the first consists of slowly varying terms $(\omega_2 t_i - \omega_2 t_j)$, while the second consists of rapidly oscillating or "double frequency" terms $(\omega_2 t_i + \omega_2 t_j)$ which largely cancel each other. Hence the second double summation can be neglected compared to the first [see Hancock and Wintz (1966), chapter 3].

The result is, with both the 1st order and 2nd order terms, a receiver that is the weighted sum of our coherent threshold receiver and our incoherent threshold receiver. The receiver, for K=1, in block diagram form, is sketched in the following figure:

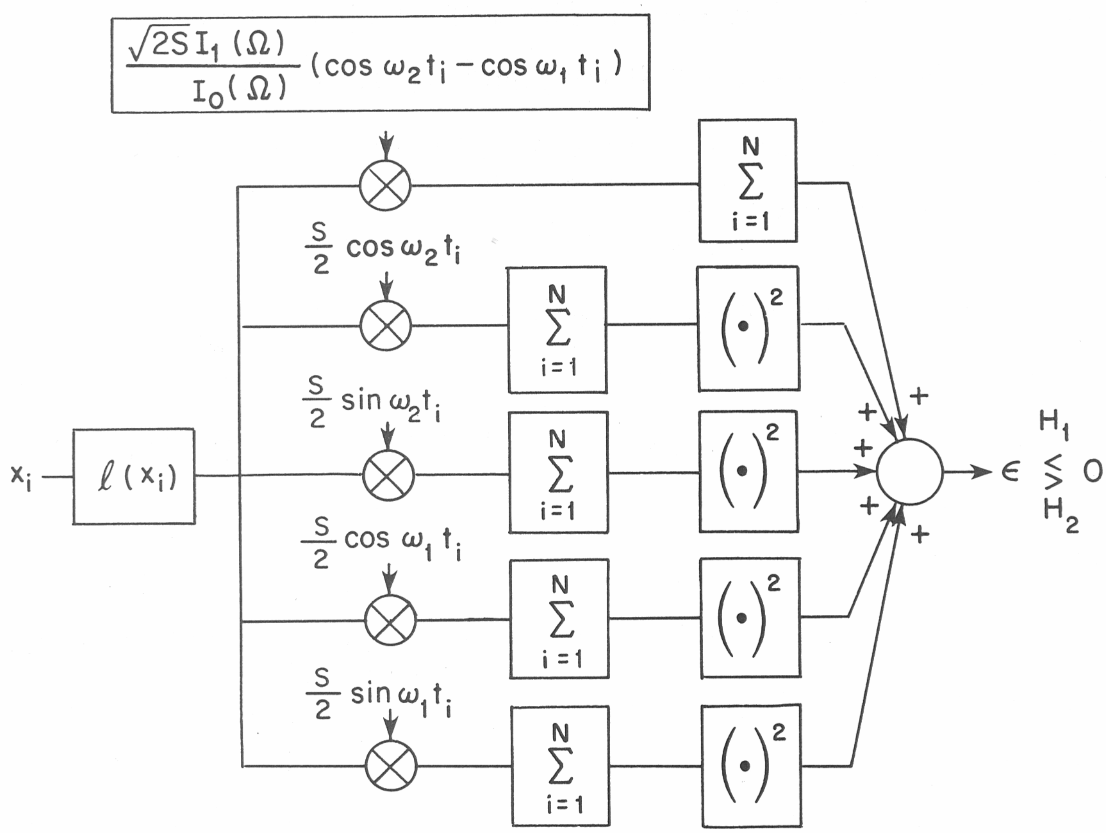

Figure 5.4. Second order threshold receiver for NCFSK with phase estimation.

In figure 5.4 we note that for $\Omega=0$, the receiver reduces to that obtained earlier (fig. 5.1) for uniformly distributed phase and as $\Omega\to\infty$, we obtain the coherent receiver also obtained earlier in section 4.4 (with the addition of the 2nd-order terms).

In studying optimum reception of coded multiple frequency keying with random variable phases, Nesenbergs (1971) has shown, for Gaussian noise, that the appropriate receiver is also a weighted sum of the standard linear (coherent) receiver and the standard quadrature (incoherent) receiver. We have shown that this is a completely canonical result in that we always get this type of LOBD receiver for partially known phase, since the results were obtained for arbitrary pdf of the interference.

