where

$$\mu_{1c} = E[Y_{1c}|H_1] \quad ,$$

$$\sigma^2_{1c} = Var[Y_{1c}|H_1] \quad ,$$

and

$$\sigma^2_{1s} = Var[Y_{1s}|H_1] \quad . \tag{5.74}$$

Also, for $\varepsilon_2 = Y^2_{2c} + Y^2_{2s}$, we have

$$p_2(\varepsilon_2) = \frac{1}{2\sigma_{2c}\sigma_{2s}} \exp\left[-\frac{(\sigma^2_{2c}+\sigma^2_{2s})\varepsilon_2}{4\,\sigma^2_{2c}\sigma^2_{2s}}\right] I_o\left(\frac{|\sigma^2_{2c}-\sigma^2_{2s}|\varepsilon_2}{4\,\sigma^2_{2c}\sigma^2_{2s}}\right) \quad , \tag{5.75}$$

where

$$\sigma^2_{2c} = Var[Y_{2c}|H_1] \quad ,$$

and

$$\sigma^2_{2s} = Var[Y_{2s}|H_1] \quad . \tag{5.76}$$

For H_2 true, we obtain completely symmetrical results, so the probability of error is given by

$$P_e = P_e|H_1 = Prob[\varepsilon_1 < \varepsilon_2]$$

$$= \int_0^\infty p_2(\varepsilon_2) \int_0^{\varepsilon_2} p_1(\varepsilon_1)\, d\varepsilon_1 d\varepsilon_2 \quad . \tag{5.77}$$

In (5.77), the integral over ε_1 is simply the distribution function of ε_1, evaluated at ε_2. This is a known result that gives an infinite series of Q_M functions (see Omura and Kailath, 1965, p. 23). The remaining integration involving products of exponentials, Q_M functions, and I_o function can also be evaluated analytically (Nuttall, 1974). These rather complex results then require numerical evaluation.

However, instead of this, we can greatly simplify these relations with the following observations, again using our small signal (LOBD) assumption.

From (5.67), we have

$$\sigma_{1c}^2 = L \sum_{i=1}^{N} \cos^2\omega_1 t_i - 2SL \sum_{i=1}^{N} \cos^4\omega_1 t_i$$

$$\approx \frac{LN}{2} \left(1 - \frac{3SL}{2}\right) . \tag{5.78}$$

Likewise, from (5.68),

$$\sigma_{1s}^2 \approx \frac{LN}{2} \left(1 - \frac{SL}{2}\right) . \tag{5.79}$$

Now L and N are large, and from our small signal assumption, SL << 1, so that

$$\sigma_{1c}^2 \approx \sigma_{1s}^2 \approx \frac{NL}{2} \equiv \sigma_1^2 , \tag{5.80}$$

and, therefore,

$$p_1(\varepsilon_1) \approx \frac{1}{2\sigma_1^2} \exp\left(- \frac{\varepsilon_1 + \mu_{1c}^2}{2\sigma_1^2}\right) I_o \left(\frac{\sqrt{\varepsilon_1}\ \mu_{1c}}{\sigma_1^2}\right) . \tag{5.81}$$

For σ_{2c} and σ_{2s}, from (5.70) and (5.72), using

$$\sum_{i=1}^{N} (\cos\omega_1 t_i + \cos\omega_2 t_i)^2 = \sum_{i=1}^{N} \frac{1}{4} \cos^2(\omega_1+\omega_2)t_i$$

$$+ \sum_{i=1}^{N} \frac{1}{4} \cos^2(\omega_1-\omega_2)t_i$$

$$+ \sum_{i=1}^{N} \frac{1}{2} \cos(\omega_1+\omega_2)t_i \cos(\omega_1-\omega_2)t_i$$

$$\approx \frac{N}{4} \quad , \tag{5.82}$$

where the third summation in (5.82) is ≈ 0, we obtain

$$\sigma_{2c}^2 \approx \frac{LN}{2} (1 - 2SL) \quad . \tag{5.83}$$

Likewise, in similar fashion,

$$\sigma_{2s}^2 \approx \frac{LN}{2} (1 - 2SL) \approx \sigma_{2c}^2 \equiv \sigma_2^2 \quad . \tag{5.84}$$

Therefore, we have

$$p_2(\varepsilon_2) \approx \frac{1}{2\sigma_2^2} \exp\left(- \frac{\varepsilon_2}{2\sigma_2^2}\right) . \tag{5.85}$$

