APPENDIX 2. VERSIONS OF THE REFRACTIVE INDEX SUBROUTINE (RINDEX)

This ray tracing program gains versatility without sacrificing speed by having several versions of some of the subroutines. For example, the 8 versions of the refractive index subroutine allow the user to decide for each ray path calculation whether to include or ignore various aspects of the propagation medium such as the earth's magnetic field or collisions between electrons and neutral air molecules.

If collisions are included, the user has the option of using the Appleton-Hartree formula (which assumes a constant collision frequency) or the Sen-Wyller formula (which assumes a Maxwell distribution of electron energies and a collision frequency proportional to energy). The Sen-Wyller formula is generally assumed to be more accurate, especially in the lower ionosphere, but the Appleton-Hartree formula can often be used with an effective collision frequency profile to save computer time.

When the effect of the earth's magnetic field is included and ray paths are calculated near vertical incidence, a spitze (Davies, 1965, p. 202) often occurs in the ray path. (At a spitze, the usual formulas for refractive index become indeterminate because the wave normal is parallel with the earth's magnetic field and the wave frequency equals the local plasma frequency.) Two versions of the refractive index subroutine have been developed to calculate ray paths through a spitze. These two versions will also work in the absence of a spitze, but the standard versions are much faster.

The input to the refractive index subroutines is through blank common and common blocks /XX/, /YY/, and /ZZ/. Output is through common block /RIN/. The refractive index subroutine is called through the entry RINDEX. The subroutine names are used only for user identification. The following 8 versions of the refractive index subroutine are listed in this appendix:

```
      SUBROUTINE AHWFWC
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C          APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR
     1             ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      EQUIVALENCE (RAY,W(1)),(F,W(6))
      LOGICAL SPACE
      REAL KR,KTH,KPH,K2
      COMPLEX N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT,
     1        POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,
     2        KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,
     3        KPHPK
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),
     1     (FIELD=1.),
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
     3     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),
     4     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),
     5     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)
     6    ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),
     7     (I=(0.,1.)),(ABSLIM=1.E-5)
      ENTRY RINDEX
      OM=PIT2*1.E6*F
      C2=C*C
      K2=KR*KR+KTH*KTH+KPH*KPH
      OM2=OM*OM
      VR =C/OM*KR
      VTH=C/OM*KTH
      VPH=C/OM*KPH
      CALL ELECTX
      CALL MAGY
      V2=VR**2+VTH**2+VPH**2
      VDOTY=VR*YR+VTH*YTH+VPH*YPH
      YLV=VDOTY/V2
      YL2=VDOTY**2/V2
      YT2=Y**2-YL2
      YT4=YT2*YT2
      CALL COLFRZ
      U=CMPLX(1.,-Z)
      UX=U-X
      UX2=UX*UX
      RAD=RAY*CSQRT(YT4+4.*YL2*UX2)
      D=2.*U*UX-YT2+RAD
      D2=D*D
      N2=1.-2.*X*UX/D
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2
      PPSPR =YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV
      PNPX=-(2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)
      PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH
      PNPVR =PNPPS*(VR *YL2/V2-YLV*YR )
      PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH)
      PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)
```

```
      PNPT=PNPX*PXPT                                                    WFWC066
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 WFWC067
      POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VDOTY*UX)                        WFWC068
      GAM=(-YT2+RAD)/(2.*UX)                                            WFWC069
      LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))                                 WFWC070
      KAY2=OM2/C2*N2                                                    WFWC071
      IF(RSTART.EQ.0.) GO TO 1                                          WFWC072
      SCALE=SQRT(REAL(KAY2)/K2)                                         WFWC073
      KR =SCALE*KR                                                      WFWC074
      KTH=SCALE*KTH                                                     WFWC075
      KPH=SCALE*KPH                                                     WFWC076
    1 CONTINUE                                                          WFWC077
C********** CALCULATES A HAMILTONIAN H                                  WFWC078
      H=.5*(C2*K2/OM2-N2)                                               WFWC079
C********** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                 WFWC080
C********** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.           WFWC081
      PHPT =-PNPT                                                       WFWC082
      PHPR =-PNPR                                                       WFWC083
      PHPTH=-PNPTH                                                      WFWC084
      PHPPH=-PNPPH                                                      WFWC085
      PHPOM=-NNP/OM                                                     WFWC086
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      WFWC087
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     WFWC088
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     WFWC089
      KPHPK=N2                                                          WFWC090
      RETURN                                                            WFWC091
         END                                                            WFWC092


      SUBROUTINE AHWFNC                                                 WFNC001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   WFNC002
C          APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS           WFNC003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                WFNC004
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               WFNC005
     1             H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHIWFNC006
     2           ,PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPIWFNC007
     3           ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN           WFNC008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WFNC009
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRWFNC010
     1          ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                      WFNC011
      COMMON /ZZ/ MODZ,Z(4)                                             WFNC012
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       WFNC013
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                WFNC014
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   WFNC015
      LOGICAL SPACE                                                     WFNC016
      REAL KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,N2,NNP,LPOLAR,LPOLRI   WFNC017
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),         WFNC018
     1     (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),         WFNC019
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), WFNC020
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),              WFNC021
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            WFNC022
     5     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   WFNC023
     6     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        WFNC024
     7     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),        WFNC025
     8     (MODZ=1H ),(U=1.)                                            WFNC026
      ENTRY RINDEX                                                      WFNC027
      OM=PIT2*1.E6*F                                                    WFNC028
      C2=C*C                                                            WFNC029
      K2=KR*KR+KTH*KTH+KPH*KPH                                          WFNC030
      OM2=OM*OM                                                         WFNC031
      VR =C/OM*KR                                                       WFNC032
      VTH=C/OM*KTH                                                      WFNC033
      VPH=C/OM*KPH                                                      WFNC034
```

```
      CALL ELECTX
      CALL MAGY
      V2=VR**2+VTH**2+VPH**2
      VDOTY=VR*YR+VTH*YTH+VPH*YPH
      YLV=VDOTY/V2
      YL2=VDOTY**2/V2
      YT2=Y**2-YL2
      YT4=YT2*YT2
      UX=U-X
      UX2=UX*UX
      RAD=RAY*SQRT(YT4+4.*YL2*UX2)
      D=2.*UX-YT2+RAD
      D2=D*D
      N2=1.-2.*X*UX/D
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV
      PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)
      NNP=N2-(2.*X*PNPX+Y*PNPY)
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPPS*PPSPR
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2
      PNPT=PNPX*PXPT
      SPACE=N2.EQ.1.
      POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)
      GAM=(-YT2+RAD)/(2.*UX)
      LPOLRI=X*SQRT(YT2)/(UX*(U+GAM))
      KAY2=OM2/C2*N2
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT(KAY2/K2)
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
  1   CONTINUE
C********* CALCULATES A HAMILTONIAN H
      H=.5*(C2*K2/OM2-N2)
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
      PHPT =-PNPT
      PHPR =-PNPR
      PHPTH=-PNPTH
      PHPPH=-PNPPH
      PHPOM=-NNP/OM
      PHPKR =C2/OM2*KR -C/OM*PNPVR
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH
      KPHPK=N2
      RETURN
         END
```

```
      SUBROUTINE AHNFWC                                                 NFWC010
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE  NFWC011
C           APPLETON-HARTREE FORMULA -- NO FIELD, WITH COLLISIONS       NFWC012
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                NFWC013
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   NFWC014
     1              PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR, NFWC015
     2              SGN                                                 NFWC016
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  NFWC017
      COMMON /YY/ MODY,Y(16)                                            NFWC018
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               NFWC019
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       NFWC020
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                NFWC021
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   NFWC022
      LOGICAL SPACE                                                     NFWC023
      REAL KR,KTH,KPH,K2                                                NFWC024
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   NFWC025
     1        KPHPK,POLAR,LPOLAR,U,I,PNPX,PNPZ,                         NFWC026
     2        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          NFWC027
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),         NFWC028
     1     (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),                 NFWC029
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            NFWC030
     3     (MODY=1H ),                                                  NFWC031
     4     (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      NFWC032
     5     (I=(0.,1.)),(ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)NFWC033
      ENTRY RINDEX                                                      NFWC034
      OM=PIT2*1.E6*F                                                    NFWC035
      C2=C*C                                                            NFWC036
      K2=KR*KR+KTH*KTH+KPH*KPH                                          NFWC037
      OM2=OM*OM                                                         NFWC038
      VR =C/OM*KR                                                       NFWC039
      VTH=C/OM*KTH                                                      NFWC040
      VPH=C/OM*KPH                                                      NFWC041
      CALL ELECTX                                                       NFWC042
      CALL COLFRZ                                                       NFWC043
      U=1.-I*Z                                                          NFWC044
      N2=1.-X/U                                                         NFWC045
      PNPX=-1./(2.*U)                                                   NFWC046
      PNPZ=-I*X/(2.*U**2)                                               NFWC047
      NNP=N2-(2.*X*PNPX+Z*PNPZ)                                         NFWC048
      PNPR =PNPX*PXPR +PNPZ*PZPR                                        NFWC049
      PNPTH=PNPX*PXPTH+PNPZ*PZPTH                                       NFWC050
      PNPPH=PNPX*PXPPH+PNPZ*PZPPH                                       NFWC051
      PNPT=PNPX*PXPT                                                    NFWC052
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 NFWC053
      KAY2=OM2/C2*N2                                                    NFWC054
      IF(RSTART.EQ.0.) GO TO 1                                          NFWC055
      SCALE=SQRT(REAL(KAY2)/K2)                                         NFWC056
      KR =SCALE*KR                                                      NFWC057
      KTH=SCALE*KTH                                                     NFWC058
      KPH=SCALE*KPH                                                     NFWC059
    1 CONTINUE                                                          NFWC060
C********* CALCULATES A HAMILTONIAN H                                   NFWC061
      H=.5*(C2*K2/OM2-N2)                                               NFWC062
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  NFWC063
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            NFWC064
      PHPT =-PNPT                                                       NFWC065
      PHPR =-PNPR                                                       NFWC066
      PHPTH=-PNPTH                                                      NFWC067
      PHPPH=-PNPPH                                                      NFWC068
      PHPOM=-NNP/OM                                                     NFWC069
      PHPKR =C2/OM2*KR                                                  NFWC070
      PHPKTH=C2/OM2*KTH                                                 NFWC071
      PHPKPH=C2/OM2*KPH                                                 NFWC072
      KPHPK=N2                                                          NFWC073
      RETURN                                                            NFWC074
         END                                                            NFWC075-
```

```
      SUBROUTINE AHNFNC
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C           APPLETON-HARTREE FORMULA -- NO FIELD, NO COLLISIONS
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,
     1              H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,
     2              PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPI
     3             ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON /YY/ MODY,Y(16)   /ZZ/ MODZ,Z(4)
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)
      EQUIVALENCE (RAY,W(1)),(F,W(6))
      LOGICAL SPACE
      REAL N2,NNP,KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,LPOLAR,LPOLRI
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),
     1     (FIELD=0.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.),
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(POLARI=1.),(LPOLAR=0.),
     4     (LPOLRI=1.),
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
     5     (MODY=1H ),(MODZ=1H ),
     6     (NNP=1.),(PNPX=-0.5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)
      ENTRY RINDEX
      OM=PIT2*1.E6*F
      C2=C*C
      K2=KR*KR+KTH*KTH+KPH*KPH
      OM2=OM*OM
      VR =C/OM*KR
      VTH=C/OM*KTH
      VPH=C/OM*KPH
      CALL ELECTX
      PNPR =PNPX*PXPR
      PNPTH=PNPX*PXPTH
      PNPPH=PNPX*PXPPH
      PNPT=PNPX*PXPT
      N2=1.-X
      SPACE=N2.EQ.1.
