5. OPTIMUM INCOHERENT RECEPTION

In the previous section we developed the optimum detector and anlyzed its performance for coherent (known phase) binary signals in class A interference. In this section we will treat the incoherent (unknown signal phase) binary case. In general, incoherent problems are much more difficult to treat than coherent ones, and the cases we are considering here (Middleton's class A interference) are no exception.

Our two hypotheses are

$$H_1: X(t) = S_1(t,\underline{\theta}) + Z(t),\ 0 \le t < T$$

$$H_2: X(t) = S_2(t,\underline{\theta}) + Z(t),\ 0 \le t < T \quad , \qquad (5.1)$$

where Z is the accompanying interference and the vector $\underline{\theta}$ denotes the unknown parameters of the signals. These unknown parameters may be phase, amplitude, frequency, or any combination of them.

The problem is now one of composite-hypothesis testing. The likelihood ratio is given by

$$\Lambda(\underline{X}) = \frac{\int_{\underline{\theta}} p(X|H_2)\ p(\underline{\theta})\ d\theta}{\int_{\underline{\theta}} p(X|H_1)\ p(\underline{\theta})\ d\theta} = \frac{p_2(\underline{X})}{p_1(\underline{X})} \underset{H_2}{\overset{H_1}{\lessgtr}} K \quad , \qquad (5.2)$$

where $p(\underline{X}|H_2)$ etc. is as before (e.g., 4.53), and $p(\underline{\theta})$ denotes the pdf of our unknown parameters $\underline{\theta}$. Since our $p(\underline{X}|H_2)$ is given by the Nth product of an infinite summation, it is unlikely that the required averages can be performed directly.

Later we will show how, in special circumstances, the averages can be performed directly, reducing the problem to one equivalent to the coherent case, so that the methods of the previous chapter can be used.

5.1 Incoherent Threshold Signals

In this section we will treat the threshold signal (LOBD) case for some representative incoherent problems.

5.1.1 Unknown Amplitude and Phase

Consider the standard fading incoherent frequency shift keying signals, with unknown amplitude and phase:

$$S_1(t,\underline{\theta}) = a\cos(\omega_1 t + \phi)$$

$$S_2(t,\underline{\theta}) = a\cos(\omega_2 t + \phi) \quad , \qquad (5.3)$$

where a denotes the unknown amplitude and ϕ the unknown phase, $\underline{\theta} = \{a,\phi\}$, a and ϕ independent. As before (e.g., 4.4), let the SNR = S, i.e., $\overline{a^2} = 2S$.

The likelihood ratio, for the discrete sampling case, is

$$\Lambda(X) = \frac{p_2(\underline{X})}{p_1(\underline{X})} = \frac{\int_\phi \int_a p_Z(\underline{X}-\underline{S_2})\, p(\phi)\, p(a)\, d\phi da}{\int_\phi \int_a p_Z(\underline{X}-\underline{S_1})\, p(\phi)\, p(a)\, d\phi da} \underset{H_2}{\overset{H_1}{\lessgtr}} K, \qquad (5.4)$$

where p(a) and p(ϕ) are the pdf's of a and ϕ, and $s_{2i} = a\cos(\omega_2 t_i + \phi)$, etc.

We start with the small signal case, where as before (4.54)

$$p_Z(\underline{X}-\underline{S_2}) = p_Z(\underline{X}) - \sum_{i=1}^{N} \frac{\partial p_Z(\underline{X})}{\partial x_i} s_{2i}$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \frac{\partial^2 p_Z(\underline{X})}{\partial x_i \partial x_k} s_{2i} s_{2k} + \ldots \quad . \qquad (5.5)$$

We can now perform the required averages. We start by letting the phase be uniformly distributed; i.e.,

$$p(\phi) = \frac{1}{2\pi}, \; -\pi \leq \phi < \pi \quad , \qquad (5.6)$$

and by letting a have an arbitrary fading distribution. We will consider two fading situations: I, slow fading, and II, fast fading.

