# Introducing Two Spatial Reference Frames for Regions of the Pacific Ocean

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**ABSTRACT:** The current realization of the North American Datum of 1983 (NAD 83) is defined in terms of a Helmert transformation from the International Terrestrial Reference Frame of 2000 (ITRF00). The parameters of this transformation were chosen, in part, so that horizontal NAD 83 velocities would be zero on average for points interior to the North American tectonic plate. Unfortunately, islands located in the Pacific Ocean move horizontally by as much as several centimeters per year relative to the North American plate. To address this motion, this document introduces two new spatial reference frames in terms of transformations from ITRF00, one frame for regions located on the Pacific tectonic plate (Hawaiian Islands, Marshall Islands, American Samoa) and one for regions located on the Mariana tectonic plate (Guam, Rota, Saipan). For each frame, points located interior to the corresponding tectonic plate are to have little or no horizontal velocity relative to this frame. The Pacific-plate-fixed frame is to be designated as NAD 83 (PACP00) and the Mariana-plate-fixed frame as NAD 83 (MARP00). These frames are defined so that their positional coordinates are consistent with corresponding positional coordinates of the current NAD 83 realization for an epoch date of 14 AUG 1993.

## Introduction

In 1986, NOAA's National Geodetic Survey (NGS) adopted the first realization of the North American Datum of 1983, which is customarily denoted NAD 83 (1986). In particular, NGS published an internally consistent set of geodetic latitudes and longitudes for more than 200,000 reference stations located in the United States and its territories (Schwarz 1989). NGS has since published several newer realizations of NAD 83 to keep pace with positioning technology (Snay and Soler 2000). The latest realization, denoted here as NAD 83 (CORS96), is based on the 3D positional coordinates (latitude, longitude, and ellipsoidal height) and the 3D velocities for several hundred reference stations contained in the National Continuously Operating Reference Station (CORS) network. Each CORS is a permanent ground-based station that collects dual-frequency Global Positioning System (GPS) data around the clock, day after day.

When a CORS first becomes operational, NGS computes its positional coordinates relative to the then current realization of the International Terrestrial Reference Frame (ITRF). The current realization as of December 2001 is denoted either as ITRF2000 or ITRF00. Moreover, NGS verifies these ITRF positional coordinates daily—using a 24-hour span of recent GPS data—to ensure that these coordinates are consistent with corresponding ITRF coordinates for other reference stations around the world. The ITRF coordinates of a CORS are updated if they are found to be inconsistent by more than 1 cm horizontally or 2 cm vertically.

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To obtain corresponding NAD 83 (CORS96) positional coordinates for a CORS, NGS has adopted a Helmert (similarity) transformation—as defined later—for each of the latest ITRF realizations, namely for ITRF96, ITRF97, and ITRF00. The Helmert transformation from ITRF96 to NAD 83 (CORS96), denoted ITRF96 → NAD 83 (CORS96), was defined so that the transformed ITRF96 latitudes and longitudes of 12 very-long-baseline-interferometry (VLBI) stations located in North America would best approximate their corresponding NAD 83 (1986) latitudes and longitudes in a least squares sense (Craymer et al. 2000). Hence, NAD 83 (CORS96) latitudes and longitudes agree on average with corresponding NAD 83 (1986) latitudes and longitudes. However, the actual latitude or longitude for a particular reference station generally differs in value between these two realizations of NAD 83, sometimes in excess of one meter.

The Helmert transformation for converting coordinates from ITRF97 to NAD 83 (CORS96), denoted ITRF97 → NAD 83 (CORS96), was set equal to the composite transformation:

ITRF97 → ITRF96 → NAD 83 (CORS96)

where ITRF97 → ITRF96 represents the Helmert transformation adopted by the International GPS

Service for converting ITRF97 coordinates to ITRF96 coordinates. Similarly, as discussed by Soler and Snay (2003), the Helmert transformation for converting coordinates from ITRF00 to NAD 83 (CORS96), denoted ITRF00 → NAD 83 (CORS96), was set equal to the composite transformation:

ITRF00 → ITRF97 → ITRF96 → NAD 83 (CORS96)

where ITRF00 → ITRF97 represents the Helmert transformation adopted by the International Earth Rotation Service (IERS) for converting ITRF00 coordinates to ITRF97 coordinates. Thus, in all cases, transformed ITRF latitudes and longitudes for the CORS are consistent "on average" with the NAD 83 (1986) realization. Moreover, because the CORS network currently forms the foundation for the National Spatial Reference System (NSRS), NAD 83 (CORS96) latitudes and longitudes of all NSRS reference stations are essentially defined in terms of a transformation from a current realization of ITRF.