      KAY2=OM2/C2*N2
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT(KAY2/K2)
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
  1   CONTINUE
C********* CALCULATES A HAMILTONIAN H
      H=.5*(C2*K2/OM2-N2)
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
      PHPT =-PNPT
      PHPR =-PNPR
      PHPTH=-PNPTH
      PHPPH=-PNPPH
      PHPOM=-NNP/OM
      PHPKR =C2/OM2*KR
      PHPKTH=C2/OM2*KTH
      PHPKPH=C2/OM2*KPH
      KPHPK=N2
      RETURN
         END
```

```
      SUBROUTINE BQWFWC                                                 BQWC001
C********* CALCULATES A HAMILTONIAN H                                   BQWC002
C********* (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)          BQWC003
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  BQWC004
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            BQWC005
C********* WITH FIELD, WITH COLLISIONS                                  BQWC006
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                BQWC007
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   BQWC008
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  BQWC009
     2             SGN                                                  BQWC010
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  BQWC011
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRBQWC012
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     BQWC013
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               BQWC014
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                BQWC015
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       BQWC016
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             BQWC017
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   BQWC018
      LOGICAL SPACE                                                     BQWC019
      REAL KR,KTH,KPH,K2,KDOTY,K4,KDOTY2                                BQWC020
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   BQWC021
     1        POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,    BQWC022
     2        KPHPK,U2,A,B,ALPHA,BETA,GAMMA,PHPX,PHPY2,PHPK2,PHPU,PHPZ, BQWC023
     3        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          BQWC024
      DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=1.),         BQWC025
     1     (FIELD=1.),                                                  BQWC026
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            BQWC027
     3     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   BQWC028
     4     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        BQWC029
     5     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)         BQWC030
     6    ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      BQWC031
     7     (I=(0.,1.)),(ABSLIM=1.E-5),(SGN=1.)                          BQWC032
      ENTRY RINDEX                                                      BQWC033
      OM=PIT2*1.E6*F                                                    BQWC034
      C2=C*C                                                            BQWC035
      K2=KR*KR+KTH*KTH+KPH*KPH                                          BQWC036
      OM2=OM*OM                                                         BQWC037
      CALL ELECTX                                                       BQWC038
      IF(X.LT..1) GO TO 2                                               BQWC039
      K4=K2*K2                                                          BQWC040
      OM4=OM2*OM2                                                       BQWC041
      C4=C2*C2                                                          BQWC042
      CALL MAGY                                                         BQWC043
      Y2=Y*Y                                                            BQWC044
      KDOTY=KR*YR+KTH*YTH+KPH*YPH                                       BQWC045
      KDOTY2=KDOTY*KDOTY                                                BQWC046
      CALL COLFRZ                                                       BQWC047
      U=CMPLX(1.,-Z)                                                    BQWC048
      U2=U*U                                                            BQWC049
      UX=U-X                                                            BQWC050
      UX2=UX*UX                                                         BQWC051
      A=UX*U2-U*Y2                                                      BQWC052
      B=-2.*U*UX2+Y2*(2.*U-X)                                           BQWC053
      ALPHA=A*C4*K4+X*KDOTY2*C4*K2                                      BQWC054
      BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2                                  BQWC055
      GAMMA=(UX2-Y2)*UX*OM4                                             BQWC056
      H=ALPHA+BETA+GAMMA                                                BQWC057
      PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+ BQWC058
     1 (-3.*UX2+Y2)*OM4                                                 BQWC059
      PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4                          BQWC060
      PHPKY2    =X*C2*(C2*K2-OM2)                                       BQWC061
      PHPU=(2.*U*UX+U2-Y2)*C4*K4+2.*(Y2-UX2-2.*U*UX)*C2*K2*OM2+(3.*UX2  BQWC062
     1 -Y2)*OM4                                                         BQWC063
      PHPZ=-I*PHPU                                                      BQWC064
      PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2                             BQWC065
```

```
      PHPT=PHPX*PXPT
      PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2  *2.*KDOTY*
     1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR) +PHPZ*PZPR
      PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2  *2.*KDOTY*
     1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT) +PHPZ*PZPTH
      PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2  *2.*KDOTY*
     1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP) +PHPZ*PZPPH
      PHPOM=(2.*BETA+4.*GAMMA)/OM
     1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2  *KDOTY2/OM -PHPZ*Z/OM
      PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2  *YR
      PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2  *YTH
      PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2  *YPH
      KAY2=K2*(-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)
C
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT((-REAL(BETA)+SGN*RAY*SQRT(REAL(BETA)**2
     1 -4.*REAL(ALPHA)*REAL(GAMMA)))/(2.*REAL(ALPHA)))
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
    1 CONTINUE
C********* THE FOLLOWING 3 CARDS USED FOR RAY TRACING IN COMPLEX SPACE
C     IF(CABS((-BETA-SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.).