I. First consider the slow fading case, where a is random, but constant over our detection time T. Then

$$\overline{s_{2i}}^{\phi} = \frac{a}{2\pi} \int_{-\pi}^{\pi} \cos(\omega_2 t_i + \phi) \, d\phi = 0 \quad , \qquad (5.7)$$

and

$$\overline{s_{2i} s_{2k}}^{\phi} = \frac{a^2}{2\pi} \int_{-\pi}^{\pi} \cos(\omega_2 t_i + \phi) \cos(\omega_2 t_k + \phi) \, d\phi$$

$$= \frac{a^2}{2} \cos(\omega_2 t_i - \omega_2 t_k) \quad , \qquad (5.8)$$

since

$$\frac{a^2}{4\pi} \int_{-\pi}^{\pi} \cos(\omega_2 t_i + \omega_2 t_k + 2\phi) \, d\phi = 0 \quad .$$

We have, therefore,

$$\overline{s_{2i}s_{2k}}^{\phi,a} = \frac{\overline{a^2}}{2} \cos(\omega_2 t_i - \omega_2 t_k) \quad . \tag{5.9}$$

Now, dividing the munerator and denominator of $\Lambda(\underline{X})$ by $p_Z(\underline{X})$ and assuming N independent samples, we obtain

$$\frac{p_2(\underline{X})}{p_Z(\underline{X})} \approx 1 + \frac{1}{2p_Z(\underline{X})} \sum_{i=1}^{N} \sum_{k=1}^{N} \frac{\partial^2 p_Z(\underline{X})}{\partial x_i \partial x_k} \frac{\overline{a^2}}{2} \cos(\omega_2 t_i - \omega_2 t_k)$$

$$\approx 1 + \frac{\overline{a^2}}{4} \sum_{\substack{i=1 \\ i \neq k}}^{N} \sum_{k=1}^{N} \ell(x_i)\ell(x_k) \cos(\omega_2 t_i - \omega_2 t_k)$$

$$+ \frac{\overline{a^2}}{4} \sum_{i=1}^{N} \frac{d^2 p_Z(x_i)}{dx_i^2} \Big/ p_Z(x_i) \quad , \tag{5.10}$$

with corresponding results for $p_1(\underline{X})$, where $\ell(x_i)$ denotes the nonlinearity obtained earlier,

$$\ell(x_i) = \frac{d}{dx_i} \ln p_Z(x_i) \quad . \tag{5.11}$$

Since

$$\sum_{i=1}^{N} \frac{p_Z''(x_i)}{p_Z(x_i)} = \sum_{i=1}^{N} \ell'(x_i) + \sum_{i=1}^{N} [\ell(x_i)]^2 \quad , \tag{5.12}$$

we obtain

$$\frac{p_2(\underline{X})}{p_Z(\underline{X})} \approx 1 + \frac{\overline{a^2}}{4} \sum_{i=1}^{N} \sum_{k=1}^{N} \ell(x_i)\ell(x_k) \cos(\omega_2 t_i - \omega_2 t_k)$$

$$+ \frac{\overline{a^2}}{4} \sum_{i=1}^{N} \ell'(x_i) \quad . \tag{5.13}$$

Therefore, since $\overline{a^2}/2 = S$, our test is given by

$$\frac{S}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \ell(x_i)\ell(x_k) \cos(\omega_2 t_i - \omega_2 t_k) + (1-K)$$

$$+ \frac{S}{2} (1-K) \sum_{i=1}^{N} \ell'(x_i) \underset{H_2}{\overset{H_1}{\gtrless}} \frac{S}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \ell(x_i)\ell(x_k) \cos(\omega_1 t_i - \omega_1 t_k). \tag{5.14}$$

We see that, for arbitrary thresholds, the receiver depends only on the average signal power S and is independent of the particular slow fading distribution. For the symmetrical case (K=1), the receiver structure is also independent of S, and we obtain, using a trigonometric identity for the cos $(\omega_2 t_i - \omega_2 t_k)$ term, the following receiver:

$$\left[\sum_{i=1}^{N} \ell(x_i) \cos \omega_2 t_i\right]^2 + \left[\sum_{i=1}^{N} \ell(x_i) \sin\omega_2 t_i\right]^2 \underset{H_2}{\overset{H_1}{\gtrless}}$$

$$\left[\sum_{i=1}^{N} \ell(x_i) \cos \omega_1 t_i\right]^2 + \left[\sum_{i=1}^{N} \ell(x_i) \sin\omega_1 t_i\right]^2. \tag{5.15}$$

Or in block diagram form:

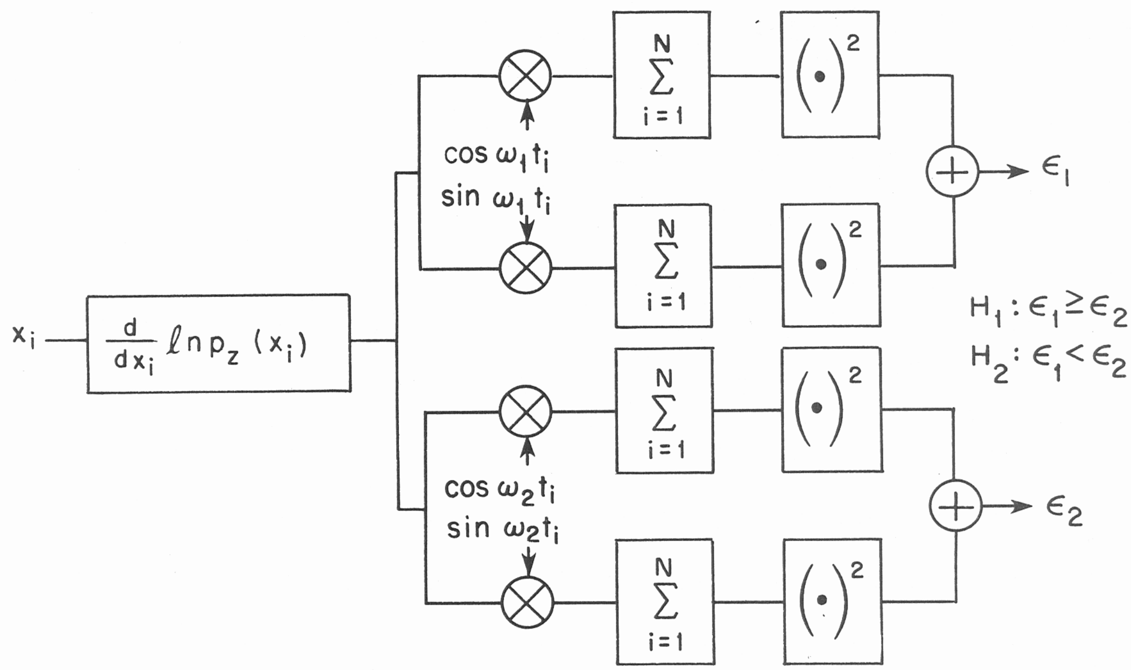

Figure 5.1. LOBD for binary NCFSK, slow fading signal, K=1.

We see this is simply the standard noncoherent binary receiver when the interference is Gaussian noise preceeded by our nonlinearity $\ell(x)$. This result corresponds to that obtained earlier for coherent signals. If the threshold is other than 1, then the receiver also depends on S and must contain a branch involving $\ell'(x)$, the derivative of our nonlinearity [via (5.14)].

II. We can also consider a fast fading signal situation where

$$s_{2i} = a_i \cos(\omega_2 t_i + \phi), \text{ etc.} \tag{5.16}$$

Then, we obtain

$$\frac{p_2(\underline{X})}{p_Z(\underline{X})} \approx 1 + \frac{1}{2p_Z(\underline{X})} \sum_{i=1}^{N} \sum_{k=1}^{N} \frac{\partial^2 p_Z(\underline{X})}{\partial x_i \partial x_k} \frac{\overline{a_i a_k}}{2} \cos(\omega_2 t_i - \omega_2 t_k) \quad . \tag{5.17}$$

If a_i and a_k are independent, $i \neq k$, then we obtain, via the above procedures

$$\frac{p_2(\underline{X})}{p_Z(\underline{X})} \approx 1 + \frac{\bar{a}^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \ell(x_i)\ell(x_k) \cos(\omega_2 t_i - \omega_2 t_k)$$

$$+ \frac{\sigma_a{}^2}{4} \left(\sum_{i=1}^{N} \ell'(x_i) + \sum_{i=1}^{N} [\ell(x_i)]^2 \right) + \frac{\bar{a}^2}{4} \sum_{i=1}^{N} \ell'(x_i) \quad , \tag{5.18}$$

with a corresponding expression for $p_1(\underline{X})$, where σ_a is the standard deviation of a. We see that if K=1, then we obtain the same receiver as before, (5.15) and (5.16), but if K≠1, the receiver for the fast fading case is different from that for the slow fading case. The fast fading receiver involves $\bar{a}^2$ as well as the signal power (i.e., $\overline{a^2}$).

Recently, Nirenberg (1975) has shown, by quite different techniques, that the above results (fig. 5.1) hold for m equiprobable incoherent signals.

