

APPENDIX 2. VERSIONS OF THE REFRACTIVE INDEX SUBROUTINE (RINDEX)

This ray tracing program gains versatility without sacrificing speed by having several versions of some of the subroutines. For example, the 8 versions of the refractive index subroutine allow the user to decide for each ray path calculation whether to include or ignore various aspects of the propagation medium such as the earth's magnetic field or collisions between electrons and neutral air molecules.

If collisions are included, the user has the option of using the Appleton-Hartree formula (which assumes a constant collision frequency) or the Sen-Wyller formula (which assumes a Maxwell distribution of electron energies and a collision frequency proportional to energy). The Sen-Wyller formula is generally assumed to be more accurate, especially in the lower ionosphere, but the Appleton-Hartree formula can often be used with an effective collision frequency profile to save computer time.

When the effect of the earth's magnetic field is included and ray paths are calculated near vertical incidence, a spitze (Davies, 1965, p. 202) often occurs in the ray path. (At a spitze, the usual formulas for refractive index become indeterminate because the wave normal is parallel with the earth's magnetic field and the wave frequency equals the local plasma frequency.) Two versions of the refractive index subroutine have been developed to calculate ray paths through a spitze. These two versions will also work in the absence of a spitze, but the standard versions are much faster.

The input to the refractive index subroutines is through blank common and common blocks /XX/, /YY/, and /ZZ/. Output is through common block /RIN/. The refractive index subroutine is called through the entry RINDEX. The subroutine names are used only for user identification. The following 8 versions of the refractive index subroutine are

listed in this appendix:

a.	Subroutine AHFWFC (Appleton-Hartree formula with field, with collisions)	93
b.	Subroutine AHWFNC (Appleton-Hartree formula with field, no collisions)	94
c.	Subroutine AHNFWC (Appleton-Hartree formula no field, with collisions)	96
d.	Subroutine AHNFNC (Appleton-Hartree formula no field, no collisions)	97
e.	Subroutine BQFWFC (Booker Quartic with field, with collisions)	98
f.	Subroutine BQWFNC (Booker Quartic with field, no collisions)	100
g.	Subroutine SWWF (Sen-Wyller formula with field)	102
h.	Subroutine SWNF (Sen-Wyller no field)	105
	Subroutine FGSW	106
	Subroutine FSW	106
	Fresnel integral function C	108
	Fresnel integral function S	108

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SUBROUTINE A4WFC                                WFWC001
CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE WFWC002
APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS WFWC003
COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN WFWC004
COMMON /RIN/ MODRIN(3),CO_L,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH, WFWC005
1 PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR WFWC006
COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX WFWC007
COMMON /YY/ MOOY,Y,PYPR,PYPH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR WFWC008
1 PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP WFWC009
COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH WFWC010
COMMON R,TH,PH,KR,KTH,KPH /WW/ ID(10),W0,W(400) WFWC011
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART WFWC012
EQUIVALENCE (RAY,W(1)),(F,W(6)) WFWC013
LOGICAL SPACE WFWC014
REAL KR,KTH,KPH,K2 WFWC015
COMPLEX N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT, WFWC016
1 POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2, WFWC017
2 KAY2,I,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH, WFWC018
3 KPHPK WFWC019
DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.), WFWC020
1 (FIELD=1.), WFWC021
2 (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.), WFWC022
3 (Y=0.),(PYPR=0.),(PYPH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.), WFWC023
4 (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.), WFWC024
5 (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.) WFWC025
6 ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.), WFWC026
7 (I=(0.,1.)),(ABSLIM=1.E-5) WFWC027
ENTRY RINDEX WFWC028
OM=PIT2*1.E6*F WFWC029
C2=C*C WFWC030
K2=KR*KR+KTH*KTH+KPH*KPH WFWC031
OM2=OM*OM WFWC032
VR =C/OM*KR WFWC033
VTH=C/OM*KTH WFWC034
VPH=C/OM*KPH WFWC035
CALL ELECTX WFWC036
CALL MAGY WFWC037
V2=VR**2+VTH**2+VPH**2 WFWC038
VDOTY=VR*YR+VTH*YTH+VPH*YPH WFWC039
YLV=VDOTY/V2 WFWC040
YL2=VDOTY**2/V2 WFWC041
YT2=Y**2-YL2 WFWC042
YT4=YT2*YT2 WFWC043
CALL COLFRZ WFWC044
U=CMPLX(1.,-Z) WFWC045
JX=U-X WFWC046
UX2=UX*UX WFWC047
RAD=RAY*CSQRT(YT4+4.*YL2*UX2) WFWC048
D=2.*U*UX-YT2+RAD WFWC049
D2=D*D WFWC050
N2=1.-2.*X*UX/D WFWC051
PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2 WFWC052
PPSPR =YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV WFWC053
PPSPH=YL2/Y*PYPH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV WFWC054
PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV WFWC055
PNPX=- (2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2 WFWC056
PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y) WFWC057
PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2 WFWC058
PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR WFWC059
PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPH WFWC060
PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH WFWC061
PNPVR =PNPPS*(VR *YL2/V2-YLV*YR) WFWC062
PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH) WFWC063
PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH) WFWC064
NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ) WFWC065

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PNPT=PNPX*PXPT
SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM
POLAR=-I*SQRT(V2)*(-YT2+RAO)/(2.*VDOOTY*UX)
GAM=(-YT2+RAD)/(2.*UX)
LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))
KAY2=OM2/C2*N2
IF(RSTART.EQ.0.) GO TO 1
SCALE=SQRT(REAL(KAY2)/K2)
KR =SCALE*KR
KTH=SCALE*KTH
KPH=SCALE*KPH
1 CONTINUE
C***** CALCULATES A HAMILTONIAN H
H=.5*(C2*K2/OM2-N2)
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
PHPT =-PNPT
PHPR =-PNPR
PHPTH=-PNPTH
PHPPH=-PNPPH
PHPOM=-NNP/OM
PHPKR =C2/OM2*KR -C/OM*PNPVR
PHPKTH=C2/OM2*KTH-C/OM*PNPVTH
PHPKPH=C2/OM2*KPH-C/OM*PNPVPH
KPHPK=N2
RETURN
END

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WFNC066
WFNC067
WFNC068
WFNC069
WFNC070
WFNC071
WFNC072
WFNC073
WFNC074
WFNC075
WFNC076
WFNC077
WFNC078
WFNC079
WFNC080
WFNC081
WFNC082
WFNC083
WFNC084
WFNC085
WFNC086
WFNC087
WFNC088
WFNC089
WFNC090
WFNC091
WFNC092

