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Unified Strain-and-Temperature Scaling Law:

Separable Parameter Set

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Why consider the separable parameter set?

- It provides the common ground for understanding and translating among the various pinning models that have been proposed.
- It provides a very useful parameterization for engineering purposes.

The separable parameter set consists of scaling parameters that depend on temperature or strain alone.
----- i.e., they are not commingled with respect to *t* and *ε*.

- Thus, they can be determined from ***independent*** strain and temperature experiments;

--- i.e., they do not require a full matrix of $J_c(B,t,\varepsilon)$ values to be initially determined.

--- The set can be built one parameter at a time, as data become available for a particular conductor.

Brief Review and Synthesis of Literature on Scaling Laws

Temperature scaling law (Fietz and Webb 1969):

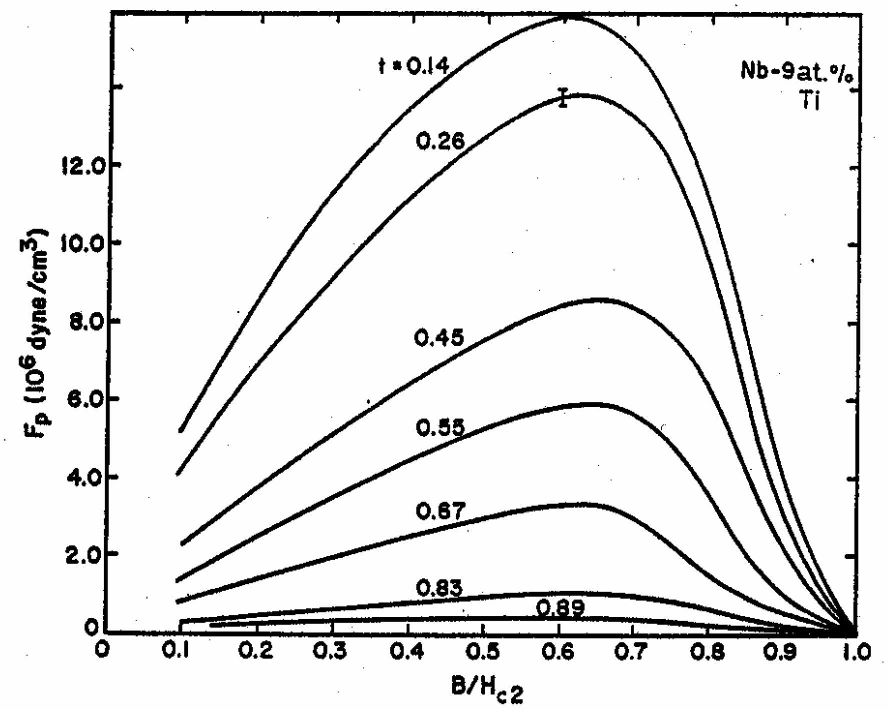
$$J_c\, B \equiv F(B, T, \varepsilon) = K(T)\; f(b)$$

Parameterization $$= [B_{c2}^{*}(T)]^{\eta}\; f(b)$$

$$b \equiv B / B_{c2}^{*}(T)$$

A correlation of a number of Nb_3Sn samples gave $\eta \approx 2.5$

(η is a constant, independent of temperature.)



Strain scaling law (Ekin 1980):

$$J_c B \equiv F(B, T, \varepsilon) = K(\varepsilon)\ f(b)$$

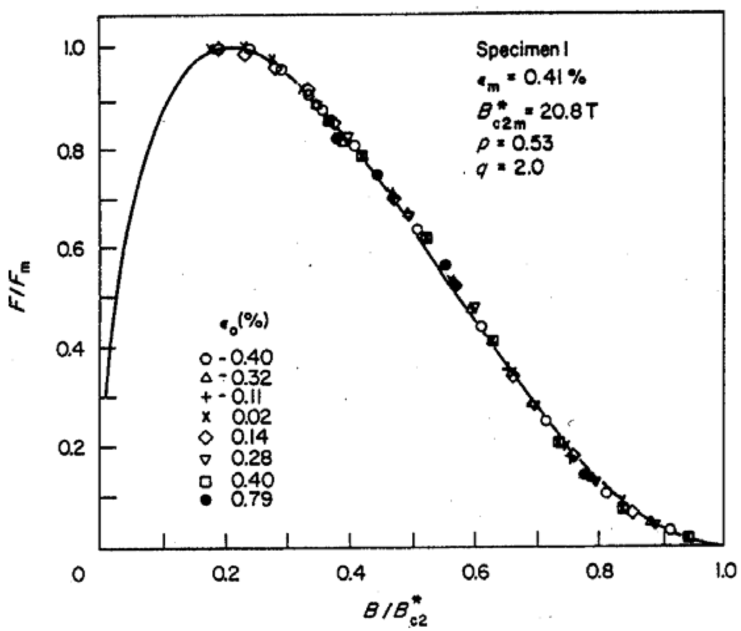
Parameterization:
(at $-0.5\% < \varepsilon_0 < \varepsilon_{0,irr}$)

