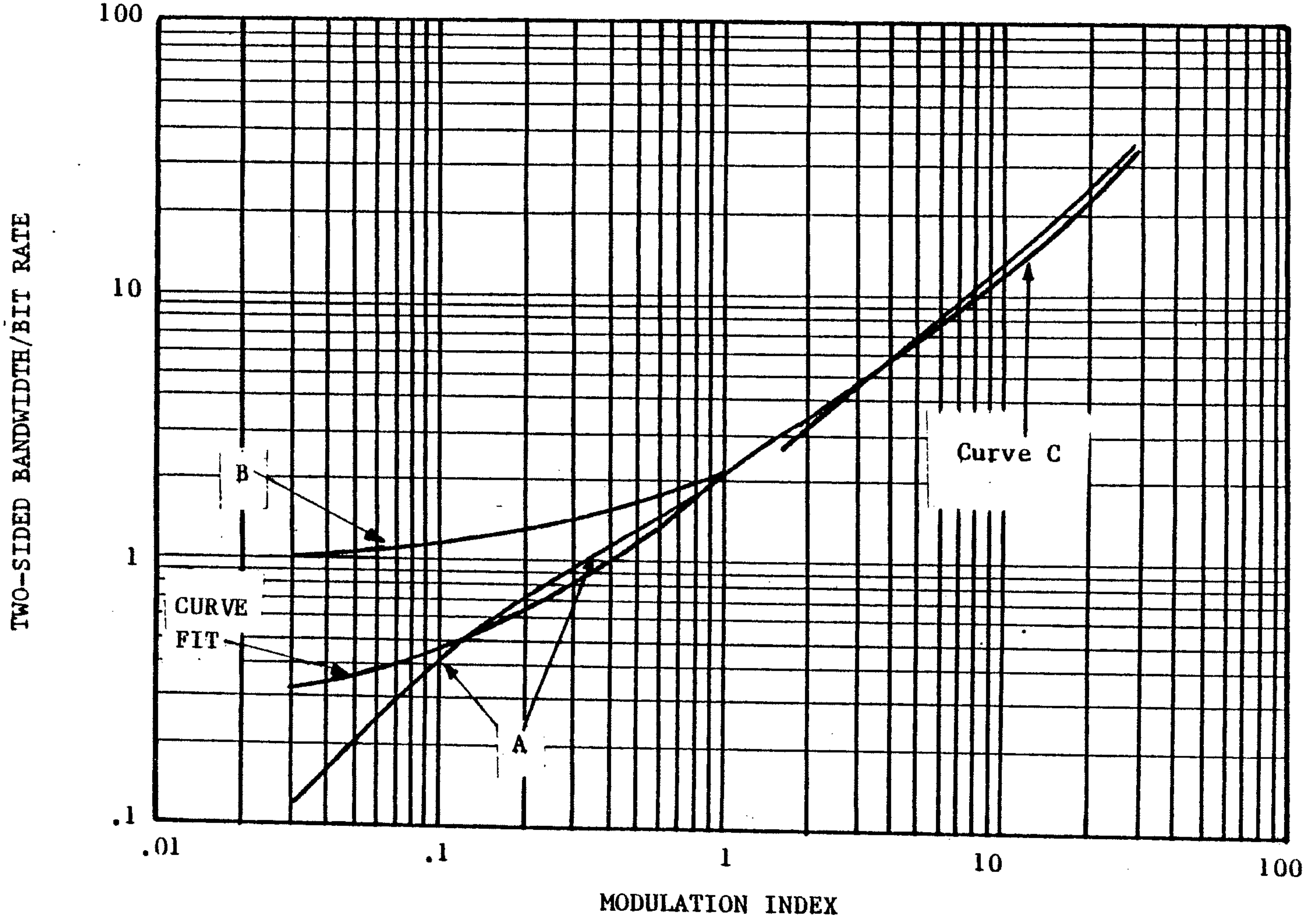

Figure 21. Necessary bandwidth of FSK modulation as a function of the modulation index. The basis for the data in the figure are: Curve A - 99% power containment data, Curve B - NTIA Annex J necessary bandwidth, and Curve C - CCIR Rec. 328-5 necessary bandwidth for F1 modulation. The Curve Fit is the composite for the necessary bandwidth over the total range of values.

Using the 99% energy data (curve A) as a basis, TABLE 2 shows two formulas, which as a composite, give the necessary bandwidth of BFSK. For modulation indices greater than 1, the NTIA Annex J necessary bandwidth formula for F1 telegraphy fits the data and is chosen as the formula for $M > 1$. Below $M = 1$, in the range $M = 0.03$ to 1, curve A was curve-fit by $Y = 1.93 M + 0.27$. The necessary bandwidth was determined using both formulas shown in TABLE 2 and is presented as curve D in Figure 21.

TABLE 2

BINARY FSK MODULATION WITH 99% ENERGY CONTAINMENT BANDWIDTH

D = frequency deviation

R = bit rate

Modulation index ($M = 2D/R$)	Necessary Bandwidth (B_n), in Hz
$0.03 < M < 1.0$	$3.86D + 0.27R$
$1.0 < M < 20$	$2.4D + 1.0R$

SECTION 7

RESULTS

Sections 4 through 6 discussed the determination of the necessary bandwidth and spectral properties of four common digital modulations. The existing necessary bandwidth formulas for digital modulation are contained in Annex J of the NTIA Manual. Based on the results given in Sections 4 through 6, the additional necessary bandwidth formulas recommended to be included in Annex J of the NTIA Manual are shown in Table 3.

