3. ELIMINATION OF CHANGEOVER VALUES

Numerical evaluations of (28) are accomplished by expanding the exponential into a series of terms involving repeated integrals of the error function with arguments β_m. The series forms the basis for a computational algorithm to calculate E/E_0 for arbitrary inputs of knife-edge heights and separation distances and is quite satisfactory for positive or zero β. For negative β the series is still suitable as long as $|\beta|$ is small; however, significant figure loss becomes a problem as any or all β's increase negatively. Here, and in what follows, we shall use the term "negative β" to mean that the angle θ in (21) has a negative value.

The original MKE attenuation computer program calculates attenuation using predetermined, critical values of β. If a knife-edge is sufficiently below the adjacent knife-edges, this results in a negative β that is less than the critical or "changeover" value. That knife-edge is then eliminated and the attenuation is calculated for the remaining knife-edges. The abrupt switch from N to N-1 knife-edges at the changeover value produces a discontinuity in attenuation which, although usually small, prevents a detailed knowledge of variation with height.

Because only negative β's cause problems in the computational procedure, it is of interest to see if the multiple integral can be reformulated into an expression containing only positive (or zero) β's. This may be accomplished in the following manner.

It is assumed that the μ^{th} knife-edge, of height h_μ, is low enough to result in a negative β, β_μ. Then the multiple integral in (19) can be partitioned into

$$\int_{h_1}^{\infty} \cdots \int_{h_\mu}^{\infty} \cdots \int_{h_N}^{\infty} \rightarrow \int_{h_1}^{\infty} \cdots \int_{-\infty}^{\infty} \cdots \int_{h_N}^{\infty} - \int_{h_1}^{\infty} \cdots \int_{-\infty}^{h_\mu} \cdots \int_{h_N}^{\infty} , \tag{29}$$

where, for notational purposes, the first multiple integral on the right hand side will be denoted by I_1 and the second by I_2.

Restricting our attention to I_1, we can write $\hat{F}_N$ from (5) as

$$\hat{F}_N = \left(\frac{ik}{2}\right)\left[\frac{(h_0-X_1)^2}{r_1} + \ldots + \frac{(X_{\mu-1}-X_{\mu+1})^2}{r_\mu+r_{\mu+1}} + \ldots + \frac{(X_N - h_{N+1})^2}{r_{N+1}} + \frac{(X_{\mu-1}-X_\mu)^2}{r_\mu} + \frac{(X_\mu-X_{\mu+1})^2}{r_{\mu+1}} - \frac{(X_{\mu-1}-X_{\mu+1})^2}{r_\mu+r_{\mu+1}}\right]$$

$$\equiv \left(\frac{ik}{2}\right) F'_{N-1} + \Omega_\mu , \tag{30}$$

$$\Omega_\mu = \frac{(X_{\mu-1}-X_\mu)^2}{r_\mu} + \frac{(X_\mu - X_{\mu+1})^2}{r_{\mu+1}} - \frac{(X_{\mu-1}-X_{\mu+1})^2}{r_\mu + r_{\mu+1}}$$

$$= \frac{X_\mu{}^2}{\rho_\mu{}^2} - 2\left(\frac{X_{\mu-1}}{r_\mu} + \frac{X_{\mu+1}}{r_{\mu+1}}\right)X_\mu + \frac{X^2_{\mu-1}}{r_\mu} + \frac{X^2_{\mu+1}}{r_{\mu+1}} - \frac{(X_{\mu-1}-X_{\mu+1})^2}{r_\mu + r_{\mu+1}}$$

$$= \left\{\frac{X_\mu}{\rho_\mu} - \left(\frac{X_{\mu-1}}{r_\mu} + \frac{X_{\mu+1}}{r_{\mu+1}}\right)\rho_\mu\right\}^2 . \tag{31}$$