Now, from (5.77), we get

$$P_e \approx \int_o^\infty \frac{1}{2\sigma_2^2} \exp\left(- \frac{\varepsilon_2}{2\sigma_2^2}\right) \left\{\int_o^{\varepsilon_2} p_1(\varepsilon_1)\, d\varepsilon_1\right\} d\varepsilon_2 \ . \tag{5.86}$$

Evaluating the ε_1 integration, where $p_1(\varepsilon_1)$ is given by (5.81), we get

$$P_e \approx \int_0^\infty \frac{1}{2\sigma_2^2} \exp\left(-\frac{\varepsilon_2}{2\sigma_2^2}\right) \left\{1 - Q\left(\frac{\mu_{1c}}{\sigma_1}, \frac{\sqrt{\varepsilon_2}}{\sigma_1}\right)\right\} d\varepsilon_2 \quad , \tag{5.87}$$

where Q is the Marcum Q function, i.e.,

$$Q(a,b) \equiv \int_b^\infty x \exp\left(-\frac{x^2+a^2}{2}\right) I_o(ax)\, dx \quad . \tag{5.88}$$

We obtain, evaluating (5.87), using (8) or (13) of Nutall (1975),

$$P_e \approx \frac{1}{1 + \sigma_1^2/\sigma_2^2} \exp\left[-\frac{\mu_{1c}^2}{2\sigma_1^2} \left(\frac{1}{1 + \sigma_2^2/\sigma_1^2}\right)\right] \quad . \tag{5.89}$$

Since SL << 1, from (5.84), we can use $\sigma_1^2 \approx \sigma_2^2 \approx NL/2$, and from (5.68),

$$\mu_{1c} \approx -\frac{NL}{2}\sqrt{2S} \quad , \tag{5.90}$$

to obtain

$$p_e \approx \frac{1}{2} \exp\left[-\frac{NLS}{4}\right], \quad (SL << 1) \quad . \tag{5.91}$$

The result (5.91) gives us our estimate of performance for the incoherent threshold receiver for the signals (5.62) (NCFSK). In section 4.1.1 we obtained corresponding results for the coherent case. The result (4.69) is for coherent antipodal signals (CPSK), i.e., $\phi = -1$, but is easily modified for CFSK signals ($\phi = 0$). Figure 5.6 shows the NCFSK performance (from 5.91) and the corresponding performance for CFSK for our case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$ (i.e., $L = 1340$).