C    1LT.CABS((-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.)
C    2 .AND.RSTART.EQ.0.) SGN=-SGN
      KPHPK=4.*ALPHA+2.*BETA
      SPACE=CABS(C2*KAY2/OM2-1.).LT.ABSLIM
      POLAR =SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY*I
      LPOLR = SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)*I
      RETURN
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C           APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS
    2 CONTINUE
      VR =C/OM*KR
      VTH=C/OM*KTH
      VPH=C/OM*KPH
      CALL MAGY
      V2=VR**2+VTH**2+VPH**2
      VDOTY=VR*YR+VTH*YTH+VPH*YPH
      YLV=VDOTY/V2
      YL2=VDOTY**2/V2
      YT2=Y**2-YL2
      YT4=YT2*YT2
      CALL COLFRZ
      U=CMPLX(1.,-Z)
      UX=U-X
      UX2=UX*UX
      RAD=SGN*RAY*CSQRT(YT4+4.*YL2*UX2)
      D=2.*U*UX-YT2+RAD
      D2=D*D
      N2=1.-2.*X*UX/D
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV
      PNPX=-(2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)
      PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2
```

```
      PNPT=PNPX*PXPT                                                     BQWC131
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                  BQWC132
      POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VDOTY*UX)                         BQWC133
      GAM=(-YT2+RAD)/(2.*UX)                                             BQWC134
      LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))                                  BQWC135
      KAY2=OM2/C2*N2                                                     BQWC136
      IF(RSTART.EQ.0.) GO TO 3                                           BQWC137
      SCALE=SQRT(REAL(KAY2)/K2)                                          BQWC138
      KR =SCALE*KR                                                       BQWC139
      KTH=SCALE*KTH                                                      BQWC140
      KPH=SCALE*KPH                                                      BQWC141
    3 CONTINUE                                                           BQWC142
      H=.5*(C2*K2/OM2-N2)                                                BQWC143
      PHPT =-PNPT                                                        BQWC144
      PHPR =-PNPR                                                        BQWC145
      PHPTH=-PNPTH                                                       BQWC146
      PHPPH=-PNPPH                                                       BQWC147
      PHPOM=-NNP/OM                                                      BQWC148
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                       BQWC149
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                      BQWC150
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                      BQWC151
      KPHPK=N2                                                           BQWC152
      RETURN                                                             BQWC153
         END                                                             BQWC154
```

```
      SUBROUTINE BQWFNC                                                  BQNC001
C********* CALCULATES A HAMILTONIAN H                                     BQNC002
C********* (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)            BQNC003
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                    BQNC004
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.              BQNC005
C********* WITH FIELD, NO COLLISIONS                                      BQNC006
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                 BQNC007
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,                BQNC008
     1             H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,BQNC009
     2             PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPIBQNC010
     3            ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN           BQNC011
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                   BQNC012
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRBQNC013
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                      BQNC014
      COMMON /ZZ/ MODZ,Z(4)                                              BQNC015
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART        BQNC016
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                 BQNC017
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                    BQNC018
      LOGICAL SPACE                                                      BQNC019
      REAL N2,NNP,LPOLAR,LPOLARI,KR,KTH,KPH,K2,KDOTY,K4,KDOTY2,          BQNC020
     1     KPHPK,KPHPKI,KAY2,KAY2I                                       BQNC021
      DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=0.),          BQNC022
     1     (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),          BQNC023
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), BQNC024
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),               BQNC025
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),             BQNC026
     5     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),    BQNC027
     6     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),         BQNC028
     7     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),         BQNC029
     8     (MODZ=1H ),(U=1.),(U2=1.)                                     BQNC030
      ENTRY RINDEX                                                       BQNC031
      OM=PIT2*1.E6*F                                                     BQNC032
      C2=C*C                                                             BQNC033
      K2=KR*KR+KTH*KTH+KPH*KPH                                           BQNC034
      OM2=OM*OM                                                          BQNC035
```

```
      CALL ELECTX
      IF(X.LT..1) GO TO 2
      K4=K2*K2
      OM4=OM2*OM2
      C4=C2*C2
      CALL MAGY
      Y2=Y*Y
      KDOTY=KR*YR+KTH*YTH+KPH*YPH
      KDOTY2=KDOTY*KDOTY
      UX=U-X
      UX2=UX*UX
      A=UX*U2-U*Y2
      B=-2.*U*UX2+Y2*(2.*U-X)
      ALPHA=A*C4*K4+X*KDOTY2*C4*K2
      BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2
      GAMMA=(UX2-Y2)*UX*OM4
      H=ALPHA+BETA+GAMMA
      PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+
     1 (-3.*UX2+Y2)*OM4
      PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4
      PHPKY2   =X*C2*(C2*K2-OM2)
      PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2
      PHPT=PHPX*PXPT
      PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2  *2.*KDOTY*
     1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR)
      PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2  *2.*KDOTY*
     1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT)
      PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2  *2.*KDOTY*
     1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP)
      PHPOM=(2.*BETA+4.*GAMMA)/OM
     1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2  *KDOTY2/OM
      PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2  *YR
      PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2  *YTH
      PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2  *YPH
      KAY2 = K2   *(-BETA+RAY*SQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT(KAY2/K2)
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
    1 CONTINUE
      KPHPK=4.*ALPHA+2.*BETA
      SPACE=KAY2.EQ.OM2/C2
      POLARI=SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY
      LPOLRI= SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)
      RETURN
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C           APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS
    2 CONTINUE
      VR =C/OM*KR
      VTH=C/OM*KTH
      VPH=C/OM*KPH
      CALL MAGY
      V2=VR**2+VTH**2+VPH**2
      VDOTY=VR*YR+VTH*YTH+VPH*YPH
      YLV=VDOTY/V2
      YL2=VDOTY**2/V2
      YT2=Y**2-YL2
      YT4=YT2*YT2
      UX=U-X
      UX2=UX*UX
      RAD=RAY*SQRT(YT4+4.*YL2*UX2)
      D=2.*UX-YT2+RAD
      D2=D*D
      N2=1.-2.*X*UX/D
```

```
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           BQNC101
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             BQNC102
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             BQNC103
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             BQNC104
      PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2   BQNC105
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   BQNC106
      NNP=N2-(2.*X*PNPX+Y*PNPY)                                         BQNC107
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPPS*PPSPR                           BQNC108
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH                          BQNC109
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH                          BQNC110
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               BQNC111
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               BQNC112
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               BQNC113
      PNPT=PNPX*PXPT                                                    BQNC114
      SPACE=N2.EQ.1.                                                    BQNC115
      POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)                           BQNC116
      GAM=(-YT2+RAD)/(2.*UX)                                            BQNC117
      LPOLRI=K*SQRT(YT2)/(UX*(J+GAM))                                   BQNC118
      KAY2=OM2/C2*N2                                                    BQNC119
      IF(RSTART.EQ.0.) GO TO 3                                          BQNC120
      SCALE=SQRT(KAY2/K2)                                               BQNC121
      KR =SCALE*KR                                                      BQNC122
      KTH=SCALE*KTH                                                     BQNC123
      KPH=SCALE*KPH                                                     BQNC124
    3 CONTINUE                                                          BQNC125
      H=.5*(C2*K2/OM2-N2)                                               BQNC126
      PHPT =-PNPT                                                       BQNC127
      PHPR =-PNPR                                                       BQNC128
      PHPTH=-PNPTH                                                      BQNC129
      PHPPH=-PNPPH                                                      BQNC130
      PHPOM=-NNP/OM                                                     BQNC131
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      BQNC132
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     BQNC133
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     BQNC134
      KPHPK=N2                                                          BQNC135
      RETURN                                                            BQNC136
         END                                                            BQNC137-
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      SUBROUTINE SWWF                                                   SWWF001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   SWWF002
C          SEN-WYLLER FORMULA -- WITH FIELD                             SWWF003
C          NEEDS SUBROUTINE FSW AND FUNCTIONS C AND S.                  SWWF004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,SEA,LOGTEN              SWWF005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   SWWF006
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  SWWF007
     2             SGN                                                  SWWF008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SWWF009
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRSWWF010
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     SWWF011
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               SWWF012
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       SWWF013
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                SWWF014
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   SWWF015
      LOGICAL SPACE                                                     SWWF016
      REAL KR,KTH,KPH,K2                                                SWWF017
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   SWWF018
     1        KPHPK,POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2, SWWF019
     2        ALPHA,BETA,GAMMA,A,B,C,TEMP1,TEMP2,TEMP3,ALPOAL,BEPOBE,   SWWF020
     3        GAPOGA,CB2,N2M1,J2,D2GA,DAL,DBET,DGAM,DADY,DADZ,DBDY,DBDZ,SWWF021
     4        DCDY,DCDZ,DUDZ,DT1DX,DT1DY,DT1DZ,DT1DPS,DT2DX,DT2DY,DT2DZ,SWWF022
     5        DT2DPS,DRADDX,DRADDY,DRADDZ,DRDDPS,DDDX,DDDY,DDDZ,DDDPX,  SWWF023
```

```
     6        UPX,N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT
      DATA (MOORIN=8H        SE,8HN-WYLLER,8H FORMULA),(COLL=1.),
     1        (FIELD=1.),(LPOLAR=(0.,0.)),
     2        (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
     3        (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),
     4        (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),
     5        (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),
     6        (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),
     7        (I=(0.,1.)),(ABSLIM=1.E-5)
      ENTRY RINDEX
      OM=PIT2*1.E6*F
      C2=SEA*SEA
      K2=KR*KR+KTH*KTH+KPH*KPH
      OM2=OM*OM
      VR =SEA/OM*KR
      VTH=SEA/OM*KTH
      VPH=SEA/OM*KPH
      CALL ELECTX
      CALL MAGY
      OPY=1.+Y
      OMY=1.-Y
      CALL COLFRZ
      Z2=Z*Z
      CALL FSW(1./Z,ALPHA,DAL)
      ALPOAL=DAL/ALPHA
      CALL FSW(OMY/Z,BETA,DBET)
      BEPOBE=DBET/BETA
      CALL FSW(OPY/Z,GAMMA,DGAM)
      GAPOGA=DGAM/GAMMA
      U=Z/ALPHA
      DUDZ=(1.+ALPOAL/Z)/ALPHA
      U2=U*U
      UX=U-X
      UPX=U+X
      B=ALPHA/BETA
      DBDY=B*BEPOBE/Z
      DBDZ=-B*(ALPOAL-OMY*BEPOBE)/Z2
      C=ALPHA/GAMMA
      DCDY=-C*GAPOGA/Z
      DCDZ=-C*(ALPOAL-OPY*GAPOGA)/Z2
      A=.5*(B+C)-1.
