4.3 Performance of Sub-optimum Receivers

The receivers in general use today are basically those designed to be optimum when the interference is white Gaussian noise. These are the well-known correlation, or matched filter receivers (Hancock and Wintz, 1966). In this section the performance of these receivers in the impulsive class A interference is given, and this performance is then compared with that of the corresponding optimum detector derived and analyzed earlier.

The performance of the correlation receiver can be obtained most easily by using a geometric representation of the receiving system (see Arthur and Dym, 1962). Halton and Spaulding (1966), using this approach, have analyzed the performance of differentially coherent phase systems (DCPSK) in impulsive noise. The performance of our three systems can be obtained from these results since our antipodal signaling is binary coherent phase shift keying (CPSK), which is a special case of the multilevel DCPSK results. We find, using (24) of Halton and Spaulding (1966), that, in general, for binary CPSK, the average probability of error is given by

$$P_e = \frac{1}{\pi} \int_{\sqrt{S}}^{\infty} p(\epsilon) \cos^{-1} \left(\frac{\sqrt{S}}{\epsilon}\right) d\epsilon \quad , \tag{4.46}$$

where $p(\epsilon)$ is the probability density of the noise <u>envelope</u> voltage, normalized to the rms noise and, as before, the SNR is given by S.

The cumulative distribution of the envelope of class A interference has been found to be given by

$$\mathrm{Prob}(\epsilon > \epsilon_o) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-\epsilon_o^2/\sigma_m^2} \quad . \tag{4.47}$$

Therefore, our required $p(\epsilon)$ is

$$p(\epsilon) = 2\epsilon\, e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!\sigma_m^2} e^{-\epsilon^2/\sigma_m^2} \quad . \tag{4.48}$$

Using (4.48) in (4.46), interchanging integration and summation, integrating by parts, and making a change of variable, we obtain

$$P_e = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \frac{e^{-S/\sigma_m^2}}{\pi} \int_o^{\infty} \frac{e^{-Sy^2/\sigma_m^2}}{y^2 + 1} dy \quad , \tag{4.49}$$

which gives us, finally,

$$P_e = \frac{e^{-A}}{2} \sum_{m=0}^{\infty} \frac{A^m}{m!} \operatorname{erfc} \sqrt{S/\sigma_m^2} \quad . \tag{4.50}$$

The results (4.50) are for binary CPSK. In general, the geometrical representation of the correlation receivers gives immediately

$$P_e = \frac{e^{-A}}{2} \sum_{m=0}^{\infty} \frac{A^m}{m!} \operatorname{erfc} \sqrt{S/k\sigma_m^2} \quad , \tag{4.51}$$

where k = 1 for antipodal signals, 2 for orthogonal signals, and 4 for OFF-ON keying.

Figure 4.8 shows the calculated performance, from (4.50), for antipodal signals for $\Gamma' = 1 \times 10^{-4}$, parametric in A, while figure 4.9 shows this calculated performance for A = 0.35, parametric in Γ'.

There have been few measurements of the performance of these receivers in narrow band interference. Figure 4.10 shows the measured performance of some systems similar to the types considered here, where the single interfering signal was one of various digital and analog types. These results were taken from Mayer (1972). The modulation types on figure 4.10 are given in standard CCIR terms; A1 is amplitude modulation pulsed, A2 is two-tone pulsed amplitude modulation, A3 is amplitude modulated telephony, A3J is single sideband voice, suppressed carrier, F1 is frequency shift keying, F3 is frequency modulated telephony, and PCM is pulse coded modulation (FM). These results are given in terms of the input signal-to-interference ratio, which does not correspond directly to the SNR of figures 4.8 and 4.9. While these interfering signal types do not necessarily correspond directly to Middleton's class A interference, the results indicate that system performance is much like that calculated (figs. 4.8 and 4.9) for narrow band interference.