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SUBROUTINE AHWFNC
C CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS
COMMON /CONST/ PI,PIT2,PI02,DEGS,RADIAN,K,C,LOGTEN
COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,
1 H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI
2 ,PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPI
3 ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN
COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR
1 ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP
COMMON /ZZ/ MODZ,Z(4)
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
COMMON R,TH,PH,KR,KTH,KPH /WW/ ID(10),W0,W(400)
EQUIVALENCE (RAY,W(1)),(F,W(6))
LOGICAL SPACE
REAL KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,N2,NNP,LPOLAR,LPOLRI
DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),
1 (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),
2 (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.),
3 (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),
4 (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
5 (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),
6 (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),
7 (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),
8 (MODZ=1H),(U=1.)
ENTRY RINDEX
OM=PIT2*1.E6*F
C2=C*C
K2=KR*KR+KTH*KTH+KPH*KPH
OM2=OM*OM
VR =C/OM*KR
VTH=C/OM*KTH
VPH=C/OM*KPH

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WFNC001
WFNC002
WFNC003
WFNC004
WFNC005
WFNC006
WFNC007
WFNC008
WFNC009
WFNC010
WFNC011
WFNC012
WFNC013
WFNC014
WFNC015
WFNC016
WFNC017
WFNC018
WFNC019
WFNC020
WFNC021
WFNC022
WFNC023
WFNC024
WFNC025
WFNC026
WFNC027
WFNC028
WFNC029
WFNC030
WFNC031
WFNC032
WFNC033
WFNC034

CALL ELECTX	WFNC035
CALL MAGY	WFNC036
V2=VR**2+VTH**2+VPH**2	WFNC037
VDOTY=VR*YR+VTH*YTH+VPH*YPH	WFNC038
YLV=VDOTY/V2	WFNC039
YL2=VDOTY**2/V2	WFNC040
YT2=Y**2-YL2	WFNC041
YT4=YT2*YT2	WFNC042
UX=U-X	WFNC043
UX2=UX*UX	WFNC044
RAD=RAY*SQRT(YT4+4.*YL2*UX2)	WFNC045
D=2.*UX-YT2+RAD	WFNC046
D2=0*D	WFNC047
N2=1.-2.*X*UX/D	WFNC048
PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2	WFNC049
PPSPR=YL2/Y*PYPR-(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV	WFNC050
PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV	WFNC051
PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV	WFNC052
PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2	WFNC053
PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)	WFNC054
NNP=N2-(2.*X*PNPX+Y*PNPY)	WFNC055
PNPR=PNPX*PXPR+PNPY*PYPR+PNPPS*PPSPR	WFNC056
PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH	WFNC057
PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH	WFNC058
PNPVR=PNPPS*(VR*YL2-VDOTY*YR)/V2	WFNC059
PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2	WFNC060
PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2	WFNC061
PNPT=PNPX*PXPT	WFNC062
SPACE=N2.EQ.1.	WFNC063
POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)	WFNC064
GAM=(-YT2+RAD)/(2.*UX)	WFNC065
LPOLRI=X*SQRT(YT2)/(UX*(U+GAM))	WFNC066
KAY2=OM2/C2*N2	WFNC067
IF(RSTART.EQ.0.) GO TO 1	WFNC068
SCALE=SQRT(KAY2/K2)	WFNC069
KR=SCALE*KR	WFNC070
KTH=SCALE*KTH	WFNC071
KPH=SCALE*KPH	WFNC072
1 CONTINUE	WFNC073
C***** CALCULATES A HAMILTONIAN H	WFNC074
H=.5*(C2*K2/OM2-N2)	WFNC075
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO	WFNC076
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.	WFNC077
PHPT=-PNPT	WFNC078
PHPR=-PNPR	WFNC079
PHPTH=-PNPTH	WFNC080
PHPPH=-PNPPH	WFNC081
PHPOH=-NNP/OM	WFNC082
PHPKR=C2/OM2*KR-C/OM*PNPVR	WFNC083
PHPKTH=C2/OM2*KTH-C/OM*PNPVTH	WFNC084
PHPKPH=C2/OM2*KPH-C/OM*PNPVPH	WFNC085
KPHPK=N2	WFNC086
RETURN	WFNC087
END	WFNC088-

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SUBROUTINE A4NFWC
C      CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C      APPLETON-HARTREE FORMULA -- NO FIELD, WITH COLLISIONS
COMMON /CONST/ PI,PIT2,PIJ2,DEGS,RADIAN,K,C,LOGTEN
COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,
1      PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,
2      SGN
COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
COMMON /YY/ MODY,Y(16)
COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
COMMON R,TH,PH,KR,KTH,KPH /WW/ ID(10),W0,W(400)
EQUIVALENCE (RAY,W(1)),(F,W(6))
LOGICAL SPACE
REAL KR,KTH,KPH,K2
COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,
1      KPHPK,POLAR,LPOLAR,U,I,PNPX,PNPZ,
2      N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,NNP,PNPT
DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),
1      (FIELD=0.), (POLAR=(0.,1.)), (LPOLAR=(0.,0.)),
2      (X=0.), (PXPR=0.), (PXPTH=0.), (PXPPH=0.), (PXPT=0.),
3      (MODY=1H),
4      (Z=0.), (PZPR=0.), (PZPTH=0.), (PZPPH=0.),
5      (I=(0.,1.)), (ABSLIM=1.E-5), (PNPVR=0.), (PNPVTH=0.), (PNPVPH=0.)
ENTRY RINDEX
OM=PIT2*1.E6*F
C2=C*C
K2=KR*KR+KTH*KTH+KPH*KPH
OM2=OM*OM
VR=C/OM*KR
VTH=C/OM*KTH
VPH=C/OM*KPH
CALL ELECTX
CALL COLFRZ
U=1.-I*Z
N2=1.-X/U
PNPX=-1./(2.*U)
PNPZ=-I*X/(2.*U**2)
NNP=N2-(2.*X*PNPX+Z*PNPZ)
PNPR=PNPX*PXPR+PNPZ*PZPR
PNPTH=PNPX*PXPTH+PNPZ*PZPTH
PNPPH=PNPX*PXPPH+PNPZ*PZPPH
PNPT=PNPX*PXPT
SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM
KAY2=OM2/C2*N2
IF(RSTART.EQ.0.) GO TO 1
SCALE=SQRT(REAL(KAY2)/K2)
KR=SCALE*KR
KTH=SCALE*KTH
KPH=SCALE*KPH
1 CONTINUE
C***** CALCULATES A HAMILTONIAN H
H=.5*(C2*K2/OM2-N2)
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
PHPT=-PNPT
PHPR=-PNPR
PHPTH=-PNPTH
PHPPH=-PNPPH
PHPOM=-NNP/OM
PHPKR=C2/OM2*KR
PHPKTH=C2/OM2*KTH
PHPKPH=C2/OM2*KPH
KPHPK=N2
RETURN
END

