Measures of Spectral Efficiency in Land Mobile Radio

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Abstract—Measures of spectral efficiency are very important to the resolution of contemporary issues in land mobile radio (LMR), because they a) allow the comparison of existing and proposed systems in terms of their spectral efficiency; b) permit estimates to be made of the ultimate capacity of various system types at different levels of development; and c) are useful in setting minimum standards for spectral efficiency. The purpose of this paper is to review the advantages, disadvantages, and limitations of various measures of spectral efficiency that have been proposed.

INTRODUCTION

MEASURES OF spectral efficiency are very important to the resolution of contemporary issues in land-mobile radio (LMR). Objective measures are needed for the following reasons.

a) *They allow the comparison of existing and proposed systems in terms of their spectral efficiency.* In the Federal Communications Commission (FCC) proceedings regarding the reallocation of 115 MHz of spectrum in the vicinity of 900 MHz for LMR (Docket 18262), there were conflicting claims regarding the relative spectral efficiencies of proposed systems. Resolution of these disputes was complicated by the lack of a precise definition of spectral efficiency. These conflicting assertions are likely to continue as proposed systems are installed and their various proponents attempt to obtain additional channel assignments from the substantial reserves being held for demonstrated future needs.

b) *They permit estimates to be made of the ultimate capacity of various system types at different levels of development.* The ultimate capacity of a LMR system for a given quality of service is directly related to its spectral efficiency. Accurate measures of spectral efficiency can be used to determine the ultimate capacity of a particular system or systems using a certain amount of spectrum. Coupled with demand growth projections, the point of spectrum saturation can be predicted.

c) *They are useful in setting minimum standards for spectral efficiency.* In frequency-congested urban areas, channels should not be assigned unless certain minimum spectral-efficiency standards are met. This approach is particularly important when services are provided by competitive systems. The standards prevent one operator from lowering costs or offering higher quality service by squandering spectrum. For example, if wireline common carriers (WCC's), radio common carriers (RCCs), and private-shared systems were all allowed to provide dispatch service, fairness would require that each should meet the same minimum level of spectrum efficiency when they are competing for the same business.

An example of one such use of a spectral efficiency measure is contained in a paper by Lane [1].

The purpose of this paper is to review the advantages, disadvantages, and limitations of various measures of spectral efficiency that have been proposed for land-mobile radio.

MOBILES/CHANNEL

The number of mobile units per channel is perhaps the simplest measure of spectral efficiency and, as a result, it has certain deficiencies.

First, the mobiles in two systems which are being compared may not generate the same amount of traffic. For example, the users of one system may generate twice as much busy-hour traffic per mobile as another system. Since they both could carry the same total traffic, the latter could then claim twice as many users per channel for the same quality of service. This would be misleading if the lower value of average traffic per user was obtained by purposely adding mobiles to the system which would generate little or no usage. In effect, the mobiles-per-channel figure would be easy to inflate. On the other hand, it would not necessarily be misleading if the operator achieved the lower figure by such techniques as a) charging on a measured-rate basis to discourage unnecessary usage, b) encouraging the use of brevity codes to shorten transmissions, or c) by employing digital transmission where it is more efficient.

A *second* problem is that the channel bandwidth may not be the same, e.g., one might employ 40-kHz channels and the other 25-kHz channels. This is a relatively minor problem since an appropriate adjustment can be made for the difference in bandwidth, or, more simply, it can be solved by going to mobiles per unit of bandwidth as the measure.

A *third* problem concerns the geographic area covered by the system. Consider, as an extreme example, a single-channel shared system serving 100 mobiles with a repeater on a very tall building. Because of the antenna height, this channel might not be reusable within a radius of 70 mi from the repeater. Contrast this with a more conventional system using a low roof- or tower-mounted antenna also serving 100 mobiles. With the lower antenna height, channel reuse might be permissible beyond only 35 miles. The former system might prevent the reuse of the channel over a whole region, while the latter might prevent its reuse in only one community, i.e., several communities could reuse the channel. The latter is obviously more spectrally efficient than the former (especially if regional coverage is not required) even though both have 100 mobiles per channel. A cellular system carries this concept

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one step further so that a given channel is reused a number of times within a community or metropolitan area. It is clear that geographic area is an important parameter in measures of spectral efficiency.