We now want to compare the performance of these suboptimum systems with that of our optimum system. Figure 4.11

shows the performance found for the optimum receiver compared with the theoretical performance of the standard correlation receiver for antipodal signaling for N = 10. Our example case A = 0.35, $\Gamma' = 0.5 \times 10^{-3}$ is used. For this system, N = 10 corresponds to a time-bandwidth product of 10, and we note that the optimum system does not perform a great deal better than the current suboptimum system for this time-bandwidth product for small P_e. Figure 4.12 shows the same comparison, now for N = 100. Now the optimum system performs substantially better than the correlation receiver for the same time bandwidth product. For example, these results indicate that if a time-bandwidth product of 100 is being used to combat interference (a not unusual situation), then the optimum system requires at least 33 dB less signal power than the correlation receiver to achieve an error rate of 10^{-5} or less. Another way of looking at this is in terms of usage of the spectrum resource. These results (fig. 4.12) indicate that for a given signal power and performance criterion ($P_e = 10^{-5}$, say), the optimum system can achieve this performance with a time-bandwidth product somewhere between 10 and 20, while the suboptimum system requires a time-bandwidth product of 100. That is, somewhere between 5 and 10 times the actual required amount of spectrum space is currently being used (in this example).

The results shown on figures 4.11 and 4.12 indicate that for large P_e($P_e \sim 1/2$) the suboptimum receiver apparently performs better than the optimum receiver. This is due to the looseness of the upper bound used to specify optimum performance at large P_e (see fig. 4.1, for example). Of course, by definition, the optimum receiver must give superior performance vis-à-vis any suboptimum receiver for the same purpose and under the same conditions.