With any ITRF realization, the positional coordinates of a point change with time because of crustal motion, especially that associated with plate tectonics. The rate of this change exceeds 9 mm/yr everywhere throughout the United States. So that NAD 83 users will not have to cope with ever changing coordinates, NGS incorporated numerical parameters from a plate tectonic model into the transformation ITRF96 → NAD 83 (1986) (Craymer et al. 2000). Accordingly, points located within the interior of the North American tectonic plate (Figure 1) experience little or no motion relative to NAD 83 (CORS96). Such is not the case for points located on other tectonic plates (e.g., Hawaii, Guam, and American Samoa) or within a few hundred kilometers of the North American plate boundary (e.g., California, Oregon, Washington, and Alaska). For example, Honolulu, HI, moves northwestward about 80 mm/yr relative to NAD 83 (CORS96). Hence, this document introduces two new spatial reference frames—one for the Pacific tectonic plate to be called NAD 83 (PACP00) and one for the Mariana tectonic plate to be called NAD 83 (MARP00)—so that users of these frames do not have to cope with changing positional coordinates for those positioning applications that are confined to the interior of one of these tectonic plates. Each of these two frames is defined in terms of a transformation from ITRF00. Unfortunately, the crustal motion occurring near the North American plate boundary is so complex that no single reference frame can be defined to "remove" this motion. NAD 83 users thus need to cope with changing coordinates at points located within a few hundred kilometers of this boundary. To help these users, NGS has developed the HTDP (horizontal time-dependent positioning) software (Snay 1999).

In the summer of 1993, NGS conducted an extensive GPS survey to determine positional coordinates for points on numerous islands in the Pacific Ocean. The computational process was performed so that the resulting positional coordinates for these points would be "consistent" with adopted NAD 83 (HARN) positional coordinates for eight VLBI stations located on the North American plate at an epoch date of 14 AUG 1993 (= 1993.62) (Frakes 1994). The unofficial name NAD 83 (HARN) has been introduced here to represent the specific realization of NAD 83, which was current when the positional coordinates of these Pacific reference stations were first computed. The results of those computations are used here to determine the seven parameters for transforming ITRF00 positional coordinates at an epoch date of 1993.62 to both NAD 83 (PACP00) and NAD 83 (MARP00) coordinates at this same epoch date. Thus, at this epoch date, these two reference frames will have positional coordinates that are consistent with NAD 83 (HARN) positional coordinates. In addition, computed ITRF00 velocities for points on several Pacific plate islands will be used to define the seven additional parameters needed to transform ITRF00 positional coordinates at an arbitrary epoch date, denoted *t*, to corresponding NAD 83 (PACP00) positional coordinates at *t*. Finally, the velocity of a point on Guam will be used to define the seven additional parameters needed to transform ITRF00 positional coordinates at *t* to corresponding NAD 83 (MARP00) positional coordinates at *t*.

## Transforming Positional Coordinates

Let $x(t)_{NAD83}$ , $y(t)_{NAD83}$ , *and* $z(t)_{NAD83}$ denote the NAD 83 positional coordinates for a point at time *t* as expressed in a 3D cartesian Earth-centered, Earth-fixed coordinate system. These coordinates are expressed as a function of time to reflect the reality of the crustal motion associated with plate tectonics, land subsidence, volcanic activity, post-glacial rebound, etc. Similarly, let $x(t)_{ITRF}$ , $y(t)_{ITRF}$ , $z(t)_{ITRF}$ denote the ITRFxx positional coordinates for this same point at time *t* and for some appropriate value of xx. The given ITRFxx coordinates are related to their corresponding NAD 83 coordinates by a Helmert transformation that is approximated by the equations:

$$x(t)_{NAD83} = T_x(t) + [1+s(t)]\cdot x(t)_{ITRF} + \omega_z(t)\cdot y(t)_{ITRF} - \omega_y(t)\cdot z(t)_{ITRF}$$
$$y(t)_{NAD83} = T_y(t) - \omega_z(t)\cdot x(t)_{ITRF} + [1+s(t)]\cdot y(t)_{ITRF} + \omega_x(t)\cdot z(t)_{ITRF}$$
$$z(t)_{NAD83} = T_z(t) + \omega_y(t)\cdot x(t)_{ITRF} - \omega_x(t)\cdot y(t)_{ITRF} + [1+s(t)]\cdot z(t)_{ITRF} \quad (1)$$

Here $T_x(t)$, $T_y(t)$, and $T_z(t)$ are translations along the $x$-, $y$-, and $z$-axis, respectively; $\omega_x(t)$, $\omega_y(t)$ and $\omega_z(t)$ are counterclockwise rotations about these same three axes; $s(t)$ is a differential scale change between ITRFxx and NAD 83. These approximate equations suffice because the three rotations usually have rather small magnitudes. Note that each of these seven quantities is represented as a function of time, because improvements in space-based geodetic techniques have enabled us to detect their time-related variations with some degree of accuracy. These time-dependent variations are assumed to be mostly linear, whereby the quantities may be expressed by the equations:

$$T_x(t) = T_x(t_0) + \dot{T}_x\cdot(t-t_0)$$
$$T_y(t) = T_y(t_0) + \dot{T}_y\cdot(t-t_0)$$
$$T_z(t) = T_z(t_0) + \dot{T}_z\cdot(t-t_0) \quad (2)$$