5.2 General Incoherent Signals

In the previous section we obtained threshold or locally optimum receivers for a variety of binary incoherent and partially coherent problems. Here, we want to look at the general composite hypothesis testing problem; i.e., without any small signal assumptions. Generally, we will not be able to obtain any obvious receiver structures, but we will be able to obtain expressions that can be useful in determining a bound on optimum performance. The techniques developed will be helpful in any composite hypothesis testing problem, such as the interference discrimination problem of section 4.4.2, as well as the previous incoherent problems (sec. 5.1). The technical difficulties lie in performing the averages required by our random parameter vector $\underline{\theta}$ in the likelihood ratio. Let us look at class A interference specifically in our effort to establish performance bounds.

Our pdf for this interference is

$$p_Z(z) = \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-z^2/2\sigma_m^2} , \qquad (5.38)$$

where

$$\sigma_m^2 = \frac{m/A + \Gamma'}{1 + \Gamma'} . \qquad (5.39)$$

This can be written as (see Appendix A)

$$p_Z(z) = \frac{e^{-A}}{2\pi} \int_{-\infty}^{\infty} e^{j\xi z} e^{-\frac{\xi^2}{2}\left(\frac{\Gamma'}{1+\Gamma'}\right)} \exp\left[Ae^{-\frac{\xi^2}{2A(1+\Gamma')}}\right] d\xi . \qquad (5.40)$$

Our Nth order distribution, with independent samples, can therefore be written as an N-fold integral. The expression (5.40) is, of course, obtained by expressing $p_Z(z)$ via the transform of its characteristic function $F_1(j\xi)$, where [Middleton (1974)]

$$F_1(j\xi) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-\frac{\xi^2\sigma_m^2}{2}} . \qquad (5.41)$$

Consider, for example, the coherent interference discrimination problem of section 4.4.2. Using (5.40), we get

$$\Lambda(\underline{X}) = \frac{p_2(\underline{X})}{p_1(\underline{X})} , \qquad (5.42)$$

where

$$p_2(\underline{X}) = \int_0^{b_o} \frac{e^{-NA}}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{j \sum_{i=1}^{N} \xi_i x_i} e^{-jb \sum_{i=1}^{N} \xi_i s_i}$$

$$\times e^{-\frac{\Gamma'}{2(1+\Gamma')} \sum_{i=1}^{N} \xi_i^2} \exp\left[A \sum_{i=1}^{N} e^{-\frac{\xi_i^2}{2A(1+\Gamma')}}\right]$$

$$\times p(b)\, d\xi \ldots d\xi_N\, db \quad , \qquad (5.43)$$

with a corresponding expression for $p_1(\underline{X})$. We can now interchange the integration operations and perform the required average over b. What we would like is for p(b) and b_o to be such that once the average is performed in (5.43), the result can again be reduced to a product of one dimensional integrals so that the algorithms developed in section 4.2 can be applied. While we could, undoubtedly, invent a p(b) and b_o that would "work," it is doubtful that such an exercise would have much physical significance. However, once a real problem of this type is given, i.e., where we know an appropriate p(b) and b_o, then the problem can be attacked either via the threhold receiver (4.76) or via (5.43). The results using (5.43) could, in principle, be used in performance calculations.

We now return to the incoherent problem (5.3) in which the phase is uniformly distributed. For this case, we have

$$p_2(\underline{X}) = \frac{1}{2\pi}\int_{\pi}^{\pi}\frac{e^{-NA}}{(2\pi)^N}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} e^{j\sum_{i=1}^{N}\xi_i x_i}$$

$$\times \quad e^{-j\sum_{i=1}^{N}\xi_i\sqrt{2S}\cos(\omega_2 t_i+\phi)}\; e^{-\frac{\Gamma'}{2(1+\Gamma')}\sum_{i=1}^{N}\xi_i{}^2}$$

$$\times \quad \exp\left[A\sum_{i=1}^{N} e^{-\frac{\xi_i{}^2}{2A(1+\Gamma')}}\right]d\xi_1\ \ldots\ d\xi_N\ d\phi\ . \quad (5.44)$$

The required average in (5.44), denoted by I, is then

$$I = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\sum_{i=1}^{N}\xi_i\sqrt{2S}\cos(\omega_2 t_i+\phi)}\,d\phi\ . \qquad (5.45)$$