Now make the change of variable

$$v_0 = \left(\frac{ik}{2}\right)^{1/2}\left\{\frac{X_\mu}{\rho_\mu} - \left(\frac{X_{\mu-1}}{r_\mu} + \frac{X_{\mu+1}}{r_{\mu+1}}\right)\rho_\mu\right\},\ dv_0 = \frac{(ik/2)^{1/2}}{\rho_\mu}\ dX_\mu, \tag{32}$$

and I_1 becomes

$$I_1 = \left(\frac{ik}{2\pi}\right)^{N/2}\left(\frac{C_N e^{\sigma''_N}}{\rho_1 \cdots \rho_N}\right)\left\{\left(\frac{2}{ik}\right)^{1/2} \rho_\mu \int_{-\infty}^{\infty} e^{-v_0^2} dv_0\right\} I'_{N-1}$$

$$= (ik/2\pi)^{(N-1)/2}(\rho_\mu C_N/\rho_1 \cdots \rho_N)e^{\sigma''_N} I'_{N-1} , \tag{33}$$

$$\text{where } I'_{N-1} = \int_{h_1}^{\infty} \cdots \int_{h_N}^{\infty} e^{-(ik/2)F'_{N-1}}\ dX_1 \cdots dX_N , \tag{34}$$

with the understanding that the integral with respect to X_μ has been deleted in (34).

Because of the relationship $(C_N/\rho_1 \cdots \rho_N) = (R_{N+1}/r_1 \cdots r_{N+1})^{1/2}$, it can be seen that (33) is identical to the expression for MKE attenuation (19) over the path if the

μ^{th} knife-edge were absent and an appropriate separation distance of $r_\mu + r_{\mu+1}$ were used. Thus, I_1 can be evaluated as a path with N-1 knife-edges and with the negative β_μ no longer present. The preceding argument will hold for $1 \le \mu \le N$ if we assume, for notational convenience, that $X_0 \equiv h_0$ and $X_{N+1} \equiv h_{N+1}$.

Returning to the second multiple integral in (29), we can apply the change of variables given by (22) to arrive at

$$I_2 = K\int_{\beta_1}^{\infty} \cdots \int_{-\infty}^{\beta_\mu} \cdots \int_{\beta_N}^{\infty} e^{-F_N}\, dx_1 \ldots dx_N$$

$$= K\int_{\beta_1}^{\infty} \cdots \int_{-\beta_\mu}^{\infty} \cdots \int_{\beta_N}^{\infty} e^{-F'_N}\, dx_1 \ldots dx_N\ , \qquad (35)$$

$$\text{where } K = (1/\pi)^{N/2} C_N e^{\sigma_N - \sigma'_N}\ , \qquad (36)$$

and F'_N is the same as F_N given by (25) except for the linear order terms involving x_μ: the factor $(x_\mu - \beta_\mu)$ is replaced by $(-x_\mu - \beta_\mu) = -(x_\mu + \beta_\mu)$. Since β_μ is negative, the multiple integral in (35) can now be expanded into a series containing repeated integrals of the error function (all with positive arguments) in exactly the same manner as the original computational algorithm. A negative sign, arising from the x_μ integration, will be associated with some of the terms, but the arguments of positive β assure satisfactory convergence.

The partitioning procedure described above can be applied sequentially until no negative β's remain. Thus, any combination of knife-edge heights can be decreased to $-\infty$ and the variation of attenuation studied. An algorithm utilizing partitioning will eliminate the sudden discontinuities of attenuation that occur in the original MKE computer program; however, a disadvantage is that increased computer time is sometimes necessary because of the extra number of multiple integrals that must be evaluated. Of course if all β's are positive, there is no difference in running time. Furthermore, in the basic series used to calculate the attenuation, fewer terms are necessary to assure a given accuracy if all β's are positive rather than some being negative. Thus, partitioning, which guarantees positive β's, will have more multiple integrals but each one will need fewer terms for evaluation. For this reason partitioning when N<7 is often more advantageous as far as computer running time is concerned.

The partition method has been incorporated into a computer program that calculates MKE attenuation for inputs of up to 10 knife-edges. This unpublished program (designated PAMKE) allows detailed study of attenuation even in the interference regions. Comparisons of the output from PAMKE with attenuations as calculated from the original computer program (also unpublished and designated FAMKE) that made use of changeover values are shown in the following examples.

