

# A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM FOR RADIO WAVES IN THE IONOSPHERE

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## PREFACE

This report documents the latest version of the three-dimensional ray tracing program originally described in "A Three-Dimensional Ray Tracing Computer Program," by R. M. Jones, ESSA Technical Report IER 17-ITSA 17, and later modified in "Modifications to the Three-Dimensional Ray Tracing Program Described in IER 17-ITSA 17," by R. M. Jones, ESSA Technical Memorandum ERLTM-ITS 134. This report replaces all of the material contained in the above two reports.



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