$$\omega_x(t) = [\varepsilon_x(t_0) + \dot{\varepsilon}_x\cdot(t-t_0)]\cdot m_r$$
$$\omega_y(t) = [\varepsilon_y(t_0) + \dot{\varepsilon}_y\cdot(t-t_0)]\cdot m_r$$
$$\omega_z(t) = [\varepsilon_z(t_0) + \dot{\varepsilon}_z\cdot(t-t_0)]\cdot m_r$$
$$s(t) = s(t_0) + \dot{s}\cdot(t-t_0)$$

where $m_r$ = $4.84813681\times10^{-9}$ ≡ conversion factor from milli-arcseconds (mas) to radians.Here, $t_0$ denotes a fixed, prespecified time of reference, commonly called the "epoch date." Hence, the seven quantities $T_x(t_0)$, $T_y(t_0)$, …, $s(t_0)$ are all constants. The seven other quantities $\dot{T}_x$, $\dot{T}_y$, …, $\dot{s}$, which represent rates of change with respect to time, are also assumed to be constants.

## Transformation at 1993.62

In 1994, NGS applied the aforementioned "Pacific Rim" GPS survey of 1993 to define positional coordinates for several points located in Hawaii, as well as on many other Pacific islands. These coordinates were intended to be consistent with existing NAD 83 (HARN) coordinates for points located on the North American continent at the 1993.62 epoch date. According to Frakes (1994), this consistency was accomplished as follows. NGS identified eight VLBI stations, located on the North American plate, which had both official NAD 83 (HARN) positional coordinates and ITRF92 positional coordinates with velocities. These eight stations (Pietown, New Mexico; Westford, Massachusetts; Bloomington, Indiana; Plattville, Colorado; Algonquin, Ontario; Maryland Point, Maryland; Gilcreek, Alaska; and Richmond, Florida) are included in Table 1 and displayed in Figure 1. NGS then applied ITRF92 velocities to compute ITRF92 positional coordinates for the eight VLBI stations at an epoch date of 1993.62. NAD 83 (HARN) coordinates for these eight stations were assumed to be constant with respect to time. NGS then estimated

| Site Location | $x_{NAD83}$, m | $y_{NAD83}$, m | $z_{NAD83}$, m |
|---|---|---|---|
| KAUAI 1311 VLBI, HI | 5543845.351 | --2054565.255 | 2387813.470 |
| MAUI 7120 VLBI (HALEAKALA), HI | -5465997.842 | --2404409.680 | 2242228.148 |
| MAUNA KEA 7617 VLBI, HI | -5464074.250 | -2495250.772 | 2148296.265 |
| KAUAI (KOKB) GPS MON, HI | -5543845.351 | -2054588.873 | 2387809.049 |
| UPOLU POINT (UPO1) GPS, HI | -5464031.073 | -2446034.292 | 2193282.447 |
| KOKOLE POINT (KOK1) GPS, HI | -5551749.130 | -2047251.873 | 2372725.972 |
| GUAM (GUAM) GPS MON, Guam | -5071312.533 | 3568361.806 | 1488903.370 |
| ASPA GPS ARP, Samoa | -6100259.451 | -996506.212 | -1567978.797 |
| PIETOWN VLBI, NM | -1640953.180 | -5014817.473 | 3575411.968 |
| WESTFORD 7209 VLBI, MA | 1492207.146 | -4458131.996 | 4296015.689 |
| BLOOMINGTON VLBI, IN | 302384.919 | -4941700.546 | 4007908.659 |
| PLATTVILLE VLBI, CO | -1240707.661 | -4720455.743 | 4094481.678 |
| ALGONQUIN VLBI, Ontario | 918035.288 | -4346133.663 | 4561971.264 |
| MARYLAND PT VLBI, MD | 1106629.863 | -4882908.728 | 3938087.170 |
| GILGREEK VLBI, AK | -2281546.675 | --1453646.134 | 5756992.740 |
| RICHMOND VLBI, FL | 961258.604 | -5674091.724 | 2740534.070 |

