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OPTIMUM RECEPTION IN AN IMPULSIVE INTERFERENCE ENVIRONMENT

A. D. SPAULDING
DAVID MIDDLETON

U.S. DEPARTMENT OF COMMERCE
Rogers C. B. Morton, Secretary

Betsy Ancker-Johnson, Ph. D.
Assistant Secretary for Science and Technology

OFFICE OF TELECOMMUNICATIONS
John M. Richardson, Acting Director

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PREFACE

The following report is another in a series of ongoing studies whose general aims are:

(1) To provide quantitative, statistical descriptions of man-made (and natural) electromagnetic interference.

(2) To suggest and to guide experiments, not only to obtain experimental data for urban and other electromagnetic environments, but to provide, in addition, standard procedures for assessing such EM environments.

(3) To determine and predict system performance in these general electromagnetic milieux for the general purposes of spectral management and the establishment of appropriate data bases thereto.

With the help of (1) and (2), the interference characteristics of selected regions of the EM spectrum can be predicted, and with the results of (3), rational criteria of performance can be established for the successful or unsuccessful operations of communication links and systems in various classes of interference. The combination of (1)-(3) provides quantitative procedures for spectral management.

The man-made EM environment, and much of the natural one as well, is basically "impulsive," i.e., has a highly structural character, with noticeable probabilities of large interference levels, unlike the normal noise processes incoherent in transmitting and receiving elements. This impulsive character of the interference can drastically degrade the performance of conventional systems, which are designed to operate most effectively against the usually assumed normal background noise processes. The present report is devoted to the evaluation of the performance of both optimum and conventional receivers in a broad class of such "impulsive" (mostly man-made) electromagnetic interference. Specifically, class A interference* is considered here, where standard digital signal communications, both coherently and incoherently received, are employed. The new results obtained provide:

(1) Structures of optimum signal processors in class A EM environments.

(2) Performance bounds for such processors and performance estimates for similar, conventional receivers for the same communication tasks.

With these results, one has a quantitative basis for system design and comparison, including estimates of sizeable spectral savings potentially available when optimum receivers are employed. In addition, such results provide essential assistance in the design and application of the measuring equipments needed for other important components of spectral management,

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* Class A interference is characterized by a bandwidth less than that of the receiver.

viz, assessment of spectral usage, as well as the determination of the general EM environments of urban and other geographical regions.

Finally, we emphasize that it is the quantitative interplay between experimentally verified, analytical model-building of the electromagnetic environment and the evaluation of system performance which provides essential tools for prediction of performance, the development of adequate and appropriate data bases, standardization, and spectral assessment needed for effective management of the spectral-use environment.

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GLOSSARY OF PRINCIPAL SYMBOLS

A	The impulsive index for class A interference
A_α	The impulsive index for class B interference
α	A parameter for class B interference
α	A parameter (exponent) used in performance determination
α^*	The optimum value of α
APD	Amplitude probability distribution, $P(\epsilon > \epsilon_o)$
ARE	Asymptotic relative efficiency
a, a_i	Random amplitude of fading signal
A1	Amplitude modulation pulsed
A2	Two-tone pulsed amplitude modulation
A3	Amplitude modulated telephony
A3J	Single sideband voice, suppressed carrier
B_o	Envelope of interference waveform in receiver
$\langle B_o{}^2\rangle, \langle B_o{}^4\rangle, \ldots$	Even moments of received envelope
b	Random level of desired signal in interference discrimination problem
b_o	Subjectively determined threshold
ξ	Characteristic function variable
CFSK	Coherent frequency shift keying
CPSK	Coherent phase shift keying
ϵ	Normalized envelope of interference
ϵ_{rms}	rms of interference envelope
ε, ε_1, ε_2	Decision variables

$\mathrm{erf}(\cdot)$	Error function
$\mathrm{erfc}(\cdot)$	Complementary error function
E	Signal energy
${}_1F_1$	Confluent hypergeometric function
$F_1(i\xi,t)$	1st order characteristic function
F1	Frequency shift keying
F3	Frequency modulated telephony
$\Gamma(\cdot)$	Gamma function
Γ'	Ratio of Gauss to impulsive noise power
H_1, H_2	Hypotheses for binary detection
$I_i(\cdot)$	ith order modified Bessel function
$J_i(\cdot)$	ith order Bessel function (1st kind)
K	Threshold in likelihood ratio test
$\underline{\lambda}$	Coordinates
$\ell(x)$	The nonlinearity $d/dx \ln p_Z(x)$
L	A parameter given by $\int_{-\infty}^{\infty} [\ell/x)]^2 p_Z(x)\, dx$
LOBD	Locially optimum Bayes detector
$\Lambda(\underline{X})$	Likelihood ratio for vector of received data $\underline{X}$
N	Number of samples
NCFSK	Noncoherent frequency shift keying
ν_T	Average rate of signal generation
ω, ω_o, ...	Angular frequency
Ω	A parameter in Tikhonov distribution for signal phase
P	Poisson (as subscript)
$P + G$	Poisson plus Gauss (as subscript)

$P(\epsilon > \epsilon_o)$	Probability envelope ϵ exceeds the level ϵ_o
$p(\cdot)$	Probability density
pdf	Probability density function
$p_Z(z)$	pdf of interference
$p(\underline{X}\|H_1)$	The conditional probability of receiving the data $\underline{X}$, given H_1
$p_1(\underline{X})$	The denominator of $\Lambda(\underline{X})$
$p_2(\underline{X})$	The numerator of $\Lambda(\underline{X})$
PCM	Pulse code modulation (FM)
ϕ	Correlation between binary signals $S_1(t)$ and $S_2(t)$
ϕ	Phase of desired signal
ϕ_{zz}	The NxN covariance matrix of the interference z
P_e	Average probability of error
$\hat{P}_e$	A bound on average probability of error
Q	$\omega_o T$
q_1, q_2	A priori probabilities
Q(a,b)	Marcum's Q function
Q_M	The Q_M function
RE	Relative efficiency
$\rho(\underline{\lambda})$	The process density
$\rho(t)$	Autocovariance function
ρ, ρ_n	A parameter used in performance determination
S	Signal power
SNR	Signal-to-noise ratio

$S_1(t)$, $S_2(t)$	Desired signals
s_{1i}, s_{2i}	Samples of desired signals
$\underline{S}_1$, $\underline{S}_2$	Vector of samples of desired signals
$\langle \cdot \rangle$	Statistical average
$\sigma_G^{\ 2}$	Variance of Gaussian process
$\sigma_m^{\ 2}$	A variance for class A interference, $\sigma_m^{\ 2} = \left(\frac{m}{A} + \Gamma'\right)/(1 + \Gamma')$
T	The detection time
T_s	Time duration of interfering signal
$\overline{T}_s$	The mean value of T_s
$\underline{\Theta}$	Set of random signal parameters
$U(t; \underline{\lambda}, \underline{\Theta})$	Generic interference waveform
$\underline{X}$	A vector of N samples of received wave-form
Z(t)	The interference process
z	Normalized random interference process