A propagation path consisting of four knife-edges arranged as shown in Figure 2 is assumed. It should be remembered that the knife-edge heights are measured from an arbitrary reference level and that it is their relative heights that are of interest. The MKE attenuation, (E/E_0) in dB, as a function of receiver height, h_R in meters, is plotted for both PAMKE (with partitions) and FAMKE (with changeover values). As the receiver rises from the shadow region into line-of-sight, the solid curve (PAMKE) shows small oscillations caused by interference from the intervening knife-edges. At larger heights, beyond the range of the figure, the curve eventually levels off at A(dB)=0 as it should.

The output from FAMKE, represented by the dashed lines, show the discontinuities in attenuation that occur when a changeover point is reached and a knife-edge is eliminated. A changeover value of $\beta_{min} = -1$ was selected for this example. The attenuations from the two programs are the same until h_R = 20m, at which point the fourth knife-edge (h_4 = 10m) is eliminated resulting in $\sim$0.5dB difference in the outputs of the two programs. The FAMKE curve is not shown from h_R = 25m to h_R = 47m because it is practically the same as the solid curve. At h_R = 47m the third knife-edge is eliminated, and at h_R = 67m the second is eliminated. The effective number of knife-edges, N_{eff}, is the number used by FAMKE over each portion of the curve. At no point is the difference in attenuations greater than 1dB.

Another example giving a comparison of the two algorithms is shown in Figure 3. In this case a propagation path consisting of five evenly spaced knife-edges (separation distances = 2km) is assumed. The heights are varied simultaneously such that the diffraction angles θ at all of the knife-edges are equal in value. A frequency, f=1908.538 MHz, was assumed, this value being chosen simply for computational convenience.

When clusters of negative θ's occur, it is sometimes difficult to determine changeover values that will assure enough significant figures being retained in the calculations entering into FAMKE. For this reason a test on the convergence of the series is also included, and if the test fails, the knife-edge with the "most negative" β is eliminated.