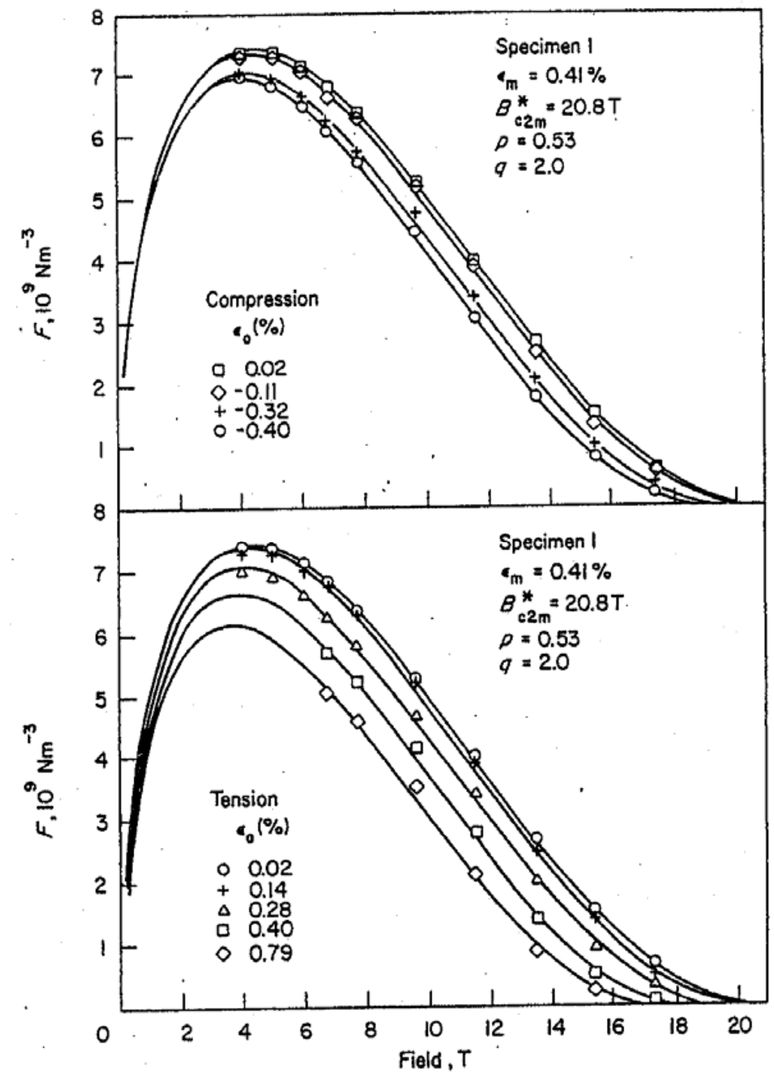
$$= [B_{c2}^{*}(\varepsilon)]^{s}\ f(b)$$

$$b \equiv B / B_{c2}^{*}(\varepsilon)$$

A correlation of a number of Nb_3Sn samples gave $s \approx 1 \pm 0.3$

(s is a constant, independent of strain.)