      DADY=.5*(DBDY+DCDY)
      DADZ=.5*(DBDZ+DCDZ)
      TEMP3=(1.-B*C)*U2+A*U*UPX
      V2=       VR**2+VTH**2+VPH**2
      VDOTY=VR*YR+VTH*YTH+VPH*YPH
      YL2=VDOTY**2/V2
      YT2=Y**2-YL2
      Y2=Y*Y
      S2PSI=YT2/Y2
      C2PSI=YL2/Y2
      UX2=UX*UX
      CB2=(C-B)**2
      TEMP1=TEMP3*S2PSI
      DT1DX= A*U*S2PSI
      DT1DY=(U*UPX*DADY-U2*(B*DCDY+C*DBDY))*S2PSI
      DT1DZ=(2.*U*DUDZ*(1.-B*C+A)+A*X*DUDZ-U2*(B*DCDZ+C*DBDZ)+U*UPX*DADZ
     1 )*S2PSI
C             (1/YLYT) D/DPSI(TEMP1)
      DT1DPS=2.*TEMP1/YT2
      TEMP2=U2*CB2*UX2*C2PSI
      DT2DX=-2.*UX*U2*CB2*C2PSI
      DT2DY=2.*U2*UX2*C2PSI*(C-B)*(DCDY-DBDY)
      DT2DZ=2.*U2*UX2*C2PSI*(C-B)*(DCDZ-DBDZ)+2.*TEMP2*(1./U+1./UX)*DUDZ
C             (1/YLYT) D/DPSI(TEMP2)
      DT2DPS=-2.*TEMP2/YL2
```

```
      RAD=RAY*CSQRT(TEMP1**2+TEMP2)
      DRADDX=(TEMP1*DT1DX +.5    *DT2DX )/RAD
      DRADDY=(TEMP1*DT1DY +    .5*DT2DY )/RAD
      DRADDZ=(TEMP1*DT1DZ +    .5*DT2DZ )/RAD
C              (1/YLYT)  D/DPSI(RAD)
      DRDDPS=(TEMP1*DT1DPS+    .5*DT2DPS)/RAD
      D=2.*U*UX*(1.+A)-TEMP1+RAD+2.*A*U*X*S2PSI
      DDDX=-2.*U-DT1DX+DRADDX+2.*A*U*S2PSI
      DDDY= 2.*U*UX*DADY-DT1DY+DRADDY+2.*U*S2PSI*DADY
      DDDZ=2.*(1.+A)*DUDZ*(U+UX)+2.*U*UX*DADZ-DT1DZ+DRADDZ+2.*X*S2PSI*
     1 (A*DUDZ+U*DADZ)
C          (1/YLYT)  D/DPSI(D)
      DDDPS=-DT1DPS+DRDDPS+2.*A*U*X/Y2
      N2M1=-2.*X*(UX+U*A*S2PSI)/D
      N2=1.+N2M1
C                N D/DX(N)
      PNPX=-(UX+U*A*S2PSI)*(1.-X*DDDX/D)/D+X/D
C                N D/DY(N)
      PNPY=-X*U*S2PSI/D*DADY-.5*N2M1/D*DDDY
C                N D/DZ(N)
      PNPZ=-X*(1.+A*S2PSI)/D*DUDZ-X*U*S2PSI/D*DADZ-.5*N2M1/D*DDDZ
C              (N/YLYT)  D/DPSI(N)
      PNPPS=-X*U*A/(D*Y2)       -.5*N2M1/D*DDDPS
      YLV=VDOTY/V2
C          (YLYT)  D/DR(PSI)
      PPSPR=YL2/Y*PYPR-(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV
C          (YLYT)  D/DTHETA(PSI)
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV
C          (YLYT)  D/DPHI(PSI)
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV
      PNPR=PNPX*PXPR+PNPY*PYPR+PNPZ*PZPR+PNPPS*PPSPR
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH
      PNPVR=PNPPS*(VR*YL2/V2-YLV*YR)
      PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH)
      PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)
      PNPT=PNPX*PXPT
      POLAR=I*(TEMP1-RAD)*Y*SQRT(V2)/(U*UX*(C-B)*VDOTY)
      COSPSI=VDOTY/(Y*SQRT(V2))
      LPOLAR=(.5*I*(C-B)*POLAR+A*COSPSI)*SQRT(S2PSI)/
     1 (POLAR*(UX*(1.+.5*I*(C-B)*COSPSI*POLAR)+A*(U-X*C2PSI)))
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM
      KAY2=OM2/C2*N2
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT(REAL(KAY2)/K2)
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
  1   CONTINUE
C********* CALCULATES A HAMILTONIAN H
      H=.5*(C2*K2/OM2-N2)
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
      PHPT =-PNPT
      PHPR =-PNPR
      PHPTH=-PNPTH
      PHPPH=-PNPPH
      PHPOM=-NNP/OM
      PHPKR =C2/OM2*KR -SEA/OM*PNPVR
      PHPKTH=C2/OM2*KTH-SEA/OM*PNPVTH
      PHPKPH=C2/OM2*KPH-SEA/OM*PNPVPH
      KPHPK=N2
      RETURN
         END
```

```
      SUBROUTINE SWNF
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C           SEN-WYLLER FORMULA -- NO FIELD
C           NEEDS SUBROUTINES FGSW AND FSW AND FUNCTIONS C AND S.
