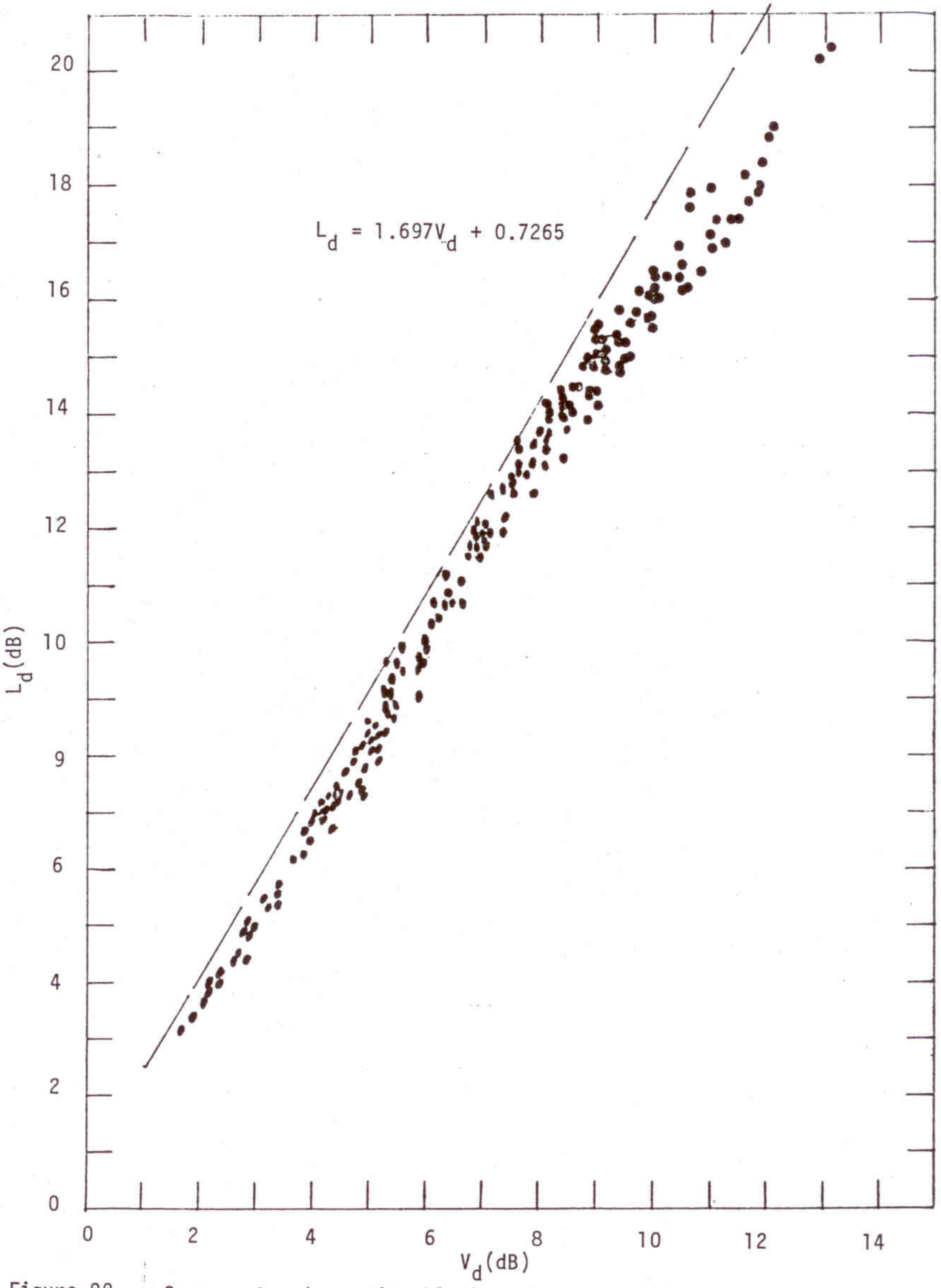

Figure 89. Season- 4 hour time block worldwide average L_d versus V_d, 200 Hz bandwidth, frequency range 13 kHz to 20 MHz.