This is a well-known integral, and so we get

$$I = J_o(\sqrt{L_c^2 + L_s^2}) \quad , \qquad (5.46)$$

where

$$L_c = \sum_{i=1}^{N}\xi_i\ \sqrt{2S}\cos\omega_2 t_i \quad ,$$

and

$$L_s = \sum_{i=1}^{N}\xi_i\ \sqrt{2S}\sin\omega_2 t_i \quad . \qquad (5.47)$$

We now remark that if the small signal approximation

$$J_o(x) \approx 1 - \frac{x^2}{4} \qquad (5.48)$$

is used, we obtain

$$p_2(\underline{X}) = \frac{e^{-NA}}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{j\sum_{i=1}^{N} \xi_i x_i} \left(1 - \frac{L_c^2 + L_s^2}{4}\right)$$

$$\times\, e^{-\frac{\Gamma'}{2(1+\Gamma')} \sum_{i=1}^{N} \xi_i^2} \exp\left[A \sum_{i=1}^{N} e^{-\frac{\xi_i^2}{2A(1+\Gamma')}}\right]$$

$$\times\, d\xi_1 \ldots d\xi_N \quad . \qquad (5.49)$$

This gives us

$$p_2(\underline{X}) = p_Z(\underline{X}) - \frac{e^{-NA}}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{j\sum_{i=1}^{N} \xi_i x_i}$$

$$\times \left[\sum_{i=1}^{N} \sum_{k=1}^{N} \xi_i \xi_k \frac{S}{2} \cos(\omega_2 t_i - \omega_2 t_k)\right] e^{-\frac{\Gamma'}{2(1+\Gamma')} \sum_{i=1}^{N} \xi_i^2}$$

$$\times \exp\left[A \sum_{i=1}^{N} e^{-\frac{\xi_i^2}{2A(1+\Gamma')}}\right] d\xi_1 \ldots d\xi_N \quad . \qquad (5.50)$$

Looking at the integrations in (5.50), we see that for i=k, we obtain for the N fold integration of the double sum

$$\sum_{i=1}^{N} \prod_{\substack{k=1 \\ k\neq i}}^{N} p_Z(x_k) \frac{e^{-A}}{2\pi} \int_{-\infty}^{\infty} \xi_k^2\, e^{j\,\xi_k x_k} F_1(j\xi_k)\, d\xi_k \qquad (5.51)$$

or

$$\sum_{i=1}^{N} \prod_{\substack{k=1\\k\neq 1}}^{N} p_Z(x_k) \frac{d^2}{dx_k^2} p_Z(x_k) \quad . \tag{5.52}$$

Likewise, for $i \neq k$ we obtain

$$- \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{\substack{\ell=1\\ \ell\neq i\\ \ell\neq k}}^{N} p_Z(x_k) \frac{e^{-A}}{2\pi} \int_{-\infty}^{\infty} \xi_i \, e^{j\,\xi_i x_i} F_1(j\,\xi_i)\, d\xi_i$$

$$\times \frac{e^{-A}}{2\pi} \int_{-\infty}^{\infty} \xi_k \, e^{j\,\xi_k x_k} F_1(j\,\xi_k)\, d\xi_k \quad , \tag{5.53}$$

or

$$\sum_{i=1}^{N} \sum_{k=1}^{N} \prod_{\substack{\ell=1\\ \ell\neq i\\ \ell\neq k}}^{N} p_Z(x_\ell) \frac{d}{dx_i} p_Z(x_i) \frac{d}{dx_k} p_Z(x_k) \quad . \tag{5.54}$$

In short, we see that upon dividing by the $p_Z(\underline{X})$ in (5.50), we obtain, as expected, the identical results (5.10) obtained in the last section for a incoherent threshold receiver.

Rather than using the weak signal approximation to $J_o(x)$ given in (5.48), we can use the steepest-decent approximation

$$J_o(x) \approx e^{-x^2/4}, \quad (\approx 1 - \frac{x^2}{4},\ x^2 << 1) \quad . \tag{5.55}$$

This will yield correct behavior for <u>both</u> large and small values of signal amplitude [via Middleton (1974), p. 27]. Now

$$L_c^2 + L_s^2 = \sum_{k=1}^{N} \sum_{i=1}^{N} \xi_i \xi_k \; 2S \cos \omega_2(t_i - t_k) \quad . \qquad (5.56)$$

For i=k,

$$L_c^2 + L_s^2 = 2S \sum_{i=1}^{N} \xi_i^2 \quad , \qquad (5.57)$$

and for $i \neq k$, we have the summation of oscillating terms. If N is large and, correspondingly, these oscillating terms contain many oscillations in the detection period [0,T], they will largely cancel and we can neglect the $i \neq k$ terms when compared to the i=k terms. Making this assumption destroys any insight into receiver structure, but will be useful in some performance calculations. Doing this, then, we obtain from (5.57) for (5.55) in (5.46) in (5.44), the incoherent result for (5.44)

$$p_2(\underline{X}) = \frac{e^{-NA}}{(2\pi)^N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{j \sum_{i=1}^{N} \xi_i x_i} \; e^{-\frac{S}{2} \sum_{i=1}^{N} \xi_i^2}$$

$$\times \; e^{-\frac{\Gamma'}{2(1+\Gamma')} \sum_{i=1}^{N} \xi_i^2} \exp\left[A \sum_{i=1}^{N} e^{-\frac{\xi_i^2}{2A(1+\Gamma')}}\right]$$

$$\times \; d\xi_1 \ldots d\xi_N \quad . \qquad (5.58)$$

This reduces to

$$p_2(\underline{X}) = \prod_{i=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!} \frac{1}{\sqrt{2\pi(\sigma_m^2 + S)}} \, e^{-x_i^2/2(\sigma_m^2+S)} . \qquad (5.59)$$

