APPENDIX A: SUPPLEMENTAL MEASUREMENT RESULTS

This appendix provides additional results. The first section shows, for the purpose of discussing anomalies, usage data for each one-hour block during the course of the measurements. The remaining two sections provide mean usage within the receiver's spatial coverage for the two frequency bands.

A.1 Usage for Each One-hour Block During the Course of the Measurements

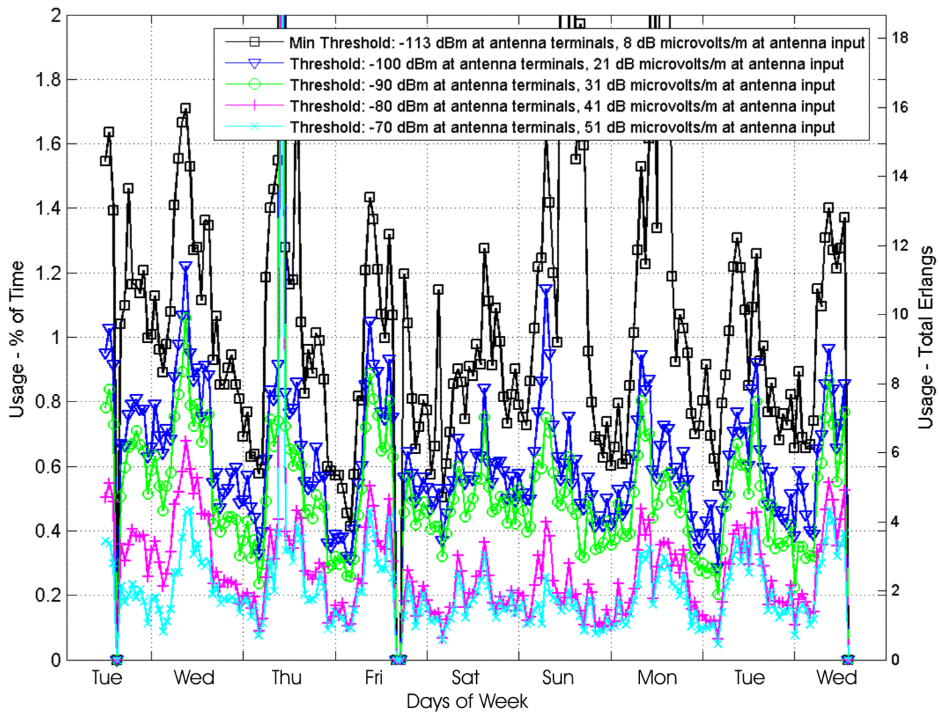

Figure A-1. *Hourly Band Usage* (percent of time and total Erlangs) during the course of the measurements for all 934 channels in the 162–174 MHz band (excluding HOCs).