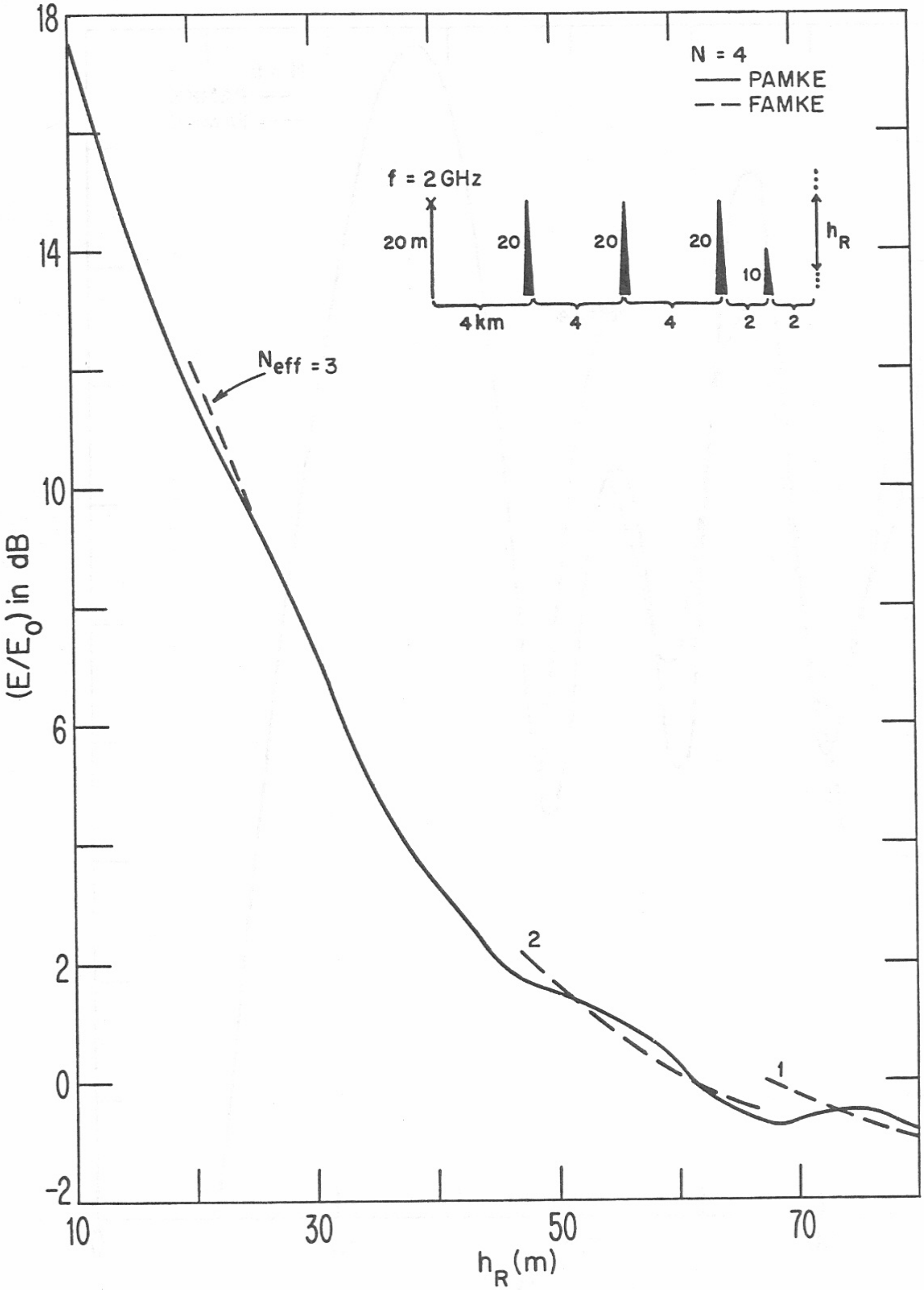

Figure 2. Comparison of attenuation by programs FAMKE and PAMKE for a four knife-edge path as receiver height is varied.