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NFWC010
NFWC011
NFWC012
NFWC013
NFWC014
NFWC015
NFWC016
NFWC017
NFWC018
NFWC019
NFWC020
NFWC021
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NFWC063
NFWC064
NFWC065
NFWC066
NFWC067
NFWC068
NFWC069
NFWC070
NFWC071
NFWC072
NFWC073
NFWC074
NFWC075-

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SUBROUTINE AHNFNC
C      CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C      APPLETON-HARTREE FORMULA -- NO FIELD, NO COLLISIONS
COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,
1      H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTI,PHPPH,PHPPHI,
2      PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPI
3      ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN
COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
COMMON /YY/ MODY,Y(16) /ZZ/ MODZ,Z(4)
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
COMMON R,TH,PH,KR,KTH,KPH /WW/ ID(10),W0,W(400)
EQUIVALENCE (RAY,W(1)),(F,W(6))
LOGICAL SPACE
REAL N2,NNP,KR,KTH,KPH,K2,KPHPK,KHPKI,KAY2,KAY2I,LPOLAR,LPOLRI
DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),
1      (FIELD=0.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),
2      (PHPTI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.),
3      (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(POLARI=1.),(LPOLAR=0.),
4      (LPOLRI=1.),
4      (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
5      (MODY=1H),(MODZ=1H),
5      (NNP=1.),(PNPX=-0.5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)
ENTRY RINDEX
OM=PIT2*1.E6*F
C2=C*C
K2=KR*KR+KTH*KTH+KPH*KPH
OM2=OM*OM
VR=C/OM*KR
VTH=C/OM*KTH
VPH=C/OM*KPH
CALL ELECTX
PNPR=PNPX*PXPR
PNPTH=PNPX*PXPTH
PNPPH=PNPX*PXPPH
PNPT=PNPX*PXPT
N2=1.-X
SPACE=N2.EQ.1.
KAY2=OM2/C2*N2
IF(RSTART.EQ.0.) GO TO 1
SCALE=SQRT(KAY2/K2)
KR=SCALE*KR
KTH=SCALE*KTH
KPH=SCALE*KPH
1 CONTINUE
C***** CALCULATES A HAMILTONIAN H
H=.5*(C2*K2/OM2-N2)
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
PHPT=-PNPT
PHPR=-PNPR
PHPTH=-PNPTH
PHPPH=-PNPPH
PHPOM=-NNP/OM
PHPKR=C2/OM2*KR
PHPKTH=C2/OM2*KTH
PHPKPH=C2/OM2*KPH
KPHPK=N2
RETURN
END

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SUBROUTINE BQWFWC
C***** CALCULATES A HAMILTONIAN H
C***** (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
C***** WITH FIELD, WITH COLLISIONS
COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,
1 PHPPH,PHPM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,
2 SGN
COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR
1 ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP
COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH
COMMON R,TH,PH,KR,KTH,KPH /WW/ ID(10),W0,W(400)
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH
EQUIVALENCE (RAY,W(1)),(F,W(6))
LOGICAL SPACE
REAL KR,KTH,KPH,K2,KDOTY,K4,KDOTY2
COMPLEX KAY2,I,PHPT,PHPR,PHPTH,PHPPH,PHPM,PHPKR,PHPKTH,PHPKPH,
1 POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,
2 KPHPK,U2,A,B,ALPHA,BETA,GAMMA,PHPX,PHPY2,PHPK2,PHPU,PHPZ,
3 N2,PNPR,PNPTH,PNPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT
DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=1.),
1 (FIELD=1.),
2 (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),(
3 (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),(
4 (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),(
5 (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)
6 ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),(
7 (I=(0.,1.)),(ABSLIM=1.E-5),(SGN=1.)
ENTRY RINDEX
OM=PIT2*1.E6*F
C2=C*C
K2=KR*KR+KTH*KTH+KPH*KPH
OM2=OM*OM
CALL ELECTX
IF(X.LT..1) GO TO 2
K4=K2*K2
OM4=OM2*OM2
C4=C2*C2
CALL MAGY
Y2=Y*Y
KDOTY=KR*YR+KTH*YTH+KPH*YPH
KDOTY2=KDOTY*KDOTY
CALL COLFRZ
U=CMPLX(1.,-Z)
U2=U*U
UX=U-X
UX2=UX*UX
A=UX*U2-U*Y2
B=-2.*U*UX2+Y2*(2.*U-X)
ALPHA=A*C4*K4+X*KDOTY2*C4*K2
BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2
GAMMA=(UX2-Y2)*UX*OM4
H=ALPHA+BETA+GAMMA
PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+
1 (-3.*UX2+Y2)*OM4
PHPY2=-U2*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4
PHPKY2 =X*C2*(C2*K2-OM2)
PHPU=(2.*U*UX+U2-Y2)*C4*K4+2.*(Y2-UX2-2.*U*UX)*C2*K2*OM2+(3.*UX2
1 -Y2)*OM4
PHPZ=-I*PHPU
PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2

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BQWC001
BQWC002
BQWC003
BQWC004
BQWC005
BQWC006
BQWC007
BQWC008
BQWC009
BQWC010
BQWC011
BQWC012
BQWC013
BQWC014
BQWC015
BQWC016
BQWC017
BQWC018
BQWC019
BQWC020
BQWC021
BQWC022
BQWC023
BQWC024
BQWC025
BQWC026
BQWC027
BQWC028
BQWC029
BQWC030
BQWC031
BQWC032
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BQWC034
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BQWC036
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BQWC042
BQWC043
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BQWC045
BQWC046
BQWC047
BQWC048
BQWC049
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BQWC052
BQWC053
BQWC054
BQWC055
BQWC056
BQWC057
BQWC058
BQWC059
BQWC060
BQWC061
BQWC062
BQWC063
BQWC064
BQWC065