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,
     2             SGN
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON /YY/ MODY,Y(16)
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)
      EQUIVALENCE (RAY,W(1)),(F,W(6))
      LOGICAL SPACE
      REAL KR,KTH,KPH,K2
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,
     1        KPHPK,POLAR,LPOLAR,PNPX,PNPZ,F1,DF,G1,DG1,
     2        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT
      DATA (MODRIN=8H       SE,8HN-WYLLER,8H FORMULA),(COLL=1.),
     1     (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
     3     (MODY=1H ),
     4     (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),
     5     (ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)
      ENTRY RINDEX
      OM=PIT2*1.E6*F
      C2=C*C
      K2=KR*KR+KTH*KTH+KPH*KPH
      OM2=OM*OM
      VR =C/OM*KR
      VTH=C/OM*KTH
      VPH=C/OM*KPH
      CALL ELECTX
      CALL COLFRZ
      CALL FGSW(1./Z,F1,DF1,G1,DG1)
      N2=1.-X*G1
      PNPX=-.5*G1
      PNPZ=.5*X*DG1/Z**2
      PNPR=PNPX*PXPR+PNPZ*PZPR
      PNPTH=PNPX*PXPTH+PNPZ*PZPTH
      PNPPH=PNPX*PXPPH+PNPZ*PZPPH
      NNP=N2-(2.*X*PNPX+Z*PNPZ)
      PNPT=PNPX*PXPT
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM
      KAY2=OM2/C2*N2
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT(REAL(KAY2)/K2)
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
  1   CONTINUE
C********* CALCULATES A HAMILTONIAN H
      H=.5*(C2*K2/OM2-N2)
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
      PHPT =-PNPT
      PHPR =-PNPR
      PHPTH=-PNPTH
      PHPPH=-PNPPH
      PHPOM=-NNP/OM
      PHPKR =C2/OM2*KR
      PHPKTH=C2/OM2*KTH
      PHPKPH=C2/OM2*KPH
      KPHPK=N2
      RETURN
          END
```

```
      SUBROUTINE FGSW (X,F,DF,G,DG)                                     FGSW001
      COMPLEX F,DF,G,DG                                                 FGSW002
      CALL FSW (X,F,DF)                                                 FGSW003
      IF(ABS(X).GT.50.) GO TO 1                                         FGSW004
      G=X*F                                                             FGSW005
      DG=F+X*DF                                                         FGSW006
      RETURN                                                            FGSW007
    1 X2=X*X                                                            FGSW008
      X3=X2*X                                                           FGSW009
      T2=2.*X2                                                          FGSW010
      T3=3.*X2                                                          FGSW011
      T4=4.*X2                                                          FGSW012
      T8=8.*X2                                                          FGSW013
      T12=12.*X2                                                        FGSW014
      T16=16.*X2                                                        FGSW015
      G=CMPLX(1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4)))/T4,       FGSW016
     12.5*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/X)         FGSW017
      DG=.5*CMPLX(35.*(1.-99./T2*(1.-585./T8*(1.-323./T3*(1.-2415./T16))FGSW018
     1 ))/X3,                                                           FGSW019
     2-5.*(1.-189./T4*(1.-715./T12*(1.-357./T4*(1.-513./T4))))/X2)      FGSW020
      RETURN                                                            FGSW021
         END                                                            FGSW 22-


      SUBROUTINE FSW (Z,F,DF)                                           FSW 001
C        F(Z) = Z*C3/2(Z) + 2.5*I*C5/2(Z)   AND   DF(Z) = DF/DZ         FSW 002
C        WHERE THE INPUT Z IS REAL AND THE OUTPUT F AND DF ARE COMPLEX. FSW 003
C       NEEDS THE SUBPROGRAMS FOR THE FRESNEL INTEGRAL FUNCTIONS S AND CFSW 004
      DIMENSION A(10),B(10),D(10)                                       FSW 005
      COMPLEX F,DF,C1,C2,C3,C8,W,TEMP,I                                 FSW 006
      DATA (I=(0.,1.)), (PI=3.1415926536),(A3=1.333333333)              FSW 007
      DATA (C2=(1.,1.)),(C3=(1.,-1.)),(C4=.79788456  ),(C6=1.333333333) FSW 008
C        C4=SQRT(2./PI)                                                 FSW 009
      DATA(A=.36230845E-02,.29579186E+00,.23193588E+01,.91355870E+01,   FSW 010
     1.25856287E+02,.60488560E+02,.12562218E+03,.24214980E+03,          FSW 011
     2.44918106E+03,.84244774E+03),                                     FSW 012
     3(B=.16747479E-02,.84796280E-01,.25285001E+00,.22665867E+00,       FSW 013
     4.83871933E-01,.13811875E-01,.98017417E-03,.26299148E-04,          FSW 014
     5.19761006E-06,.18781476E-09),                                     FSW 015