5.1.2 ON-OFF Incoherent Signals

The ON-OFF incoherent signaling case turns out to be a special situation. Consider the two hypotheses:

$$H_1: \quad X(t) = Z(t) \qquad 0 \le t < T$$

$$H_2: \quad X(t) = Z(t) + S(t,\underline{\theta}), \quad 0 \le t < T \quad , \tag{5.19}$$

where

$$S(t,\underline{\theta}) = \sqrt{2S}\cos(\omega t + \phi) \quad ,$$

and ϕ is uniformly distributed. This problem has been investigated by Algazi and Lerner (1964), but an oversight in their analysis led to a meaningless result, namely, that the threshold receiver is given by

$$\left[\sum_{i=1}^{N} \ell(x_i)\cos\omega t_i\right]^2 + \left[\sum_{i=1}^{N} \ell(x_i)\sin\omega t_i\right]^2 \overset{H_1}{\underset{H_2}{\lessgtr}} 0 \ , \tag{5.20}$$

i.e., H_2 is always chosen! This result came about by not recognizing the $\ell'(x_i)$ terms, cf. (5.14).

For K=1, we see, via (5.5), that the required statistical test is given by

$$\frac{1}{2p_Z(\underline{X})}\sum_{i=1}^{N}\sum_{k=1}^{N}\frac{\partial^2 p_Z(\underline{X})}{\partial x_i \partial x_k} S\cos(\omega t_i - \omega t_k) \overset{H_1}{\underset{H_2}{\lessgtr}} 0 \ . \tag{5.21}$$

In (5.21), as earlier, only 2nd order terms have been used. This gives us the receiver, in block diagram form:

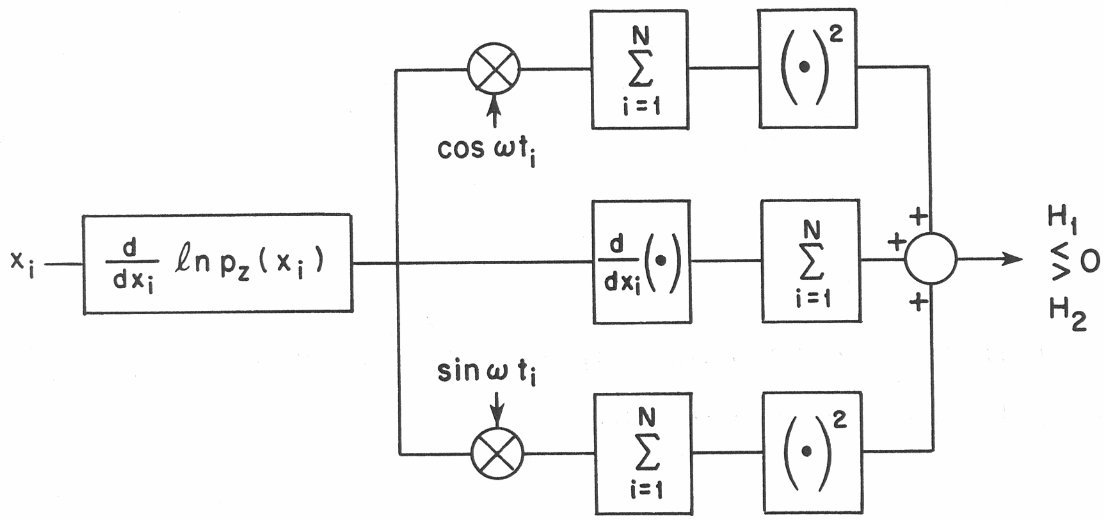

Figure 5.2. A threshold receiver for ON-OFF binary incoherent signaling.

Note, however, in (5.21) and figure 5.2, under H_1, neither H_1, the receiver structure, nor the threshold depend on the signal size S. That is, under H_1, performance is independent of signal size. This is not too satisfying physically, and we reasonably expect, that, for OFF-ON signals, the threshold, at least, should depend on signal size.

We can see how this came about by looking at the Gauss case, where the optimum receiver is well-known.