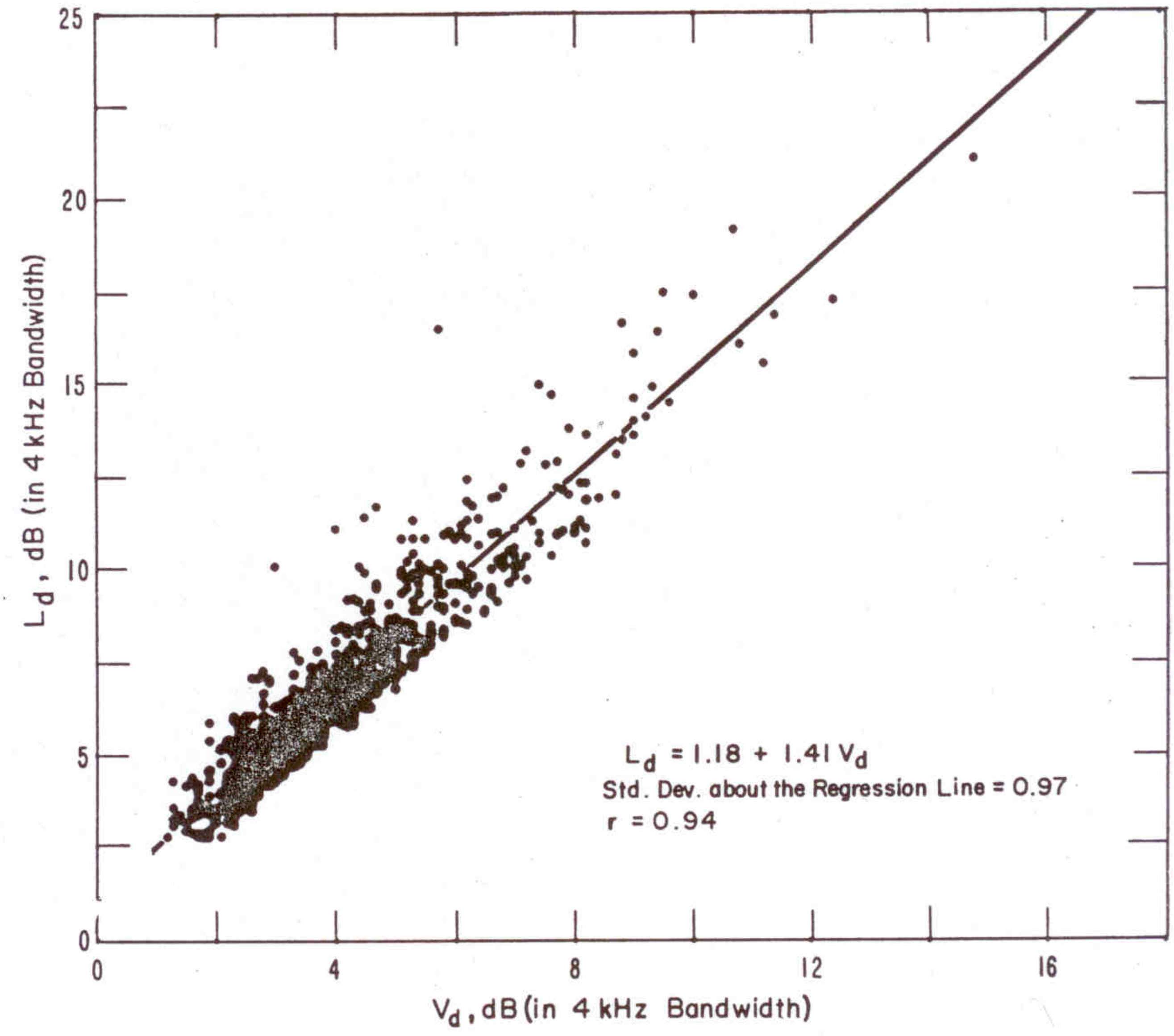

Figure 90. Correlation of V_d and L_d for man-made noise in the frequency range 250 kHz-250 MHz.

V_d as its input and is for the "standard" set of APD's using, basically, the relationship (29) for atmospheric radio noise. For other V_d, L_d combinations, the corresponding X, C, and A parameters can be obtained from Figures 84, 85, and 86. We will show how these "nonstandard" sets of parameters can be used in conjunction with the algorithms presented next.

4.2 Geometry of the Three-Section APD Curve

The APD model of Crichlow et al. (1960a) is shown on Figure 91 (and Figure 82). For small y values the curve coincides with a straight line, L_1, having the same slope as the Rayleigh distribution, i.e., -0.5. For large y values the curve coincides with another straight line L_2, having a steeper (negative) slope. These two straight lines are connected by the circular arc C.

We denote the points where the circular arc is tangent to straight lines L_1 and L_2 by (x_1,y_1) and (x_2,y_2), respectively, as shown in Figure 91. We also denote the center of the circle by (x_c,y_c) and its radius by r. Then, the three-section curve can be expressed by

$$L_1: \quad y = m_1 x + b_1 \tag{34}$$

for $x \geq x_1$ and $y \leq y_1$;

$$C: \quad (x - x_c)^2 + (y - y_c)^2 = r^2 \tag{35}$$

for $x_1 \geq x \geq x_2$ and $y_1 \leq y \leq y_2$;

$$L_2: \quad y = m_2 x + b_2 \tag{36}$$

for $x \leq x_2$ and $y \geq y_2$.