It should be noted that this area problem interacts with the previous one. If the systems employ frequency modulation, an increase in channel bandwidth permits a larger modulation index to be used which, in turn, reduces the system's susceptibility to interference. Systems then can be spaced closer together and the number of users in an area may increase, even though the number of users per unit of bandwidth on an individual system may decrease. The objective is to increase the total number of users in a given block of spectrum in a congested area, rather than just increasing the number of users in the block of spectrum. The latter does not always imply the former; thus a system with 100 subscribers per 25-kHz channel is not necessarily more spectrally efficient than a system with 100 subscribers per 50-kHz channel, and, consequently, channel splitting is not always spectrally efficient.

The seriousness of the above objections to the mobiles-per-channel definition of spectral efficiency depends upon use to which the definition is being put. Certainly the first deficiency (unequal traffic) is not a problem, and the second (unequal bandwidths) can be handled if the theoretical efficiencies of proposed systems are being compared. Likewise, the last deficiency (unequal coverage) can be handled in this application. All three objections are much more serious if the definition is being used in assignment standards.

ERLANGS (USAGE)/CHANNEL OR ERLANGS/MHZ

Using erlangs[1] per megahertz as a measure of spectral efficiency gets around some of the objections described above. An erlang is a measure of traffic intensity; it measures the quantity of traffic on a channel or group of channels per unit time and, as a ratio of time, it is dimensionless. One erlang of traffic would occupy one channel full time and 0.02 erlang would occupy it two percent of the time. Thus the traffic intensity is directly related to channel usage—an easily measured parameter, either from within the system or by external monitoring. Other things being equal, a system yielding a higher usage for a given number of channels is more spectrally efficient. The "other things being equal" in this case refers to the blocking probability—the chance that an arriving call would find all channels busy. This is the basic measure of the quality of service from the user's standpoint. It is very important that the blocking probabilities be equal when comparing the spectral efficiencies of two systems. This is easiest to see through the use of the following examples.

Consider two mobile telephone systems (A and B), each using 12 trunked channels. Suppose that each mobile generated 0.03 erlang of busy-hour traffic, and suppose that System A was loaded with 196 mobiles on its 12 channels and that System B was loaded with 265 mobiles. On the surface, System B with 22 mobiles (0.66 erlang) per channel appears to be more efficient than System A with 16 mobiles (0.48 erlang) per channel, but standard telephone-engineering tables show that the corresponding blocking probabilities would be five percent and one percent, respectively. Thus System B is not inherently more efficient than System A. If systems are to be compared in terms of spectral efficiency, recognition of and/or adjustments for differences in performance must be made. As an example of where this comparison can become a problem, it is sometimes implied that certain new systems are not as spectrally efficient as existing systems. The argument is that these existing systems are often loaded with several hundred mobiles per channel, while the proposed systems would be loaded with only 40 or 50. The problem is that the old systems often yield much higher blocking probabilities. If the existing system parameters were adjusted to give this same performance or if service on the new system were allowed to deteriorate to that of the old system, the conclusion would be the opposite.

The previous paragraph digresses somewhat from the question of evaluating erlangs per megahertz as a measure of spectral efficiency, but it does demonstrate the importance of considering quality when comparing systems. It also indicates that using erlangs or usage (rather than the number of mobiles) eases the problem of adjusting for this quality factor. This advantage and the advantage of ease in measurement are the principal features of this definition.

A principal disadvantage is that the geographic area (spatial efficiency) factor is still not included. The remaining disadvantages relate to its use as an assignment standard. Again, there is a possibility of abuse, although not so great as with the mobiles-per-channel standard. The problem is that fictitious traffic could be generated to increase channel usage. This could be done by keying mobile transmitters on occasion. This would be difficult to detect, even if monitoring were used to spot check. This abuse could be discouraged by requiring charges on common-carrier or common-user systems to be on a measured-rate basis and then modifying the definition to be on the basis of paid usage (erlangs) per megahertz. Since the operator's own mobile usage would not count and since a subscriber would be unlikely to generate fictitious traffic because he would ultimately have to pay for it in usage charges, the chances for abuse would be even further reduced. Furthermore, it would provide an auditing link between spectral efficiency and the firm's regular accounting records. This scheme would not solve the problem for purely private systems. However, it is anticipated that these will be largely in the public safety and related categories which would have slightly different standards in any event.

ERLANGS/MHZ/MI^2

This definition of spectral efficiency, (paid) erlangs per megahertz per square mile, attempts to get around the principal objection to the definition offered in the previous section, i.e., the objection that it did not include the frequency-reuse

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[1] "An erlang is a unit of communication traffic load equal to the traffic load whose calls, if placed end to end, will keep one path continuously occupied," *Dictionary of Scientific and Technical Terms.* New York: McGraw-Hill, 1974.