Unified strain and temperature scaling relation (USL) (Ekin 1980):

$$\mathrm{F}(B, T, \varepsilon) = K(T, \varepsilon_0)\ f(b)$$

Postulated shape invariance of $f(b)$ with strain and temperature.

with variables:

$\varepsilon_0 \equiv \varepsilon - \varepsilon_m$ Intrinsic strain

$t \equiv T / T_c^*(\varepsilon_0)$ Reduced temperature

$b \equiv B / B_{c2}^*(t,\varepsilon_0)$ Reduced magnetic field

where: $T_c^*(\varepsilon_0) / T_c^*(0) = [B_{c2}^*(0,\varepsilon_0) / B_{c2}^*(0,0)]^{1/w}$

$$\frac{B_{c2}^*(t,\varepsilon_0)}{B_{c2}^*(0,0)} = \frac{B_{c2}^*(0,\varepsilon_0)}{B_{c2}^*(0,0)} \frac{B_{c2}^*(t,0)}{B_{c2}^*(0,0)}$$

Postulated separability of B_{c2}^* into strain and temperature components.

and w, $T_c^*(0)$, and $B_{c2}^*(0,0)$ are scaling constants.

(A correlation of a number of Nb_3Sn samples gave the constant: $w \approx 3$.)

- This expression, the variables, and the separable $B_{c2}^*(t,\varepsilon_0)$ expression are utilized in most pinning force model expressions.
- There appears to be general consensus adoption of the Unified Scaling Law and its formalism, as well as separability of $B_{c2}^*(t,\varepsilon_0)$.
- Where the differences arise is in how to *parameterize* $B_{c2}^*(t,\varepsilon_0)$ and $K(T, \varepsilon_0)$.

Postulated separable expression for $B_{c2}^{*}(t,\varepsilon_0)$

$$\frac{B_{c2}^{*}(t,\varepsilon_0)}{B_{c2}^{*}(0,0)} = \underbrace{\frac{B_{c2}^{*}(0,\varepsilon_0)}{B_{c2}^{*}(0,0)}}_{\text{Strain part}} \; \underbrace{\frac{B_{c2}^{*}(t,0)}{B_{c2}^{*}(0,0)}}_{\text{Temp. part}}$$

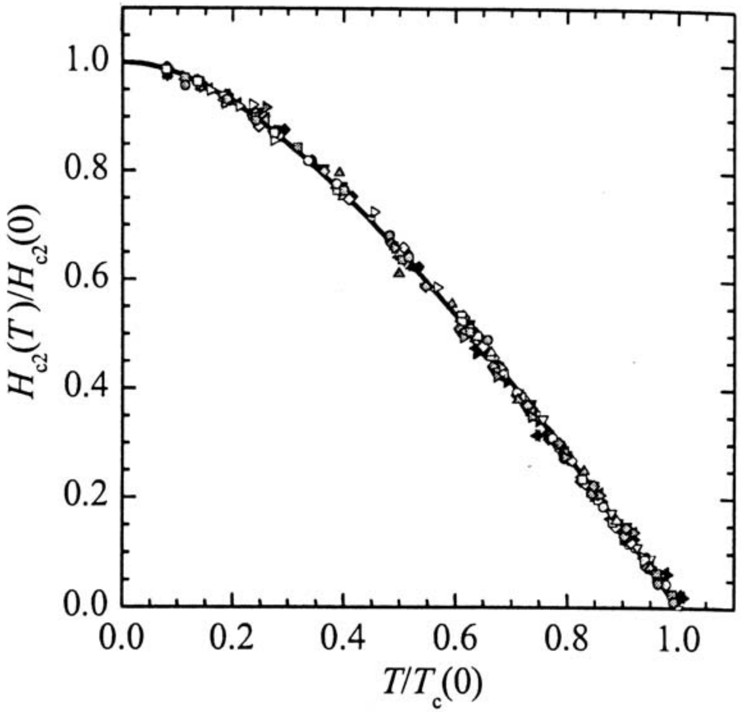
Parameterization of temperature part: $B_{c2}^{*}(t,0)/B_{c2}^{*}(0,0)$