We denote the third straight line that is tangent to the arc at its midpoint by

$$L_3: \quad y = m_3 x + b_3 \quad . \tag{37}$$

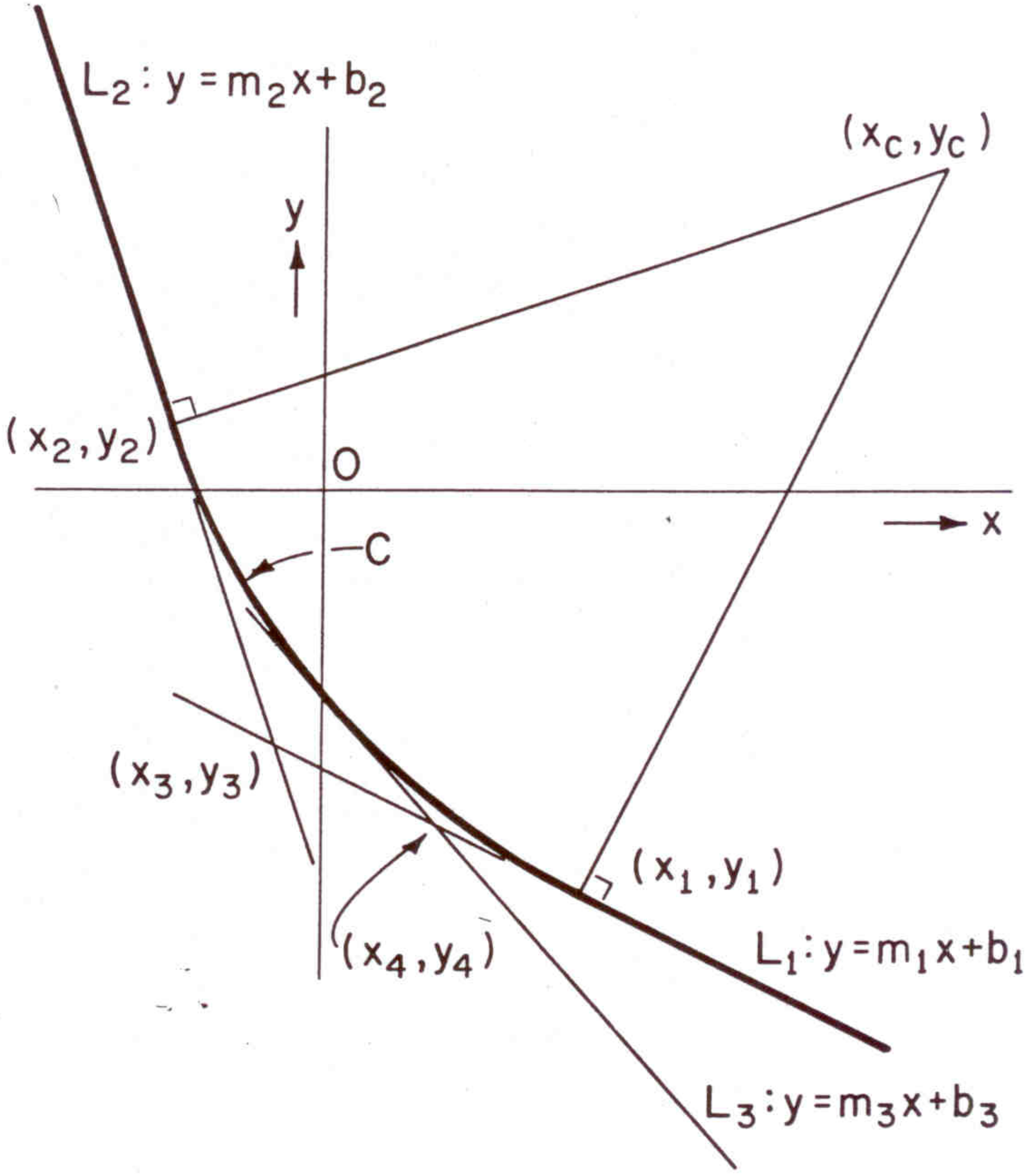

Figure 91. Three-section curve for the APD function.

Since L_3 is parallel to the bisector of the angle between L_1 and L_2, m_3 is related to m_1 and m_2 by

$$m_3 = \tan\left(\frac{\mathrm{Tan}^{-1}m_1 + \mathrm{Tan}^{-1}m_2}{2}\right) . \tag{38}$$

If we denote the direction tangent of a bisector of the angle between L_1 and L_3 by m_4, it is given by

$$m_4 = \tan\left(\frac{\mathrm{Tan}^{-1}m_1 + \mathrm{Tan}^{-1}m_3}{2}\right) . \tag{39}$$

We further denote the point of intersection between L_1 and L_2 by (x_3,y_3), and between L_1 and L_3 by (x_4,y_4), as shown in Figure 91. Then, these coordinates are given as

$$x_3 = \frac{b_2 - b_1}{m_1 - m_2} , \tag{40}$$

$$y_3 = \frac{m_1 b_2 - m_2 b_1}{m_1 - m_2} , \tag{41}$$

$$x_4 = \frac{b_3 - b_1}{m_1 - m_3} , \tag{42}$$

$$y_4 = \frac{m_1 b_3 - m_3 b_1}{m_1 - m_3} . \tag{43}$$

With these values, the coordinates x_c and y_c are given by

$$x_c = \frac{m_3(x_4 + m_4 y_4) - m_4(x_3 + m_3 y_3)}{m_3 - m_4} , \tag{44}$$

$$y_c = \frac{(x_3 + m_3 y_3) - (x_4 + m_4 y_4)}{m_3 - m_4} . \tag{45}$$

The coordinates x_1, y_1, and y_2 are given by

$$x_1 = \frac{x_c + m_1(y_c - b_1)}{1 + m_1^2} \quad , \tag{46}$$

$$y_1 = \frac{b_1 + m_1 x_c + m_1^2 y_c}{1 + m_1^2} \quad , \tag{47}$$

$$x_2 = \frac{x_c + m_2(y_c - b_2)}{1 + m_2^2} \quad , \tag{48}$$

$$y_2 = \frac{b_2 + m_2 x_c + m_2^2 y_c}{1 + m_2^2} \quad . \tag{49}$$

Finally, the radius of the circular arc is expressed by

$$r = \sqrt{(x_c - x_1)^2 + (y_c - y_1)^2} \quad . \tag{50}$$

It follows from this geometry that if five constants, m_1, m_2, b_1, b_2, and b_3, are specified, all the necessary values for determining the curve can be uniquely determined by following steps from (34) to (50). It was noted earlier in (28) that $B = 1.5\ (X - 1)$ where $X = m_2/m_1$. This means that the line L_3 can be expressed by

$$y - y_3 = m_3(x - x_3) + 1.5\ (m_2/m_1 - 1) \quad . \tag{51}$$

From this we have

$$b_3 = y_3 - m_3 x_3 + 1.5\ (m_2/m_1 - 1) \quad . \tag{52}$$

Also L_1 is a Rayleigh distribution so that

$$m_1 = -0.5. \tag{53}$$

Because of these two conditions, we have only three parameters, m_2, b_1, and b_2, that we need to specify to construct the desired curve. Later (Section 4.4), a computer algorithm to do this is specified. This algorithm used an interpolation procedure to obtain m_2, b_1, and b_2 from the input V_d. The resulting APD is the CCIR 322 "standard" APD for this value of V_d (slightly modified for small values of V_d, as noted earlier). This set of APD's is shown on Figure 92. For other "nonstandard" APD's, the following relationships can be used to specify m_2, b_1, and b_2 in terms of X, C, and A:

$$\begin{aligned} m_2 &= -X/2 \quad , \\ b_1 &= -A + 1.598, \text{ and} \\ b_2 &= 8.23 + C - A - 6.63X \quad . \end{aligned} \tag{54}$$