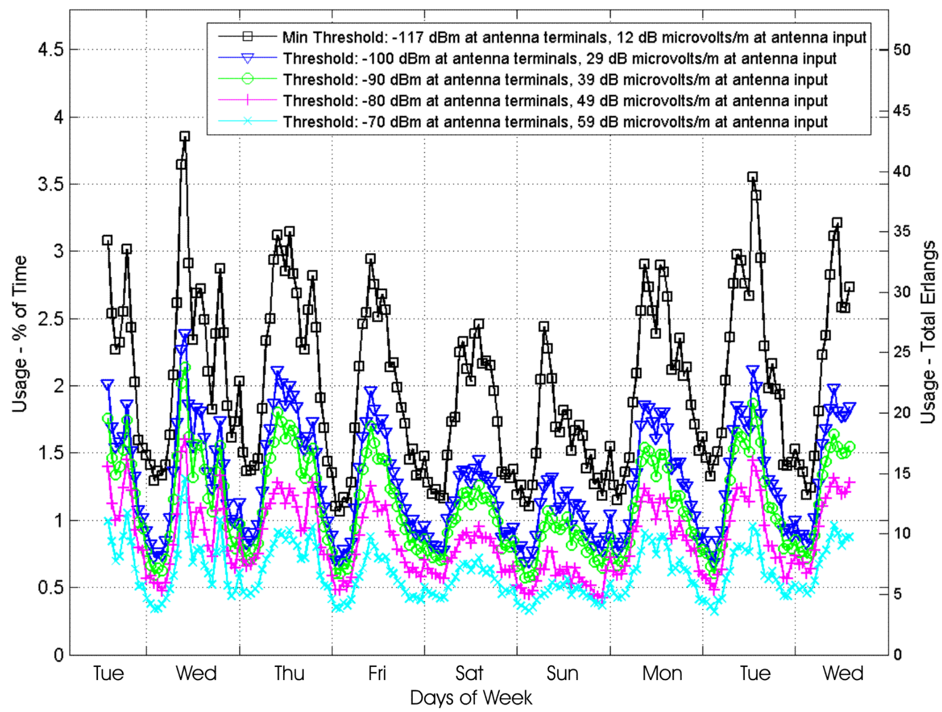

Figure A-2. *Hourly Band Usage* (percent of time and total Erlangs) during the course of the measurements for all 1113 channels in the 406–420 MHz band (excluding HOCs).

A.2 Percent Band Usage for the 162–174 MHz Band

Tables A-1 through A-4 contain the *Percent Band Usage* data for the 162–174 MHz band for the *entire week*, *weekdays* only, *weekends* only and *Election Day* only. As described in greater detail in Section 5.6, the data is presented in terms of a 24 hour period and a period of 8 AM to 5 PM for each table for that measurement time. Note that for the *entire week* and *weekdays only* tables, data is included for the *Average of Busiest Usage by Hour*. That information is not relevant to the *weekend* only and *Election Day* tables and is not included for those measurement times since the busiest times happen during the weekdays.

Table A-1. *Percent Band Usage* and *Average of Busiest Usage by Hour* During the Entire Week for All 934 Channels in the 162–174 MHz Band (Excluding HOCs)

All Days	**Mean Detection Threshold**				
Threshold (dBm)	-113	-100	-90	-80	-70
Threshold (dBμV/m)	8	21	31	41	51
Timescale	**Percent Band Usage**				
Percent Band Usage (24 hours)	1.08±0.01[17]	0.63	0.5	0.27	0.21
Average of Busiest Usage by Hour	2.72±0.08[17]	Not applicable			
Percent Band Usage (8am–5pm)	1.42±0.02[17]	0.76	0.6	0.37	0.3
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 10.09 ((1.08 / 100) * 934).					

[17] 99% confidence level - assuming an **average** message length of no greater than 5 seconds.

Table A-2. *Percent Band Usage* and *Average of Busiest Usage by Hour* During Weekdays Only for All 934 Channels in the 162–174 MHz Band (Excluding HOCs)

Weekdays Only	**Mean Detection Threshold**				
Threshold (dBm)	-113	-100	-90	-80	-70
Threshold (dBμV/m)	8	21	31	41	51
Timescale	**Percent Band Usage**				
Percent Band Usage (24 hours)	1.11±0.01[18]	0.63	0.52	0.29	0.22
Average of Busiest Usage by Hour	3.14±0.10[18]	Not applicable			
Percent Band Usage (8am–5pm)	1.53±0.02[18]	0.79	0.66	0.4	0.32
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 10.37 ((1.11 / 100) * 934).					

[18]99% confidence level - assuming an **average** message length of no greater than 5 seconds.

Table A-3. *Percent Band Usage* During *Weekend Days* for All 934 Channels in the 162–174 MHz Band (Excluding HOCs)

Weekend Days	**Mean Detection Threshold**				
Threshold (dBm)	-113	-100	-90	-80	-70
Threshold (dBµV/m)	8	21	31	41	51
Time Scale	**Percent Band Usage**				
Percent Band Usage (24 hrs)	1.16±0.02[19]	0.59	0.49	0.2	0.16
Percent Band Usage (8am–5pm)	1.62±0.04[19]	0.64	0.53	0.24	0.2
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 10.83 ((1.16 / 100) * 934).					