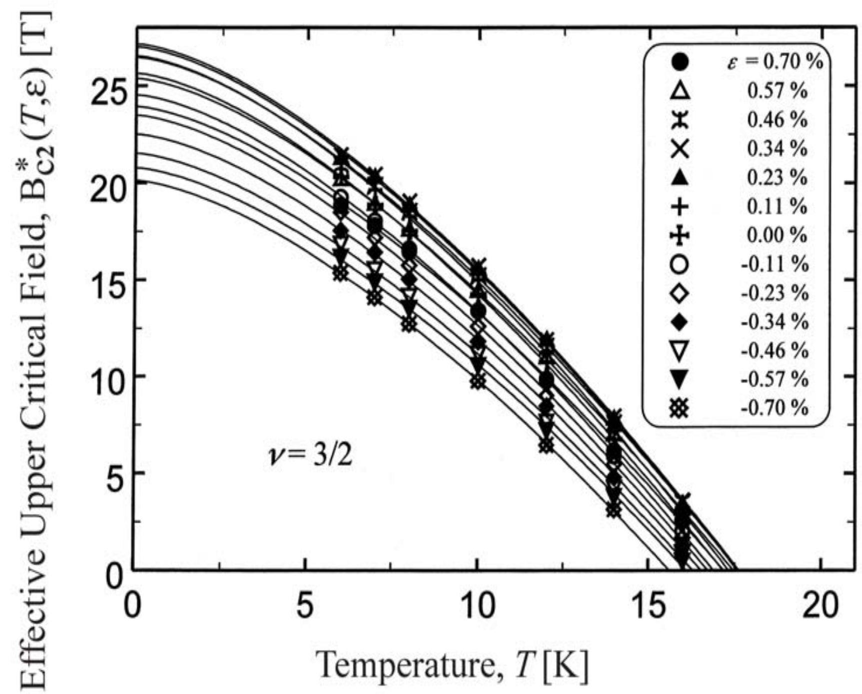
In 1980 no temperatures measurements of kappa were available, so acknowledging that it was deliberately omitted and to obtain a demonstration expression for the temperature part, used

$$\frac{B_{c2}^{*}(t,0)}{B_{c2}^{*}(0,0)} = (1-t^{2})$$

Many years later, measurements by Cheggour and Hampshire, and Goedeke et al. have shown that, to a good approximation, the temperature dependence of kappa can be included simply by

$$\frac{B_{c2}^{*}(t,0)}{B_{c2}^{*}(0,0)} = (1-t^{\nu}), \qquad \text{where } \nu \approx 1.5.$$





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Normalized temperature part of Hc2 for Nb_3Sn samples with a range of Nb-Sn phases. Solid line is for ν = 1.5.
(From Godeke et al. 2005)

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$B_{c2}{}^*(t,\varepsilon_0)$ as a function of temperature and applied strain for bronze-processed Nb3Sn. Curves are for ν = 1.5.
(From Cheggour and Hampshire 2002)

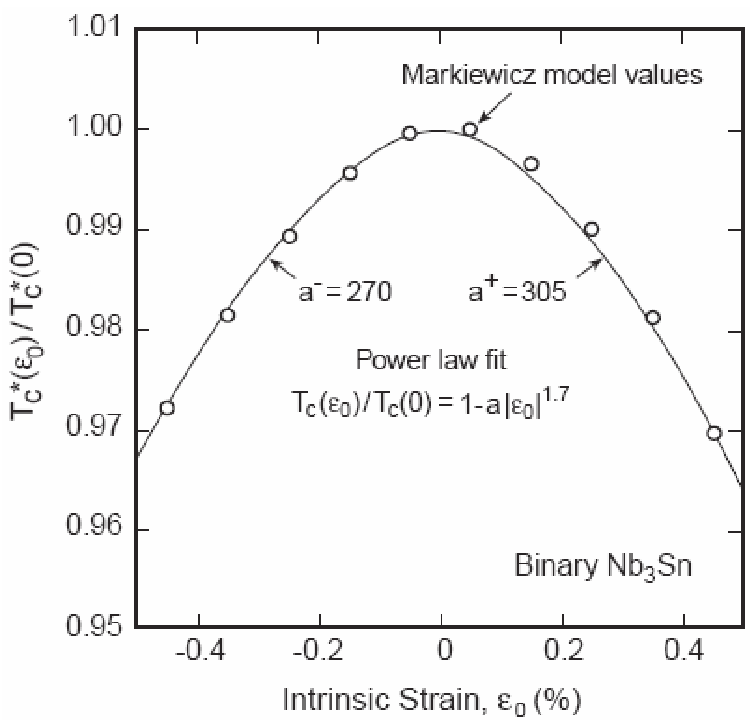
$$\boxed{\frac{B^*_{c2}(t,0)}{B^*_{c2}(0,0)} = (1-t^{\nu}), \quad \text{where } \nu \approx 1.5.}$$