The algorithms given in Section 4.4 also can be used to compute the pdf of the envelope, i.e., the derivative of P(E), (27), as well as the APD, or D(E) (27). All levels are given relative to the rms level of the envelope.

4.3 Bandwidth Conversion of the APD

As noted earlier, the parameters V_d and L_d (and the APD) are a function of receiver bandwidth and the worldwide V_d estimates given by CCIR Report 322 are for a 200-Hz bandwidth. Spaulding et al. (1962) developed a method to convert the APD to other bandwidths. These results gave a "new" V_d value as a function of the "old" V_d value and the bandwidth ratio. The analysis that led to these results was based on earlier results of Fulton (1961) and used various idealized assumptions. These bandwidth conversion results became the "standard" and are given in CCIR Report 322. Numerous measurements since the publication of CCIR Report 322 in 1964 have shown that these bandwidth conversion relationships can give quite wrong results, especially if the bandwidth ratio is at all large. The bandwidth conversion relationship given by CCIR Report 322 is one of the main deficiencies of Report 322.

Recently Herman and DeAngelis (1983) conducted an extensive study in order to develop a better V_d-bandwidth relationship. A large data base of wideband (100 kHz), high dynamic range (120 dB) digital recordings was used. This data base was for MF and was recorded in a remote location in Nevada during Autumn 1980, Winter 1981, and Summer 1981. Digital filtering and analysis allowed determination of the noise characteristics in bandwidths from 100 kHz down to 6 Hz. Figure 93 shows the

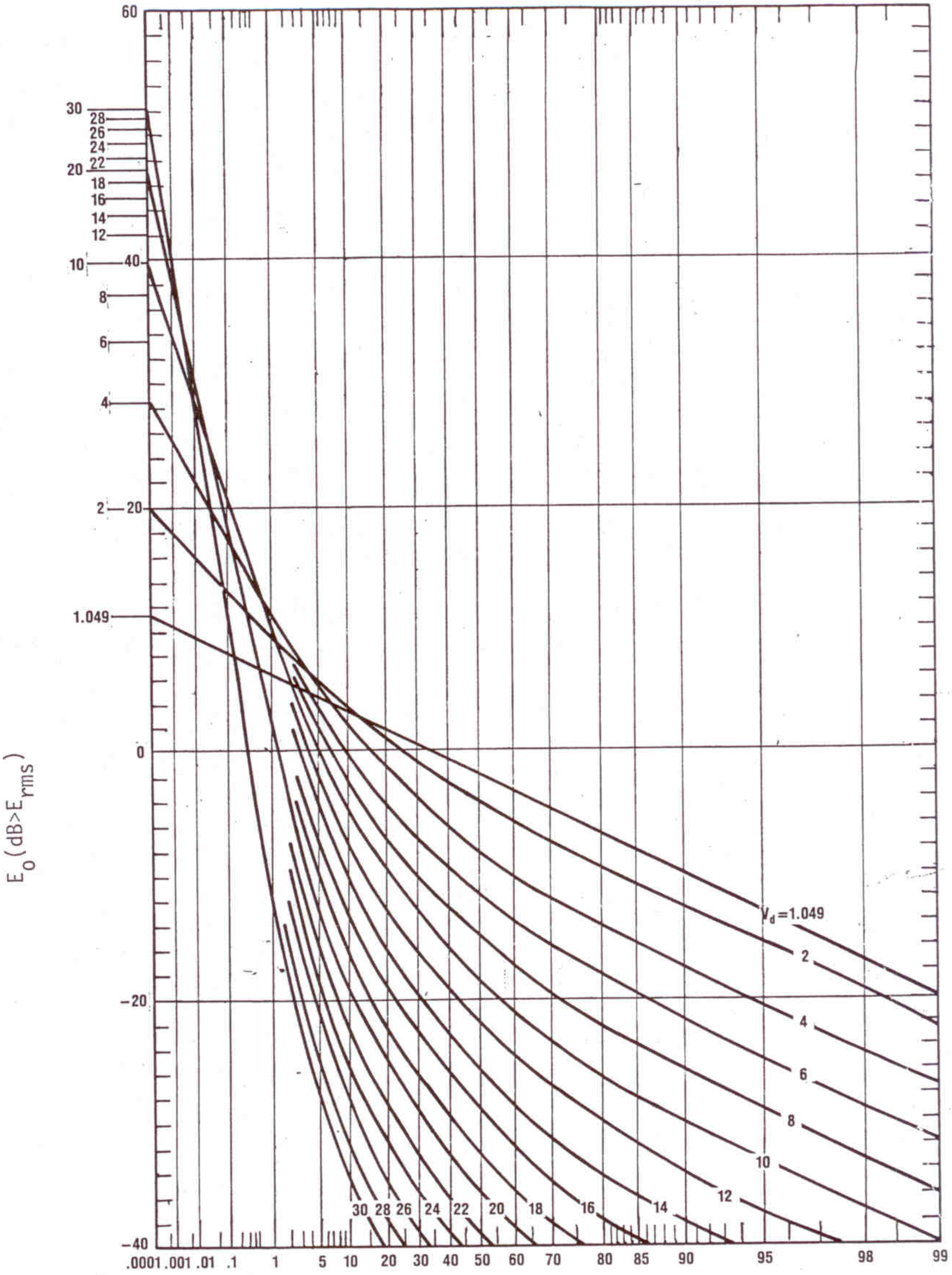

Figure 92. "New" set of amplitude probability distributions for atmospheric radio noise for various values of V_d.

