5. TRANSMISSION LOSS PREDICTION METHODS FOR WITHIN-THE-HORIZON PATHS

Ground wave propagation over a smooth spherical earth of uniform ground conductivity and dielectric constant, and with a homogeneous atmosphere, has been studied extensively. Some of the results were presented in CCIR Atlases [1955, 1959]. Recent work by Bachynski [1959, 1960, 1963], Wait [1963], Furutsu [1963], and others considers irregularities of electrical ground constants and of terrain. A distinction is made here between the roughness of terrain which determines the proportion between specular and diffuse reflection of radio waves, and large scale irregularities whose average effect is accounted for by fitting a straight line or curve to the terrain.

A comprehensive discussion of the scattering of electromagnetic waves from rough surfaces is given in a recent book by Beckmann and Spizzichino [1963]. Studies of reflection from irregular terrain as well as absorption, diffraction, and scattering by trees, hills, and man-made obstacles have been made by Beckmann [1957], Biot [1957 a, b], Kalinin [1957, 1958], Kühn [1958], McGavin and Maloney [1959], McPetrie and Ford [1946], McPetrie and Saxton [1942], Saxton and Lane [1955], Sherwood and Ginzton [1955], and many other workers. Examples of studies of reflection from an ocean surface may be found in papers by Beard, Katz and Spetner [1956], and Beard [1961].

A semi-empirical method for predicting transmission loss for within-the-horizon paths is given in annex I.

Reflections from hillsides or obstacles off the great circle path between two antennas sometimes contribute a significant amount to the received signal. Discrimination against such off-path reflections may reduce multipath fading problems, or in other cases antenna beams may be directed away from the great circle path in order to increase the signal level by taking advantage of off-path reflection or knife-edge diffraction. For short periods of time, over some paths, atmospheric focusing or defocusing will lead to somewhat smaller or much greater values of line-of-sight attenuation than the long-term median values predicted for the average path by the methods of this section.

If two antennas are intervisible over the effective earth defined in section 4, ray optics may be used to estimate the attenuation A relative to free space, <u>provided that the great circle path terrain visible to both antennas will support a substantial amount of reflection and that it is reasonable to fit a straight line or a convex curve of radius a to this portion of the terrain.</u>

5.1 Line-of-Sight Propagation Over Irregular Terrain

Where ray optics formulas, described in section 5.2, are not applicable a satisfactory estimate of line-of-sight transmission loss may sometimes be made by one of the following methods:

1. If a slight change in the position of either antenna results in a situation where ray optics formulas may be used, then A may be estimated by extrapolation or interpolation.

2. Instead of a single curve fit to terrain as in 5.2 the method may, in some cases, be extended to multiple curve fits and multiple reflections from these curves.

3. If terrain is so irregular it cannot be reasonably well approximated by a single curve, the line-of-sight knife-edge formulas of section 7 may be applicable.

4. Interpolation between curves in an atlas, or standard propagation curves such as those given in appendix I, may provide a satisfactory estimate. A useful set of calculations for $\theta = 0$ is given by Domb and Pryce [1947].

5. Empirical curves drawn through data appropriate for the problem of interest may be useful. For example, the dashed curves of figures I.1-I.3 show how values of attenuation relative to free space vary with distance and frequency for a large sample of recordings of television signals over random paths. The data shown in figures I.1-I.4 correspond to a more careful selection of receiving locations and to a greater variety of terrain and climatic conditions.

5.2 Line-of-Sight Propagation Over a Smooth or Uniformly Rough Spherical Earth

The simplest ray optics formulas assume that the field at a receiving antenna is made up to two components, one associated with a direct ray having a path length r_o, and the other associated with a ray reflected from a point on the surface, with equal grazing angles ψ. The reflected ray has a path length $r_1 + r_2$. The field arriving at the receiver via the direct ray differs from the field arriving via the reflected ray by a phase angle which is a function of the path length difference, $\Delta r = r_1 + r_2 - r_o$, illustrated in figure 5.1. The reflected ray field is also modified by an effective reflection coefficient R_e and associated phase lag $(\pi - c)$, which depend on the conductivity, permittivity, roughness, and curvature of the reflecting surface, as well as upon the ratio of the products of antenna gain patterns in the directions of direct and reflected ray paths.

Let g_{01} and g_{02} represent the directive gain for each antenna in the direction of the other, assuming antenna polarizations to be matched. Similar factors g_{r_1} and g_{r_2} are defined for each antenna in the direction of the point of ground reflection. The effective reflection coefficient R_e is then

$$R_e = DR \left(\frac{g_{r1} g_{r2}}{g_{o1} g_{o2}} \right)^{1/2} \exp \left(\frac{-0.6\, \sigma_h \sin \psi}{\lambda} \right) \tag{5.1}$$

where the divergence factor D allows for the divergence of energy reflected from a curved surface, and may be approximated as

$$D = \left[1 + \frac{2d_1 d_2}{a\, d \tan \psi} \right]^{-1/2} \tag{5.2}$$

A more exact expression for the divergence factor, D, based on geometric optics was derived by Riblet and Barker [1948]. The term R represents the magnitude of the theoretical coefficient, $R \exp[-i(\pi - c)]$, for reflection of a plane wave from a smooth plane surface of a given conductivity and dielectric constant. In most cases c may be set equal to zero and R is very nearly unity. A notable exception for vertical polarization over sea water is discussed in annex III. Values of R and c vs ψ are shown on figures III.1 to III.8 for both vertical and horizontal polarization over good, average, and poor ground, and over sea water.