Parameterization of strain part: $B_{c2}^*(0,\varepsilon_0)/B_{c2}^*(0,0)$: Moderate strain range ($-0.5\% < \varepsilon_0 < \varepsilon_{0,irr}$)

Data correlations of many conductors have shown a simple power law to work well:

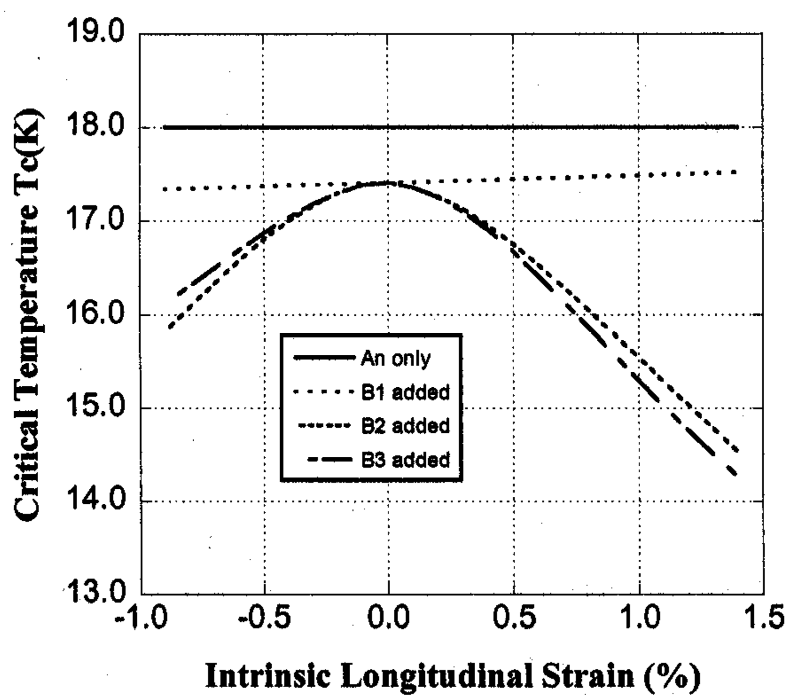
$$\frac{B_{c2}^*(0,\varepsilon_0)}{B_{c2}^*(0,0)} = \boxed{(1 - a\,|\varepsilon_0|^u), \quad \text{where } u = 1.7}$$

Physical basis: the power law expression and the exponent u = 1.7 arise from the second invariant of the deviatoric strain tensor



Model results from W. D. Markiewicz, *Cryogenics* **44**, p. 767 (2004).