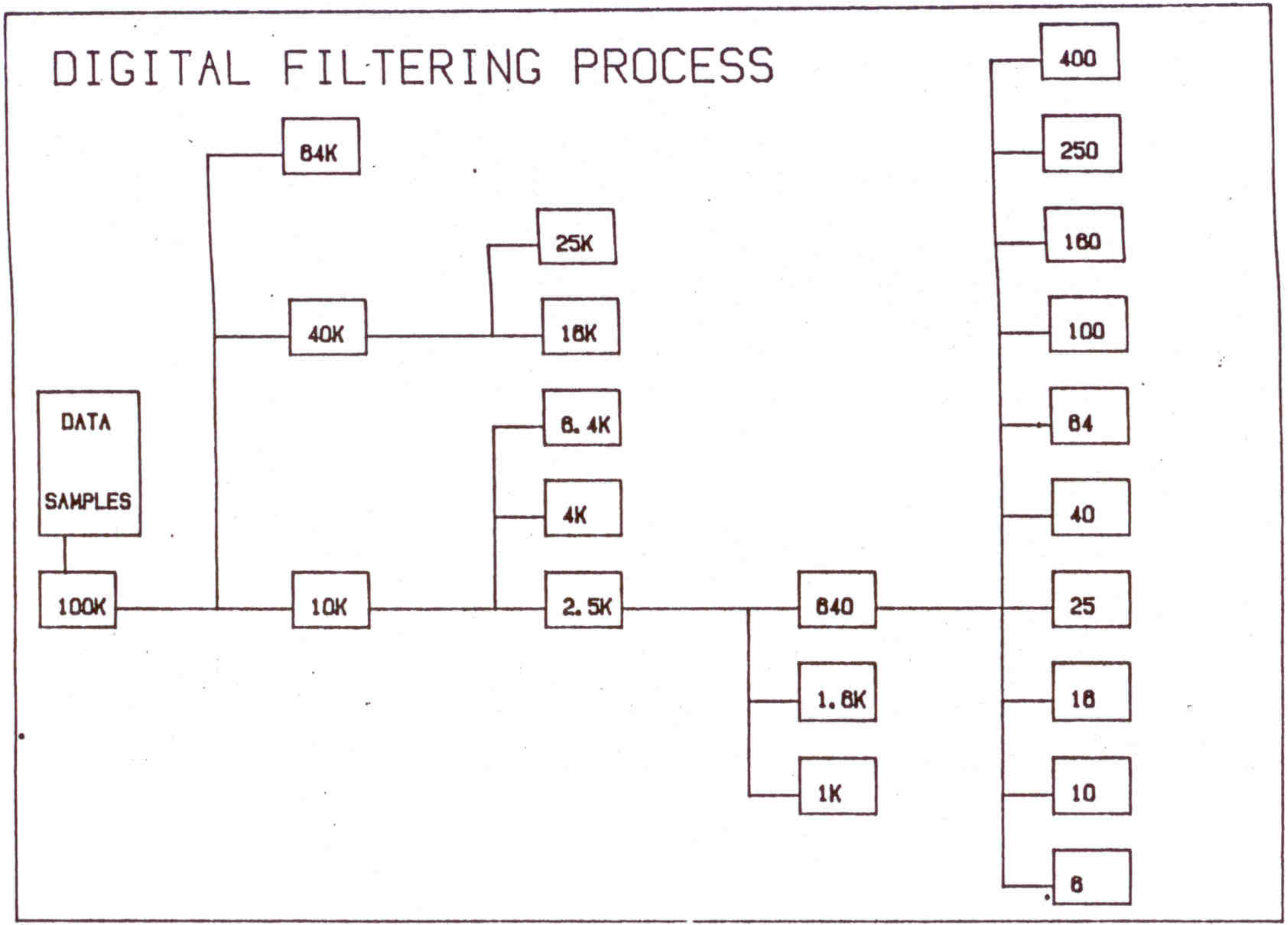

Figure 93. The digital filtering process illustrating the bandwidths used in the V_d versus bandwidth atmospheric noise analysis (Herman and DeAngelis, 1983).

digital filtering process and the resulting bandwidths. In all cases, the resulting APD's could be represented by the model given here by Figure 92 (or the CCIR 322 model). That is, the APD-V_d relationship is valid. Figure 94 gives some of the results of this study, giving means to convert the 200 Hz V_d to other bandwidths. The results of Herman and DeAngelis (1983) depend on the starting bandwidth, whereas the CCIR 322 bandwidth conversion results do not depend on the starting bandwidth, but only on the bandwidth ratio. Herman and DeAngelis's results indicate the proper bandwidth conversion relationships depend on starting bandwidth. The results of Figure 94 are strictly valid only for MF, even though the APD's obtained were equivalent to those obtained at other frequencies. The results of Figure 94 need to be verified at other frequencies (e.g., HF), but should represent a substantial improvement over the idealized results given by CCIR Report 322. The results of Figure 94 are given mathematically as

$$V_{dn} = V_{de} + (0.4679 + 0.2111\ V_{de}) \log \mathrm{BWR} \quad , \tag{55}$$

where V_{dn} is the desired value of V_d and V_{de} is the starting V_d value in a 200-Hz bandwidth, and BWR is the bandwidth ratio, i.e., desired bandwidth/200 Hz.