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PHPT=PHPX*PXPT
PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2 *2.*KDOTY*
1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR) +PHPZ*PZPR
PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2 *2.*KDOTY*
1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT) +PHPZ*PZPTH
PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2 *2.*KDOTY*
1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP) +PHPZ*PZPPH
PHPOM=(2.*BETA+4.*GAMMA)/OM
1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2 *KDOTY2/OM -PHPZ*Z/OM
PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2 *YR
PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2 *YTH
PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2 *YPH
<AY2=K2*(-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)
C
IF(RSTART.EQ.0.) GO TO 1
SCALE=SQRT((-REAL(BETA)+SGN*RAY*SQRT(REAL(BETA)**2
1 -4.*REAL(ALPHA)*REAL(GAMMA)))/(2.*REAL(ALPHA)))
KR =SCALE*KR
KTH=SCALE*KTH
KPH=SCALE*KPH
1
CONTINUE
C***** THE FOLLOWING 3 CARDS USED FOR RAY TRACING IN COMPLEX SPACE
C IF(CABS((-BETA-SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.))
C 1LT.CABS((-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.)
C 2 .AND.RSTART.EQ.0.) SGN=-SGN
KPHPK=4.*ALPHA+2.*BETA
SPACE=CABS(C2*KAY2/OM2-1.) .LT.ABSLIM
POLAR =SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY*I
LPOLR = SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)*I
RETURN
C CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C APPLETON-HARTREE FORMULA WITH FIELDS, WITH COLLISIONS
2
CONTINUE
VR =C/OM*KR
VTH=C/OM*KTH
VPH=C/OM*KPH
CALL MAGY
V2=VR**2+VTH**2+VPH**2
VDOTY=VR*YR+VTH*YTH+VPH*YPH
YLV=VDOTY/V2
YL2=VDOTY**2/V2
YT2=Y**2-YL2
YT4=YT2*YT2
CALL COLFRZ
U=CMPLX(1.,-Z)
UX=U-X
UX2=UX*UX
RAD=SGN*RAY*CSQRT(YT4+4.*YL2*UX2)
D=2.*U*UX-YT2+RAD
D2=D*D
N2=1.-2.*X*UX/D
PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2
PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPT+VPH*PYPPR)*YLV
PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV
PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV
PNPX=- (2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2
PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)
PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2
NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)
PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR
PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH
PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH
PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2
PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2
PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2
BQWC066
BQWC067
BQWC068
BQWC069
BQWC070
BQWC071
BQWC072
BQWC073
BQWC074
BQWC075
BQWC076
BQWC077
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BQWC122
BQWC123
BQWC124
BQWC125
BQWC126
BQWC127
BQWC128
BQWC129
BQWC130

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	PNPT=PNPX*PKPT	BQWC131
	SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM	BQWC132
	POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VOOTY*UX)	BQWC133
	GAM=(-YT2+RAD)/(2.*UX)	BQWC134
	LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))	BQWC135
	KAY2=OM2/C2*N2	BQWC136
	IF(RSTART.EQ.0.) GO TO 3	BQWC137
	SCALE=SQRT(REAL(KAY2)/K2)	BQWC138
	KR =SCALE*KR	BQWC139
	KTH=SCALE*KTH	BQWC140
	KPH=SCALE*KPH	BQWC141
3	CONTINUE	BQWC142
	H=.5*(C2*K2/OM2-N2)	BQWC143
	PHPT =-PNPT	BQWC144
	PHPR =-PNPR	BQWC145
	PHPTH=-PNPTH	BQWC146
	PHPPH=-PNPPH	BQWC147
	PHPOM=-NNP/OM	BQWC148
	PHPKR =C2/OM2*KR -C/OM*PNPVR	BQWC149
	PHPKTH=C2/OM2*KTH-C/OM*PNPVTH	BQWC150
	PHPKPH=C2/OM2*KPH-C/OM*PNPVPH	BQWC151
	KPHPK=N2	BQWC152
	RETURN	BQWC153
	END	BQWC154

	SUBROUTINE BQWFNC	BQNC001
	C***** CALCULATES A HAMILTONIAN H	BQNC002
	C***** (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)	BQNC003
	C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO	BQNC004
	C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.	BQNC005
	C***** WITH FIELD, NO COLLISIONS	BQNC006
	COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN	BQNC007
	COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,	BQNC008
1	H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,	BQNC009
2	PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPI	BQNC010
3	KPHPK,KHPKI,POLAR,POLARI,LPOLAR,LPOLARI,SGN.	BQNC011
	COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX	BQNC012
	COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR	BQNC013
1	PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP	BQNC014
	COMMON /ZZ/ MODZ,Z(4)	BQNC015
	COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART	BQNC016
	COMMON R,TH,PH,KR,KTH,KPH /WW/ ID(10),W0,W(400)	BQNC017
	EQUIVALENCE (RAY,W(1)),(F,W(6))	BQNC018
	LOGICAL SPACE	BQNC019
	REAL N2,NNP,LPOLAR,LPOLARI,KR,KTH,KPH,K2,KDOTY,K4,KDOTY2,	BQNC020
1	KPHPK,KHPKI,KAY2,KAY2I	BQNC021
	DATA (MODRIN=8HBOOKER Q,8HUARTIC ,8HS=0, C=1),(COLL=0.),	BQNC022
1	(FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),	BQNC023
2	(PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.),	BQNC024
3	(PHPKPI=0.),(KHPKI=0.),(POLAR=0.),(LPOLAR=0.),	BQNC025
4	(X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),	BQNC026
5	(Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),	BQNC027
6	(PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),	BQNC028
7	(PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),	BQNC029
8	(MODZ=1H),(U=1.),(U2=1.)	BQNC030
	ENTRY RINDEX	BQNC031
	OM=PIT2*1.E6*	BQNC032
	C2=C*C	BQNC033
	K2=KR*KR+KTH*KTH+KPH*KPH	BQNC034
	OM2=OM*OM	BQNC035