Table A-4. *Percent Band Usage* During *Election Day* for All 934 Channels in the 162–174 MHz Band (Excluding HOCs)

Election Day	**Mean Detection Threshold**				
Threshold (dBm)	-113	-100	-90	-80	-70
Threshold (dBµV/m)	8	21	31	41	51
Percent Band Usage (24 hours)	0.90±0.02[19]	0.56	0.45	0.25	0.22
Percent Band Usage(8am–5pm)	1.10±0.04[19]	0.72	0.62	0.37	0.33
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 8.41 ((0.9 / 100) * 934).					

[19] 99% confidence level - assuming an **average** message length of no greater than 5 seconds.

A.3 Percent Band Usage for the 406–420 MHz Band

Tables A-5 through A-8 contain the *Percent Band Usage* data for the 406–420 MHz band for the *entire week*, *weekdays* only, *weekends* only and *Election Day* only. As described in greater detail in Section 5.6, the data is presented in terms of a 24 hour period and a period of 8 AM to 5 PM for each table for that measurement time. Note that for the *entire week* and *weekdays* only tables, data is included for the *Average of Busiest Usage by Hour*. That information is not relevant to the *weekend* only and *Election Day* tables and is not included for those measurement times since the busiest times happen during the weekdays.

Table A-5. *Percent Band Usage* and *Average of Busiest Usage by Hour* During the Entire Week for All 1113 Channels in the 406–420 MHz Band (Excluding HOCs)

All Days	**Mean Detection Threshold**				
Threshold (dBm)	-117	-100	-90	-80	-70
Threshold (dBµV/m)	12	29	39	49	59
Timescale	**Percent Usage**				
Percent Band Usage (24 hours)	2.00±0.01[20]	1.3	1.1	0.84	0.6
Average of Busiest Usage by Hour	3.79±0.04[20]	Not applicable			
Percent Band Usage (8am–5pm)	2.55±0.01[20]	1.6	1.41	1.04	0.75
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 22.26 ((2.0 / 100) * 1113).					

[20] 99% confidence level - assuming an **average** message length of no greater than 5 seconds.

Table A-6. *Percent Band Usage* and *Average of Busiest Usage by Hour* During Weekdays for All 1113 Channels in the 406–420 MHz Band (Excluding HOCs)

Weekdays Only	**Mean Detection Threshold**				
Threshold (dBm)	-117	-100	-90	-80	-70
Threshold (dBμV/m)	12	29	39	49	59
Timescale	**Percent Usage**				
Percent Band Usage (24 hours)	2.08±0.01[21]	1.3	1.14	0.89	0.63
Average of Busiest Usage by Hour	4.29±0.05[21]	Not applicable			
Percent Band Usage (8am–5pm)	2.67±0.01[21]	1.7	1.49	1.1	0.79
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 23.15 ((2.08 / 100) * 1113).					

[21] 99% confidence level - assuming an **average** message length of no greater than 5 seconds.

Table A-7. *Percent Band Usage* During *Weekend* Days for All 1113 Channels in the 406–420 MHz Band (Excluding HOCs)

Weekend Days	**Mean Detection Threshold**				
Threshold (dBm)	-117	-100	-90	-80	-70
Threshold (dBµV/m)	12	29	39	49	59
Time Scale	**Percent Band Usage**				
Percent Band Usage (24 hours)	1.76±0.01[22]	1.11	0.96	0.72	0.52
Percent Band Usage (8am–5pm)	2.19±0.02[22]	1.37	1.18	0.87	0.63
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 19.59 ((1.76 / 100) * 1113).					

Table A-8. *Percent Band Usage* During *Election* Day for All 1113 Channels in the 406–420 MHz Band (Excluding HOCs)

Election Day	**Mean Detection Threshold**				
Threshold (dBm)	-117	-100	-90	-80	-70
Threshold (dBµV/m)	12	29	39	49	59
Time Scale	**Percent Band Usage**				
Percent Band Usage (24 hours)	2.17±0.02[22]	1.3	1.15	0.88	0.59
Percent Band Usage (8am–5pm)	2.92±0.03[22]	1.79	1.59	1.21	0.79
Note: Erlangs can be calculated by multiplying the Percent Band Usage for any threshold by the number of channels. For example, Mean Erlangs for a threshold of -113 dBm is 24.15 ((2.17 / 100) * 1113).					