**Table 1.** NAD 83 (HARN) positional coordinates at epoch 1993.62.

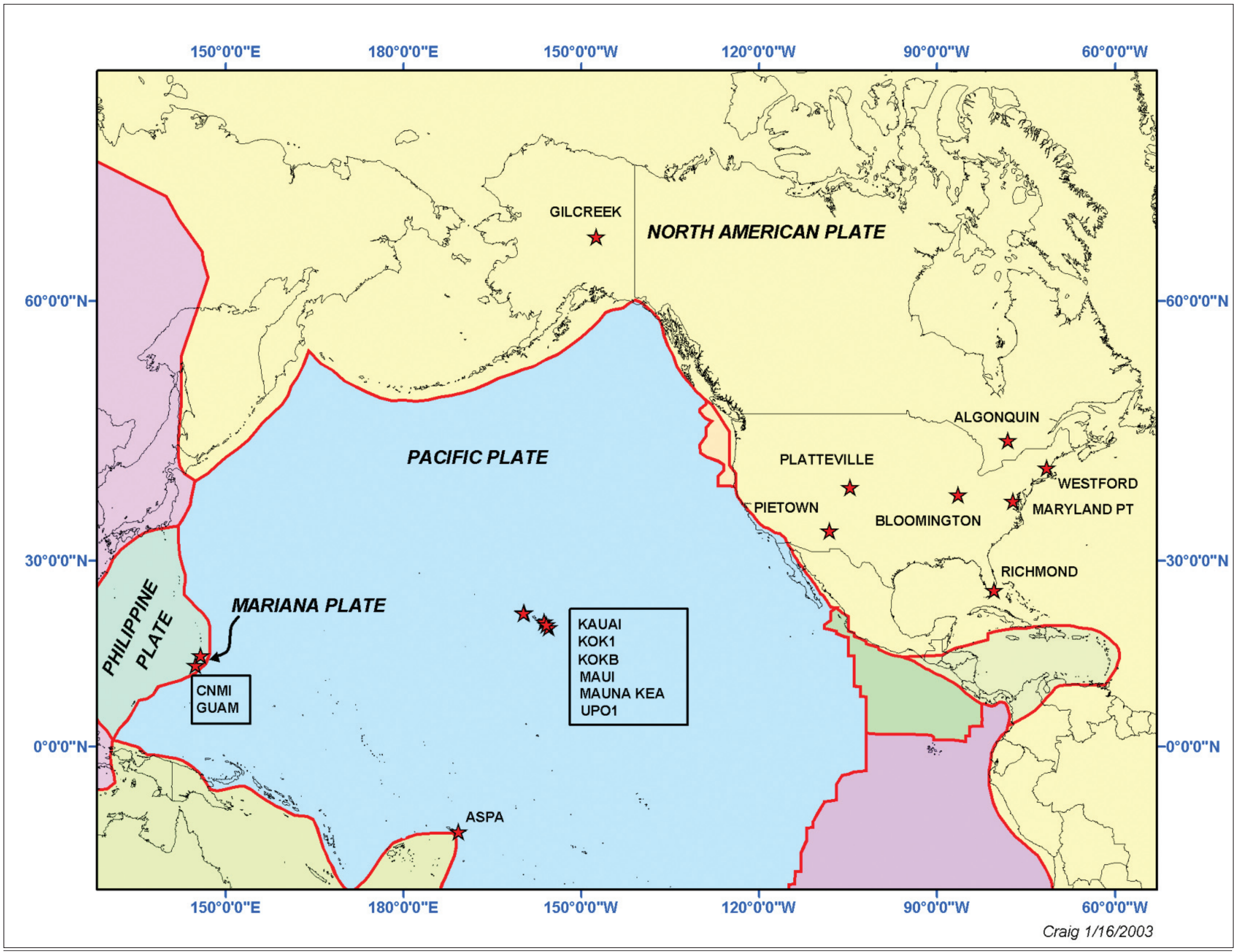


**Figure 1.** VLBI and GPS reference marks involved in defining NAD 83 (PACP00) and NAD 83 (MARP00).

the seven parameters—$T_x$*(1993.62),* $T_y$*(1993.62), …, s(1993.62)*—that best transformed these eight ITRF92 positional coordinates to corresponding NAD 83 (HARN) positional coordinates, in a least square sense, according to Equation (1) for the special case when $t$ is fixed to the value of $t_o$ = 1993.62. This transformation was used to transform official ITRF92 positional coordinates at two VLBI stations in Hawaii—Kauai (Kokee Park) and Maui (Haleakala)—to consistent NAD 83 (HARN) coordinates at the 1993.62 epoch date. The reference marks associated with these two VLBI stations had been occupied as part of the 1993 Pacific Rim GPS survey. NGS thus constrained positional coordinates for these two reference marks to their derived NAD 83 (HARN) values when the agency performed a network adjustment of the 1993 GPS survey to obtain positional coordinates for other reference stations involved in the survey.

The approach applied in 1994 is emulated here to estimate seven parameters for transforming ITRF00 positional coordinates to NAD 83 (HARN) coordinates at the 1993.62 epoch date. Table 1 presents currently adopted NAD 83 (HARN) positional coordinates at the 1993.62 epoch date for several pertinent VLBI and GPS stations. Table 2 presents the ITRF00 positional coordinates adopted for these same stations, but at an epoch date of 1997.00 (1 JAN 1997). Table 2 also presents corresponding ITRF00 velocities of these stations.

Table 3 presents equivalent ITRF00 positional coordinates at the 1993.62 epoch date. The $x$-coordinate at 1993.62, for example, was computed by the equation:

$$x(1993.62) = x(1997.00) + v_x \cdot (1993.62 - 1997.00) \quad (3)$$

where $v_x$ denotes the $x$-component of the station's velocity. The $y$- and $z$-coordinates at 1993.62 were computed in a similar fashion.