     6(D=.10080653E-03,.46117941E-01,.38507643E+00,.68507885E+00,       FSW 016
     7.42648105E+00,.10742102E+00,.10985920E-01,.40924533E-03,          FSW 017
     8.41881263E-05,.54513142E-08),(G=1.5045055)                        FSW 018
      C1=2./3.*I                                                        FSW 019
      C8=C2*A3*SQRT(PI/2.)                                              FSW 020
      X=Z                                                               FSW 021
      X2=X*X                                                            FSW 022
      X3=X2*X                                                           FSW 023
      IF(ABS(X).GT.50.) GO TO 500                                       FSW 024
      IF(ABS(X).GT.6.) GO TO 1                                          FSW 025
      IF(ABS(X).LT..05)GO TO 200                                        FSW 026
C                                FRESNEL                                FSW 027
      IF(X.GT.0.) GO TO 300                                             FSW 028
100    Y=C4*SQRT(-X)                                                    FSW 029
      X2=X*X                                                            FSW 030
      W=(COS(X)+I*SIN(X))*(1.-C3*(C(Y)+I*S(Y)))                         FSW 031
      F   =C1+C6*(X+C3*X*X/Y*W)                                         FSW 032
      DF=A3*CMPLX(1.,X)+CMPLX(1.5,X)*A3*C3*X/Y*W                        FSW 033
       RETURN                                                           FSW 034
300   Y=C4*SQRT(X)                                                      FSW 035
      X2=X*X                                                            FSW 036
      W=(COS(X)+I*SIN(X))*(1.-C2*(C(Y)-I*S(Y)))                         FSW 037
      F   =C1+C6*(X-C2*X*X/Y*W)                                         FSW 038
      DF=A3*CMPLX(1.,X)-CMPLX(1.5,X)*A3*C2*X/Y*W                        FSW 039
      RETURN                                                            FSW 040
```

```
C                                 POWER SERIES
  200 X=ABS(Z)
      X2=X*X
      X3=X2*X
      X4=X*X3
      X5=X*X4
      TEMP=-C8* SQRT(X)*CEXP(I*X)
      F=CMPLX(4./3.*X-16./9.*X3+64./315.*X5,2./3.+8./3.*X2-32./45.*X4)
     1   +TEMP*X
      DF=CMPLX(4./3.-16./3.*X2+64./63.*X4,16./3.*X-128./45.*X3
     1   +256./945.*X5)
     2   +TEMP*CMPLX(1.5,X)
      IF(Z.GE.0.) RETURN
      F=-CONJG(F)
      DF=CONJG(DF)
      RETURN
C                                      HERMITE
    1 XQ = X**2
      X2=XQ
      FR = 0.
      FI = 0.
      DFR = 0.
      DFI = 0.
      DO 2 J = 1,10
      SS = A(J) + XQ
      SB = B(J)/SS
      SD = D(J)/SS
      FR = FR + SB
      FI = FI + SD
      DFR = DFR + SB/SS
    2 DFI = DFI + SD/SS
      F = CMPLX(X*FR,FI)*G
      DF = G*(FR - 2.*X* CMPLX(X*DFR,DFI))
      RETURN
C                                 ASYMPTOTIC
  500 X2=X*X
      X3=X2*X
      X4=X3*X
      X5=X4*X
      T2=2.*X2
      T3=3.*X2
      T4=4.*X2
      T8=8.*X2
      T16=16.*X2
      T28=28.*X2
      F=CMPLX((1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4))))/X
     1,5.*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/T2)
      DF=-CMPLX((1.-105./T4*(1.-165./T4*(1.-273./T4*(1.-2907./T28))))/X2
     1,5.*(1.-63./T2*(1.-429./T8*(1.-255./T3*(1.-1995./T16))))/X3)
      RETURN
         END
```

```
      FUNCTION C(X)
      DOUBLEPRECISION   PIH, XD, Y, V, A, QZ, QN, Q, Z
      DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)
      PIH = 1.570796326794897
      XA = ABS(X)
      IF (XA.GT.4.)   GOTO 20
C
      XD = X
      Y = PIH*XD*XD
      V = Y*Y
      A = 1.D0
      Z = A
      M = 15.*(XA + 1.)
      DO 10   I = 1, M
      KZ=2*(I-1)
      KV=4*(I-1)
      QZ = KV + 1
      QN = (KZ + 1)*(KZ + 2)*(KV + 5)
       Q = QZ/QN
      A = -A*Q*V
   10 Z = Z + A
      Z = Z*XD
      C = Z
      RETURN
C
   20  W = PIH*X*X
      XV=XA**4
      C=0.5+(A1-B1/XV)*SIN(W)/XA-(A2-B2/XV)*COS(W)/XA**3
      IF (X.LT.0.)   C = -C
      RETURN
         END
```

```
      FUNCTION S(X)
      DOUBLEPRECISION   PIH, XD, Y, V, A, QZ, QN, Q, Z
      DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)
      PIH = 1.570796326794897
C
      XA = ABS(X)
      IF (XA.GT.4.)   GOTO 20
C
      XD = X
      Y = PIH*XD*XD
      V = Y*Y
      A = Y/3.D0
      Z = A
      M = 15.*(XA + 1.)
      DO 10   I = 1, M
      KZ=2*(I-1)
      KV=4*(I-1)
      QZ = KV + 3
      QN = (KZ + 2)*(KZ + 3)*(KV + 7)
       Q = QZ/QN
       A = -A*Q*V
   10 Z = Z + A
      Z = Z*XD
      S = Z
      RETURN
C
   20  W = PIH*X*X
      XV=XA**4
      S=0.5-(A1-B1/XV)*COS(W)/XA-(A2-B2/XV)*SIN(W)/XA**3
      IF (X.LT.0.)   S = -S
      RETURN
         END
```