[22] 99% confidence level - assuming an **average** message length of no greater than 5 seconds.

APPENDIX B: CONFIDENCE INTERVALS FOR OCCUPANCY MEASUREMENTS

B.1 Introduction

This appendix describes a method for calculating confidence intervals for channel occupancy measurements. Typically, a channel occupancy measurement involves random samples of the signal strength. The channel is deemed occupied if a predetermined threshold is exceeded and hence each measurement is a realization of a binary random variable. The probability of exceeding the threshold gives the fraction of time the channel is occupied.

B.2 Statistical Character of Channel Occupancy Measurements

We wish to estimate the probability p that given a random observation, the signal strength exceeds our predetermined threshold. Let the binary random variable ξ_i represent the i^{th} observation, then

$$\xi_i = \begin{cases} 1 & \text{with prob, } p \\ 0 & \text{with prob, } q = 1 - p \ . \end{cases}$$

To estimate channel occupancy, we make n observations and obtain a realization of the random variable

$$\upsilon = \sum_{i=1}^{n} \xi_i \tag{1}$$

and estimate of channel occupancy $\hat{p} = \upsilon/n$. Note that this estimate has the desirable property of being unbiased (i.e. $\mathcal{E}\{\hat{p}\} = p$).

To further understand the statistical nature of our estimate it is useful to determine its probability distribution. Some simplifying assumptions are needed to make this exercise tractable as described below.

B.2.1 Characterization of the Process

The sequence of observations is a realization of a discrete random process. We assume that in general the samples are not independent and are reasonably characterized as a regular first-order Markov chain as discussed in [1].

For our purposes, the regular Markov chain is characterized by a 2 x 2 transition matrix **P** and the relation

$$(q,p)\mathbf{P} = (q,p). \tag{2}$$

In terms of p, q and the transition probability $\eta = P\{\xi_i = 0 \mid \xi_{i-1} = 1\}$ we have

$$\mathbf{P} = \begin{pmatrix} 1 - \eta p/q & \eta p/q \\ \eta & 1 - \eta \end{pmatrix}.$$

When n is large, we can use the Central Limit Theorem for Markov Chains [2], which gives the limiting normal distribution for υ

$$P\{r < \frac{\upsilon - np}{\sqrt{n\beta}} < s\} \rightarrow \frac{1}{\sqrt{2\pi}} \int_r^s e^{-x^2/2}\, dx \tag{3}$$

where $n\beta$ is the *limiting variance* for the number of times that $\xi_i = 1$ and

$$\beta = pqL \tag{4}$$

where

$$L \approx \left(\frac{1 + \lambda}{1 - \lambda} \right)$$

and $\lambda = 1 - \eta = P\{\xi_i = 1 \mid \xi_{i-1} = 1\}$ and $p << 1$.

The limiting distribution of the channel occupancy estimate is

$$P\{r < \frac{\hat{p} - p}{\sqrt{\beta/n}} < s\} \rightarrow \frac{1}{\sqrt{2\pi}} \int_r^s e^{-x^2/2}\, dx. \tag{5}$$

B.2.2 Confidence Intervals

Since our measurement is an estimate, we would like to make some intelligent remarks about its accuracy. A commonly used methodology is to calculate the endpoints of an interval that with probability $1 - \epsilon$ contains the actual value of the population statistic. Thus, given a small quantity ϵ we need to calculate r and s so that

$$P\{p + r\sqrt{\beta/n} \le \hat{p} \le p + s\sqrt{\beta/n}\} = 1 - \epsilon, \tag{6}$$

where it is customary to set $s = -r = \gamma_\epsilon$. By writing $\hat{p} = p \pm \gamma_\epsilon\sqrt{pqL/n}$ and solving for p, it can be shown that the probability that $\hat{p}$ lies between the limits $p \pm \gamma_\epsilon\sqrt{pqL/n}$ is equivalent to the probability that p lies between the limits