The XYZ3 software developed by Dennis Milbert of NGS was used to estimate (in a least squares sense) the seven parameters—$T_x$*(1993.62),* $T_y$*(1993.62), …, s(1993.62)*—needed to transform the ITRF00 positional coordinates listed in Table 3 to the NAD 83 (HARN) coordinates listed in

| Site | x (m) | y (m) | z (m) | $v_x$ (mm/yr) | $v_y$ (mm/yr) | $v_z$ (mm/yr) |
|---|---|---|---|---|---|---|
| KAUAI 1311 VLBI | -5543846.063 | -2054563.643 | 2387814.111 | -9.5 | 63.0 | 29.8 |
| MAUI 7120 VLBI (HALEAKALA) | -5465998.562 | -2404408.039 | 2242228.751 | -13.8 | 60.7 | 31.1 |
| MAUNA KEA 7617 VLBI | -5464074.965 | -2495249.118 | 2148296.845 | -13.5 | 63.1 | 29.6 |
| KAUAI (KOKB) GPS MON | -5543838.118 | -2054587.260 | 2387809.705 | -9.5 | 63.0 | 29.8 |
| UPOLU POINT (UPO1) GPS | -5464031.768 | -2446032.649 | 2193283.030 | -12.4 | 63.2 | 29.3 |
| KOKOLE POINT (KOK1) GPS | -5551749.829 | -2047250.258 | 2372726.612 | -8.5 | 64.2 | 29.0 |
| GUAM (GUAM) GPS MON | -5071312.849 | 3568363.526 | 1488904.330 | 4.2 | 10.5 | 3.0 |
| ASPA GPS ARP | -6100259.943 | -996503.926 | -1567978.119 | -19.3 | 59.7 | 29.8 |
| PIETOWN VLBI | -1640953.713 | -5014816.027 | 3575411.878 | -14.7 | -0.6 | -8.4 |
| WESTFORD 7209 VLBI | 1492206.588 | -4458130.518 | 4296015.541 | -15.6 | -1.3 | 2.6 |
| BLOOMINGTON VLBI | 302384.405 | -4941699.065 | 4007908.512 | -12.9 | 1.1 | -1.5 |
| PLATTVILLE VLBI | -1240708.205 | -4720454.351 | 4094481.613 | -15.3 | 1.7 | -7.7 |
| ALGONQUIN VLBI | 918034.742 | -4346132.271 | 4561971.166 | -16.1 | -4.1 | 2.6 |
| MARYLAND PT VLBI | 1106629.297 | -4882907.158 | 3938086.959 | -15.6 | 3.4 | -2.0 |
| GILCREEK VLBI | -2281547.309 | -1453645.086 | 5756993.162 | -22.2 | -3.6 | -9.2 |
| RICHMOND VLBI | 961258.052 | -5674090.060 | 2740533.809 | -9.7 | -1.3 | 1.3 |

**Table 2.** ITRF00 positional coordinates at epoch 1997.00 with ITRF00 velocities.

Table 1, according to Equation (1) for the special case when $t$ is fixed to the value of $t_o$ = 1993.62. Suitable values for these seven parameters could be found only if the stations GUAM and ASPA (American Samoa) were excluded. If either of these two stations were included, then residuals (differences between the input NAD 83 (HARN) coordinates and the transformed ITRF00 coordinates) of several decimeters were encountered. Table 4 exhibits the residuals for GUAM and ASPA when six parameters are constrained to those obtained in the solution using only the 14 geodetic stations located in North America and Hawaii and when the seventh parameter, *s(1993.62)*, is constrained to zero. This constraint was applied so that the scale of the new reference frames would equal the scale of ITRF00. Table 4 also displays the derived values for these seven parameters.

The results displayed in Table 4 indicate that the existing NAD 83 (HARN) coordinates in Hawaii are excellent! Unfortunately, these results also indicate a problem with the NAD 83 (HARN) coordinates derived for Guam and American Samoa using the 1993 GPS survey. It follows that previously derived

| Site | x (m) | y (m) | z (m) |
|---|---|---|---|
| KAUAI 1311 VLBI | -5543846.031 | -2054563.856 | 2387814.010 |
| MAUI 7120 VLBI (HALEAKALA) | -5465998.515 | -2404408.245 | 2242228.646 |
| MAUNA KEA 7617 VLBI | -5464074.919 | -2495249.332 | 2148296.745 |
| KAUAI (KOKB) GPS MON | -5543838.086 | -2054587.473 | 2387809.604 |
| UPOLU POINT (UPO1) GPS | -5464031.725 | -2446032.863 | 2193282.931 |
| KOKOLE POINT (KOK1) GPS | -5551749.800 | -2047250.475 | 2372726.514 |
| GUAM (GUAM) GPS MON | -5071312.814 | 3568363.456 | 1488904.306 |
| ASPA GPS ARP | -6100259.878 | -996504.128 | -1567978.220 |
| PIETOWN VLBI | -1640953.663 | -5014816.025 | 3575411.906 |
| WESTFORD 7209 VLBI | 1492206.641 | -4458130.514 | 4296015.532 |
| BLOOMINGTON VLBI | 302384.449 | -4941699.069 | 4007908.517 |
| PLATTVILLE VLBI | -1240708.153 | -4720454.357 | 4094481.639 |
| ALGONQUIN VLBI | 918034.796 | -4346132.257 | 4561971.157 |
| MARYLAND PT VLBI | 1106629.350 | -4882907.170 | 3938086.966 |
| GILCREEK VLBI | -2281547.234 | -1453645.074 | 5756993.193 |
| RICHMOND VLBI | 961258.085 | -5674090.056 | 2740533.805 |