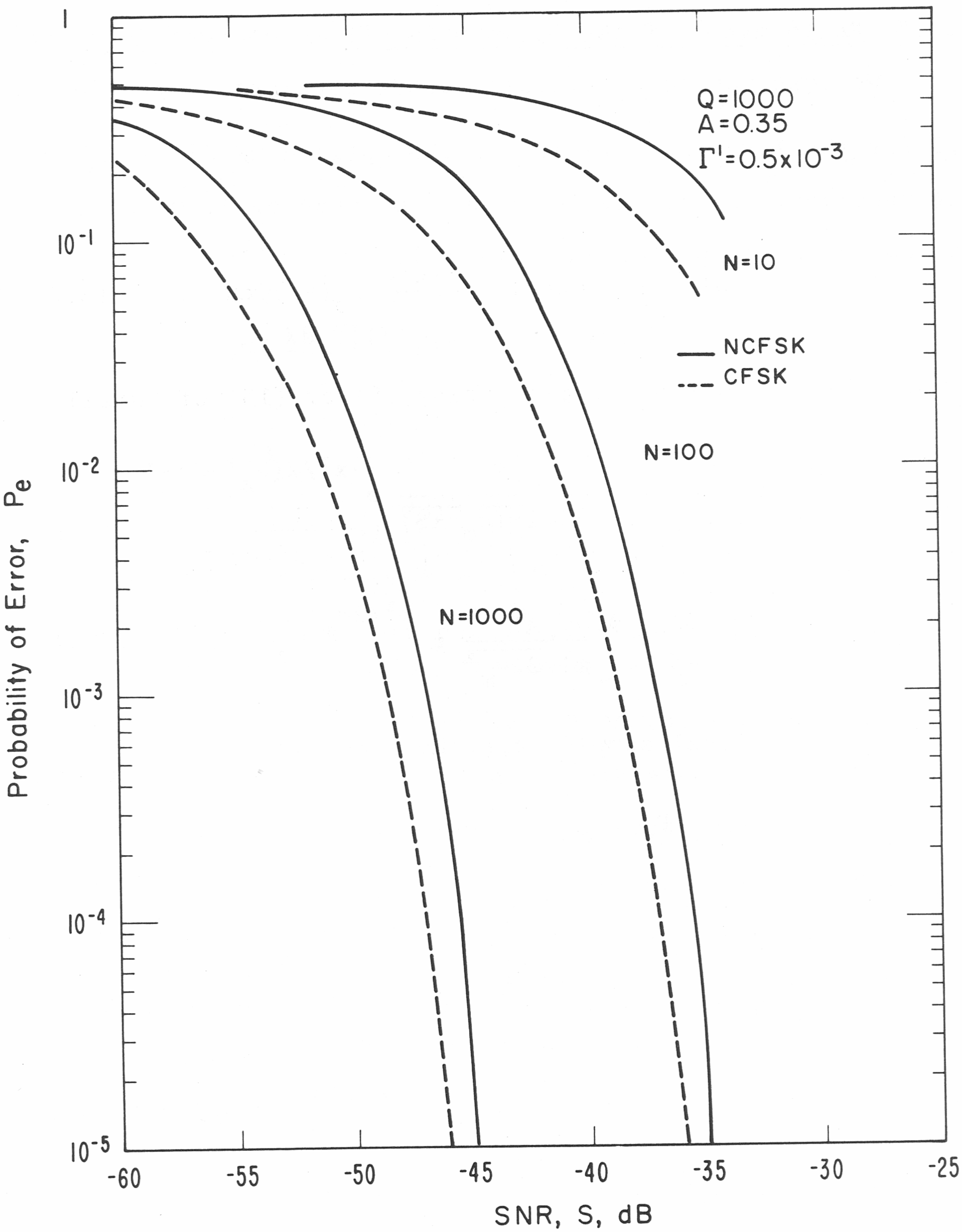

Figure 5.6. Performance of threshold receiver for binary NCFSK (from 5.91) and for binary CFSK (from sec. 4.1.1) for the class A interference case A = 0.35 and $\Gamma' = 0.5 \times 10^{-3}$.

5.3.2 Performance of the Optimum ON-OFF Incoherent Receiver

We turn now to the result (5.60) obtained in section 5.2 for the likelihood ratio for the optimum ON-OFF incoherent receiver. Using the techniques of chapter 4, we obtain for our bound on performance for K=1,

$$P_e \le \frac{1}{2} \int_{\underline{X}} [p_2(\underline{X})]^{1-\alpha} \, [p_1(\underline{X})]^{\alpha} \, d\underline{X} \quad , \tag{5.92}$$

or

$$P_e \le \frac{1}{2} \prod_{i=1}^{N} \int_{-\infty}^{\infty} \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi(\sigma_m{}^2+S)}} \, e^{-x^2/2(\sigma_m{}^2+S)} \right]^{1-\alpha}$$

$$\times \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m{}^2}} \, e^{-x^2/2\sigma_m{}^2} \right]^{\alpha} dx \ . \tag{5.93}$$

Since now all signal samples are the same, we have

$$P_e \le \frac{1}{2}[I_{\alpha*}(S)]^N \quad , \tag{5.94}$$

where

$$I_{\alpha*}(S) = 2 \int_{0}^{\infty} \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi(\sigma_m{}^2+S)}} \, e^{-x^2/2(\sigma_m{}^2+S)} \right]^{1-\alpha*}$$

$$\times \left[\sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m{}^2}} \, e^{-x^2/2\sigma_m{}^2} \right]^{\alpha*} dx \quad , \tag{5.95}$$

and $\alpha*$ is the value of α that minimizes the right hand side of (5.93). The proper $\alpha*$ depends on S, so that the computer evaluation also involves finding $\alpha*$ for each S. The following table gives $\alpha*$ for various S, obtained by computer search.

Table 5.1. The Value of α^* for Various Signal Levels S

S(dB)	α^*	S(dB)	α^*	S(dB)	α^*
30	.30	0	.57	-30	.45
25	.38	- 5	.54	-35	.47
20	.46	-10	.50	-40	.485
15	.53	-15	.47	-45	.49
10	.58	-20	.44	-50	.495
5	.59	-25	.43		

Figure 5.7 shows the estimate P_e of performance for N=10 from (5.94). Also shown is the performance bound $\hat{P}_e$ obtained in chapter 4 for coherent ON-OFF signaling (table 4.1). Figure 5.8 shows the performance bound and performance estimate (coherent and incoherent) for N=10, 100, and 1000. On these figures the SNR is given by S/2; i.e., the average signal power in the two signals, one signal being zero. The results indicate that the incoherent optimum receiver performs substantially worse than the coherent optimum detector. However, our estimate P_e of performance for the incoherent case is quite likely much cruder than the coherent bound $\hat{P}_e$, especially for small N.