$$\left(1 + \frac{\gamma_\epsilon^2 L}{n}\right)^{-1}\left(\hat{p} + \frac{\gamma_\epsilon^2 L}{2n} \pm \gamma_\epsilon\sqrt{\frac{\hat{p}\hat{q}L}{n} + \frac{\gamma_\epsilon^2 L^2}{4n^2}}\right) \tag{7}$$

as described in [3]. For large n, we can use the normal distribution given in Equation 5 to calculate γ_ϵ

$$\frac{1}{\sqrt{2\pi}}\int_{\gamma_\epsilon}^{\infty} e^{-x^2/2}\,\mathrm{d}x = \frac{\epsilon}{2}. \tag{8}$$

Note that the determination of L in Equation 7 requires that we know the transition probability λ. A method for estimating λ is given in [1]. This method assumes that the time duration T of a transmitted signal is random with an exponential distribution. Denoting the mean duration as t_0 and the time between samples as τ, the following expression is used to estimate the transition probability:

$$\lambda \approx P\{T \ge \tau\} = \frac{1}{t_0}\int_{\tau}^{\infty} e^{-t/t_0}\,\mathrm{d}t = e^{-\tau/t_0}. \tag{9}$$

B.2.3 Example

Assume we obtain $n = 4000$ samples and observe that $\hat{p} = 0.02$ using a sample rate that is twice the average time duration of the transmitted signal $(\tau = t_0/2)$. For the 90% confidence level we have $\epsilon = 0.1$ and using Equation 8, $\gamma_\epsilon = 1.64$. From Equation 9, $\lambda = 0.6$ and $L = 4$. Substituting into Equation 7 gives an upper limit of 0.0286 and a lower limit of 0.0139.

B.3 Average Occupancy for a Band of Channels

Previous sections have addressed calculating the occupancy of a single channel. We now turn our attention to calculating the average occupancy of a group or band of N channels, the calculation of which is described as follows

$$\bar{p} = \frac{1}{N}\sum_{i=1}^{N} \hat{p}_i \tag{10}$$

where $\hat{p}_i$ is the probability of occupancy of each channel as described above. When the number of measurements for the i^{th} channel n_i is large, $\hat{p}_i$ is approximately normal with variance $p_i q_i L_i / n_i$.

Assuming the individual channels are independent, $\bar{p}$ is approximately normal with variance

$$\sigma^2 = \frac{1}{N^2}\sum_{i=1}^{N} \frac{p_i q_i L_i}{n_i}\,. \tag{11}$$

Since we have sampled from several different populations, it is difficult to obtain exact confidence intervals. However, for large n_i the observed values of the i^{th} channel statistics

$(\hat{p}_i, \hat{q}_i = 1 - \hat{p}_i, \text{and } \hat{L}_i \text{ from estimates of } \lambda_i)$ can be used to approximate the variance

$$\sigma^2 \approx \frac{1}{N^2}\sum_{i=1}^{N} \frac{\hat{p}_i \hat{q}_i \hat{L}_i}{n_i}\,. \tag{12}$$

We can then say with $(1 - \epsilon)100\%$ confidence that the mean probability of occupancy over a band is between the following limits

$$\bar{p} \pm \gamma_\epsilon \sigma \tag{13}$$

where γ_ϵ is calculated as before and σ is calculated using Equation 12.

B.4 References

[1] A.D. Spaulding, and G.H. Hagn, “On the definition and estimation of spectrum occupancy,” *IEEE Trans. on EMC*, vol. 19, no. 3, pp. 269-280, Aug. 1977.

[2] J.G. Kemeny, and J.L. Snell (1960), *Finite Markov Chains*, Princeton, New Jersey: D. Van Nostrand Company, Inc., 1960, p. 89.

[3] H. Cramer, *Mathematical Methods of Statistics*, Princeton, New Jersy: Princeton University Press, 1946, pp. 514-515.