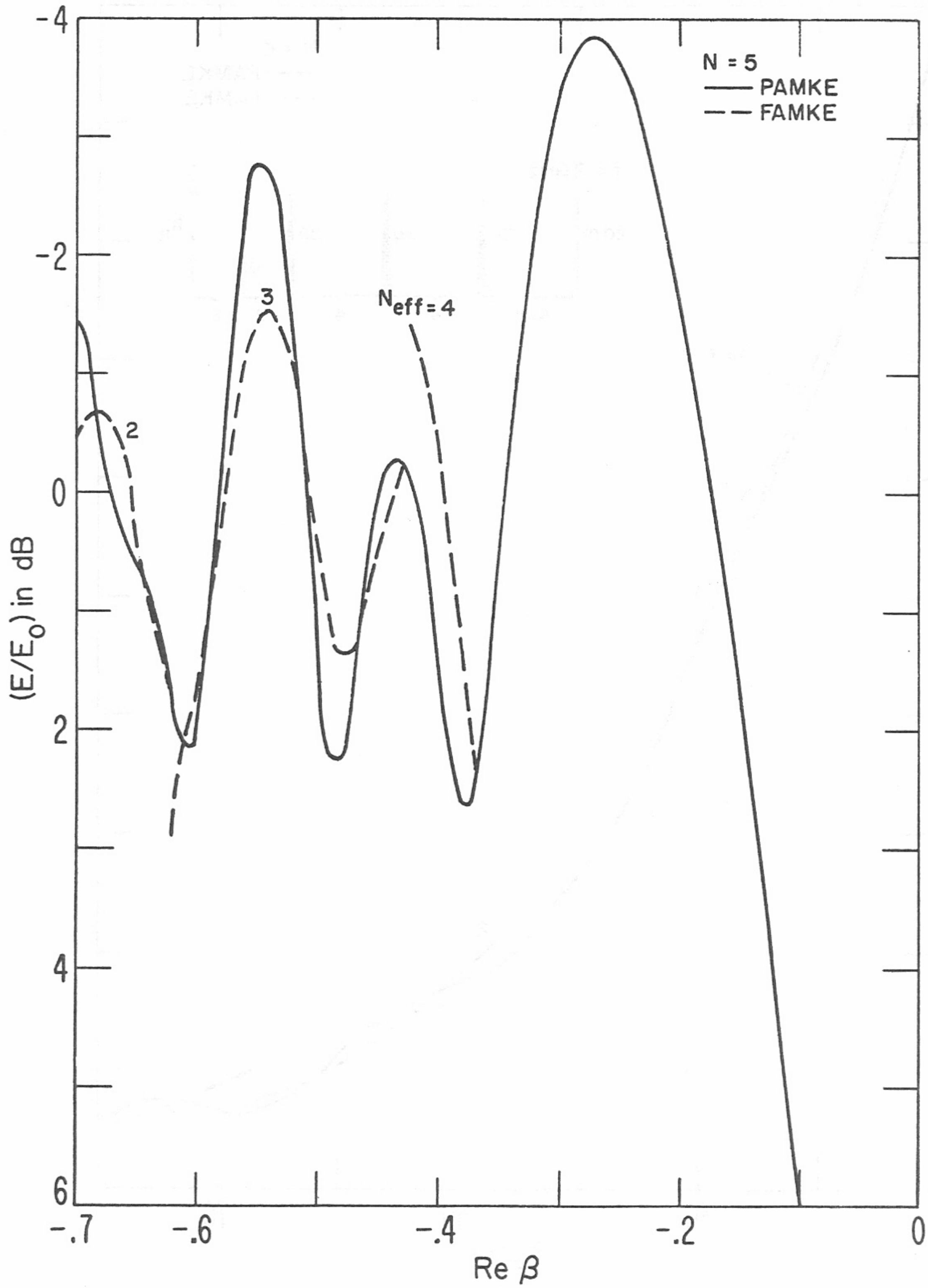

Figure 3. Comparison of attenuation by programs FAMKE and PAMKE for a five knife-edge path in which all θ's are equal.

In Figure 3 the outputs of PAMKE and FAMKE are the same for Reβ >-0.37. At this point the convergence test fails and FAMKE then calculates the attenuation for one less knife-edge (the portion of the dashed curve labeled $N_{eff} = 4$). Similar failures occur at -0.42 and -0.62, resulting in the discontinuities at these values. It can be seen that differences in attenuation from the exact curve as calculated by PAMKE are always less than 1.5 dB.

The partitioning procedure described in this section provides a means of calculating the MKE attenuation function (equation (28)) for any knife-edge heights without concern for the problems of discontinuities or significant figure loss. Details of attenuation variation even in the interference region can thus be shown for any combination of knife-edges.

4. DISCUSSION

The derivation of the MKE function by Fresnel-Kirchhoff theory as used in this paper implies certain restrictive conditions regarding the physics of the problem, e.g., perfectly absorbing half-screens, plane wave propagation, kr>>1, and path difference approximations. However, as a mathematical function, its numerical evaluation may be accomplished even when the geometric parameters (h and r) tend to unrealistic physical values. For instance the use of partitioning provides valid answers for any knife-edge heights subject only to computer limitations. With regard to separation distances, the limitations are more inherent in the computational algorithm.

Mathematically, the MKE function can be evaluated for separation distances ranging from zero to infinity. This is apparent when closed form solutions of the function are available. For example, Vogler (1982) has derived the attenuation for three knife-edges under the condition $\theta_1=\theta_2=\theta_3=0$:

$$E/E_0 = (1/8)[1+(2/\pi)\ \tan^{-1}a_1 + \tan^{-1}a_2 + \tan^{-1}a_3\ ], \tag{37}$$

$$a_1 = [r_1(r_3+r_4)/r_2r_t]^{1/2},\ a_2 = [(r_1+r_2)r_4/r_3r_t]^{1/2}\ ,$$

$$a_3 = [r_1r_4/(r_2+r_3)r_t]^{1/2}\ ,\ r_t = r_1+r_2+r_3+r_4\ .$$

For r_2 or $r_3 = 0$, (37) reduces to the known expression for a double knife-edge. For r_2 and $r_3 = 0$, $(E/E_0)=1/2$ which is the value for a single knife-edge.