TABLE 3

NECESSARY BANDWIDTH

Emission	Formula	Sample Calculation	Emission
BPSK	$B_n = 2RK$	Digital modulation used to send 10 Mb/s by use of PSK with 2 signaling states $R = 10 \times 10^6$ b/s $K = 1$; $S = 2$; $B_n = 20$ MHz	20M0G7DDT
MSK	$B_n = 1.18R$ (1)	Digital modulation used to send 2 Mb/s using 2-ary minimum or sinusoidal frequency shift keying (SFSK); $B_n = 2.36$ MHz (1) $B_n = 4.4$ MHz (2)	2M36G1DBN
SFSK	$B_n = 2.20R$ (2)		4M40G1DBN
FSK (2 states)	$3.86D + .27R$ $0.03 < M < 1.0$ $2.4D + 1.0R$ $1.0 < M < 20$	Digital modulation used to send 1 Mb/s by FSK with 2 signaling states and 0.75 MHz peak deviations of carrier $R = 1 \times 10^6$ b/s; $D = 0.75 \times 10^6$ Hz $B_n = 2.8$ MHz	2M80FIDBC

LIST OF REFERENCES

Amoroso, Frank [1976] "Pulse and Spectrum Manipulation in the Minimum (Frequency) Shift Keying (MSK) Format," IEEE Transactions on Communications, March.

Amoroso, Frank [1980] "The Bandwidth of Digital Data Signals," IEEE Communications Magazine, November.

CCIR [1978] Document P/17, United Kingdom, "Examples of Necessary Bandwidth, "Input Document to Special Preparatory Meeting, CCIR, WARC-79,Geneva, Switzerland.

CCIR [1982a] Recommendation 328-5, "Spectra and Bandwidth of Emissions," Volume I, Recommendations and Reports of the CCIR, Geneva, Switzerland.

CCIR [1982b] Report 179-1, Bandwidth of Radiotelegraph Emissions A1A, A1B, and F1B, XVth Plenary Assembly, Vol. I, Geneva, Switzerland.

CCIR [1982c] Report 378-3, "Characteristics and Performance Objectives for Digital Radio-Relay Systems," Volume IX, XVth Plenary Assembly, Recommendations and Reports of the CCIR, Geneva, Switzerland.

Department of Commerce [1981] Organization Order, DOO 25-7, Subject: NTIA.

ITU [1982] "International Telecommunication Union Radio Regulations," Geneva, Switzerland.

Korn, I. [1980] "Error Probability and Bandwidth of Digital Modulation," IEEE Transactions on Communications, Vol. COM-28, February.

LaRosa, Richard; Thomas J. Marynowski; Kenneth J. Henrich [1982] "A Simple Modulator for Sinusoidal Frequency Shift Keying," IEEE Transactions on Communications, Vol COM-30, No. 5, May.

Mathwich,H. [1974] Joseph F. Balcewicz and Martin Hecht, "The Effect of Tandem Band and Amplitude Limiting on the E_b/N_o Performance of Minimum (Frequency) Shift Keying (MSK)," IEEE Transaction on Communications, October.

NTIA [1981] "Manual of Regulations and Procedures for Federal Radio Frequency Management," National Telecommunications and Information Administration, Washington, DC, January.

Nyquist, H. [1928] "Certain Topics in Telegraph Transmission Theory," AIEE Transaction, Vol. 47, April.

Papoulis, A. [1965] "Probability, Random Variables and Stochastic Processes," McGraw Hill, New York, NY.

Parks, John H. [1969] "Effect of Bandlimiting on the Detection of Binary Signals," IEEE Transaction on Aerospace and Electronic Letters, September.

Pasupathy, Subbarayan [1979] "Minimum Shift Keying: A Spectrally Efficient Modulation," IEEE Communications Magazine, July.

Pelchat, M.G. [1964] "The Autocorrelation Function and Power Spectrum of PCM/FM with Random Binary Modulating Waveforms," IEEE Transactions on Space Electronics and Telemetry, March.

Shalvi, Sidney [1980] "Digital DTWS Users Manual," ECAC-UM-80-030, DoD ECAC, Annapolis, MD, January.

Shamugam, K.S. [1979] "Digital and Analog Communication Systems," John Wiley and Sons, New York, NY.

Shelton, William [1984] David Cohen, G. Crandall, G. Hurt, and W. Speights, "Assessment of Narrowband Modulation Technologies for Government Land Mobile Operations," NTIA Report 84-156, National Telecommunications and Information Administration, Washington, DC.

Skylar [1983] "A Structural Overview of Digital Communications - A Tutorial Review - Part I," IEEE Communications Magazine, August.

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The National Telecommunications and Information Administration (NTIA) and the Interdepartment Radio Advisory Committee (IRAC) require, for their management of the spectrum, technical guidance concerning the necessary bandwidth and spectral characteristics of digital modulation. The necessary bandwidth is the minimum emission bandwidth required for an acceptable quality of service. The NTIA Manual includes all guidance and criteria relating to necessary bandwidth in Annex J entitled "Determination of Necessary Bandwidth." In this report, the concept of necessary bandwidth is applied to digital modulation. The necessary bandwidths for several digital modulations are considered in detail. These are phase shift keying (PSK), minimum shift keying (MSK), and frequency shift keying (FSK). The techniques used to determine the necessary bandwidths are general and could be extended to other digital modulations. This report shows that for digital modulation systems, the necessary bandwidth, signal power, and bit error rate are all interrelated and tradeoffs among these factors are possible.

16. Key Words *(Alphabetical order, separated by semicolons)*

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