CALL ELECTX	BQNC036
IF(X.LT..1) GO TO 2	BQNC037
K4=K2*K2	BQNC038
OM4=OM2*OM2	BQNC039
C4=C2*C2	BQNC040
CALL MAGY	BQNC041
Y2=Y*Y	BQNC042
KDOTY=KR*YR+KTH*YTH+KPH*YPH	BQNC043
KDOTY2=KDOTY*KDOTY	BQNC044
UX=U-X	BQNC045
UX2=UX*UX	BQNC046
A=UX*U2-U*Y2	BQNC047
B=-2.*U*UX2+Y2*(2.*U-X)	BQNC048
ALPHA=A*C4*K4+X*KDOTY2*C4*K2	BQNC049
BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2	BQNC050
GAMMA=(UX2-Y2)*UX*OM4	BQNC051
H=ALPHA+BETA+GAMMA	BQNC052
PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+	BQNC053
1 (-3.*UX2+Y2)*OM4	BQNC054
PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4	BQNC055
PHPKY2 =X*C2*(C2*K2-OM2)	BQNC056
PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2	BQNC057
PHPT=PHPX*PXPT	BQNC058
PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2 *2.*KDOTY*	BQNC059
1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR)	BQNC060
PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2 *2.*KDOTY*	BQNC061
1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT)	BQNC062
PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2 *2.*KDOTY*	BQNC063
1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP)	BQNC064
PHPOH=(2.*BETA+4.*GAMMA)/OM	BQNC065
1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2 *KDOTY2/OM	BQNC066
PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2 *YR	BQNC067
PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2 *YTH	BQNC068
PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2 *YPH	BQNC069
KAY2 = K2 *(-BETA+RAY*SQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)	BQNC070
IF(RSTART.EQ.0.) GO TO 1	BQNC071
SCALE=SQRT(KAY2/K2)	BQNC072
KR =SCALE*KR	BQNC073
KTH=SCALE*KTH	BQNC074
KPH=SCALE*KPH	BQNC075
1 CONTINUE	BQNC076
KPHPK=4.*ALPHA+2.*BETA	BQNC077
SPACE=KAY2.EQ.OM2/C2	BQNC078
POLARI=SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY	BQNC079
LPOLRI= SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)	BQNC080
RETURN	BQNC081
C CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE	BQNC082
C APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS	BQNC083
2 CONTINUE	BQNC084
VR =C/OM*KR	BQNC085
VTH=C/OM*KTH	BQNC086
VPH=C/OM*KPH	BQNC087
CALL MAGY	BQNC088
V2=VR**2+VTH**2+VPH**2	BQNC089
VDOTY=VR*YR+VTH*YTH+VPH*YPH	BQNC090
YLV=VDOTY/V2	BQNC091
YL2=VDOTY**2/V2	BQNC092
YT2=Y**2-YL2	BQNC093
YT4=YT2*YT2	BQNC094
JX=U-X	BQNC095
UX2=UX*UX	BQNC096
RAD=RAY*SQRT(YT4+4.*YL2*UX2)	BQNC097
D=2.*UX-YT2+RAD	BQNC098
D2=D*D	BQNC099
N2=1.-2.*X*UX/D	BQNC100

	PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2	BQNC101
	PPSPR=YL2/Y*PYPR-(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV	BQNC102
	PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV	BQNC103
	PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV	BQNC104
	PNPX=- (2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2	BQNC105
	PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)	BQNC106
	NNP=N2-(2.*X*PNPX+Y*PNPY)	BQNC107
	PNPR=PNPX*PXPR+PNPY*PYPR+PNPPS*PPSPR	BQNC108
	PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH	BQNC109
	PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH	BQNC110
	PNPVR=PNPPS*(VR*YL2-VDOTY*YR)/V2	BQNC111
	PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2	BQNC112
	PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2	BQNC113
	PNPT=PNPX*PXPT	BQNC114
	SPACE=N2.EQ.1.	BQNC115
	POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)	BQNC116
	GAM=(-YT2+RAD)/(2.*UX)	BQNC117
	LPOLRI=X*SQRT(YT2)/(UX*(J+GAM))	BQNC118
	KAY2=OM2/C2*N2	BQNC119
	IF(RSTART.EQ.0.) GO TO 3	BQNC120
	SCALE=SQRT(KAY2/K2)	BQNC121
	KR=SCALE*KR	BQNC122
	KTH=SCALE*KTH	BQNC123
	KPH=SCALE*KPH	BQNC124
3	CONTINJE	BQNC125
	A=.5*(C2*K2/OM2-N2)	BQNC126
	PHPT=-PNPT	BQNC127
	PHPR=-PNPR	BQNC128
	PHPTH=-PNPTH	BQNC129
	PHPPH=-PNPPH	BQNC130
	PHPOM=-NNP/OM	BQNC131
	PHPKR=C2/OM2*KR-C/OM*PNPVR	BQNC132
	PHPKTH=C2/OM2*KTH-C/OM*PNPVTH	BQNC133
	PHPKPH=C2/OM2*KPH-C/OM*PNPVPH	BQNC134
	KPHPK=N2	BQNC135
	RETURN	BQNC136
	END	BQNC137-

	SUBROUTINE SHWF	SHWF001
C	CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE	SHWF002
C	SEN-WYLLER FORMULA -- WITH FIELD	SHWF003
C	NEEDS SUBROUTINE FSW AND FUNCTIONS C AND S.	SHWF004
	COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,SEA,LOGTEN	SHWF005
	COMMON /RIN/ MODRIN(3),CJLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,	SHWF006
1	PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,	SHWF007
2	SGN	SHWF008
	COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX	SHWF009
	COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR	SHWF010
1	,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP	SHWF011
	COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH	SHWF012
	COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART	SHWF013
	COMMON R,TH,P1,KR,KTH,KPH /HW/ ID(10),W0,W(400)	SHWF014
	EQUIVALENCE (RAY,W(1)),(F,W(6))	SHWF015
	LOGICAL SPACE	SHWF016
	REAL KR,KTH,KPH,K2	SHWF017
	COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,	SHWF018
1	KPHPK,POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,	SHWF019
2	ALPHA,BETA,GAMMA,A,B,C,TEMP1,TEMP2,TEMP3,ALPOAL,BEPOBE,	SHWF020
3	GAPOGA,CB2,N2M1,J2,D2GA,DAL,OBET,OGAM,DAOY,DAOZ,OBDOY,OBDOZ,	SHWF021
4	DCOY,DCOZ,DUDZ,DT1OX,DT1OY,DT1OZ,DT1OPS,DT2DX,DT2DY,DT2DZ,	SHWF022
5	DT2OPS,DRADDX,DRADDOY,DRADDOZ,DRDOOPS,DOOX,DOOY,DOOZ,DOOPX,	SHWF023