The grazing angle ψ and the other geometrical parameters d, d_1, d_2, and a are shown on figure 5.1. The terrain roughness factor, σ_h, defined in section 5.2.2, and the radio wave length, λ, are expressed in the same units. The exponent $(\sigma_h \sin \psi)/\lambda$ is Rayleigh's criterion of roughness.

If the product DR $\exp(-0.6\,\sigma_h \sin\psi/\lambda)$ is less than $\sqrt{\sin\psi}$, and is less than 0.5, ground reflection may be assumed to be entirely diffuse and R_e is then expressed as

$$R_e = \left[\frac{g_{r1} g_{r2}}{g_{01} g_{02}} \sin\psi \right]^{1/2} \tag{5.3}$$

where terrain factors D, R and σ_h are ignored. The factor $g_{r1} g_{r2} / g_{01} g_{02}$ in (5.3) makes R_e approach zero when narrow-beam antennas are used to discriminate against ground reflections.

For a single ground reflection, the attenuation relative to free space may be obtained from the general formula

$$A = -10 \log \left\{ g_{01} g_{02} \left[1 + R_e^2 - 2 R_e \cos\left(\frac{2\pi \Delta r}{\lambda} - c \right) \right] \right\} + G_p + A_a \text{ db} \tag{5.4}$$

where the path antenna gain G_p may not be equal to the sum of the maximum antenna gains. Losses A_a due to atmospheric absorption, given by (3.4), may be important at frequencies above 1 GHz. The basic transmission loss L_b is

$$L_b = 32.45 + 20 \log f + 20 \log r + A. \tag{5.5}$$

Over a smooth perfectly-conducting surface, $R_e = 1$ and $c = 0$. Assuming also that free space antenna gains are realized, so that $G_p = 10 \log(g_{01} g_{02})$, the attenuation relative to free space is

$$A = -6 - 10 \log \sin^2 (\pi \Delta r/\lambda) \text{ db}. \tag{5.6}$$

Exact formulas for computing Δr are given in annex III. The appropriate approximations given in (5.9) to (5.13) suffice for most practical applications. If Δr is less than 0.12λ, (5.4) may underestimate the attenuation and one of the methods of section 5.1 should be used.

Section 5.2.1 shows how to define antenna heights h_1' and h_2' above a plane earth, or above a plane tangent to the earth at the point of reflection. The grazing angle ψ is then defined by

$$\tan\psi = h_1'/d_1 = h_2'/d_2 \tag{5.7}$$

where heights and distances are in kilometers and d_1 and d_2 are distances from each antenna to the point of specular reflection:

$$d_1 + d_2 = d, \qquad d_1 = d(1 + h_2'/h_1')^{-1}, \qquad d_2 = d(1 + h_1'/h_2')^{-1}, \tag{5.8a}$$

The distances d_1 and d_2 may be approximated for a spherical earth by substituting antenna heights h_1 and h_2 above the earth for the heights h_1' and h_2' in (5.8a). Then these heights may be calculated as

$$h_1' = h_1 - d_1^2/(2a), \qquad h_2' = h_2 - d_2^2/(2a) \tag{5.8b}$$

for an earth of effective radius a, and substituted in (5.8a) to obtain improved estimates of d_1 and d_2. Iterating between (5.8a) and (5.8b), any desired degree of accuracy may be obtained.

The path length difference between direct and ground reflected rays is

$$\Delta r = \sqrt{d^2 + \left(h_1' + h_2'\right)^2} - \sqrt{d^2 + \left(h_1' - h_2'\right)^2} \cong 2 h_1' h_2'/d \tag{5.9}$$

where the approximation in (5.9) is valid for small grazing angles.

Referring to (5.5) the greatest distance, d_o, for which A is zero, (assuming that $R_e = 1$ and that free space gains are realized) occurs when $\Delta r = \lambda/6$. From (5.9) $\Delta r \cong 2 h_1' h_2'/d$; therefore:

$$d_o = 12\, h_1' h_2'/\lambda . \tag{5.10a}$$

This equation may be solved graphically, or by iteration, choosing a series of values for d_o, solving (5.8) for h_1', h_2', and testing the equality in (5.10a).