Using

$$p_Z(z) = \frac{1}{\sqrt{\pi}} e^{-z^2} \quad , \qquad (5.22)$$

we obtain the likelihood ratio

$$\Lambda(\underline{X}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[L_c \cos\phi - L_s \sin\phi - E]d\phi \quad , \qquad (5.23)$$

where

$$L_c = 2\sqrt{2S} \sum_{i=1}^{N} x_i \cos \omega t_i \quad ,$$

$$L_s = 2\sqrt{2S} \sum_{i=1}^{N} x_i \sin \omega t_i \quad , \text{ and}$$

$$E = 2S \sum_{i=1}^{N} \cos^2 \omega t_i \quad .$$

This gives, for our optimum receiver,

$$I_o\left(\sqrt{L_c^2 + L_s^2}\right) \underset{H_2}{\overset{H_1}{\lessgtr}} e^E \quad . \tag{5.24}$$

The corresponding optimum threshold receiver is obtained by using the small signal assumptions, and writing

$$I_o(x) \approx 1 + \frac{X^2}{4} \quad , \tag{5.25}$$

to obtain for (5.24)

$$1 + 2S\left(\sum_{i=1}^{N} x_i \cos\omega t_i\right)^2 + 2S\left(\sum_{i=1}^{N} x_i \sin\omega t_i\right)^2$$

$$\underset{H_2}{\overset{H_1}{\lessgtr}} 1 + 2S \sum_{i=1}^{N} \cos^2 \omega t_i + \ldots \quad . \tag{5.26}$$

If all signal terms above degree 2 are dropped, we obtain

$$\left(\sum_{i=1}^{N} x_i \cos\omega t_i\right)^2 + \left(\sum_{i=1}^{N} x_i \sin\omega t_i\right)^2 \underset{H_2}{\overset{H_1}{\lessgtr}} \sum_{i=1}^{N} \cos^2 \omega t_i , \tag{5.27}$$

which is, of course, identical to the result obtained by substituting (5.22) into (5.21) or figure 5.2, using the fact that

$$\sum_{i=1}^{N} \cos^2\omega t_i \approx \frac{N}{2} \quad . \tag{5.28}$$

So we see that in order to obtain a physically meaningful threshold receiver for the incoherent ON-OFF case, we must include higher order terms, at least in the determination of the threshold. This is a special instance of insuring "consistency" (i.e., $P_e \to 0$ as $N \to \infty$). [See comments on use of the LOBD approach in chapter 3 and section 4.4.]

Returning to the case of impulsive interference, the next nonzero terms in the expression (5.21) are the 4th order terms, namely

$$\frac{1}{4!p_Z(\underline{X})} \sum_{i_4=1}^{N} \sum_{i_3=1}^{N} \sum_{i_2=1}^{N} \sum_{i_1=1}^{N} \frac{\partial^4 p_Z(\underline{X})}{\partial x_{i_1} \partial x_{i_2} \partial x_{i_3} \partial x_{i_4}} \overline{s_{i_1} s_{i_2} s_{i_3} s_{i_4}} \quad . \tag{5.29}$$

If we discard all the above 4th order terms except those that contribute to the threshold, that is, all terms except for $i_1 = i_2 = i_3 = i_4$, we obtain the factor

$$\frac{1}{4!} \sum_{i=1}^{N} \frac{d^4 p_Z(x_i)}{dx_i^4} \overline{s_i^4} \quad , \tag{5.30}$$

which tells us, after evaluating $\overline{s_i^4}$, that we could replace the zero threshold in (5.21) by

$$- \frac{3S^2}{2.4!} \sum_{i=1}^{N} \frac{d^4 p_Z(x_i)}{dx_i^{\,4}} \quad . \tag{5.31}$$

Now the threshold depends on the signal size. Whether using (5.31) resolves the problem or not can only be determined by a study of the ARE, etc.

5.1.3 Threshold Receiver when Phase Estimation is Used

In all the above, we assumed that the phase was uniformly distributed. It is common in receiving systems to employ phase tracking loops to obtain an estimate of the phase. We might term this a partially coherent receiver. When such phase estimation is used, it is common to use the Tikhonov distribution for the distribution of the phase angle or phase error. That is, we use

$$p(\phi;\Omega) = \frac{\exp[\Omega \cos\phi]}{2\pi I_o(\Omega)}, \quad -\pi \le \phi < \pi \quad , \tag{5.32}$$

where Ω is a parameter that controls the spread of the density and which has an important physical significance in the study of phase estimation (Van Trees, 1971, chapter II.2). In (5.32), $\Omega=0$, gives the uniform distribution case and as $\Omega \to \infty$, we approach the coherent, or completely known phase case.