[Note that $p_2(\underline{X})$ is a proper pdf, i.e., integrates to 1.]

Using (5.59), we find that the likelihood ratio for the ON-OFF incoherent case now becomes

$$\Lambda(\underline{X}) = \frac{\prod_{i=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!} \frac{1}{\sqrt{2\pi(\sigma_m^2+S)}} e^{-x_i^2/2(\sigma_m^2+S)}}{\prod_{i=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!} \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{-x_i^2/2\sigma_m^2}} \underset{H_2}{\overset{H_1}{\lessgtr}} K. \quad (5.60)$$

We will use (5.60) in the next section to compute performance for the ON-OFF incoherent case (for K=1).

5.3 Determination of Incoherent Performance

Having obtained LOBD receiver structures for arbitrary interference, we now wish to evaluate the performance of these receivers. Canonical results for the performance of the LOBD structures can be obtained, and we then will apply these results to our case of class A interference and compare performance with that obtained in chapter 4 for coherent LOBD receivers. We obtained an approximation to the likelihood ratio (5.60) for the optimum incoherent ON-OFF system. We will use this to determine performance and then compare this performance with that of the corresponding coherent system. The performance of the suboptimum incoherent correlation receivers can also be computed for our class A interference.

5.3.1 Performance of Threshold Receiver for Binary NCFSK

In this section we use the receiver structures and results obtained in sections 5.1 and 5.2 to determine the performance of our incoherent receiver. We start with the detection situation

$$H_1: \quad X(t) = Z(t) + S_1(t,\phi), \quad 0 \le t < T$$
$$H_2: \quad X(t) = Z(t) + S_2(t,\phi), \quad 0 \le t < T \quad , \tag{5.61}$$

where

$$S_1(t,\phi) = \sqrt{2S}\cos(\omega_1 t + \phi)$$
$$S_2(t,\phi) = \sqrt{2S}\cos(\omega_2 t + \phi) \quad , \tag{5.62}$$

and the phase ϕ is uniformly distributed. For the threshold K=1, the appropriate threshold (small signal) receiver is sketched in the following figure.

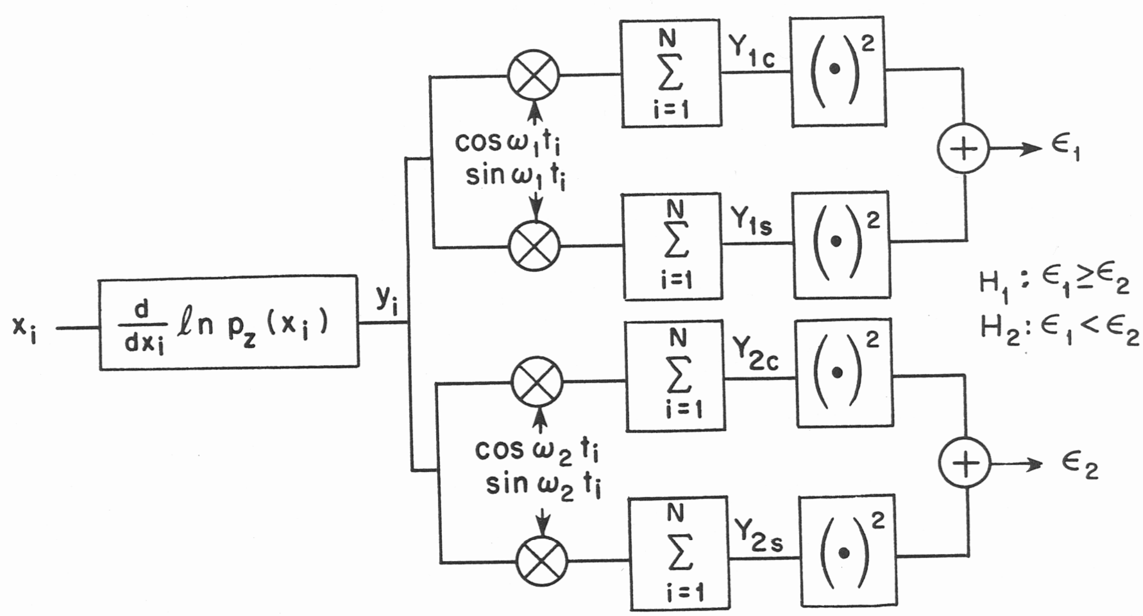

Figure 5.5. LOBD for binary NCFSK signals (5.62), K=1.