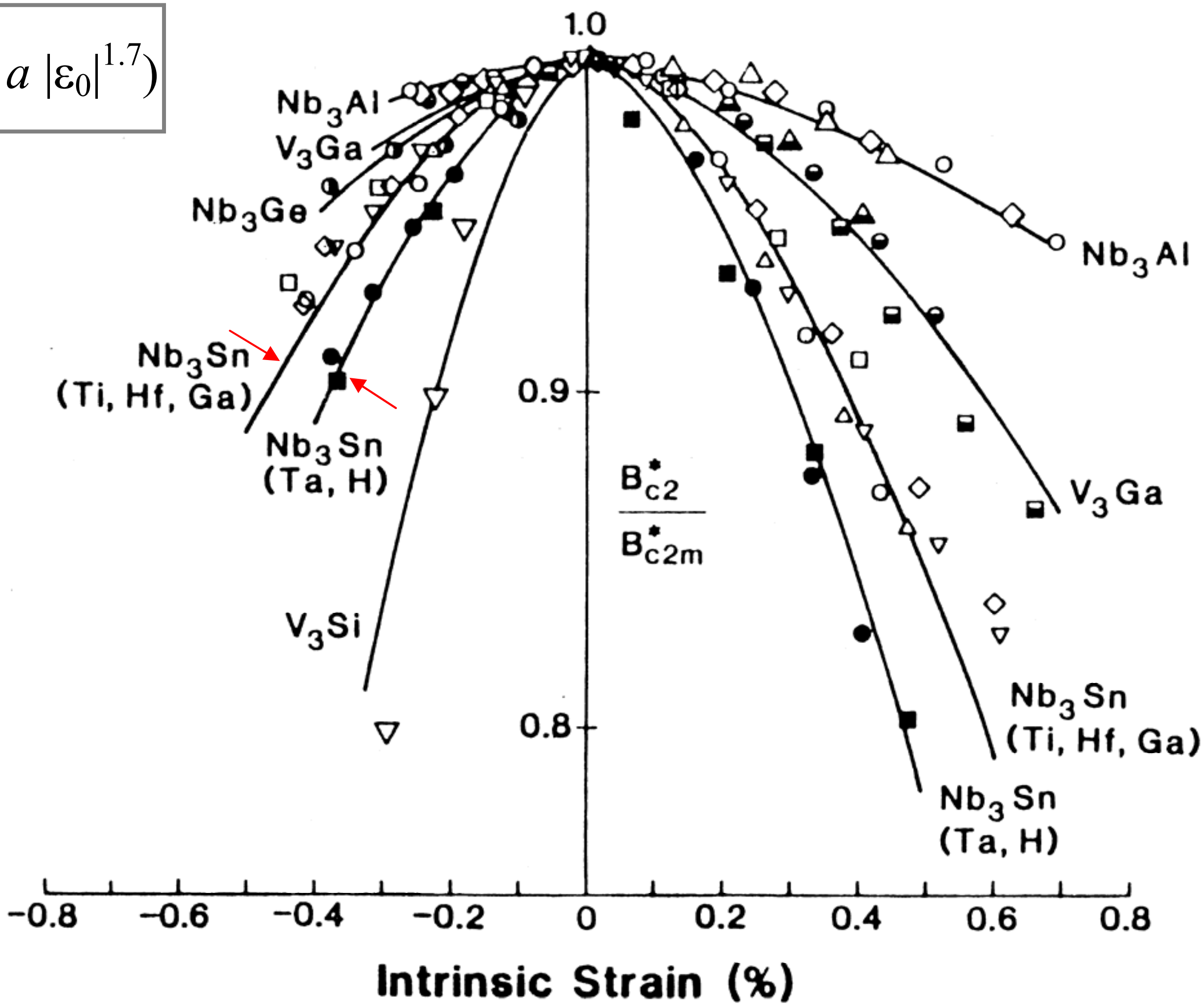
The second invariant, B2, is the main determinant of the power law over the moderate-strain range.



From W. D. Markiewicz, *IEEE Trans. Appl. Supercond.* **15**, 3368 (2005).

- Data correlations show that the curvature parameter *u* equals 1.7 across the A15 spectrum.
- Parameter "*a*" is simple scalar index of strain sensitivity.
- With one scalar parameter, can instantly characterize the strain sensitivity of a conductor.

$$\frac{B^*_{c2}(0,\varepsilon_0)}{B^*_{c2}(0,0)} = (1 - a\,|\varepsilon_0|^{1.7})$$



From Ekin (1984), *Adv. Cryo. Eng.* 30, pp. 823-836.

Scaling parameters for low-T_c multifilamentary wire superconductors at 4.2 K

Note that values of "*a*" in Nb_3Sn depend on doping. For all conductors u = 1.7

Superconductor	Crystal Structure	Magnetic-Field Dependence of I_c			Strain Dependence of I_c ‡ * (–0.5% < ε_0 < +0.4%)			Ref.
		p †	q †	B^*_{c2} at 4.2K,ε_0=0 [T]	a^- (ε_0<0)	a^+ (ε_0>0)	s	
Strain-dependent Superconductors								
Nb_3Al (RHQT)	A15	0.5	~2.0	26	370		~2.5	a
V_3Ga	"	0.4	1.0	21	450	650	1.4	b
Nb_3Ge	"	0.6	1.9	25	500	—	~2	c
Nb_3Sn*	"	0.5	2.0	21	900	1250	1	d
Nb_3Sn +0.6at%Ti*	"	0.6	1.7	23	900	1250	1.1	e
Nb_3Sn +1.85at%Ti*	"	0.5	1.5	25	1100	1450	1.2	e,f
Nb_3Sn +0.6at%Ta*	"	0.5	1.4	24	900	1250	1.0	e
Nb_3Sn +2.2at%Ta*	"	0.5	1.4	24	1350	1800	~1	e,g
V_3Si	"	0.5	1.7	16	3500	—	~1	a
$PbMo_6S_8$	Chevrel	0.3	6	63	—	1900	~2	h
Strain-independent Superconductors								
NbN	B1	1.2	2.4	24	0	0	—	i
NbCN	"	1.4	2.5	17	—	0	—	j
V_2(Hf,Zr)	C15	0.7	0.6	20	—	0	—	k

