FURTHER INVESTIGATIONS OF THE MULTIPLE KNIFE-EDGE ATTENUATION FUNCTION

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The multiple knife-edge (MKE) attenuation function is derived from Fresnel-Kirchhoff theory and compared with the original derivation from Furutsu's generalized residue series. It is shown that the two methods give complex attenuations with the same absolute magnitude but differing in phase. The analytical basis for an improved computational procedure is developed that eliminates the changeover values and abrupt discontinuities of attenuation inherent in the original MKE computer program. A brief discussion of previous MKE diffraction results is presented and an example comparison is made with the Geometrical Theory of Diffraction and the approximations of Epstein-Peterson and Deygout.

Key words: attenuation calculations; electromagnetic wave propagation; multiple knife-edge diffraction

1. INTRODUCTION

The purpose of this paper is to present further developments concerning the multiple knife-edge (MKE) attenuation function that has been derived recently by Vogler (1981, 1982). The MKE function is the basis for a computer program that calculates attenuation over a propagation path that may consist of up to 10 knife-edges.

The original derivation started from a generalized residue series for diffraction over a sequence of rounded obstacles developed by Furutsu (1963). It is not at all obvious that the MKE function derived in this manner is the same as one derived from Fresnel-Kirchhoff theory such as was used by Millington et al. (1962) for double knife-edge diffraction. In fact, the complex attenuations of the two methods differ in phase although their absolute values are equal. In Section 2 the MKE function will be derived from Fresnel-Kirchhoff theory and an explicit expression for the phase difference factor will be given.

Another aspect to be discussed in the present work concerns the numerical evaluation of the MKE function. Attenuation values are obtained from a series whose terms involve repeated integrals of the error function. As noted in the original

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paper (Vogler, 1981), certain limitations on the series could cause an abrupt discontinuity in the attenuation as the height of a knife-edge was decreased. In Section 3 an analysis of the problem is presented which leads to a means of eliminating the discontinuities.

2. THE FRESNEL-KIRCHHOFF DERIVATION

The geometry associated with the knife-edge problem is shown in Figure 1. It is assumed that the knife-edges are perfectly absorbing screens placed normal to the direction of propagation and extending to infinity in both horizontal directions and vertically downwards. For a path consisting of N knife-edges and two antenna terminals with heights h_m (above a reference plane) and with separation distances r_m, the diffraction angles θ, for θ_m small, can be approximated by

$$\theta_m = \frac{h_m - h_{m-1}}{r_m} + \frac{h_m - h_{m+1}}{r_{m+1}} = \frac{h_m}{\rho_m^2} - \frac{h_{m-1}}{r_m} - \frac{h_{m+1}}{r_{m+1}} , \tag{1}$$

$$\text{where } \rho_m = [r_m r_{m+1}/(r_m + r_{m+1})]^{1/2}, \quad m=1, 2, \ldots, N. \tag{2}$$

In the application of Fresnel-Kirchhoff theory to MKE diffraction, elements of the wavefront are formed in the aperture above each knife-edge and the assumption is made that the field at any particular element arises solely from the total field over the preceding aperture. Only spatial phase change effects are considered significant and the usual condition is made that the separation distances are large compared with the knife-edge heights and wavelength λ. Furthermore, because of knife-edge symmetry in the horizontal direction (y-axis) and because factors obtained by integrating with respect to y cancel out in forming the attenuation (Millington et al., 1962), only phase differences in the plane of propagation are required.

The path length between two points, P and Q, above adjacent knife-edges, m and m+1, minus the distance between knife-edges is

$$\overline{PQ} - r_{m+1} = [r_{m+1}^2 + (X_m - X_{m+1})^2]^{1/2} - r_{m+1} \simeq (X_m - X_{m+1})^2/2r_{m+1} , \tag{3}$$

where X_m denotes the vertical coordinates of the points.

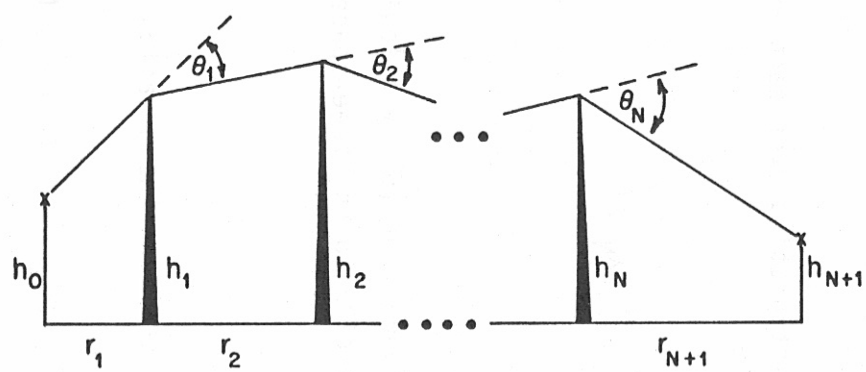

Figure 1. Geometry for multiple knife-edge diffraction.