Using this phase distribution in the threshold receiver development, with

$$s_{2i} = \sqrt{2S} \cos(\omega_2 t_i + \phi) \quad , \tag{5.33}$$

we find that the 1st order terms in the LOBD are no longer zero, and we obtain

$$\overline{s_{2i}} = \sqrt{2S} \int_{-\pi}^{\pi} \cos(\omega_2 t_i + \phi) \frac{\exp[\Omega \cos\phi]}{2\pi I_o(\Omega)} d\phi \quad ,$$

$$\overline{s_{2i}} = \frac{2\sqrt{2S} \cos\omega_2 t_i}{2\pi I_o[\Omega]} \int_{o}^{\pi} \cos\phi \, e^{\Omega \cos\phi} \, d\phi \quad ,$$

$$\overline{s_{2i}} = \frac{I_1(\Omega)}{I_o(\Omega)} \sqrt{2S} \cos \omega t_i \quad . \qquad (5.34)$$

We note, $\overline{s_{2i}} = 0$, for $\Omega = 0$, as before, and that $\overline{s_{2i}} = \sqrt{2S}$ $\cos\omega_2 t_i$, as $\Omega \to \infty$, since

$$\lim_{\Omega \to \infty} \frac{I_1(\Omega)}{I_o(\Omega)} = 1 \quad ,$$

that is, the previous result for the coherent case. Our statistical test is, with only the 1st order terms,

$$1 - \frac{\sqrt{2S}\, I_o(\Omega)}{I_o(\Omega)} \sum_{i=1}^{N} \ell(x_i) \cos\omega_2 t_i \overset{H_1}{\underset{H_2}{\lessgtr}} 1$$

$$- \frac{\sqrt{2S}\, I_1(\Omega)}{I_o(\Omega)} \sum_{i=1}^{N} \ell(x_i) \cos\omega_1 t_i \quad , \qquad (5.35)$$

or

$$\sum_{i=1}^{N} \ell(x_i) \cos\omega_2 t_i \overset{H_1}{\underset{H_2}{\lessgtr}} \sum_{i=1}^{N} \ell(x_i) \cos\omega_1 t_i \quad . \qquad (5.35a)$$

Or we obtain the receiver, in block diagram form, as in the following figure:

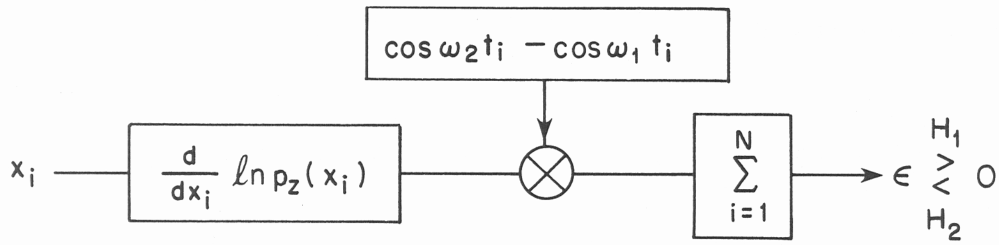

Figure 5.3. First order threshold receiver for NCFSK with phase estimation ($\Omega \neq 0$).

That is, the same threshold receiver as we obtained in section 4.4 for coherent signals (coherent FSK in this case). The receiver is independent of the parameter $\Omega(\Omega \neq 0)$.

For the 2nd order terms, we have

$$\overline{s_{2i}s_{2j}} = \int_{-\pi}^{\pi} 2S \cos(\omega_2 t_i + \phi) \cos(\omega_2 t_j + \phi) \frac{\exp[\Omega \cos\phi]}{2\pi I_o(\Omega)} d\Phi \quad . \quad (5.36)$$

Expanding and integrating, we obtain

$$\overline{s_{2i}s_{2j}} = S \cos(\omega_2 t_i - \omega_2 t_j) + \frac{S\ I_2(\Omega)}{I_o(\Omega)} \cos(\omega_2 t_i + \omega_2 t_j), (5.37)$$

where we have used

$$\int_{-\pi}^{\pi} e^{\Omega \cos\phi} \sin 2\phi \ d\phi = 0 \quad ,$$

and

$$\frac{1}{\pi} \int_{0}^{\pi} e^{\Omega \cos\phi} \cos 2\phi \ d\phi = I_2(\Omega) \quad .$$

In our threshold (or LOBD) expansion, the 2nd order terms give a sum of two double summations from (5.5) and (5.37),