For the special case of equal antenna heights over a spherical earth of radius a, the distance d_o may be obtained as follows:

$$\Delta r = \lambda/6 = \frac{2}{d_o}\left[h - d_o^2/(8a)\right]^2 = 2h^2/d_o - h d_o/(2a) + d_o^3/(32a^2) \tag{5.10b}$$

where

$$d_1 = d_2 = d/2, \qquad h_1 = h_2 = h, \quad \text{and} \quad h' = h - d_o^2/(8a).$$

For this special case where $h_1 = h_2$ over a smooth spherical earth of radius a, the angle ψ may be defined as

$$\tan \psi = 2\, h/d - d/(4a) \tag{5.11a}$$

and

$$\Delta r = d(\sec \psi - 1) = d\left[\sqrt{1 + \tan^2 \psi} - 1\right] . \tag{5.11b}$$

Let θ_h represent the angle of elevation of the direct ray r_o relative to the horizontal at the lower antenna, h_1, assume that $h_1 << h_2$, $h_1 << 9\, a\, \psi^2/2$, and that the grazing angle, ψ, is small; then, over a spherical earth of effective radius a,

$$\Delta r \cong 2\, h_1 \sin\psi \cong h_1\left[\sqrt{\theta_h^{\,2} + 4 h_1/(3a)} + \theta_h\right] \tag{5.12}$$

whether θ_h is positive or negative. For $\theta_h = 0$, $d_1 \cong 2\, h_1/(3\psi)$.

Two very useful approximations for Δr are

$$\Delta r \cong 2\, \psi^2\, d_1 d_2/d \cong 2\, h_1 \sin \psi \text{ kilometers} \tag{5.13}$$

and the corresponding expressions for the path length difference in electrical radians and in electrical degrees are

$$2\pi\Delta r/\lambda = 41.917\, f\, h_1' h_2'/d = 41.917\, f\, \psi^2\, d_1 d_2/d \cong 42\, f\, h_1 \sin\psi \quad \text{radians} \tag{5.14a}$$

$$360\Delta r/\lambda = 2401.7\, f\, h_1' h_2'/d = 2401.7\, f\, \psi^2\, d_1 d_2/d \cong 2402\, f\, h_1 \sin\psi \quad \text{degrees} \tag{5.14b}$$

where f is the radio frequency in MHz and all heights and distances are in kilometers. The last approximation in (5.13) should be used only if h_1 is small and less than $h_2/20$, as it involves neglecting $d_1^2/(2a)$ relative to h_1 in (5.8) and assuming that $d_2 \cong d$.

As noted following (5.5), ray optics formulas are limited to grazing angles such that $\Delta r > 0.06\lambda$. With this criterion, and assuming $R_e = 1$, the attenuation A is 15 dB for the corresponding minimum grazing angle

$$\psi_m \cong \sqrt{0.03\, \lambda\, d/(d_1 d_2)} \text{ radians}$$

where antennas are barely intervisible. A comparison with the CCIR Atlas of smooth-earth diffraction curves shows that the attenuation relative to free space varies from 10 to 20 decibels for a zero angular distance ($\theta = 0$, $\psi = 0$) except for extremely low antennas.

Figure 5.1a shows how rays will bend above an earth of actual radius a_o = 6370 kilometers, while figure 5.1b shows the same rays drawn as straight lines above an earth of effective radius a. Antenna heights above sea level, h_{ts} and h_{rs}, are usually slightly greater than the effective antenna heights h'_1 and h'_2, defined in 5.2.1. This difference arises from two circumstances: the smooth curve may be a curve-fit to the terrain instead of representing sea level, and straight rays above an effective earth overestimate the ray bending at high elevations. This latter correction is insignificant unless d is large.

5.2.1 A Curve-Fit to Terrain

A smooth curve is fitted to terrain visible from both antennas. It is used to define antenna heights h'_1 and h'_2, as well as to determine a single reflection point where the angle of incidence of a ray r_1 is equal to the angle of reflection of a ray r_2 in figure 5.1. This curve is also required to obtain the deviation, σ_h, of terrain heights used in computing R_e in (5.1). Experience has shown that both h'_1 and h'_2 should exceed 0.16λ for the following formulas to be applicable. One of the prediction methods listed in subsection 5.1 may be used where these formulas do not apply.

First, a straight line is fitted by least squares to equidistant heights $h_i(x_i)$ above sea level, and $x_i^2/(2a)$ is then subtracted to allow for the sea level curvature $1/a$ illustrated in figure 6.4. The following equation describes a straight line $h(x)$ fitted to 21 equidistant values of $h_i(x_i)$ for terrain between $x_i = x_0$ and $x_i = x_{20}$ kilometers from the transmitting antenna. The points x_0 and x_{20} are chosen to exclude terrain adjacent to either antenna which is not visible from the other:

$$h(x) = \bar{h} + m(x - \bar{x}) \tag{5.15a}$$

$$\bar{h} = \frac{1}{21}\sum_{i=0}^{20} h_i, \quad \bar{x} = \frac{x_0 + x_{20}}{2}, \quad m = \frac{2\sum_{i=0}^{20} h_i(i-10)}{77(x_{20} - x_0)}. \tag{5.15b}$$

Smooth modified terrain values given by

$$y(x) = h(x) - x^2/(2a) \tag{5.16}$$

will then define a curve of radius a which is extrapolated to include all values of x from $x = 0$ to $x = d$, the positions of the antennas.