4.4 Computer Software

In this section we simply present computer software (FORTRAN) to use with the results above. The first program (PROGRAM APD) is an example program. It takes a 200-Hz V_d (VD200) of 7.0 dB and a bandwidth ratio (BWR) of 100, and produces the corresponding APD. The bandwidth conversion of V_d is accomplished via (55), which is given by FUNCTION VDC(VD200,BWR) called by PROGRAM APD. The main software given here is FUNCTION APDAN(VD,K,DB) which actually produces the APD value (or the corresponding envelope pdf value) for a given V_d(VD) at a specified level (DB) relative to the rms level. Other details are well covered by the comment statements in the programs and FUNCTION statements below. The standard set of APD's given on Figure 92 was obtained from FUNCTION APDAN(VD,K,DB) using a program similar to the PROGRAM APD given below. Table 45 is a sample of the output for a V_d of 20 dB.

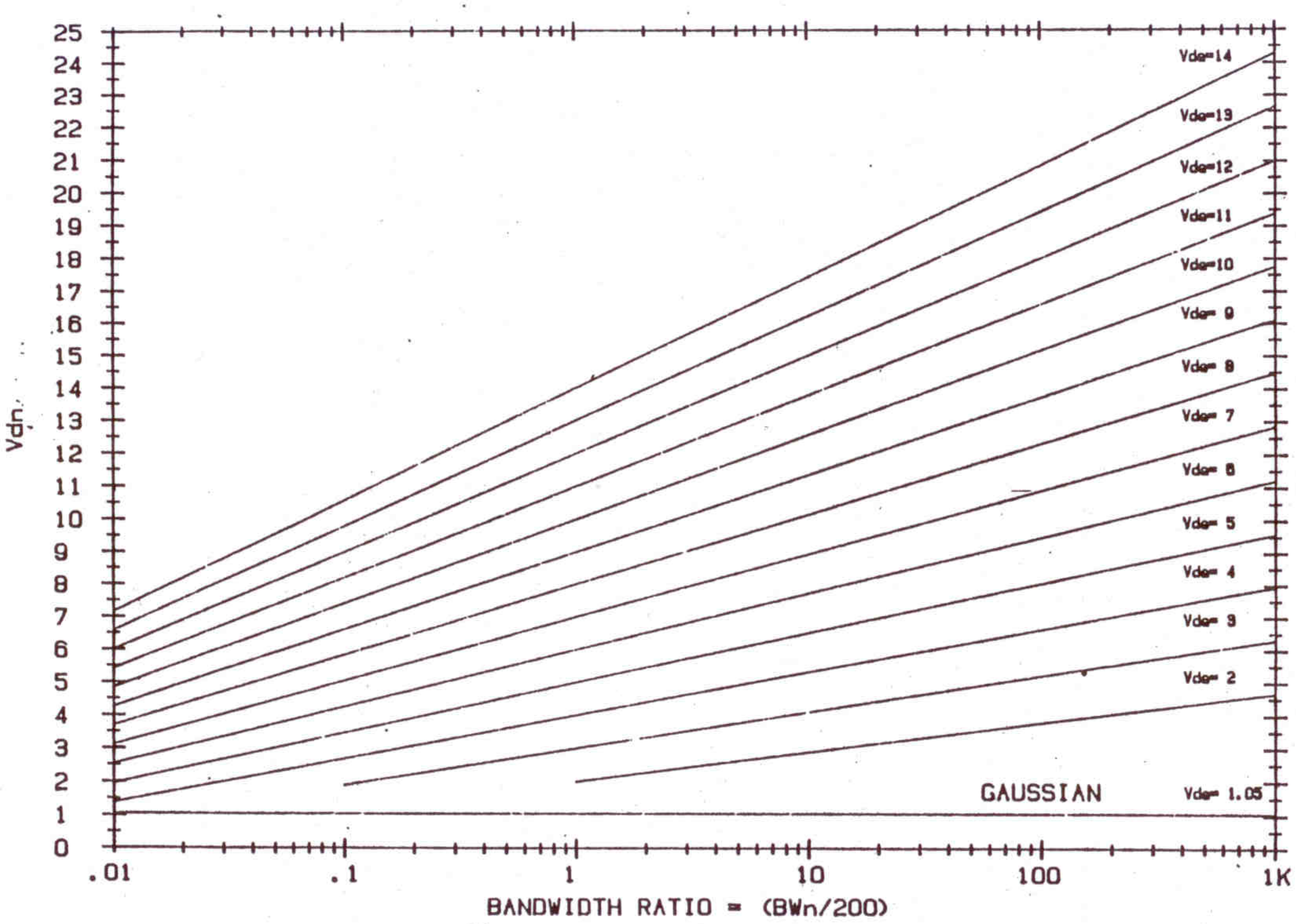

Figure 94. Translation of a 200-Hz bandwidth V_d, V_{de}, to other bandwidths, BWn.

```
1         PROGRAM APD (INPUT,OUTPUT)
 C        RADIO NOISE FOR THE PARAMETER  VD.  VD200 IS THE CCIR 322 (OR OTHER)
 C        EXTIMATED VALUE FOR A 200 HZ BANDWIDTH.
          PRINT 3
          VD200 = 7.0
          BWR = 100.
 C        OBTAIN VD FOR THE PROPER BANDWIDTH  (BWR * 200.).
          VD = VDC (VD200,BWR)
          PRINT 4, VD
 C        COMPUTE EXCEEDENCE PROBABILITY (K=1) OR PROBABILITY DENSITY (K=2) OF
 C        NOISE ENVELOPE.
 C        LEVELS ARE RELATIVE TO RMS LEVEL AND IN DB.
       10 I = 0      $       $K = 1
       20 I = I + 1
          DB = -(I-1) * 2.
          P = APDAN (VD,K,DB)
          PRINT 5, DB, P
          IF (P.LE.0.99)  GO TO 20
       30 J = 0      $ DB = 0.
       40 J = J + 1
          DB =   J * 2
          P = APDAN (VD,K,DB)
          PRINT 5, DB,P
          IF (P.GE.1.0E-6)  GO TO 40
 C        FORMAT STATEMENTS
        3 FORMAT(1H1)
        4 FORMAT(10X,*  VD = *,F5.2,/)
        5 FORMAT(10X,F10.1,2X,1PE10.3)
          CALL EXIT
          END


          FUNCTION VDC(VD200,BWR)
 C        OBTAINS THE NOISE PARAMETER -VD-,FOR THE SPECIFIED BANDWIDTH
 C        FROM THE CCIR REPORT 322 (OR OTHER) 200HZ BANDWITH -VD- (VD200).
 C       -BWR- IS THE BANDWITH RATIO (REQUIRED BANDWIDTH/200 HZ BANDWIDTH).
          IF(VD200.LE.1.049)  GO TO 50
          VDO = VD200 + (0.4679 + 0.2111 * VD200) * ALOG10(BWR)
          IF(VDO.LE.1.049)  GO TO 50
          VDC = VDO
          RETURN
       50 VDC = 1.049
          RETURN
          END
```

```
C
      FUNCTION APDAN(VD,K,DB)
C      AMPLITUDE PROBABILITY DISTRIBUTION (APD) OF THE ENVELOPE OF ATMOSPHERIC
C      NOISE
C      VD = CCIR REPORT 322 NOISE PARAMETER
C          = RATIO OF RMS TO AVERAGE OF NONE ENVELOPE IN DB, FOR THE APPROPRIATE
C      RECEIVER BANDWITH.
C      NOTE: FUNCTION VDC CONVERTS CCIR 200HZ VD(VD200) TO OTHER BANDWIDTHS.
C      K = 1 FOR APD (EXCEEDENCE PROBABILITY)
C        = 2 FOR PROBABILITY DENSITY FUNCTION OF ENVELOPE.
C      DB = LEVEL RELATIVE TO RMS IN DB.
      DIMENSION VVD(24), BB1(24), BB2(24), MM2(24)
      REAL MM2, M1, M2, M3, M4, M1SQP1,M2SQP1
      DATA VVD/ 1.0491,  1.1779,  1.3215,  1.4803,  1.6549,  1.8466,
     1          2.2831,  2.7973,  3.3941,  4.0796,  4.8567,  5.7218,
     2          6.6744,  7.7069,  8.8107,  9.9740, 12.9794, 16.0528,
     3         22.1551, 28.2294, 34.2720, 40.2839, 46.2711, 52.2264/
      DATA BB1/0.0000,  -0.4329,  -0.8909,  -1.3751,  -1.8867,  -2.4269,
     1        -3.5913,  -4.8927,  -6.3195,  -7.8868,  -9.5991, -11.4490,