6	UPX,N2,PNPR,PNPT4,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT	SWWF024
	DATA (MOORIN=8H SE,84N-WYLLER,8H FORMULA),(COLL=1.),	SWWF025
1	(FIELD=1.), (LPOLAR=(0.,0.)),	SWWF026
2	(X=0.), (PXPR=0.), (PXPTH=0.), (PXPPH=0.), (PXPT=0.),	SWWF027
3	(Y=0.), (PYPR=0.), (PYPTH=0.), (PYPPH=0.), (YR=0.), (PYRPR=0.),	SWWF028
4	(PYRPT=0.), (PYRPP=0.), (YTH=0.), (PYTPR=0.), (PYTPT=0.),	SWWF029
5	(PYTPP=0.), (YPH=0.), (PYPPR=0.), (PYPPT=0.), (PYPPP=0.),	SWWF030
6	(Z=0.), (PZPR=0.), (PZPTH=0.), (PZPPH=0.),	SWWF031
7	(I=(0.,1.)), (ABSLIM=1.E-5)	SWWF032
	ENTRY RINDEX	SWWF033
	OM=PIT2*1.E6*F	SWWF034
	C2=SEA*SEA	SWWF035
	K2=KR*KR+KTH*KTH+KPH*KPH	SWWF036
	OM2=OM*OM	SWWF037
	VR =SEA/OM*KR	SWWF038
	VTH=SEA/OM*KTH	SWWF039
	VPH=SEA/OM*KPH	SWWF040
	CALL ELECTX	SWWF041
	CALL MAGY	SWWF042
	OPY=1.+Y	SWWF043
	OMY=1.-Y	SWWF044
	CALL COLFRZ	SWWF045
	Z2=Z*Z	SWWF046
	CALL FSW(1./Z,ALPHA,DAL)	SWWF047
	ALPOAL=DAL/ALPHA	SWWF048
	CALL FSW(OMY/Z,BETA,DBET)	SWWF049
	BEPOBE=DBET/BETA	SWWF050
	CALL FSW(OPY/Z,GAMMA,DGAM)	SWWF051
	GAPOGA=DGAM/GAMMA	SWWF052
	U=Z/ALPHA	SWWF053
	DUDZ=(1.+ALPOAL/Z)/ALPHA	SWWF054
	U2=U*U	SWWF055
	UX=U-X	SWWF056
	JPX=U+X	SWWF057
	B=ALPHA/BETA	SWWF058
	DBDY=B*BEPOBE/Z	SWWF059
	DBDZ=-B*(ALPOAL-OMY*BEPOBE)/Z2	SWWF060
	C=ALPHA/GAMMA	SWWF061
	DCDY=-C*GAPOGA/Z	SWWF062
	DCDZ=-C*(ALPOAL-OPY*GAPOGA)/Z2	SWWF063
	A=.5*(B+C)-1.	SWWF064
	DADY=.5*(DBDY+DCDY)	SWWF065
	DAOZ=.5*(DBDZ+DCDZ)	SWWF066
	TEMP3=(1.-B*C)*U2+A*U*UPX	SWWF067
	V2= VR**2+VTH**2+VPH**2	SWWF068
	VDOTY=VR*YR+VTH*YTH+VPH*YPH	SWWF069
	YL2=VDOTY**2/V2	SWWF070
	YT2=Y**2-YL2	SWWF071
	Y2=Y*Y	SWWF072
	S2PSI=YT2/Y2	SWWF073
	C2PSI=YL2/Y2	SWWF074
	UX2=UX*UX	SWWF075
	CB2=(C-B)**2	SWWF076
	TEMP1=TEMP3*S2PSI	SWWF077
	DT1DX= A*U*S2PSI	SWWF078
	DT1DY=(U*JPX*DADY-U2*(B*DCDY+C*DBDY))*S2PSI	SWWF079
	DT1DZ=(2.*U*DUDZ*(1.-B*C+A)+A*X*DUDZ-U2*(B*DCDZ+C*DBDZ)+U*UPX*DAOZ	SWWF080
1)*S2PSI	SWWF081
C	(1/YLYT) D/DPSI(TEMP1)	SWWF082
	DT1DPS=2.*TEMP1/YT2	SWWF083
	TEMP2=U2*CB2*JX2*C2PSI	SWWF084
	DT2DX=-2.*UX*U2*CB2*C2PSI	SWWF085
	DT2DY=2.*U2*UX2*C2PSI*(C-B)*(DCDY-DBDY)	SWWF086
	DT2DZ=2.*U2*UX2*C2PSI*(C-B)*(DCDZ-DBDZ)+2.*TEMP2*(1./U+1./UX)*DUDZ	SWWF087
C	(1/YLYT) D/DPSI(TEMP2)	SWWF088
	DT2DPS=-2.*TEMP2/YL2	SWWF089

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RAD=RAY*CSQRT(TEMP1**2+TEMP2)
DRADDX=(TEMP1*DT1DX+.5*DT2DX)/RAD
DRADDDY=(TEMP1*DT1DY+.5*DT2DY)/RAD
DRADDDZ=(TEMP1*DT1DZ+.5*DT2DZ)/RAD
C (1/YLYT) D/DPSI(RAD)
DRDDPS=(TEMP1*DT1DPS+.5*DT2DPS)/RAD
D=2.*U*UX*(1.+A)-TEMP1+RAD+2.*A*U*X*S2PSI
DDD=-2.*U-DT1DX*DRADDX+2.*A*U*S2PSI
DDDY=2.*U*UX*DADY-DT1DY*DRADDDY+2.*U*S2PSI*DADY
DDDZ=2.*(1.+A)*DUDZ*(U+UX)+2.*U*UX*DAOZ-DT1DZ*DRADDDZ+2.*X*S2PSI*
1 (A*DUDZ+J*JADZ)
C (1/YLYT) D/DPSI(D)
DDDDPS=-DT1DPS+DRDDPS+2.*A*U*X/Y2
N2M1=-2.*X*(UX+U*A*S2PSI)/D
N2=1.+N2M1
C N D/OX(N)
PNPX=-(JX+U*A*S2PSI)*(1.-X*DDD/D)/D+X/D
C N D/DY(N)
PNPY=-X*U*S2PSI/D*DADY-.5*N2M1/D*DDDY
C N D/OZ(N)
PNPZ=-X*(1.+A*S2PSI)/D*DUDZ-X*U*S2PSI/D*DAOZ-.5*N2M1/D*DDDZ
C (N/YLYT) D/DPSI(N)
PNPPS=-X*U*A/(D*Y2) -.5*N2M1/D*DDDDPS
YLV=VDOTY/V2
C (YLYT) D/DR(PSSI)
PPSPR=YL2/Y*PYPR-(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV
C (YLYT) J/DTHETA(PSSI)
PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV
C (YLYT) D/DPHI(PSSI)
PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV
PNPR=PNPX*PXPR+PNPY*PYPR+PNPZ*PZPR+PNPPS*PPSPR
PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH
PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH
PNPVR=PNPPS*(VR*YL2/V2-YLV*YR)
PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH)
PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)
NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)
PNPT=PNPX*PXPT
POLAR=I*(TEMP1-RAD)*Y*SQRT(V2)/(U*UX*(C-B)*VDOTY)
COSPSI=VDOTY/(Y*SQRT(V2))
LPOLAR=(.5*I*(C-B)*POLAR+A*COSPSI)*SQRT(S2PSI)/
1 (POLAR*(JX*(1.+5*I*(C-B)*COSPSI*POLAR)+A*(U-X*C2PSI)))
SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM
KAY2=OM2/32*N2
IF(RSTART.EQ.0.) GO TO 1
SCALE=SQRT(REAL(KAY2)/K2)
KR=SCALE*KR
KTH=SCALE*KTH
KPH=SCALE*KPH
1 CONTINUE
C***** CALCULATES A HAMILTONIAN H
H=.5*(C2*K2/OM2-N2)
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
PHPT=-PNPT
PHPR=-PNPR
PHPTH=-PNPTH
PHPPH=-PNPPH
PHPOM=-NNP/OM
PHPKR=C2/OM2*KR-SEA/OM*PNPVR
PHPKTH=C2/OM2*KTH-SEA/OM*PNPVTH
PHPKPH=C2/OM2*KPH-SEA/OM*PNPVPH
KHPK=N2
RETURN
END
SWWF090
SWWF091
SWWF092
SWWF093
SWWF094
SWWF095
SWWF096
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SWWF100
SWWF101
SWWF102
SWWF103
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SWWF145
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SWWF151
SWWF152
SWWF153
SWWF154-