From Appendix A10.2, *Expt'l Techniques in Low Temp. Measurements*, Oxford Univ. Press, 2006

Summary and comments on the parameterization of $B_{c2}^*(t,\varepsilon_0)$

- At high compressive strains ($\varepsilon_0 < -0.5\%$), the simple power law expression does not hold (ten Haken 1994, Keys & Hampshire 2003, Taylor & Hampshire 2005)

- Then need more parameters than just "*a*", (empirical expressions: polynomial, deviatoric, or indicator function in book)

- For now, to demonstrate the separable parameter set, we use the simple moderate-strain parameterization for B_{c2}^*.

So:

$$\frac{B_{c2}^*(t,\varepsilon_0)}{B_{c2}^*(0,0)} = \underbrace{\frac{B_{c2}^*(0,\varepsilon_0)}{B_{c2}^*(0,0)}}_{\text{Strain part}} \; \underbrace{\frac{B_{c2}^*(t,0)}{B_{c2}^*(0,0)}}_{\text{Temp. part}}$$

can be parameterized simply at moderate strains as:

$$\boxed{\frac{B_{c2}^*(t,\varepsilon_0)}{B_{c2}^*(0,0)} = (1 - a\,|\varepsilon_0|^{1.7})\;(1 - t^{1.5}) \qquad \text{(moderate strain range } -0.5\% < \varepsilon_0 < \varepsilon_{0,irr}\text{)}}$$

At any given strain, only three parameters: *a*, $B_{c2}^*(0,0)$, and $T_c^*(0)$!

Turning to the prefactor $K(T, \varepsilon_0)$

Unified strain and temperature scaling relation (USL) (Ekin 1980):

$$F(B, T, \varepsilon) = K(T, \varepsilon_0)\ f(b)$$

Shape invariance of $f(b)$ with strain and temperature.

Parameterization of $K(T, \varepsilon_0)$

$$K(T, \varepsilon) = g(\varepsilon_0) \times h(t)$$

⟵ Again, in 1980 postulated representing K as separable function

Useful postulate – It facilitates the practical application of the unified scaling law, and enables the definition of a separable parameter set.

Simplest parameterization of $g(\varepsilon_0)$ and $h(t)$ is in terms of the ε_0 and t components of B_{c2}* (where η is the Fietz and Webb parameter, and s the strain-scaling-law parameter):

$$K(T,\varepsilon_0) = C \left[\frac{B^*_{c2}(0,\varepsilon_0)}{B^*_{c2}(0,0)}\right]^s \left[\frac{B^*_{c2}(t,0)}{B^*_{c2}(0,0)}\right]^\eta$$

→ Note that ***s*** is determined only by strain data, and **η** only by temperature data (i.e. separable).

Example: Over the moderate strain range $(-0.5\% < \varepsilon_0 < \varepsilon_{0,irr})$

$$K(T,\varepsilon_0) = C \underbrace{(1 - a\,|\varepsilon_0|^{1.7})^s}_{g(\varepsilon_0)} \underbrace{(1 - t^{1.5})^\eta}_{h(t)}$$

This is an empirical parameterization.

Other empirical representations of $K(T, \varepsilon_0)$ have been proposed more recently based on various pinning force models.

It turns out that all fit the available $J_c(B,T,\varepsilon_0)$ data about equally well (within experimental error).

Furthermore, the alternative parameterizations can be separated into *ε* and *t* parts.

Example (Keys and Hampshire 2003)

$$K(T,\varepsilon_0) = C \ \alpha(\varepsilon)[T_c^*(\varepsilon)]^2 (1 - t^2)^2 [B_{c2}^*(t, \varepsilon)]^{n-2},$$

where $B_{c2}^*(t, \varepsilon) = B_{c2}^*(0, \varepsilon)\,(1 - t^{\nu})$.