**Table 3.** ITRF00 positional coordinates at epoch 1993.62.

| Site | $r_x$ | $r_y$ | $r_z$ | $r_{north}$ | $r_{east}$ | $r_{up}$ |
|---|---|---|---|---|---|---|
| HAWAII | | | | | | |
| KAUAI 1311 VLBI | 0.0131 | -0.0075 | -0.0013 | 0.0024 | 0.0116 | -0.0095 |
| MAUI VLBI (HALEAKALA) | 0.0101 | 0.0042 | 0.0097 | 0.0130 | 0.0002 | -0.0068 |
| MAUNA KEA 7617 VLBI | 0.0142 | -0.0041 | 0.0046 | 0.0082 | 0.0096 | -0.0090 |
| KAUAI (KOKB) GPS MON | 0.0061 | -0.0065 | 0.0137 | 0.0140 | 0.0082 | 0.0019 |
| UPOLU POINT (UPO1) GPS | 0.0315 | -0.0088 | 0.0017 | 0.0103 | 0.0208 | -0.0230 |
| KOKOLE POINT (KOK1) GPS | 0.0242 | -0.0102 | -0.0007 | 0.0065 | 0.0180 | -0.0181 |
| | | | | | | |
| MARIANAS | | | | | | |
| GUAM (GUAM) GPS MON | 0.7313 | 0.0938 | -0.3739 | -0.2356 | -0.4976 | -0.6167 |
| | | | | | | |
| SAMOA | | | | | | |
| ASPA GPS ARP | 0.5109 | 0.1478 | -0.1404 | -0.2667 | -0.0636 | -0.4769 |
| | | | | | | |
| NORTH AMERICA | | | | | | |
| PIETOWN VLBI, NM | 0.0073 | 0.0176 | 0.0038 | 0.0139 | 0.0014 | -0.0136 |
| WESTFORD 7209 VLBI, MA | -0.0226 | -0.0003 | -0.0168 | -0.0077 | -0.0216 | -0.0164 |
| BLOOMINGTON VLBI, IN | 0.0028 | 0.0124 | 0.0083 | 0.0141 | 0.0035 | -0.0042 |
| PLATTVILLE VLBI, CO | -0.0126 | 0.0091 | 0.0049 | 0.0073 | -0.0145 | -0.0011 |
| ALGONQUIN VLBI, ONT | -0.0171 | -0.0108 | -0.0105 | -0.0124 | -0.0190 | -0.0027 |
| MARYLAND PT VLBI, MD | -0.0340 | 0.0442 | -0.0228 | 0.0135 | -0.0234 | -0.0538 |
| GILGREEK VLBI, AK | -0.0008 | -0.0319 | -0.0138 | -0.0219 | 0.0265 | -0.0050 |
| RICHMOND VLBI, FL | -0.0203 | -0.0073 | 0.0205 | 0.0169 | -0.0212 | 0.0123 |

Parameters for "best" (ITRF00 --> NAD 83(HARN)) transformation at epoch 1993.62.

$T_x(1993.62) = 0.9102$ meters

$T_y(1993.62) = -2.0141$ meters

$T_z(1993.62) = -0.5602$ meters

$\varepsilon_x(1993.62) = 29.039$ mas

$\varepsilon_y(1993.62) = 10.065$ mas

$\varepsilon_z(1993.62) = 10.101$ mas

$s(1993.62) = 0.0$ (unit-less)

**Table 4.** Residuals to "best" (ITRF00 --> NAD 83(HARN)) transformation for an epoch date of 1993.62. All units are in meters.

NAD 83 (HARN) coordinates for points located on some Pacific Islands (other than Guam, American Samoa, and the Hawaiian Islands) may also be in error, but no reference stations currently exist to help in estimating the errors in these NAD 83 (HARN) coordinates. This problem with coordinate errors reflects certain limitations associated with GPS technology in 1993. The people responsible for collecting the 1993 GPS data, for processing these data, and for adjusting these data did the best job possible during the 1993-1994 time frame. The 1994 network adjustment of the 1993 GPS survey used essentially only two VLBI stations in Hawaii as positional constraints to determine coordinates for the reference marks involved in the 1993 GPS survey, and some of these marks are located several thousand kilometers away from Hawaii. A better job adjusting these data can be performed today using coordinates of recently established CORS as constraints and using GPS measurements performed among points on these islands since 1993.