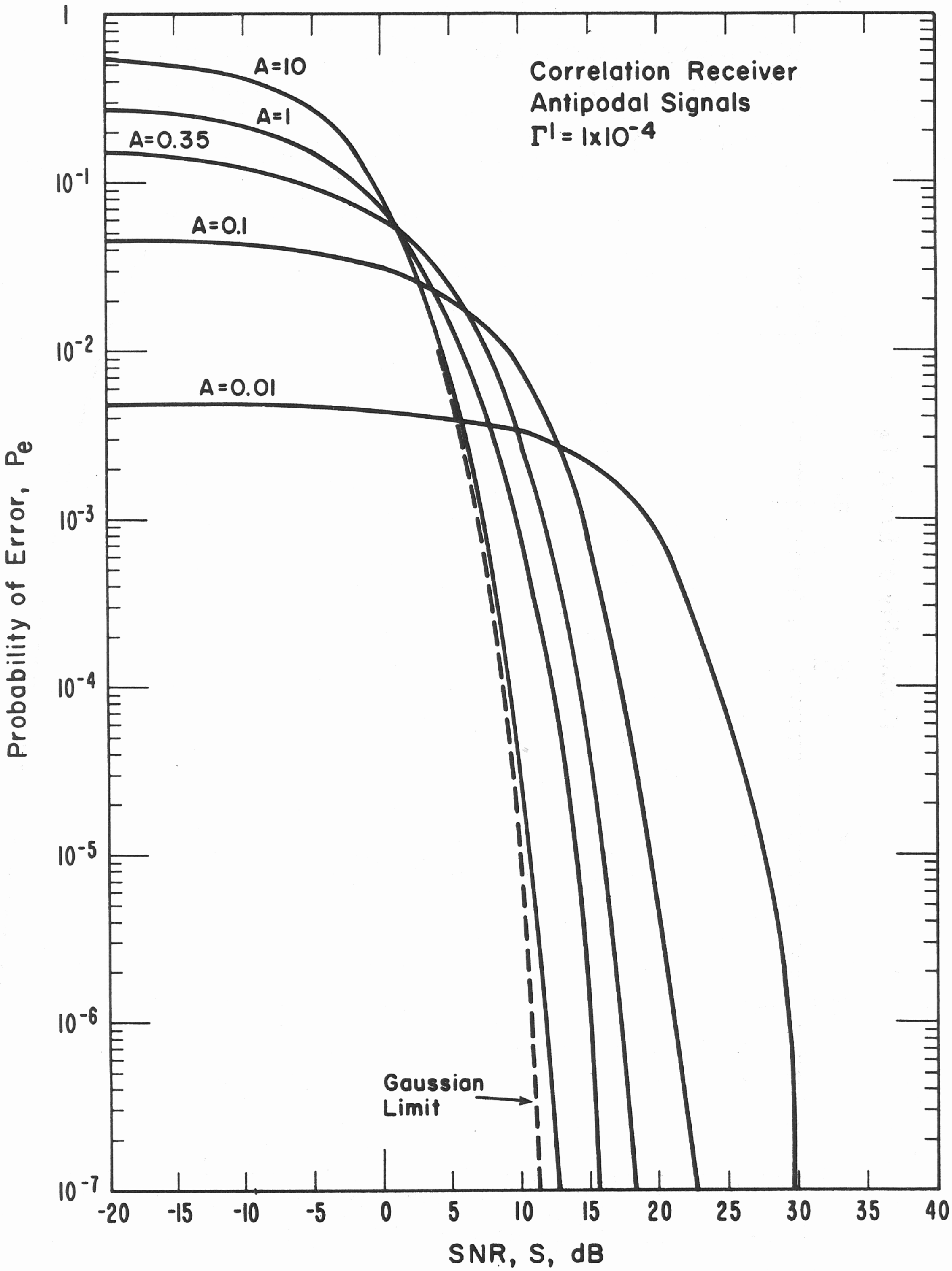

Figure 4.8. Performance of the correlation receiver (Gaussian optimum receiver) in class A interference (4.50) for $\Gamma' = 10^{-4}$ for various values of the impulsive index A.