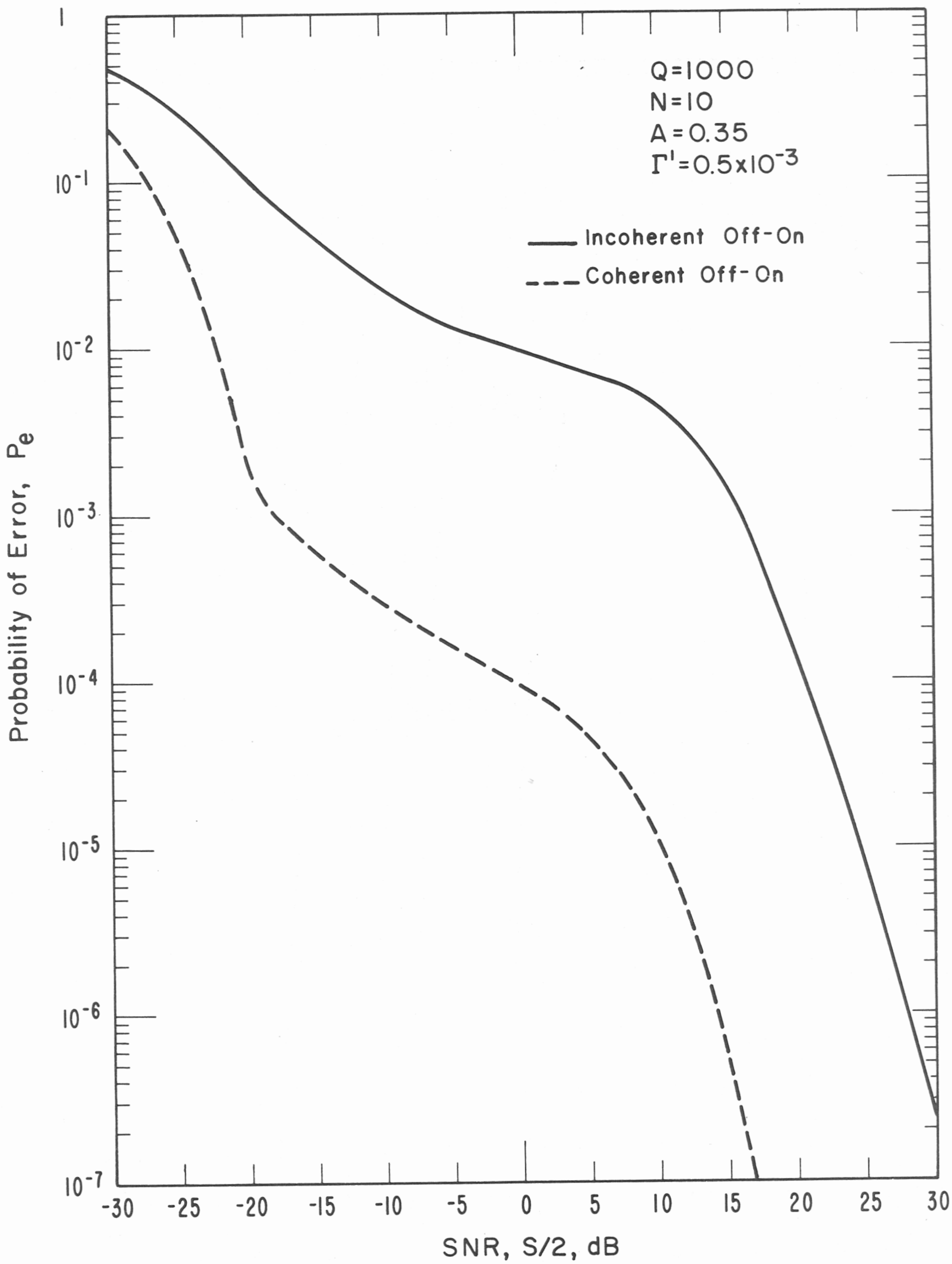

Figure 5.7. Performance estimate for the optimum incoherent ON-OFF receiver (from 5.94) and the performance bound for the optimum coherent ON-OFF receiver (from table 4.1) for the interference case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$ and for $N = 10$.