The heights of the antennas above this curve are

$$h'_1 = h_{ts} - h(0), \qquad h'_2 = h_{rs} - h(d) \tag{5.17}$$

If h'_1 or h'_2 is greater than one kilometer, a correction term, Δh, defined by (6.12) and shown on figure 6.7 is used to reduce the value given by (5.17).

Where terrain is so irregular that it cannot be reasonably well approximated by a single curve, σ_h is large and $R_e = 0$, not because the terrain is very rough, but because it is irregular. In such a situation, method 3 of section 5.1 may be useful.

5.2.2 The terrain roughness factor, σ_h

The terrain roughness factor σ_h in (5.1) is the root-mean-square deviation of modified terrain elevations, y_i, relative to the smooth curve defined by (5.16), within the limits of the first Fresnel zone in the horizontal reflecting plane. The outline of a first Fresnel zone ellipse is determined by the condition that

$$r_{11} + r_{21} = r_1 + r_2 + \lambda/2$$

where $r_{11} + r_{21}$ is the length of a ray path corresponding to reflection from a point on the edge of the Fresnel zone, and $r_1 + r_2$ is the length of the reflected ray for which angles of incidence and reflection are equal. Norton and Omberg [1947] give general formulas for determining a first Fresnel zone ellipse in the reflecting plane. Formulas are given in annex III for calculating distances x_a and x_b from the transmitter to the two points where the first Fresnel ellipse cuts the great circle plane.

A particularly interesting application of some of the smooth-earth formulas given in this section is the work of Lewin [1962] and others in the design of space-diversity configurations to overcome phase interference fading over line-of-sight paths. Diffraction theory may be used to establish an optimum antenna height for protection against long-term power fading, choosing for instance the minimum height at which the attenuation below free space is 20 db for a horizontally uniform atmosphere with the maximum positive gradient of refractivity expected to be encountered. Then the formulas of this section will determine the optimum diversity spacing required to provide for at least one path a similar 20 db protection against multipath from direct and ground-reflected components throughout the entire range of refractivity gradients expected. In general, the refractive index gradient will vary over wider ranges on over-water paths [Ikegami, 1964].

5.3 Some Effects of Cluttered Terrain

The effects of refraction, diffraction, and absorption by trees, hills, and man-made obstacles are often important, especially if a receiving installation is low or is surrounded by obstacles. Absorption of radio energy is probably the least important of these three factors except in cases where the only path for radio energy is directly through some building material or where a radio path extends for a long distance through trees.

Studies made at 3000 MHz indicate that stone buildings and groups of trees so dense that the sky cannot be seen through them should be regarded as opaque objects around which diffraction takes place [McPetrie and Ford, 1946]. At 3000 MHz the loss through a 23-centimeter thick dry brick wall was 12 db and increased to 46 db when the wall was thoroughly soaked with water. A loss of 1.5 db through a dry sash window, and 3 db through a wet one were usual values.

The only objects encountered which showed a loss of less than 10 db at 3000 MHz were thin screens of leafless branches, the trunk of a single tree at a distance exceeding 30 meters, wood-framed windows, tile or slate roofs, and the sides of light wooden huts. Field strengths obtained when a thick belt of leafless trees is between transmitter and receiver are within about 6 db of those computed assuming Fresnel diffraction over an obstable slightly lower than the trees. Loss through a thin screen of small trees will rarely exceed 6 db if the transmitting antenna can be seen through their trunks. If sky can be seen through the trees, 15 db is the greatest expected loss.

The following empirical relationship for the rate of attenuation in woods has been given by Saxton and Lane [1955]:

$$A_w = d(0.244 \log f - 0.442) \text{ decibels}, \qquad (f > 100 \text{ MHz}) \qquad (5.18)$$

where A_w is the absorption in decibels through d meters of trees in full leaf at a frequency f megahertz.

The situation with a high and a low antenna in which the low antenna is located a small distance from and at a lower height than a thick stand of trees is quite different from the situation in which both antennas may be located in the woods. Recent studies at approximately 500 MHz show the depression of signal strengths below smooth earth values as a function of clearing depth, defined as the distance from the lower antenna to the edge of the woods [Head, 1960]. The following empirical relation is established:

$$\Delta_c = 52 - 12 \log d_c \text{ decibels} \qquad (5.19)$$

where Δ_c is the depression of the field strength level below smooth earth values and d_c is the clearing depth in meters.

5.4 Sample Calculation of Line-of-Sight Predictions

Attenuation relative to free space is predicted for a short line-of-sight path shown in figure 5.2. Measurements at a frequency of 100 MHz were made using vertical polarization. The transmitting and receiving antennas are 4 meters and 9 meters, respectively, above ground.

