4. OPTIMUM COHERENT RECEPTION

In this section we will obtain the optimum coherent detectors for a variety of signaling situations. The term "coherent" means that the phase of the desired signal is completely known to the receiver. Other parameters of the signal may or may not be known to the receiver. We will first test the purely coherent case where the desired signals are completely known, and in section 4.4, where threshold signaling situations (LOBD's) are treated, we will also investigate the problem of interference discrimination. In this problem the signal is completely known except for its amplitude, and the receiver is required to decide whether or not the signal is "large enough" to be useful.

Once we have these optimum detectors, we will analyze their performance and compare this optimum performance against the performance of the corresponding current suboptimum receivers (i.e., those optimum in Gaussian noise).

4.1 The Optimum Coherent Receiver

In this section we will first develop the optimum coherent detector for the three basic classes of coherent binary digital transmission when the interfering noise is Middleton's class A interference; i.e., where the emission bandwidths of the interferers are less than that of the receiver.

The problem of obtaining an optimum decision algorithm is one of simple-hypothesis testing, following the Bayes' strategy (see chapter 3). We have the two hypotheses

$$H_1: \quad X(t) = S_1(t) + Z(t), \; 0 \leq t < T$$

$$H_2: \quad X(t) = S_2(t) + Z(t), \; 0 \leq t < T \quad , \qquad (4.1)$$

where $X(t)$ is our received waveform, $S_1(t)$ or $S_2(t)$ is the completely known desired signal, and $Z(t)$ is the interference process.

We will consider the three basic signal sets; <u>antipodal</u>,

$$S_1(t) = \sqrt{2S} \cos(\omega_o t), \; 0 \leq t < T$$

$$S_2(t) = -\sqrt{2S} \cos(\omega_o t), \; 0 \leq t < T \quad , \qquad (4.2)$$

<u>orthogonal</u>,

$$S_1(t) = \sqrt{2S} \sin(\omega_o t), \; 0 \leq t < T$$

$$S_2(t) = \sqrt{2S} \cos(\omega_o t), \; 0 \leq t < T \quad , \qquad (4.3)$$

and <u>ON-OFF keyed</u>,

$$S_1(t) = \sqrt{2S} \cos(\omega_o t), \; 0 \leq t < T$$

$$S_2(t) = 0, \quad 0 \leq t < T \quad , \qquad (4.4)$$

where $\omega_o T >> 1$, and S is the signal power.

We represent (4.1) in vector form

$$\underline{X} = \underline{S}_j + \underline{Z}, \; j = 1,2 \quad , \qquad (4.5)$$

where $\underline{X}$ is a vector of N samples of our received waveform $X(t)$, $x_n = X(t_n)$, and we assume that the sample times $\{t_n\}$ are such that the noise sample $\{z_n\}$ are statistically independent. The ramifications of this postulation of independence have been discussed in chapter 3 (sec. 3.2).

In the Bayes' test, we take $q_1 = q_2 = 1/2$ and the threshold $K = 1$. This is the usual situation in digital transmission systems and, in any case, all the results of this chapter can be easily and directly modified for arbitrary K. That is, using the above simplifies the notation, but results in no loss of generality.

The likelihood ratio for our case, with the assumption of independent samples, is

$$\Lambda(\underline{X}) = \frac{\prod_{n=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-(x_n - s_{2n})^2/2\sigma_m^2}}{\prod_{n=1}^{N} \sum_{m=0}^{\infty} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-(x_n - s_{1n})^2/2\sigma_m^2}} \underset{S_2}{\overset{S_1}{\lessgtr}} 1 \quad , \quad (4.6)$$

where s_{1n} and s_{2n} are samples of the signals $S_1(t)$ and $S_2(t)$ and x_n are the samples of our received waveform. Taking logarithms gives a somewhat more useful form of the algorithm,

$$\sum_{n=1}^{N} \ln \left(\sum_{m=0}^{\infty} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-(x_n - s_{2n})^2/2\sigma_m^2} \right)$$

$$- \sum_{n=1}^{N} \ln \left(\sum_{m=0}^{\infty} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-(x_n - s_{1n})^2/2\sigma_m^2} \right) \underset{S_2}{\overset{S_1}{\lessgtr}} 0 \quad . \quad (4.7)$$

Neither (4.6) nor (4.7) can, in general, be simplified further. We will show later how a small signal assumption can be used greatly to reduce (4.6) or (4.7) to obtain a physically realizable receiver which is optimum for threshold (small) signals but which, in general, becomes suboptimum. The physical operations required of a receiver are obvious from (4.6), but it is unlikely one would actually want to build such a receiver.