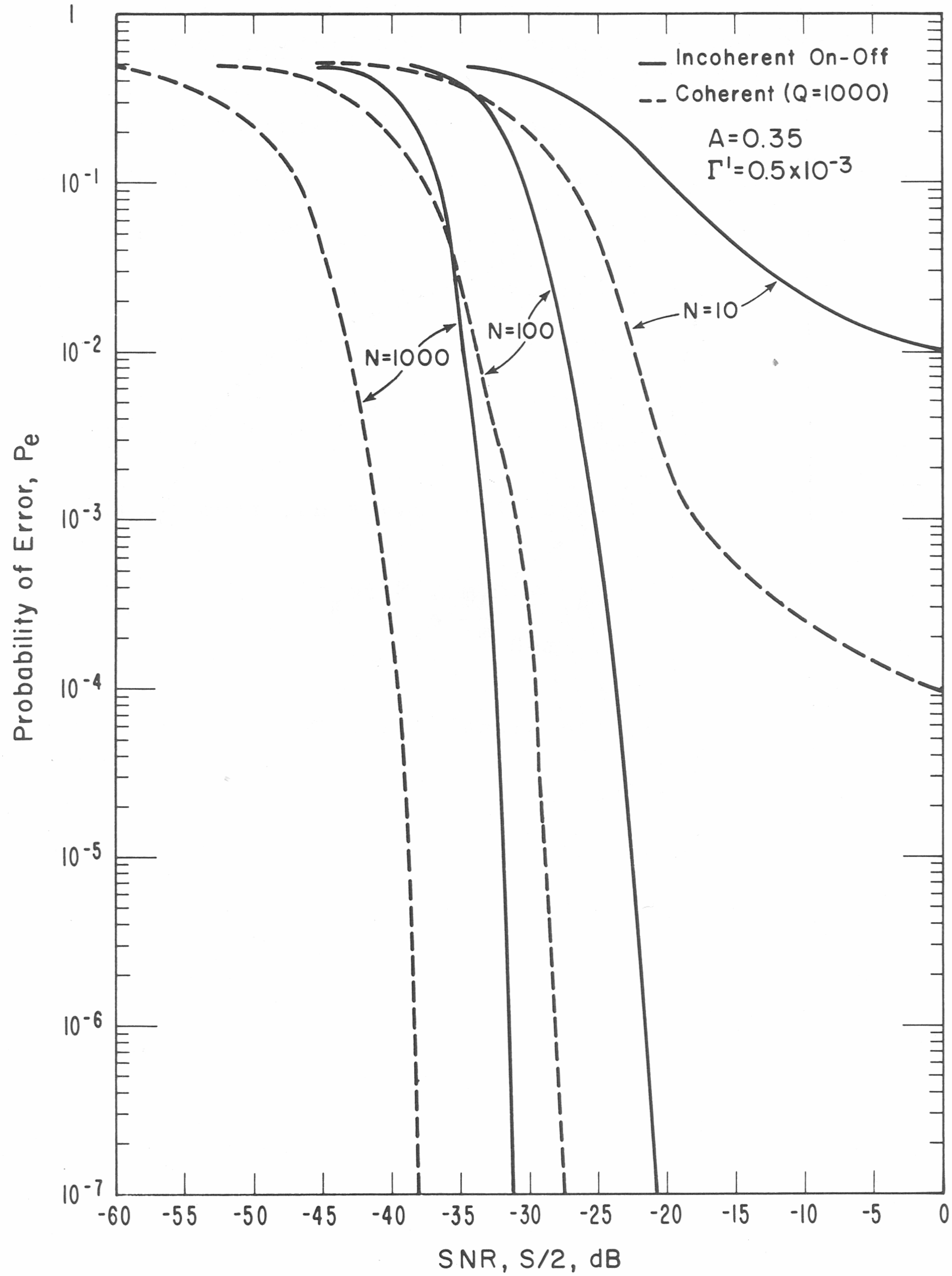

Figure 5.8. Performance estimate for the optimum incoherent ON-OFF receiver (from 5.94) and the performance bound for the optimum coherent ON-OFF receiver (from table 4.1) for the interference case $A = 0.35$, $\Gamma' = 0.5 \times 10^{-3}$ and for $N = 10$, 100, and 1000.

5.3.3 Performance of Noncoherent Correlation Receivers in Class A Interference

It remains now to compute the performance of the current suboptimum correlation receivers in class A "impulsive" interference. The performance for binary NCFSK is quite easy to obtain. For arbitrary interference, Montgomery (1954) has shown that the probability of error is given by

$$P_e = \frac{1}{2}\,\text{Prob[rms noise envelope > rms signal amplitude]}. \quad (5.96)$$

In our class A interference, this gives us, from (4.47),

$$P_e = \frac{1}{2} e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-S/2\sigma_m^2} , \qquad (5.97)$$

where, as before, S is the signal power (and also the SNR because of our normalization).