     2       -13.4448, -15.5800, -17.8472, -20.2380, -26.3694, -32.6321,
     3       -44.9001, -57.0708, -69.2146, -81.3777, -93.6426,-105.8298/
      DATA BB2/ 0.0000, -0.7529,  -1.5309,  -2.3305,  -3.1667,  -4.0269,
     1         -5.8383, -7.7827,  -9.8695, -12.1068, -14.4991, -17.0495,
     2        -19.7548,-22.6100, -25.6072, -28.7380, -37.0919, -46.0824,
     3        -65.6023,-86.8042,-109.4042,-133.2062,-158.0634,-183.8612/
      DATA MM2/  -0.5,  -0.6,  -0.7,  -0.8,  -0.9, -1.0,
     1           -1.2,  -1.4,  -1.6,  -1.8,  -2.0, -2.2,
     2           -2.4,  -2.6,  -2.8,  -3.0,  -3.5,  -4.0,
     3           -5.0,  -6.0,  -7.0,  -8.0,  -9.0, -10./
      DATA VDPV/ 0.0/
      DBO =  DB
      M1 = -0.5
      M1SQP1 = 1.25
      C1 = 0.2302585093
      C2 = C1/2.
   10 VDO = VD
      KO = K
      IF (VDO.LT.1.049)   GO TO 90
   20 IF (VDO.GE.1.05)    GO TO 22
   21 L = 1
      PP = EXP(C1 * DBO)
      GO TO 65
   22 IF (VD.EQ.VDPV)  GO TO 60
C     LOCATE VDO AND INTERPOLATE FOR B1,B2, AND M2.
   30 VDPV = VDO
      IF (VDO.LT.VVD(3))      GO TO 35
      IF (VDO.GE.VVD(22))     GO TO 36
      IMN = 4
```

```
      IMX = 22
   31 I = (IMN+IMX)/2
      IF (VDO.GE.VVD(I))      GO TO 33
   32 IMX = I
      GO TO 34
   33 IMN = I + 1
   34 IF (IMX.GT.IMN)         GO TO 31
      I = IMX
      GO TO 40
   35 I = 3
      GO TO 40
   36 I = 23
   40 CONTINUE
      V1 = VVD(I-2) - VDO
      V2 = VVD(I-1) - VDO
      V3 = VVD(I) - VDO
      V4 = VVD(I+1) - VDO
      V43 = V4 - V3
      V42 = V4 - V2
      V41 = V4-V1
      V21 = V2 - V1
      V32 = V3 - V2
      V31 = V3 - V1
      A1 = V41 * V31 * V21
      A1 = V4 * V3 * V2 / A1
      A2 = V42 * V32 * V21
      A2 = -V4 * V3 * V1 / A2
      A3 = V43 * V32 * V31
      A3 = V4 * V2 * V1 / A3
      A4 = V43 * V42 * V41
      A4 = -V3 * V2 * V1 / A4
      B1 = A1*BB1(I-2) + A2*BB1(I-1) + A3*BB1(I) + A4*BB1(I+1)
      B2 = A1*BB2(I-2) + A2*BB2(I-1) + A3*BB2(I) + A4*BB2(I+1)
      M2 = A1*MM2(I-2) + A2*MM2(I-1) + A3*MM2(I) + A4*MM2(I+1)
C     GEOMETRY
   50 SF = M2/M1
      M2SQP1 = M2 * M2 + 1.0
      DM = M1 - M2
      SM = M1 + M2
      X3 = (B2-B1) / DM
      Y3 = (M1*B2 - M2*B1) / DM
      T = (1.0-M1*M2)/SM
      M3 = -T - SQRT(T*T+1.0)
      B3 = Y3 - M3 * X3 + 1.5 * (SF-1.0)
      DM = M1 - M3
      SM = M1 + M3
      X4 = (B3-B1) / DM
      Y4 = (M1*B3 - M3*B1) / DM
```

```
      T = (1.0 - M1*M3)  / SM
      M4 = -T - SQRT(T*T+1.0)
      DM = M3 - M4
      X3 = X3 + M3 * Y3
      X4 = X4 + M4 * Y4
      XC = (M3*X4-M4*X3) / DM
      YC = (X3-X4) / DM
      DBMN = (B1+M1*(XC+M1*YC)) / M1SQP1
      DBMX = (B2+M2*(XC+M2*YC)) / M2SQP1
      RSQ = (YC-DBMN)**2 * M1SQP1
      PRINT 99, B1,B2,M2,DBMN,DBMX,YC
   99 FORMAT (6(F10.0))
C     COMPUTATION OF THE FUNCTION
   60 IF (DBO.GE.DBMX)         GO TO 63
      IF (DBO.GT.DBMN)         GO TO 62
   61 L = 1
      PP = EXP(C1*(DBO-B1))
      GO TO 65
   62 L = 2
      DY = YC - DBO
      DX = SQRT(RSQ-DY*DY)
      PP = EXP(C2*(DX-XC))
      GO TO 65
   63 L = 3
      PP = EXP(C1*(DBO-B2)/SF)
   65 APDAN = EXP(-PP)
      IF (KO.EQ.1)                     RETURN
      GO TO (69,67,68), L
   67 APDAN = 0.5 * DY * APDAN / DX
      GO TO 69
   68 APDAN = APDAN / SF
   69 APDAN = APDAN * PP * C1
      RETURN
C     ERROR EXIT
   90 PRINT 2090
      GO TO 95
   95 PRINT 2095, VDO,KO,DBO
      RETURN
C     FORMAT STATEMENTS
 2090 FORMAT(1X/21H  888   VD TOO SMALL./)
 2095 FORMAT(*   VD =  *,E12.3,8X,* K =*,I3,8X,*DB =* E12.3/
     1*  ERROR DETECTED IN ROUTINE  -APDAN-*)
      END
```