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SUBROUTINE SWNF
C      CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C      SEM-WYLLER FORMULA -- NO FIELD
C      NEEDS SUBROUTINES FGSW AND FSW AND FUNCTIONS C AND S.
COMMON /CONST/ PI,PIT2,PIJ2,DEGS,RADIAN,K,C,LOGTEN
COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,
1      PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,
2      SGN
COMMON /XX/ MOOX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
COMMON /YY/ MOOY,Y(16)
COMMON /ZZ/ MOOZ,Z,PZPR,PZPTH,PZPPH
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
COMMON R,TH,PH,KR,KTH,KPH /WW/ IO(10),W0,W(400)
EQUIVALENCE (RAY,W(1)),(F,W(6))
LOGICAL SPACE
REAL KR,KTH,KPH,K2
COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,
1      KPHPK,POLAR,LPOLAR,PNPX,PNPZ,F1,DF,G1,DG1,
2      N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT
DATA (MODRIN=8H SE,84N-WYLLER,8H FORMULA),(COLL=1.),
1      (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),
2      (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
3      (MOOY=1H ),
4      (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),
5      (ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)
ENTRY RINJEX
OM=PIT2*1.E6*F
C2=C*C
K2=KR*KR+KTH*KTH+KPH*KPH
OM2=OM*OM
VR =C/OM*KR
VTH=C/OM*KTH
VPH=C/OM*KPH
CALL ELECTX
CALL COLFRZ
CALL FGSW(1./Z,F1,DF1,G1,DG1)
N2=1.-X*G1
PNPX=-.5*G1
PNPZ=.5*X*DG1/Z**2
PNPR=PNPX*PXPR+PNPZ*PZPR
PNPTH=PNPX*PXPTH+PNPZ*PZPTH
PNPPH=PNPX*PXPPH+PNPZ*PZPPH
NNP=N2-(2.*X*PNPX+Z*PNPZ)
PNPT=PNPX*PXPT
SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM
KAY2=OM2/C2*N2
IF(RSTART.EQ.0.) GO TO 1
SCALE=SQRT(REAL(KAY2)/K2)
KR =SCALE*KR
KTH=SCALE*KTH
KPH=SCALE*KPH
1 CONTINUE
C***** CALCULATES A HAMILTONIAN H
H=.5*(C2*K2/OM2-N2)
C***** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C***** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
PHPT =-PNPT
PHPR =-PNPR
PHPTH=-PNPTH
PHPPH=-PNPPH
PHPOM=-NNP/OM
PHPKR =C2/OM2*KR
PHPKTH=C2/OM2*KTH
PHPKPH=C2/OM2*KPH
KPHPK=N2
RETURN
END
SWNF001
SWNF002
SWNF003
SWNF004
SWNF005
SWNF006
SWNF007
SWNF008
SWNF009
SWNF010
SWNF011
SWNF012
SWNF013
SWNF014
SWNF015
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SWNF064
SWNF065
SWNF066
SWNF067-

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SUBROUTINE FGSW (X,F,DF,G,DG)
COMPLEX F,DF,G,DG
CALL FSW (X,F,DF)
IF (ABS(X).GT.50.) GO TO 1
G=X*F
DG=F+X*DF
RETURN
1 X2=X*X
X3=X2*X
T2=2.*X2
T3=3.*X2
T4=4.*X2
T8=8.*X2
T12=12.*X2
T16=16.*X2
G=CMPLX(1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4)))/T4,
12.5*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4)))/X)
DG=.5*CMPLX(35.*(1.-99./T2*(1.-585./T8*(1.-323./T3*(1.-2415./T16)
1 ))/X3,
2-5.*(1.-189./T4*(1.-715./T12*(1.-357./T4*(1.-513./T4)))/X2)
RETURN
END

```

FGSW001
FGSW002
FGSW003
FGSW004
FGSW005
FGSW006
FGSW007
FGSW008
FGSW009
FGSW010
FGSW011
FGSW012
FGSW013
FGSW014
FGSW015
FGSW016
FGSW017
FGSW018
FGSW019
FGSW020
FGSW021
FGSW 22-