Figure 5.9 shows the performance of the standard (i.e., optimum in Gauss) NCFSK receiver, from (5.97), for $\Gamma' = 1\times10^{-4}$ and A = 0.01, 0.1, 1, and 10. Also shown is the standard CFSK performance from (4.51) of chapter 4, k=2. As expected, for the standard receivers, NCFSK results only in a small degradation of performance compared to CFSK for large S (small P_e).

The performance of the suboptimum incoherent ON-OFF system is more difficult to determine. The situation is shown in the following diagram.

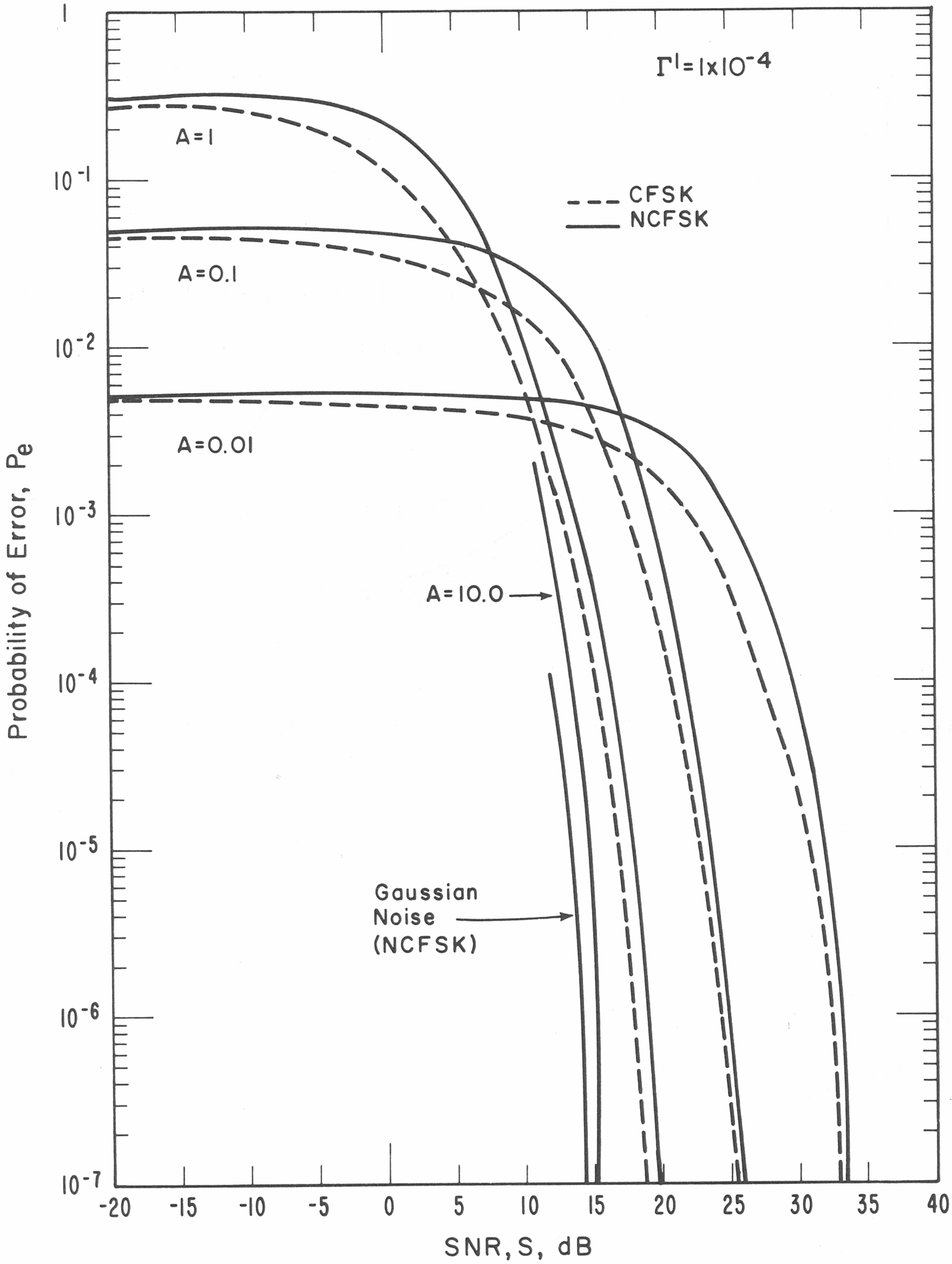

Figure 5.9. Performance of NCFSK correlation receiver (from 5.97) and CFSK correlation receiver (from 4.51) in class A interference for $\Gamma' = 1 \times 10^{-4}$ and A = 0.01, 0.1, 1, and 10.