FORM **NTIA-29** (4-80)		U.S. DEPARTMENT OF COMMERCE NAT'L. TELECOMMUNICATIONS AND INFORMATION ADMINISTRATION

BIBLIOGRAPHIC DATA SHEET

1. PUBLICATION NO.	2. Government Accession No.	3. Recipient's Accession No.
TR-07-448		

4. TITLE AND SUBTITLE	5. Publication Date
Measurements to Characterize Land Mobile Channel Occupancy for Federal Bands 162-174 MHz and 406-420 MHz in the Washington, D.C., Area	July 2007
	6. Performing Organization Code NTIA/ITS
7. AUTHOR(S) J. Randy Hoffman, Robert J. Matheson, and Roger A. Dalke	9. Project/Task/Work Unit No.
8. PERFORMING ORGANIZATION NAME AND ADDRESS NTIA/ITS.M U.S. Department of Commerce 325 Broadway Boulder, CO 80305	10. Contract/Grant No.
11. Sponsoring Organization Name and Address	12. Type of Report and Period Covered

14. SUPPLEMENTARY NOTES

15. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

This report describes field measurements to characterize Land Mobile Radio (LMR) channel occupancy of Federal bands 162–174 MHz and 406–420 MHz at a single central location in the Washington, D.C., area. This is part of the National Telecommunications and Information Administration effort to evaluate the spectrum efficiency in the Federal frequency bands. Measurements of the received signal levels in LMR frequency bands 162–174 MHz and 406–420 MHz were performed over an eight day period for the purpose of determining channel usage within the measurement system's coverage area of approximately 100-km radius for base stations, 50-km radius for mobile units, and 25-km radius for portable units. The measurements were made using new equipment and techniques that digitize as much as a 5-MHz segment of spectrum and process it to obtain simultaneous signal levels of up to 400 individual LMR channels. These techniques provided faster measurements, but also allowed enhanced post-processing of the data to remove effects of impulsive noise.

16. Key Words (Alphabetical order, separated by semicolons)

channel occupancy; channel usage; Federal radio usage; Land Mobile Radio (LMR); measurements; spectrum efficiency

17. AVAILABILITY STATEMENT	18. Security Class. (This report)	20. Number of pages
UNLIMITED.		81
	19. Security Class. (This page)	21. Price:

NTIA FORMAL PUBLICATION SERIES

NTIA MONOGRAPH (MG)

A scholarly, professionally oriented publication dealing with state-of-the-art research or an authoritative treatment of a broad area. Expected to have long-lasting value.

NTIA SPECIAL PUBLICATION (SP)

Conference proceedings, bibliographies, selected speeches, course and instructional materials, directories, and major studies mandated by Congress.

NTIA REPORT (TR)

Important contributions to existing knowledge of less breadth than a monograph, such as results of completed projects and major activities. Subsets of this series include:

NTIA RESTRICTED REPORT (RR)

Contributions that are limited in distribution because of national security classification or Departmental constraints.

NTIA CONTRACTOR REPORT (CR)

Information generated under an NTIA contract or grant, written by the contractor, and considered an important contribution to existing knowledge.

JOINT NTIA/OTHER-AGENCY REPORT (JR)

This report receives both local NTIA and other agency review. Both agencies' logos and report series numbering appear on the cover.

NTIA SOFTWARE & DATA PRODUCTS (SD)

Software such as programs, test data, and sound/video files. This series can be used to transfer technology to U.S. industry.

NTIA HANDBOOK (HB)

Information pertaining to technical procedures, reference and data guides, and formal user's manuals that are expected to be pertinent for a long time.

NTIA TECHNICAL MEMORANDUM (TM)

Technical information typically of less breadth than an NTIA Report. The series includes data, preliminary project results, and information for a specific, limited audience.

For information about NTIA publications, contact the NTIA/ITS Technical Publications Office at 325 Broadway, Boulder, CO, 80305 Tel. (303) 497-3572 or e-mail info@its.bldrdoc.gov.

This report is for sale by the National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22161,Tel. (800) 553-6847.