(spatial-efficiency) factor. An example of the geographic reuse of a frequency was given earlier. Now consider another example of a 70-channel mobile telephone system (MTS) and suppose we are comparing two system forms. With full duplex operation, the 70 channels might occupy 3.5 MHz of spectrum and they could carry approximately 54 erlangs of traffic at a blocking probability of two percent. In a conventional system, a single 70-channel base station on a tall building might provide coverage over a radius of say 40 mi or an area of roughly 5000 mi^2. Thus the ultimate spectral efficiency using the current definition is 54 erlangs/3.5 MHz/5000 mi^2, or 0.003 erlang/MHz/mi^2.

In a cellular system using these 70 channels with a seven-cell repeating pattern, a cell would have an average of 10 channels or a capacity of 4.6 erlangs per cell (two-percent blocking probability). In the AT&T cellular system [2] the largest hexagonal cell size proposed had an exterior radius of 4.2 mi and the smallest a radius of 1.05 mi. The corresponding areas of the hexagonal cells are 45.8 mi^2 and 2.9 mi^2, respectively. With the former, there would be 109 cells in the 5000 mi^2 capable of carrying a total of 500 erlangs (4.6 × 109) of traffic. The spectral efficiency is then 500 erlangs/3.5 MHz/5000 mi^2 or 0.031 erlang/MHz/mi^2. With the smallest cell, there would be 1724 cells with a total capacity of 7930 erlangs (4.6 × 1724). Thus the spectral efficiency is 7930 erlangs/3.5 MHz/5000 mi^2 or 0.45 erlang/MHz/mi^2. In summary form, then, the following table can be constructed:

System	Spectral Efficiency (erlang/MHz/mi^2)
Conventional	0.003
Cellular (largest cell)	0.031
Cellular (smallest cell)	0.45.

Use of this definition properly indicates the increasing spectral efficiencies of these three systems. If a comparison had been made strictly on the basis of the traffic capacity per megahertz or per channel, a very misleading result would have been obtained. In the conventional system, there are 15.4 erlangs/MHz (i.e., 54 erlangs/3.5 MHz), while in the first cellular system there are only 9.2 erlangs/MHz (i.e., 4.6 erlangs/0.5 MHz) in any given cell. While it is less efficient on a cell-by-cell comparison basis (the conventional system can be considered to have one large cell), the conventional system is clearly less efficient and has a considerably smaller ultimate capacity in the 5000-mi^2 region.

At least one problem still remains, however. The problem is associated with the geographic distribution of the mobiles (actually the communications traffic) to be served. Consider an extreme example where all of the traffic to be carried is concentrated in only seven adjacent cells in the cellular system composed of the smallest feasible cells. In this case, no frequency reuse is possible, and the cellular system is not as efficient as a conventional system. This is because the channels are trunked in groups of ten in the cellular system while, in the conventional system, all 70 are trunked in a single group. In a more realistic case, the demand may drop off sufficiently fast on the outskirts of a metropolitan area such that extensive frequency reuse is not required there. The ultimate capacity would then be determined by the capacity of the seven center cells, and it would occur at an average frequency-reuse factor less than the theoretical maximum. It should be noted that the cellular system would still present some advantages in spectral efficiency over and above the reuse multiple actually achieved. First, as total demand increased so that capacity at the geographic peak was exceeded, the performance (i.e., blocking probability) in only these few high traffic cells would deteriorate—not the performance over the entire region as is the case for a conventional system. Second, channels not used in the suburbs could be used for other services. Such reuse would be precluded by a conventional system.

In summary, the definition of spectral efficiency as erlangs per megahertz per square mile appears to be a useful one as long as the geographic distribution of demand is considered when comparing systems or projecting ultimate capacity. One remaining question concerns the spectral efficiency in using cellular systems for "all-call" or "group-call" dispatch service. This question is beyond the scope of this paper.

As a final note, it may be more effective to limit transmitter power and antenna height than to implicitly specify a reuse factor as part of a standard. If the maximums were low enough, it would force reuse to obtain reasonable coverage. This approach is essentially the method in use today, although in the bands affected, the limits do not force any appreciable frequency reuse. As congestion grows in a given area, the limits would have to be reduced appropriately.

REFERENCES

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