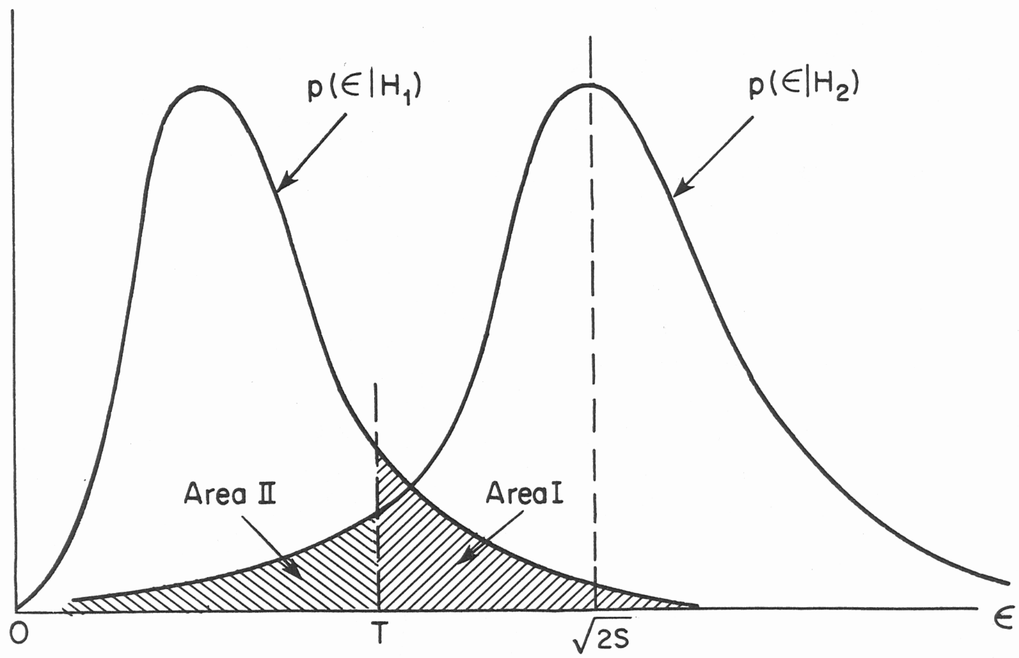

Figure 5.10. Probability of error for incoherent ON-OFF correlation receiver.

In figure 5.10 the probability of error is given by

$$P_e = \frac{1}{2}\,[\text{Area I} + \text{Area II}] \quad , \tag{5.98}$$

where the threshold T depends on signal size S and is set to minimize P_e. From (4.48) we have

$$p(\epsilon|H_1) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left(\frac{2\epsilon}{\sigma_m^2}\right) e^{-\epsilon^2/\sigma_m^2} \quad , \tag{5.99}$$

and for the pdf of the envelope of signal plus noise, we get

$$p(\epsilon|H_2) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left[\frac{2\epsilon}{\sigma_m^2}\, e^{-\frac{\epsilon^2+2S}{\sigma_m^2}}\, I_o\left(\frac{2\epsilon\sqrt{2S}}{\sigma_m^2}\right)\right] \quad . \tag{5.100}$$

Therefore,

$$\text{Area I} = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-T^2/\sigma_m^2} , \qquad (5.101)$$

and

$$\text{Area II} = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left(\frac{2}{\sigma_m^2}\right) e^{-2S/\sigma_m^2} \int_0^T \epsilon \, e^{-\epsilon^2/\sigma_m^2}$$

$$\times I_0 \left(\frac{2\epsilon\sqrt{2S}}{\sigma_m^2}\right) d\epsilon . \qquad (5.102)$$

Using (10) of Nuttall (1972) to evaluate the integral in (5.102), substituting (5.101) and (5.102) into (5.98) and simplifying, we obtain

$$P_e = \frac{1}{2}\left\{1 + e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left[e^{-T^2/\sigma_m^2} - Q\left(\frac{2\sqrt{S}}{\sigma_m}; \frac{T\sqrt{2}}{\sigma_m}\right)\right]\right\}. \qquad (5.103)$$

The calculation of performance for the suboptimum ON-OFF system requires numerical evaluation of (5.103). [We will not do such computations here.]

However, to compare performance of the standard suboptimum incoherent ON-OFF receiver with that of our optimum receiver, we will compare the performance of the coherent ON-OFF correlation receiver with our optimum incoherent ON-OFF receiver. This is a reasonable approximation (for lack of numerical results from (5.103)), since we have seen that for the normal correlation receivers in class A interference, there is little difference between coherent and incoherent performance for small P_e (e.g., fig. 5.9).