A straight line is fitted by least squares to the terrain visible from both antennas. Terrain near the transmitter is excluded because it is shadowed by high foreground terrain. Twenty-one equidistant points $x_i = x_0, x_1, \ldots x_{20}$ are chosen as shown on figure 5.2a, and the corresponding terrain heights, h_i, are read. From (5.15) the average terrain height $\bar{h}$ is 1531.8 m, the average distance $\bar{x}$ is 13.0 km, and the slope m is -6.0 meters per kilometer. The equation for the straight line is then

$$h(x) = 1531.8 - 6(x-13) \text{ m} = 1.5318 - 6(x-13) \cdot 10^{-3} \text{ km}.$$

An effective earth's radius, a, is obtained using figure 4.1 and equations (4.3) and (4.4). For this area in Colorado N_s is 280 and a = 8200 km. From (5.16) the adjustment to allow for the sea level curvature is

$$y(x) = h(x) - x^2/(16,400) \text{ km}.$$

Figure 5.2b shows the curve y(x) vs x and terrain which has been modified to allow for the sea level curvature.

At the transmitter, x = 0 and h(x = 0) is 1609.5 m. At the receiver, x = d = 19.75 km and h(x = 19.75) is 1491.4 m. From (5.17) the antenna heights above the smooth reflecting plane are then:

$$h_1' = h_{ts} - h(0) = 1647.1 - 1609.5 = 37.6 \text{ m} = 0.0376 \text{ km},$$

$$h_2' = h_{rs} - h(d) = 1524.0 - 1491.4 = 32.6 \text{ m} = 0.0326 \text{ km},$$

where $h_{ts} = 1647.1$ m and $h_{rs} = 1524.0$ m are the heights above sea level at the transmitter and receiver respectively. At 100 MHz ($\lambda = 3$ m), the criterion that both h_1' and h_2' must exceed $0.16\,\lambda$ is met. Neither h_1' nor h_2' exceeds one kilometer, so no correction factor Δh, is required. From (5.6) and (5.7) the distances d_1 and d_2 from each antenna to the point of specular reflection are

$$d_1 = d(1 + h_2'/h_1')^{-1} = 10.58 \text{ km}, \quad d_2 = d(1 + h_1'/h_2')^{-1} = 9.17 \text{ km},$$

and the grazing angle ψ is

$$\tan\psi = h_1'/d_1 = h_2'/d_2 = 0.003554$$

$$\psi = 0.003554 \text{ radians}.$$

From (5.9) the path length difference, Δr, between direct and reflected rays is

$$\Delta r = \left[d^2 + (h'_1 + h'_2)^2\right]^{\frac{1}{2}} - \left[d^2 + (h'_1 - h'_2)^2\right]^{\frac{1}{2}} = 1.2413 \times 10^{-4} \text{ km.}$$

The approximation

$$\Delta r \simeq 2h'_1 h'_2/d = 1.2413 \times 10^{-4} \text{ km} = 0.124 \text{ m} \cong 0.04\lambda$$

is also valid in this case. Note that Δr is less than 0.12λ and optical methods including a divergence factor may underestimate the attenuation.

One should note that important reflections might occur from the high ground near the transmitter. In this case the reflecting plane would correspond to the foreground terrain giving $h'_1 = 4$ m, $h'_2 = 50$ m, $d_1 = 1.53$ km, $d_2 = 18.22$ km and $\Delta r = 0.02$ m which is much less than 0.16λ. Optical methods would not be applicable here.

The attenuation relative to free space may be estimated using one of the methods described in subsection 5.1. Of these, methods 4 and 5 would apply in this case. Choosing heights $h_1 = 4$ m, $h_2 = 25$ m, as heights above foreground terrain, the theoretical smooth earth curves in the CCIR Atlas [1959] show the predicted field to be about 36 db below the free space value. The "standard" propagation curves, annex I, figure I.7, drawn for 100 MHz and $h_1 = h_2 = 30$ meters show the median basic transmission loss to be about 15 dB below the free space loss. Greater attenuation would be expected with lower antennas over irregular terrain. Method 5 using the empirical curve through data recorded at random locations, annex I, figure I.1 shows the attenuation to be about 20 dB below free space. These data were recorded with an average transmitter height of about 250 m, and a receiver height of 10 m.

For the very low antennas used on this Colorado path one would expect the losses to exceed the values shown on figures I.7 and I.1, and also to exceed the theoretical smooth earth value of $A \simeq 36$ db obtained from the CCIR Atlas. Spot measurements yield a value of about 40 db.

If a prediction were desired for transmission over the same path at 300 MHz, $\lambda = 1$ m, then $\Delta r = 0.124$ m is slightly greater than 0.12λ and optical methods could be used. Using the value $\Delta r = 0.124$ m the path length difference in electrical radians $2\pi\Delta r/\lambda = 0.7805$ radians. As a check, this quantity may be computed using (5.14a):

$$\begin{aligned} 2\pi\Delta r/\lambda = 41.917\, f h'_1 h'_2/d &= 0.7805 \text{ radians} \\ &\simeq 44.7 \text{ degrees.} \end{aligned}$$