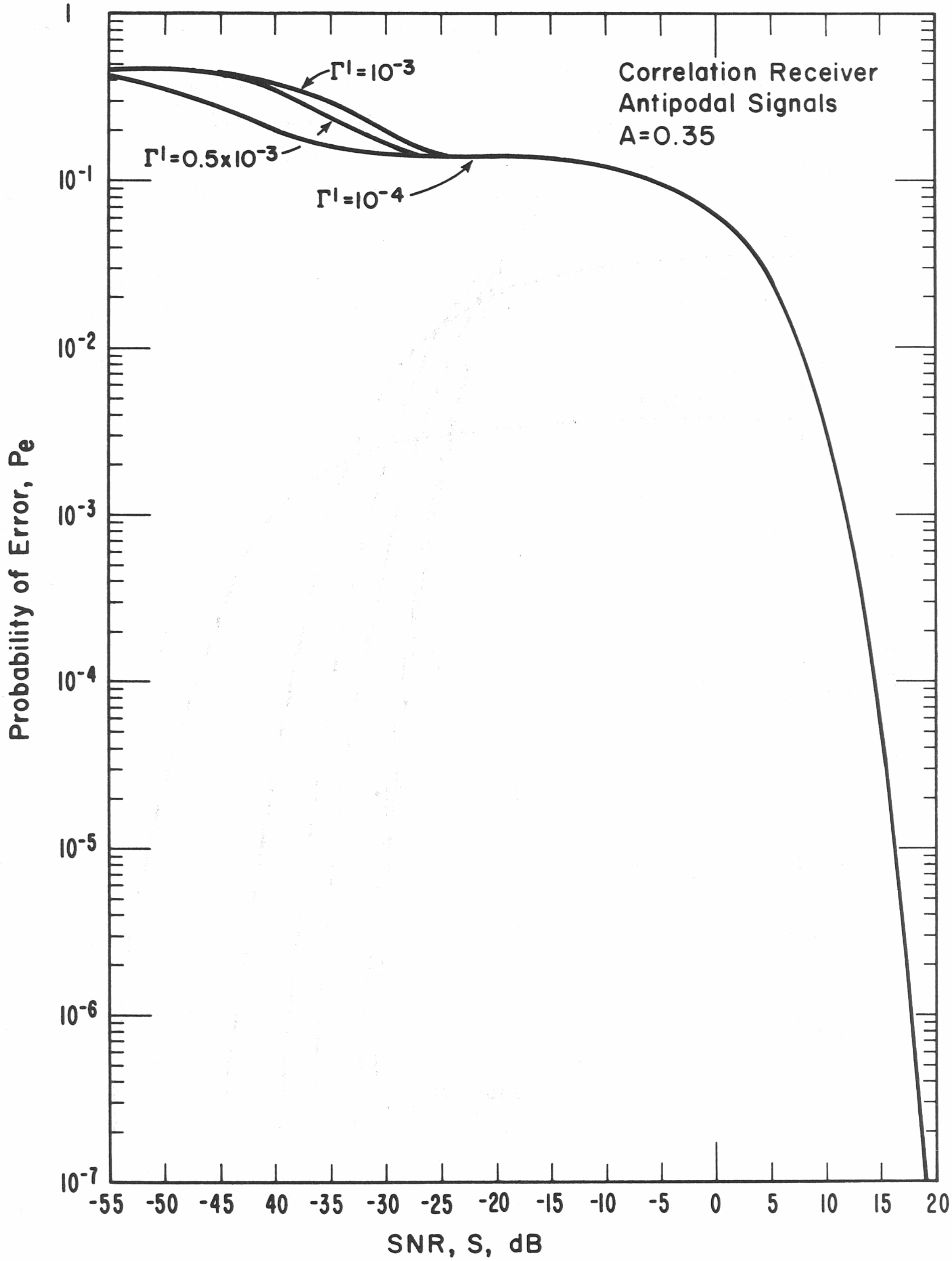

Figure 4.9. Performance of the correlation receiver (Gaussian optimum receiver) in class A interference for A = 0.35 for various values of the parameter Γ'.

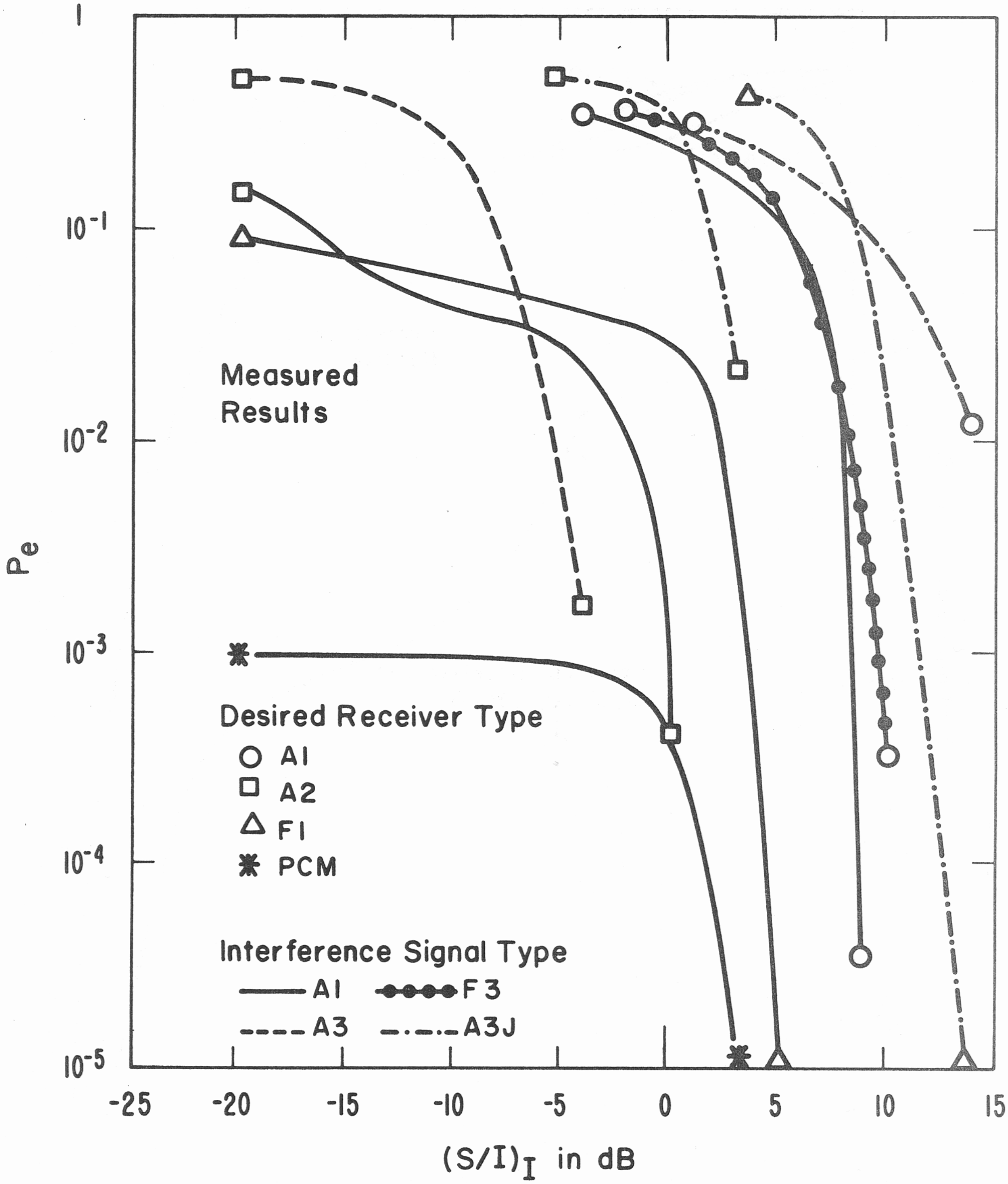

Figure 4.10. Measured performance of various receivers with various types of interfering signals as a function of input desired signal to interfering signal ratio (from Mayer, 1972).