## Transformation from ITRF00 to NAD 83 (PACP00)

The 18$^{th}$ century Swiss mathematician, Leonard Euler (1707–1783), proved that the rigid motion of any area confined to the surface of a sphere can be defined by a rotation of that area about a pole passing through the center of the sphere.

Accordingly, Beavan et al. (2002) used horizontal ITRF00 velocities for 11 reference stations located on the Pacific tectonic plate to demonstrate that the horizontal motion of this plate can be adequately described by a rotation rate of $\dot{\Omega}$=.677 deg/Myr = 2.437 mas/yr about a pole that pierces the Earth's surface at spherical latitude φ = S63.75 deg and spherical longitude λ = E110.86 deg. These three parameters may be converted to three equivalent parameters $\dot{\Omega}_x, \dot{\Omega}_y$, and $\dot{\Omega}_z$ that represent counter-clockwise rotation rates about the *x*-axis, *y*-axis, and *z*-axis, respectively. This conversion is defined by the equations:

$$\begin{aligned}\dot{\Omega}_x &= \dot{\Omega}\cdot\cos\varphi\cdot\cos\lambda = -0.384 mas/yr\\ \dot{\Omega}_y &= \dot{\Omega}\cdot\cos\varphi\cdot\sin\lambda = 1.007 mas/yr\\ \dot{\Omega}_z &= \dot{\Omega}\cdot\sin\varphi = -2.186 mas/yr\end{aligned} \quad (4)$$

Accordingly, the mathematical equations

$$\begin{aligned}v_{x_{ITRF}} &= (-\dot{\Omega}_z\cdot y_{ITRF} + \dot{\Omega}_y\cdot z_{ITRF})\cdot m_r\\ v_{y_{ITRF}} &= (\dot{\Omega}_z\cdot x_{ITRF} - \dot{\Omega}_x\cdot z_{ITRF})\cdot m_r\\ v_{z_{ITRF}} &= (-\dot{\Omega}_y\cdot x_{ITRF} + \dot{\Omega}_x\cdot y_{ITRF})\cdot m_r\end{aligned} \quad (5)$$

provide approximate horizontal ITRF00 velocities for those points located on the Pacific tectonic plate which are not experiencing regional or local deformation.

Velocities referred to a NAD 83 reference frame can be obtained from their corresponding ITRF00 velocities by taking the derivatives of Equation (1) with respect to time. After neglecting second-order terms, the following equations are obtained:

$$\begin{aligned}v_{x_{NAD83}} &= v_{x_{ITRF}} + \dot{T}_x + \dot{s}\cdot x(t)_{ITRF} + \dot{\omega}_z\cdot y(t)_{ITRF} - \dot{\omega}_y\cdot z(t)_{ITRF}\\ v_{y_{NAD83}} &= v_{y_{ITRF}} + \dot{T}_y + \dot{s}\cdot y(t)_{ITRF} - \dot{\omega}_z\cdot x(t)_{ITRF} + \dot{\omega}_x\cdot z(t)_{ITRF}\\ v_{z_{NAD83}} &= v_{z_{ITRF}} + \dot{T}_z + \dot{s}\cdot z(t)_{ITRF} + \dot{\omega}_y\cdot x(t)_{ITRF} - \dot{\omega}_x\cdot y(t)_{ITRF}\end{aligned} \quad (6)$$

where:

$$\begin{aligned}\dot{\omega}_x &= \dot{\varepsilon}_x\cdot m_r\\ \dot{\omega}_y &= \dot{\varepsilon}_y\cdot m_r\\ \dot{\omega}_z &= \dot{\varepsilon}_z\cdot m_r\end{aligned} \quad (7)$$

Hence, adopting the following values:

$$\begin{aligned}\dot{\varepsilon}_x &= \dot{\Omega}_x = -0.384 mas/yr\\ \dot{\varepsilon}_y &= \dot{\Omega}_y = 1.007 mas/yr\\ \dot{\varepsilon}_z &= \dot{\Omega}_z = -2.186 mas/yr\\ \dot{T}_x &= \dot{T}_y = \dot{T}_z = \dot{s} = 0.0\end{aligned} \quad (8)$$

together with the values for $T_x(1993.62)$, $T_y(1993.62)$, …, $s(1993.62)$—as given in Table 4—defines a transformation from ITRF00 to NAD 83 (PACP00), in accordance with Equations (1) and (2), such that points which do not move relative to the stable part of the Pacific plate will have a horizontal velocity of zero relative to NAD 83 (PACP00).