Table 45. Sample Output of PROGRAM APD for V_d = 20 dB. E_o is Envelope Voltage (dB>E_{rms}) and P is Probability of Level E_o Being Exceeded

```
    Eo            P                     Eo            P

    0.0      1.413E-02                 2.0      1.138E-02
   -2.0      1.736E-02                 4.0      9.067E-03
   -4.0      2.112E-02                 6.0      7.140E-03
   -6.0      2.546E-02                 8.0      5.554E-03
   -8.0      3.091E-02                10.0      4.266E-03
  -10.0      3.803E-02                12.0      3.233E-03
  -12.0      4.727E-02                14.0      2.416E-03
  -14.0      5.919E-02                16.0      1.779E-03
  -16.0      7.439E-02                18.0      1.289E-03
  -18.0      9.358E-02                20.0      9.196E-04
  -20.0      1.175E-01                22.0      6.446E-04
  -22.0      1.467E-01                24.0      4.438E-04
  -24.0      1.819E-01                26.0      2.998E-04
  -26.0      2.234E-01                28.0      1.985E-04
  -28.0      2.713E-01                30.0      1.287E-04
  -30.0      3.252E-01                32.0      8.167E-05
  -32.0      3.844E-01                34.0      5.063E-05
  -34.0      4.477E-01                36.0      3.063E-05
  -36.0      5.135E-01                38.0      1.806E-05
  -38.0      5.801E-01                40.0      1.037E-05
  -40.0      6.455E-01                42.0      5.788E-06
  -42.0      7.079E-01                44.0      3.136E-06
  -44.0      7.656E-01                46.0      1.647E-06
  -46.0      8.175E-01                48.0      8.374E-07
  -48.0      8.626E-01
  -50.0      9.006E-01
  -52.0      9.315E-01
  -54.0      9.555E-01
  -56.0      9.717E-01
  -58.0      9.820E-01
  -60.0      9.886E-01
  -62.0      9.928E-01
```