As was discussed in chapter 4, the action of the nonlinearity $\ell(x)$ allows us to obtain an estimate of performance via the Central Limit Theorem. That is, we can allow Y_{1c}, Y_{1s}, Y_{2c}, and Y_{2s} in figure 5.5 to be (asymptotically) normally distributed (see sec. 4.4.1).

Suppose H_1 is true, so that $x_i = z_i + s_{1i}$, $s_{1i} = \sqrt{2S} \cos(\omega_1 t_i + \phi)$, and $P_e|H_1 = \text{Prob}[\varepsilon_1 < \varepsilon_2]$. The squaring and summation operations in the receiver (fig. 5.5) result in receiver operation being independent of the unknown phase ϕ. We therefore can set $\phi = 0$. We now use the results of chapter 4, equations (4.61) and (4.65), namely,

$$E[y_i|H_1] = -s_{1i} L \quad , \tag{5.63}$$

and

$$\text{Var}[y_i|H_1] = L - s_{1i}^2 L^2 \quad , \tag{5.64}$$

where L was defined and evaluated in chapter 4, (4.62) and (4.70). For the top branch of the receiver, we have

$$\begin{aligned} E[Y_{1c}|H_1] &= -L \sum_{i=1}^{N} s_{1i} \cos \omega_1 t_i \\ &= -L \sqrt{2S} \sum_{i=1}^{N} \cos^2\omega_1 t_i \quad , \end{aligned} \tag{5.65}$$

and

$$\text{Var}[Y_{1c}|H_1] = \sum_{i=1}^{N} (L - 2SL^2\cos^2\omega_1 t_i) \cos^2\omega t_i \quad . \tag{5.66}$$

For Y_{1s}, we have

$$E[Y_{1s}|H_1] = -L\sqrt{2S} \sum_{i=1}^{N} \cos\omega_1 t_i \sin\omega_1 t_i$$

$$= -\frac{L\sqrt{2S}}{2} \sum_{i=1}^{N} \sin 2\omega_1 t_i \approx 0 \quad , \tag{5.67}$$

and

$$Var[Y_{1s}|H_1] = \sum_{i=1}^{N} (L - 2SL^2\cos^2\omega_1 t_i) \sin^2\omega_1 t_i$$

$$= \sum_{i=1}^{N} (L \sin^2\omega_1 t_i - SL^2\sin^2 2\omega_1 t_i) \ . \tag{5.68}$$

For the lower, ω_2, branch of the receiver, we get

$$E[Y_{2c}|H_1] = -L\sqrt{2S} \sum_{i=1}^{N} \cos\omega_1 t_i \cos\omega_2 t_i \approx 0 \quad , \tag{5.69}$$

$$Var[Y_{2c}|H_1] = \sum_{i=1}^{N} (L - 2SL^2 \cos^2\omega_1 t_i) \cos^2\omega_2 t_i \quad , \tag{5.70}$$

$$E[Y_{2s}|H_1] = -L\sqrt{2S} \sum_{i=1}^{N} \cos\omega_1 t_i \sin\omega_2 t_i \approx 0 \quad , \tag{5.71}$$

and

$$Var[Y_{2s}|H_1] = \sum_{i=1}^{N} (L - 2SL^2\cos^2\omega_1 t_i) \sin^2\omega_2 t_i \quad . \tag{5.72}$$

Now $\varepsilon_1 = Y_{1c}^2 + Y_{1s}^2$, so that (Omura and Kailath, 1965, p. 69)

$$p_1(\varepsilon_1) = \frac{1}{2\sigma_{1c}\sigma_{1s}} \exp\left(-\frac{\varepsilon_1 + \mu_{1c}^2}{2\sigma_{1c}^2}\right) \sum_{j=o}^{\infty} \frac{\Gamma(\frac{1}{2}+j)}{j!\,\Gamma(\frac{1}{2})} \left[\frac{\sqrt{\varepsilon_1}(\sigma_{1s}^2 - \sigma_{1s}^2)}{\mu_{1c}\sigma_{1s}^2}\right]^j$$

$$\times I_j\left(\frac{\sqrt{\varepsilon_1}\mu_{1c}}{\sigma_{1c}^2}\right) \ , \tag{5.73}$$