Equation (5.4) shows the attenuation relative to free space assuming a single ground reflection from the smooth curve $y(x)$, figure 5.2b. Assuming that free space gains are realized so that $G_p = 10 \log g_{01} g_{02}$ the equation may be written

$$A = -10 \log\left[1 + R_e^2 - 2R_e \cos\left(\frac{2\pi\Delta r}{\lambda} - c\right)\right]$$

where R_e is the effective reflection coefficient defined by (5.1):

$$R_e = DR\left(\frac{g_{r1}\, g_{r2}}{g_{o1}\, g_{o2}}\right)^{\frac{1}{2}} \exp\left(\frac{-0.6\, \sigma_h \sin\psi}{\lambda}\right).$$

With $f = 300$ MHz, and $\tan\psi = 0.003554$, figure III.3, annex III shows the theoretical reflection coefficient $R = 0.97$ and the phase shift $c = 0$ for vertical polarization over average ground. The angle between the direct and the reflected ray is small so the ratio of gains in (5.1) may be considered to be unity. The divergence factor D and effective reflection coefficient R_e are

$$D = \left(1 + \frac{2d_1 d_2}{ad \tan\psi}\right)^{-\frac{1}{2}} = 0.865$$

$$R_e = 0.839 \exp\left(\frac{-0.6\, \sigma_h \sin\psi}{\lambda}\right).$$

The terrain roughness factor, σ_h, is the root-mean-square deviation of modified terrain elevations relative to the curve $y(x)$ within the limits of the first Fresnel zone in the horizontal reflecting plane. The first Fresnel ellipse cuts the great circle plane at two points x_a and x_b kilometers from the transmitter. The distances x_a and x_b may be computed using equations (III.18) or (III.19) to (III.21) of annex III,

$$B = 0.135, \quad x_0 = 10.02, \quad x_1 = 9.12$$

$$x_a = x_0 - x_1 = 0.90 \text{ km}, \quad x_b = x_0 + x_1 = 19.14 \text{ km}$$

The first Fresnel zone cuts the great circle plane at points 0.9 and 19.14 km from the transmitter with an intervening distance of 18.24 km. Equidistant points are chosen at $x = 1, 2, \ldots 19$ and corresponding modified terrain heights and values of $y(x)$ are obtained. With height differences in kilometers:

$$\sigma_h^2 = \sum_{j=1}^{19} (y_j - h_j)^2 / 19, \quad \sigma_h = 0.008222.$$

The effective reflection coefficient is then

$$R_e = 0.839 \exp - 0.01753 = 0.824$$

which is greater than 0.5 and greater than $\sqrt{\sin \psi}$. The predicted attenuation relative to free space A is then

$$- 10 \log \left[1 + R_e^2 - 2 R_e \cos \left(\frac{2\pi \Delta r}{\lambda} - c \right) \right] = - 10 \log \left[1.6793 - 1.6484 \cos 0.7805 \right] \simeq 3 \text{ db}.$$

Due to diffraction effects over irregular terrain, the attenuation A is often observed to be much greater than the values corresponding to the ray theory calculations illustrated in this example. Ray theory is most useful to identify the location and depth of nulls in an interference pattern in the region visible to two antennas. Figure 5.3 shows an interference pattern from an aircraft at 10,000 ft., transmitting on 328.2 MHz. Measured values compared with theoretical curves based on ray theory are shown on the figure.