Assuming plane wave propagation and exp($i\omega t$) time dependence, an element of field strength dE at the receiver due to a particular path from the source can be written as

$$dE \propto \exp[-\hat{F}_N] dX_1 \ldots dX_N \,, \tag{4}$$

$$\text{where } \hat{F}_N = \frac{ik}{2}\left[\frac{(h_0-X_1)^2}{r_1} + \sum_{m=1}^{N-1} \frac{(X_m-X_{m+1})^2}{r_{m+1}} + \frac{(X_N-h_{N+1})^2}{r_{N+1}}\right] . \tag{5}$$

Applying Huygens' principle and integrating over each aperture, we find that the total field is then

$$E = K \int_{h_1}^{\infty} \cdot \cdot \cdot \int_{h_N}^{\infty} e^{-\hat{F}_N} dX_1 \cdot \cdot \cdot dX_N \,, \tag{6}$$

where K is an unknown constant.

The free-space field E_0 is found from (6) by allowing the heights h_m to approach $-\infty$. The attenuation relative to the free-space field is then given by

$$(E/E_0) = E(h_m)/E(-\infty) \,, \tag{7}$$

where, for convenience, we have introduced the notation

$$E(h_m) = \int_{h_1}^{\infty} \ldots \int_{h_N}^{\infty} e^{-\hat{F}_N} dX_1 \cdots dX_N \,, \; m = 1, \ldots , N \,, \tag{8}$$

and $E(-\infty)$ is (8) with the lower limits replaced by $-\infty$.

It appears, that (8) can be integrated in closed form only in certain special cases, one of which is the free-space condition $E(-\infty)$. To show this we first define the function

$$C_k^2 \equiv [r_2 \ldots r_k R_{k+1}/(r_1+r_2)(r_2+r_3)\ldots(r_k+r_{k+1})]$$

$$= C_{k-1}^2 - \alpha_{k-1}^2 C_{k-2}^2, \; (k \geq 2), \; C_0 \equiv 1, \; C_1 \equiv 1 \,, \tag{9}$$

where $R_{k+1} = r_1 + \cdots + r_{k+1}$, (10)

and $\alpha_m = [r_m r_{m+2}/(r_m+r_{m+1})(r_{m+1}+r_{m+2})]^{1/2} = \rho_m\rho_{m+1}/r_{m+1}$. (11)

Next, we make the change of variables

$$v_m = \left(\frac{ik}{2}\right)^{1/2}\left[\frac{C_m(X_m-h_0)}{C_{m-1}\rho_m} - \frac{C_{m-1}\rho_m(X_{m+1}-h_0)}{C_m r_{m+1}}\right], \ m = 1, \ldots, N, \tag{12}$$

successively, so that

$$dv_m = (ik/2)^{1/2}(C_m/C_{m-1}\rho_m)dX_m , \tag{13}$$

and with the understanding that $X_{N+1} \equiv h_{N+1}$. Lower limits of the integrals in (8) are given by

$$\upsilon_m = \left(\frac{ik}{2}\right)^{1/2}\left[\frac{C_m(h_m-h_0)}{C_{m-1}\rho_m} - \tau_m\right], \tag{14a}$$

$$\tau_m = \tau_m(v_{m+1}, \ldots, v_N), \ \tau_N = C_{N-1}\rho_N(h_{N+1}-h_0)/C_N r_{N+1} . \tag{14b}$$

Explicit expressions for υ_m and τ_m in terms of knife-edge heights and separation distances are

$$\upsilon_m = (ik/2)^{1/2}\left[(R_{m+1}/r_{m+1}R_m)^{1/2}(h_m-h_0) - \tau_m\right], \ m = 1, \ldots, N, \tag{15a}$$

$$\tau_k = (r_{k+2}R_k/r_{k+1}R_{k+2})^{1/2}\left[(2/ik)^{1/2}v_{k+1} + \tau_{k+1}\right], \ k = N-1, \ldots, 1, \tag{15b}$$

$$\tau_N = (R_N/r_{N+1}R_{N+1})^{1/2}(h_{N+1}-h_0) ,$$

with R_m defined by (10). Notice that $\upsilon_m \to -\infty$ when $h_m \to -\infty$.

With the definitions (12), the function $\hat{F}_N$ becomes

$$\hat{F}_N = v_1^2 + \ldots + v_N^2 + (ik/2)(h_{N+1}-h_0)^2/R_{N+1} , \tag{16}$$

where R_{N+1} is the total path distance from source to receiver along the reference plane (see Figure 1). For free-space conditions, (8) is now easily integrated to obtain

$$\begin{aligned} E(-\infty) &\propto \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} e^{-(v_1^2 + \cdots + v_N^2)} dv_1 \ldots dv_N \\ &= 2^N \left[\int_0^{\infty} e^{-t^2} dt \right]^N = 2^N (\sqrt{\pi}/2)^N = \pi^{N/2} , \end{aligned} \tag{17}$$

and the attenuation (7) becomes

$$E/E_0 = (1/\pi)^{N/2} \int_{\upsilon_1}^{\infty} \ldots \int_{\upsilon_N}^{\infty} e^{-(v_1^2 + \ldots + v_N^2)} dv_1 \ldots dv_N , \tag{18}$$

where the lower limits are given by (14).