Figure 5.11 shows, for N=100, the performance of the coherent ON-OFF correlation receiver from (4.51) versus the estimated performance of the optimum incoherent receiver from section 5.3.2 for our example $\Gamma' = 0.5 \times 10^{-3}$ and $A = 0.35$. As in the coherent cases, these results indicate that substantial improvement can be achieved. For the example of figure 5.11, this improvement is on the order of 25 dB.

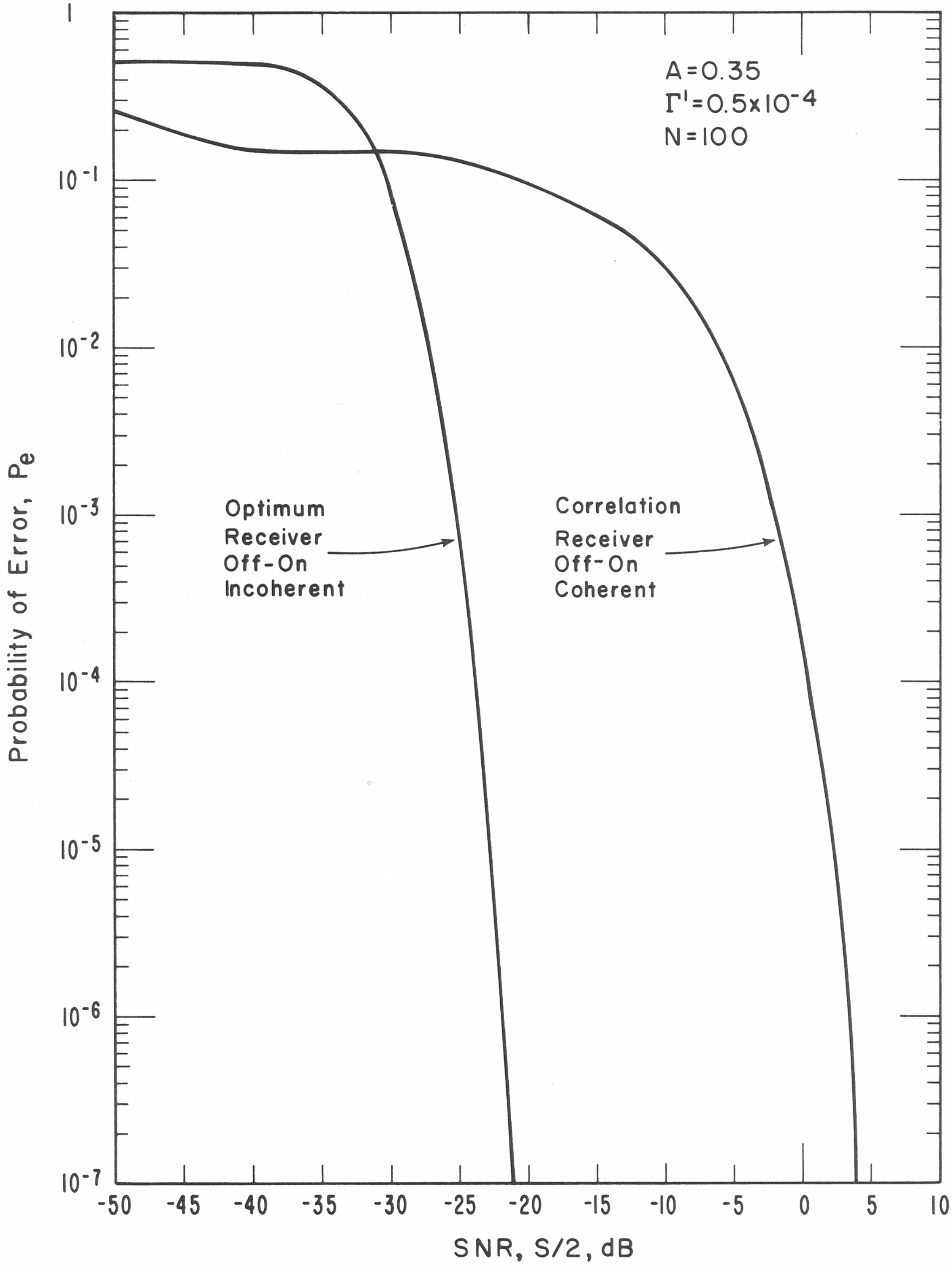

Figure 5.11. Performance of the suboptimum coherent ON-OFF correlation receiver (from 4.51) and the estimated performance of the optimum incoherent ON-OFF receiver (from sec. 5.3.2) for the class A interference case A = 0.35, $\Gamma' = 0.5 \times 10^{-3}$, for N = 100.