## Transformation from ITRF00 to NAD 83 (MARP00)

As discussed in the previous section, the horizontal motion of a rigid tectonic plate is completely defined by three parameters: (a) the rotation rate about a pole through the Earth's center with the spherical latitude and longitude of the point where this pole intersects the Earth's surface or (b) three rotation rates—$\dot{\Omega}_x, \dot{\Omega}_y$, and $\dot{\Omega}_z$. Consequently, one needs the horizontal ITRF00 velocity of at least two reference stations to uniquely resolve the values for either set of three parameters. Unfortunately, the Mariana plate contains only one reference station whose horizontal ITRF00 velocity is well determined; namely, the CORS known as GUAM with:

$$v_{north} = 0.0023 \text{ m/yr} \quad \text{and} \quad v_{east} = -0.0110 \text{ m/yr} \quad (9)$$

as was adopted by both IERS and NGS.

To determine unique values for the three plate–motion parameters, it was assumed that the CORS known as CNMI, which is located on Saipan, has the same horizontal ITRF00 velocity as GUAM.

It follows from Equation (5) that:

$$\begin{aligned}v_{north}/m_r = {} & [(z\cdot\sin\varphi\cdot\sin\lambda)+(y\cdot\cos\varphi)]\cdot\dot{\Omega}_x\\ & -[(z\cdot\sin\varphi\cdot\cos\lambda)+(x\cdot\cos\varphi)]\cdot\dot{\Omega}_y\\ & +[(y\cdot\sin\varphi\cdot\cos\lambda)-(x\cdot\sin\varphi\cdot\sin\lambda)]\cdot\dot{\Omega}_z\end{aligned} \quad (10)$$

and

$$v_{east}/m_r = [-z\cdot\cos\lambda]\cdot\dot{\Omega}_x - [z\cdot\sin\lambda]\cdot\dot{\Omega}_y + [(y\cdot\sin\lambda)+(x\cdot\cos\lambda)]\cdot\dot{\Omega}_z \quad (11)$$

for the point with *x, y, and z* as its cartesian Earth-centered, Earth-fixed coordinates and with φ and λ as its corresponding geodetic latitude and longitude, respectively.

Using the ITRF00 positional coordinates of GUAM given in Table 2 and applying the ITRF00 positional coordinates for CNMI as:

$$\begin{aligned}x &= -5{,}088{,}866 \text{ m}\\ y &= 3{,}464{,}934 \text{ m}\\ z &= 1{,}662{,}431 \text{ m}\end{aligned} \quad (12)$$

the following rotation rates:

$$\begin{aligned}\dot{\Omega}_x &= -0.020 \text{ mas/yr}\\ \dot{\Omega}_y &= 0.105 \text{ mas/yr}\\ \dot{\Omega}_z &= -0.347 \text{ mas/yr}\end{aligned} \quad (13)$$

were obtained as the "best" solution to Equations (10) and (11), in a least squares sense, which yields the adopted horizontal ITRF00 velocities of GUAM and CNMI. Hence, adopting the following values:

$$\dot{\varepsilon}_x = \dot{\Omega}_x = -0.020 mas/yr$$
$$\dot{\varepsilon}_y = \dot{\Omega}_y = 0.105 mas/yr$$
$$\dot{\varepsilon}_z = \dot{\Omega}_z = -0.347 mas/yr$$
$$\dot{T}_x = \dot{T}_y = \dot{T}_z = \dot{s} = 0.0 \quad (14)$$

together with the values for $T_x$*(1993.62)*, $T_y$*(1993.62), …, s(1993.62)*—as specified in Table 4—defines a transformation from ITRF00 to NAD 83 (MARP00), in accordance with Equations (1) and (2), such that GUAM and CNMI will each have a horizontal velocity of zero relative to NAD 83 (MARP00).

## Conclusion

Two new reference frames were introduced for regions of the Pacific Ocean—one frame for locations on the Pacific tectonic plate and the other for locations on the Mariana tectonic plate. As a result, users of these frames will not need to cope with changing positional coordinates in the corresponding regions, except for those changes associated with such local phenomena as earthquakes, volcanic activity, and land subsidence. These new reference frames were defined so as to be consistent with NAD 83 realizations at the 1993.62 epoch. Otherwise, these frames are distinct from all NAD 83 realizations. For instance, the NAD 83 (PACP00) positional coordinates for a point in Hawaii would equal this point's NAD 83 (CORS96) positional coordinates for an epoch date of 1993.62, but not for any other epoch date. The horizontal speed of this Hawaiian point would equal approximately 0 mm/yr relative to NAD 83 (PACP00), whereas its horizontal speed would equal approximately 80 mm/yr relative to NAD 83 (CORS96).

The adopted transformation parameters have been encoded into version 2.7 of the HTDP software, which is accessible via the "Geodetic Tool Kit" on the NGS web site at http://www.ngs.noaa.gov. Hence, HTDP users can transform positional coordinates and velocities from any of several popular reference frames to either of these two new frames, and vice versa. The list of popular reference frames currently encoded into HTDP includes all official realizations of ITRF, all official realizations of the World Geodetic System of 1984, and NAD 83 (CORS96).

ACKNOWLEDGMENTS

The author thanks Nikki Case, Stephen Frakes, Jim Ray, Tomás Soler, and Maralyn Vorhauer for suggestions that helped improve the presentation of this paper. Cindra Craig created the graphic in Figure 1.

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