Eastwood and Lugannami (1973) developed a series algorithm, using a method given by Schwartz (1969), for problems of the above type. This algorithm, as given below for our problem, is, perhaps, more easily implemented physically, but cannot be used to compute performance except by direct computer simulation. Also, this series algorithm, for our problem, only applies to the one-sample case (N=1). After specifying the series algorithm, we will, therefore, use (4.6) to compute performance, i.e., the average probability of error for our three signal classes.

The Eastwood-Lugannani algorithm is based on having an infinite series representation of the likelihood ratio and using a truncated series. Instead of considering the test statistic

$$\Lambda(\underline{X}) = \frac{p(\underline{X}|H_2)}{p(\underline{X}|H_1)} \quad ,$$

consider the completely equivalent test statistic

$$R(\underline{X}) = p(\underline{X}|H_2) - p(\underline{X}|H_1) \underset{H_2}{\overset{H_1}{\lessgtr}} 0 \quad . \tag{4.8}$$

In our case, for N=1, we have

$$\left| p(x|H_2) - \sum_{m=0}^{q} e^{-A} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} e^{-(x-s_2)^2/2\sigma_m^2} \right| < B_2(q,x) \quad , \tag{4.9}$$

where as $q \to \infty$, the truncation error bound $B_2(q,x) \to 0$ for all x. From (4.9) we form a truncated test statistic, $R_q(x)$. The truncation error bound is then

$$|R(x) - R_q(x)| < B_2(q,x) + B_1(q,x) \equiv B(q,x) \quad . \tag{4.10}$$

An algorithm for optimal detection using $R_q(x)$ and $B(q,x)$ is:

(a) $q=1$,

(b) calculate $R_q(x)$, and

(c) if $|R_q(x)| > B(q,x)$, so that $\text{sgn}\,[R_q(x)] = \text{sgn}\,[R(x)]$, then decide H_2 if $R_q(x) > 0$; H_1 otherwise and stop.

(d) If $|R_q(x)| \le B(q,x)$, set $q = q+1$ and go to (a).

This algorithm will always terminate (Eastwood and Lugannani, 1973).

In our case, since

$$\sigma_m^{\ 2} = \frac{m/A + \Gamma'}{1 + \Gamma'}$$

and the exponentials are bounded above by 1, we can use

$$B_1(q,x) = B_2(q,x) = \frac{e^{-A}}{\sqrt{2\pi\sigma_{q+1}^{\ 2}}} \sum_{m=q+1}^{\infty} \frac{A^m}{m!} , \qquad (4.11)$$

and the series is bounded above by exp(A) $A^{q+1}/(q+1)!$, so that a valid bound for the test statistic is

$$B(q,x) = \frac{2A^{q+1}}{(q+1)!\sqrt{2\pi\sigma_{q+1}^{\ 2}}} . \qquad (4.12)$$

Performance for this detector can be determined only by computer simulation of the above algorithm. Eastwood and Lugannani (1973) attempted such a simulation, but since the probabilities of error of interest are small, typically 10^{-4} or less, a very large number of samples would be required to obtain any statistical significance for the low probabilities of error. This quickly results in too much computer time for even the fastest computers, and this is only for the N=1 case! As mentioned previously, in order to gain any advantage over "normal" receivers, we require N to be large.

Eastwood and Lugannani (1973) attempt to obtain an approximation for the multidimensional distribution of Z by use of an N dimensional characteristic function and using the same approximation used by Middleton in the one dimensional case to obtain our basic pdf. Their results are, for our case,

$$p(\underline{X}|H_1) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \frac{e^{-\frac{1}{2}(\underline{X}-\underline{S_1})^T \Phi_{zz}^{-1}(\underline{X}-\underline{S_1})}}{\sqrt{(2\pi)^N \, |\Phi_{zz}|}} , \qquad (4.13)$$

where Φ_{zz} is the NxN covariance matrix for the N noise samples; i.e., for N=1, $\Phi_{zz} = \sigma_m^2 I$, etc. The above series algorithm can be applied to (4.13) exactly as in the one dimensional case. Note, however, that for independent samples, (4.13) gives us

$$p(\underline{X}|H_1) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \prod_{n=1}^{n} \frac{e^{-(x_n - s_{1n})^2/2\sigma_m^2}}{\sqrt{2\pi\sigma_m^2}} , \qquad (4.14)$$

and this is no way similar to the product of the first-order densities, as in (4.6), as we should reasonably expect.

In order to apply this series algorithm to our cases of interest, where N is large, we would need to represent our Nth order distribution as an infinite summation. While it is, in principle, always possible to represent the product of infinite summations as a double summation, this does not appear to be feasible for our distribution (4.6). Even so, we would undoubtedly obtain a result which would be quite impractical to construct physically. We can therefore conclude that the series algorithm approach will not be helpful in developing realizable receiver structures or in determining optimum performance.