```

SUBROUTINE FSW (Z,F,DF)
C F(Z) = Z*C3/2(Z) + 2.5*I*C5/2(Z) AND DF(Z) = DF/DZ
C WHERE THE INPUT Z IS REAL AND THE OUTPUT F AND DF ARE COMPLEX.
C NEEDS THE SUBPROGRAMS FOR THE FRESNEL INTEGRAL FUNCTIONS S AND C
DIMENSION A(10),B(10),D(10)
COMPLEX F,DF,C1,C2,C3,C8,W,TEMP,I
DATA (I=(0.,1.)), (PI=3.1415926536), (A3=1.333333333)
DATA (C2=(1.,1.)), (C3=(1.,-1.)), (C4=.79788456 ), (C6=1.333333333)
C C4=SQRT(2./PI)
DATA(A=.36230845E-02,.29579186E+00,.23193588E+01,.91355870E+01,
1.25856287E+02,.60488560E+02,.12562218E+03,.24214980E+03,
2.44918106E+03,.84244774E+03),
3(B=.16747479E-02,.84796280E-01,.25285001E+00,.22665857E+00,
4.83871933E-01,.13811875E-01,.98017417E-03,.26299148E-04,
5.19761006E-06,.18781476E-09),
6(D=.10080653E-03,.46117941E-01,.38507643E+00,.68507885E+00,
7.42648105E+00,.10742102E+00,.10985920E-01,.40924533E-03,
8.41881263E-05,.54513142E-08),(G=1.5045055)
C1=2./3.*I
C8=C2*A3*SQRT(PI/2.)
X=Z
X2=X*X
X3=X2*X
IF (ABS(X).GT.50.) GO TO 500
IF (ABS(X).GT.5.) GO TO 1
IF (ABS(X).LT..05) GO TO 200
C FRESNEL
IF (X.GT.0.) GO TO 300
100 Y=C4*SQRT(-X)
X2=X*X
W=(COS(X)+I*SIN(X))*(1.-C3*(C(Y)+I*S(Y)))
F =C1+C6*(X+C3*X*X/Y*W)
DF=A3*CMPLX(1.,X)+CMPLX(1.5,X)*A3*C3*X/Y*W
RETURN
300 Y=C4*SQRT(X)
X2=X*X
W=(COS(X)+I*SIN(X))*(1.-C2*(C(Y)-I*S(Y)))
F =C1+C6*(X-C2*X*X/Y*W)
DF=A3*CMPLX(1.,X)-CMPLX(1.5,X)*A3*C2*X/Y*W
RETURN

```

FSW 001
FSW 002
FSW 003
CFSW 004
FSW 005
FSW 006
FSW 007
FSW 008
FSW 009
FSW 010
FSW 011
FSW 012
FSW 013
FSW 014
FSW 015
FSW 016
FSW 017
FSW 018
FSW 019
FSW 020
FSW 021
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FSW 031
FSW 032
FSW 033
FSW 034
FSW 035
FSW 036
FSW 037
FSW 038
FSW 039
FSW 040

C		POWER SERIES		FSW 041
	200	X=ABS(Z)		FSW 042
		X2=X*X		FSW 043
		X3=X2*X		FSW 044
		X4=X*X3		FSW 045
		X5=X*X4		FSW 046
		TEMP=-C8* SQRT(X)*CEXP(I*X)		FSW 047
		F=CMPLX(4./3.*X-16./9.*X3+64./315.*X5,2./3.+8./3.*X2-32./45.*X4)		FSW 048
	1	+TEMP*X		FSW 049
		DF=CMPLX(4./3.-16./3.*X2+64./63.*X4,16./3.*X-128./45.*X3		FSW 050
	1	+256./945.*X5)		FSW 051
	2	+TEMP*CMPLX(1.5,X)		FSW 052
		IF(Z.GE.0.) RETURN		FSW 053
		F=-CONJG(F)		FSW 054
		JF=CONJG(DF)		FSW 055
		RETURN		FSW 056
C		HERMITE		FSW 057
	1	XQ = X**2		FSW 058
		X2=XQ		FSW 059
		FR = 0.		FSW 060
		FI = 0.		FSW 061
		DFR = 0.		FSW 062
		DFI = 0.		FSW 063
		DO 2 J = 1,10		FSW 064
		SS = A(J) + XQ		FSW 065
		SB = B(J)/SS		FSW 066
		SD = D(J)/SS		FSW 067
		FR = FR + SB		FSW 068
		FI = FI + SD		FSW 069
		DFR = DFR + SB/SS		FSW 070
	2	DFI = DFI + SD/SS		FSW 071
		F = CMPLX(X*FR,FI)*G		FSW 072
		DF = G*(FR - 2.*X*CMPLX(X*DFR,DFI))		FSW 073
		RETURN		FSW 074
C		ASYMPTOTIC		FSW 075
	500	X2=X*X		FSW 076
		X3=X2*X		FSW 077
		X4=X3*X		FSW 078
		X5=X4*X		FSW 079
		T2=2.*X2		FSW 080
		T3=3.*X2		FSW 081
		T4=4.*X2		FSW 082
		T8=8.*X2		FSW 083
		T16=16.*X2		FSW 084
		T28=28.*X2		FSW 085
		F=CMPLX((1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4))))/X		FSW 086
		1,5.*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/T2		FSW 087
		DF=-CMPLX((1.-105./T4*(1.+165./T4*(1.-273./T4*(1.-2907./T28))))/X2		FSW 088
		1,5.*(1.-63./T2*(1.-429./T8*(1.-255./T3*(1.-1995./T16))))/X3		FSW 089
		RETURN		FSW 090
		END		FSW 91-

```

FUNCTION C(X)
DOUBLEPRECISION  PIH, XD, Y, V, A, QZ, QN, Q, Z
DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)
PIH = 1.570796326794897
XA = ABS(X)
IF (XA.GT.4.) GOTO 20
C
XD = X
Y = PIH*XD*XD
V = Y*Y
A = 1.D0
Z = A
M = 15.*(XA + 1.)
DO 10 I = 1, M
KZ=2*(I-1)
KV=4*(I-1)
QZ = KV + 1
QN = (KZ + 1)*(KZ + 2)*(KV + 5)
Q = QZ/QN
A = -A*Q*V
10 Z = Z + A
Z = Z*XD
C = Z
RETURN
C
20 W = PIH*X*X
XV=XA**4
C=0.5+(A1-B1/XV)*SIN(W)/XA-(A2-B2/XV)*COS(W)/XA**3
IF (X.LT.0.) C = -C
RETURN
END
C

```

```

FUNCTION S(X)
DOUBLEPRFCISION  PIH, XD, Y, V, A, QZ, QN, Q, Z
DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)
PIH = 1.570796326794897
C
XA = ABS(X)
IF (XA.GT.4.) GOTO 20
C
XD = X
Y = PIH*XD*XD
V = Y*Y
A = Y/3.D0
Z = A
M = 15.*(XA + 1.)
DO 10 I = 1, M
KZ=2*(I-1)
KV=4*(I-1)
QZ = KV + 3
QN = (KZ + 2)*(KZ + 3)*(KV + 7)
Q = QZ/QN
A = -A*Q*V
10 Z = Z + A
Z = Z*XD
S = Z
RETURN
C
20 W = PIH*X*X
XV=XA**4
S=0.5-(A1-B1/XV)*COS(W)/XA-(A2-B2/XV)*SIN(W)/XA**3
IF (X.LT.0.) S = -S
RETURN
END
S

```