GEOMETRY FOR WITHIN-THE-HORIZON PATHS

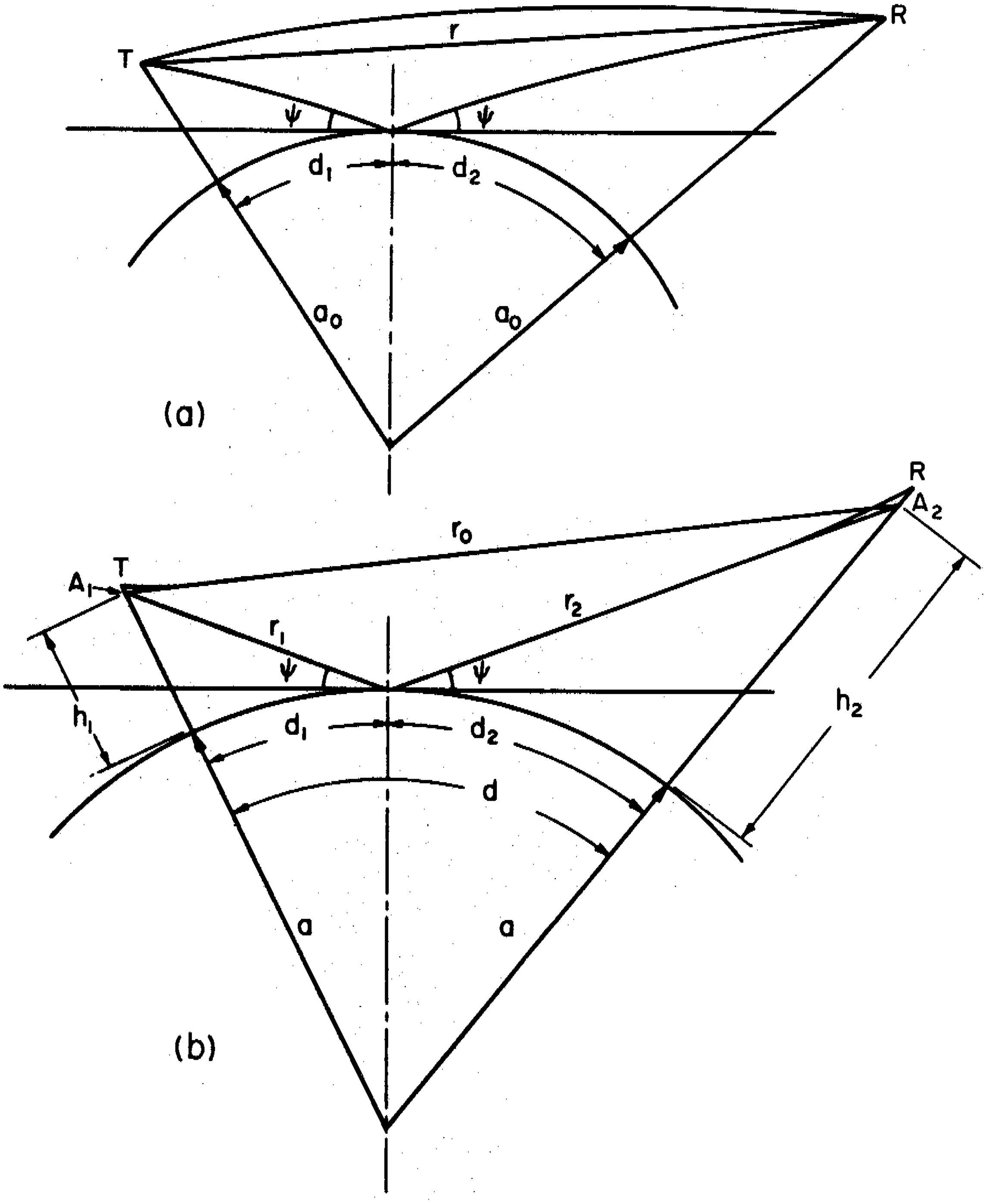

Figure 5.1

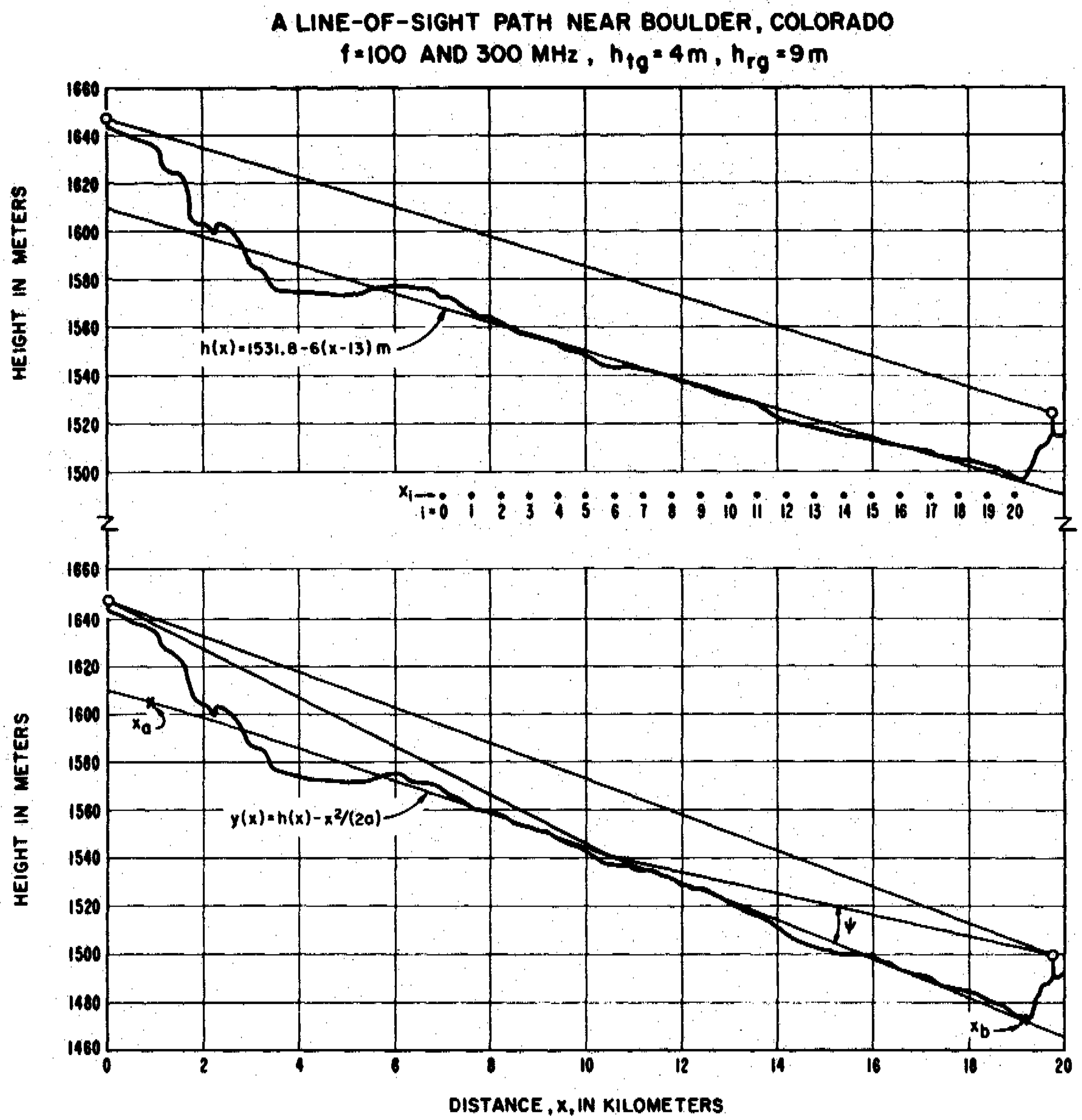
A LINE-OF-SIGHT PATH NEAR BOULDER, COLORADO
f=100 AND 300 MHz, h_{tg}=4m, h_{rg}=9m
HEIGHT IN METERS
h(x)=1531.8-6(x-13) m
x_i
i=0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
y(x)=h(x)-x²/(2a)
x_a
x_b
ψ
DISTANCE, x, IN KILOMETERS