For N=1, (18) is the familiar single knife-edge attenuation function. For N=2 and $h_0=h_3=0$, (18) is equal to the conjugate of the double knife-edge attenuation function given by equation (12) in Millington et al. (1962). The conjugate relationship arises simply because of the different time dependence conventions used.

The Fresnel-Kirchhoff formulation of MKE attenuation is easily expressed in terms of the heights and separation distances of Figure 1 by applying relationships (12) through (14) to equation (18). The result is

$$E/E_0 = (ik/2\pi)^{N/2}(C_N/\rho_1 \ldots \rho_N)e^{\sigma_N''} \int_{h_1}^{\infty} \ldots \int_{h_N}^{\infty} e^{-\hat{F}_N} dX_1 \ldots dX_N , \tag{19}$$

where $\sigma_N'' = (ik/2)(h_{N+1}-h_0)^2/R_{N+1}$, (20)

and $\hat{F}_N$, C_N, and ρ_m are defined in (5), (9), and (2) respectively.

The "Fresnel-Kirchhoff" MKE function given by (19) now can be related to the "residue series" MKE function of the original derivation (Vogler, 1981) through the following considerations. First we define the parameter

$$\beta_m = (ik/2)^{1/2}\rho_m\theta_m = (ik/2)^{1/2}\left[\frac{h_m}{\rho_m} - \left(\frac{h_{m-1}}{r_m} + \frac{h_{m+1}}{r_{m+1}}\right)\rho_m\right] . \quad (21)$$

Then with the change of variables

$$x_m = (ik/2)^{1/2}\left[\frac{X_m}{\rho_m} - \left(\frac{h_{m-1}}{r_m} + \frac{h_{m+1}}{r_{m+1}}\right)\rho_m\right], \; m = 1, \ldots, N, \quad (22)$$

we have that

$$dx_m = (ik/2)^{1/2}(dX_m/\rho_m), \; (x_m-\beta_m) = (ik/2)^{1/2}(X_m-h_m)/\rho_m , \quad (23)$$

and the lower limit of the new variable x_m is β_m.

The exponent in (19) now becomes

$$\hat{F}_N - \sigma_N'' = F_N + \sigma_N' - \sigma_N , \quad (24)$$

$$\text{where } F_N = x_1^2 + \ldots + x_N^2 - 2\sum_{m=1}^{N-1} \alpha_m(x_m-\beta_m)(x_{m+1}-\beta_{m+1}) , \quad (25)$$

$$\sigma_N' = (ik/2)\left[\sum_{m=1}^{N+1} \frac{(h_{m-1}-h_m)^2}{r_m} - \frac{(h_{N+1}-h_0)^2}{R_{N+1}}\right], \quad (26)$$

$$\sigma_N = \beta_1^2 + \ldots + \beta_N^2 , \quad (27)$$

and α_m and β_m are defined in (11) and (21). Finally, the attenuation in terms of the variables x_m is given by

$$E/E_0 = (1/\pi)^{N/2} C_N e^{\sigma_N - \sigma_N'} \int_{\beta_1}^{\infty} \cdots \int_{\beta_N}^{\infty} e^{-F_N} dx_1 \ldots dx_N \quad , \tag{28}$$

which is identical to the original derivation obtained from the residue series except for the phase factor $\exp(-\sigma_N')$. (See equation (29) in Vogler (1981).)

The difference in phase arises from the fact that the reference free-space path in the residue series derivation consists of the path segments connecting the tops of the knife-edges, whereas the reference path in the present derivation is the total distance along the reference plane (see Figure 1). It should be noticed that the double knife-edge derivation of Millington et al. (1962) is restricted to the condition that the source and receiver are the same distance from the reference plane ($h_0=h_3$).

Numerical comparisons of double knife-edge diffraction by the method of Furutsu (1963) and by the method of Millington et al. (1962) show that values of the complex attenuation are the same in absolute value but differ in phase. The phase difference is just the factor $\exp(-\sigma_2')$ where σ_2' is given by (26) with $h_0=h_3=0$.

As a knife-edge tends to $-\infty$, the resultant attenuation should agree both in magnitude and phase with the attenuation that would be calculated without that knife-edge. The use of R_{N+1} as the reference distance for the free-space field assures this result, whereas a reference distance obtained by connecting the tops of the knife-edges does not. Furthermore, in the application of the MKE diffraction to actual radio propagation paths, the geometry of Figure 1 is more convenient than if the reference base were the straight line connecting the tops of the antennnas. Simple expressions that take into account the effects of earth curvature and atmospheric refraction are available to calculate effective heights of terrain features and the receiving antenna relative to the reference plane.

Although the attenuation as given by (19) is in a form more easily recognizable geometrically, the expression in (28) is the one used to obtain numerical evaluations of MKE attenuation. Further discussions on this subject are covered in the next section.