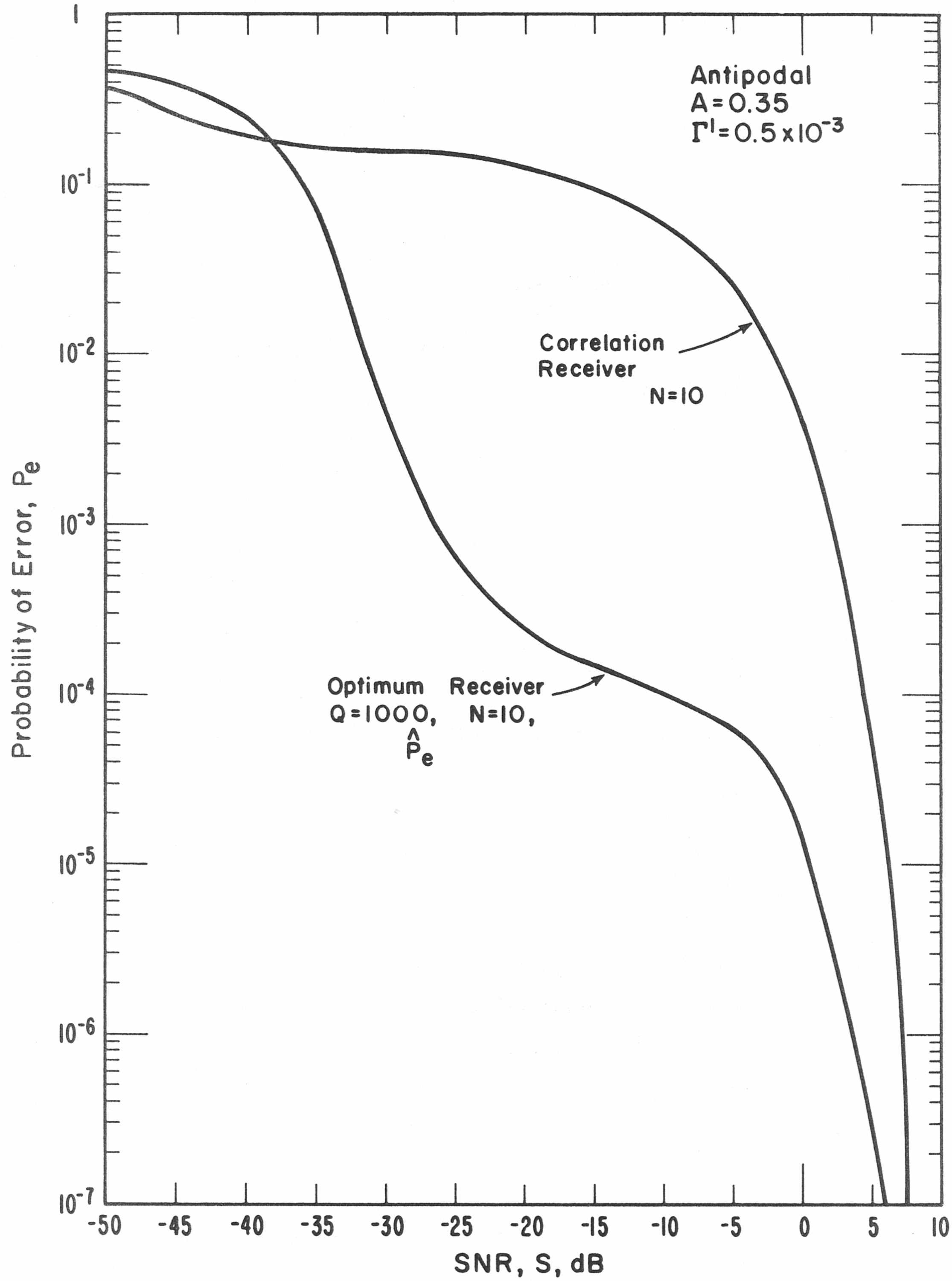

Figure 4.11. Comparison of performance of correlation receiver in class A interference (4.50) with the upper bound on optimum performance (4.44) for Q = 1000 and N = 10 for antipodal signals.