Figure 5.2

OBSERVED INPUT VOLTAGE VARIATION AT GROUND STATION RECEIVER FROM AN AIRCRAFT AT 10,000 FEET TRANSMITTING ON 328.2 MHz

TRANSMITTER POWER: 6 WATTS; TRANSMITTING AND RECEIVING ANTENNA GAIN: 2.15 db (RELATIVE TO AN ISOTROPIC)
GROUND ANTENNA HEIGHT: 75 FEET; TRANSMISSION OVER WATER; 6 db COMMUNICATIONS SYSTEM LOSS ASSUMED FOR THEORETICAL CURVES

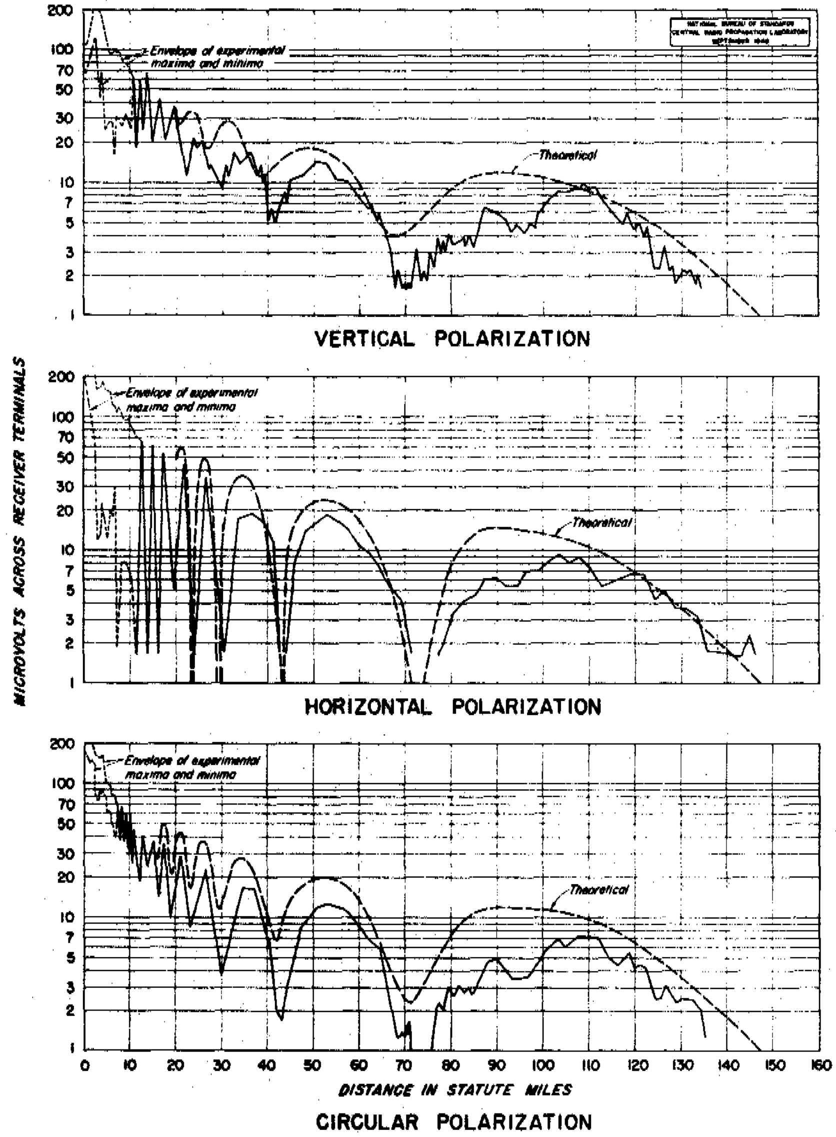

Figure 5.3