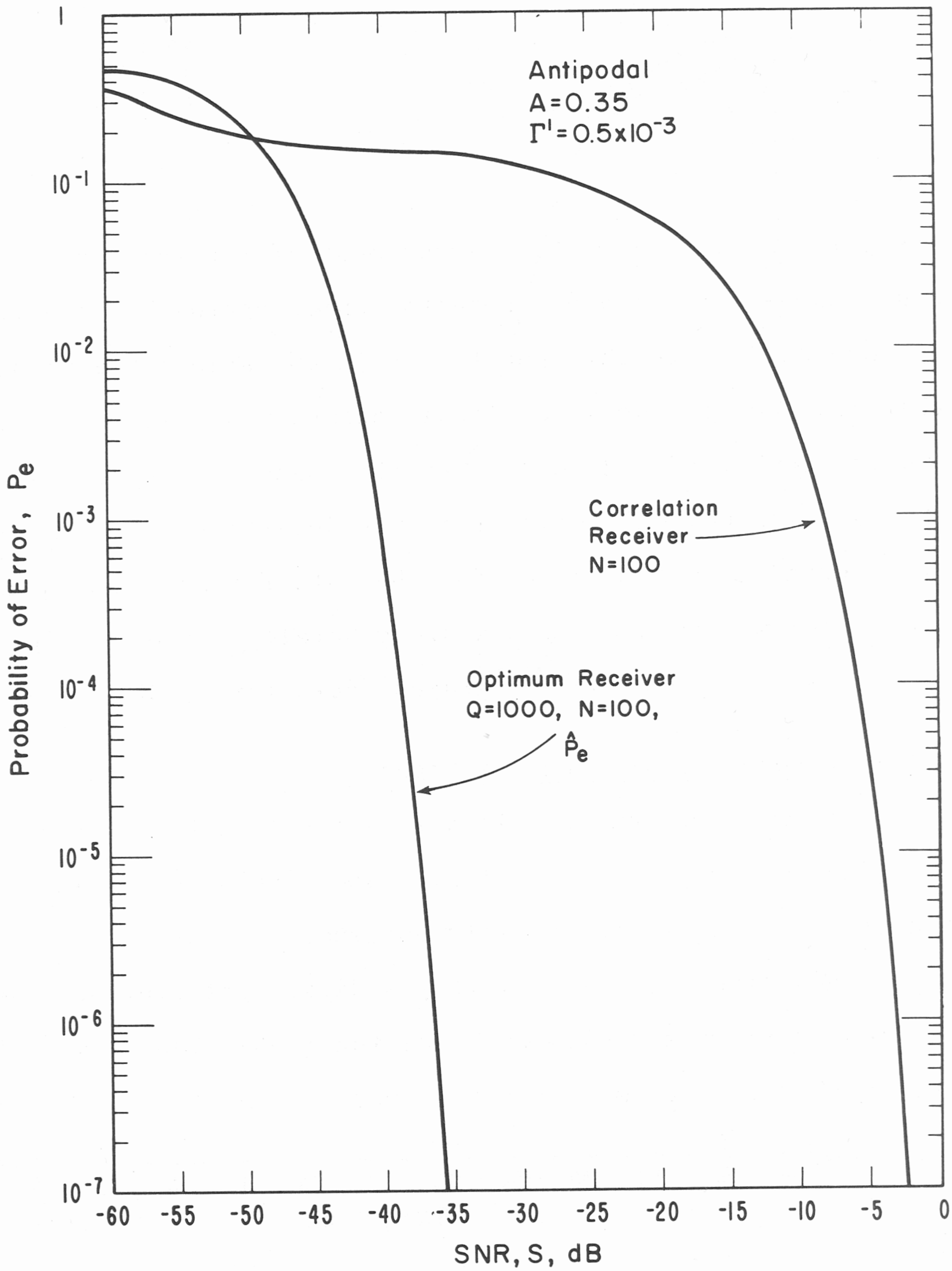

Figure 4.12. Comparison of performance of correlations receiver in class A interference (4.50) with the upper bound on optimum performance (4.44) for Q = 1000 and N = 100 for antipodal signals.