Substituting and collecting terms into ε and *t* groupings:

$$K(T,\varepsilon_0) = C \ \underbrace{\alpha(\varepsilon)[T_c^*(\varepsilon)]^2 [B_{c2}^*(0, \varepsilon)]^{n-2}}_{g(\varepsilon_0)} \quad \underbrace{(1 - t^2)^2 \ (1 - t^{\nu})^{n-2}}_{h(t)}$$

“Rosetta Stone” Table for Translating Among Parameter Schemes

[Note: Terms for $g(\varepsilon)$ in the table are given in their most general form, since further simplified expressions may be invalid at high compressive strains.]

Separable form of the Unified Scaling Relation (USL) F =		$g(\varepsilon)$ ×	$h(t)$ ×	$f(b)$	$B_{c2}^*(t,\varepsilon)/B_{c2}^*(0,0)$
Ekin (1980)	= C	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]^s$	$[B_{c2}^*(t,0)/B_{c2}^*(0,0)]^\eta$	$f(b)$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ [B_{c2}^*(t,0)/B_{c2}^*(0,0)]$
	C	$(1-a\|\varepsilon_0\|^u)^s$	$(1-t^2)^\eta$	$b^p(1-b)^q$	$(1-a\|\varepsilon_0\|^u)\quad(1-t^2)$
(Book 2006)	C	$(1-a\|\varepsilon_0\|^u)^s + f(g_1,g_2)$	$(1-t^\nu)^\eta$	$b^p(1-b)^q$	$(1-a\|\varepsilon_0\|^u)+f(a_1,a_2)\quad(1-t^\nu)$
Summers (1991)	= C	$(1-900\|\varepsilon_0\|^{1.7})^1$	$(1-t^2)^{1.5}[1-0.31\,t^2(1-1.77\ln t)]^{-1/2}$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\,(1-t^2)[1-0.31\,t^2(1-1.77\ln t)]$
Cheggour & Hamp-	=	$A(\varepsilon)[B_{c2}^*(0,\varepsilon)]^n$	$(1-t^\nu)^n$	$b^p(1-b)^q$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^\nu)$
-shire (1999, 2002)		$A(\varepsilon)[B_{c2}^*(0,\varepsilon)]^n$	$(1-t^{1.5})^n$	$b^p(1-b)^q$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.5})$
Keys & Hampshire	= C	$\alpha(\varepsilon)[T_c^*(\varepsilon)]^2\,[B_{c2}^*(0,\varepsilon)]^{n-2}$	$(1-t^2)^2\ (1-t^\nu)^{n-2}$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^\nu)$
(2003)	C	$\alpha(\varepsilon)[T_c^*(\varepsilon)]^2\,[B_{c2}^*(0,\varepsilon)]^{n-2}$	$(1-t^2)^2\ (1-t^{1.37})^{0.5}$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.37})$
Godeke et al.	= C	$[B_{c2}^*(0,\varepsilon)]^n$	$(1-t^2)^\nu\,[1-0.31\,t^2(1-1.77\ln t)]^{\nu-\gamma}$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\,(1-t^2)[1-0.31\,t^2(1-1.77\ln t)]$
(1999)	C	$[B_{c2}^*(0,\varepsilon)]$	$(1-t^2)^2\,[1-0.31\,t^2(1-1.77\ln t)]$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\,(1-t^2)[1-0.31\,t^2(1-1.77\ln t)]$
Godeke et al.	= C	$[B_{c2}^*(0,\varepsilon)]^{\nu-\alpha\gamma}$	$(1-t^2)^\gamma\ (1-t^{1.52})^{\nu-\gamma}$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.52})$
(2006)	C	$[B_{c2}^*(0,\varepsilon)]$	$(1-t^2)\ (1-t^{1.52})$	$b^{0.5}(1-b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.52})$

[In the third line of the table, $f(c_1,c_2)$ is defined as $f(c_1,c_2) \equiv H(\varepsilon_{01}-\varepsilon_0)\,c_1\,|\varepsilon_{01}-\varepsilon_0|^{c2}$, where $H(x)$ is the Heavyside or indicator function (0 for $x < 0$ and 1 for $x > 0$), and ε_{01} is the compressive strain beyond which the term is applied: $\varepsilon_{01} = -0.5\%$ for Nb_3Sn.]

Fortunate to have so many separable model expressions that provide some level of fundamental justification for the separability of $K(t,\varepsilon)$ and the simple separable parameter set.

By comparing terms, the table also gives equivalences among the various scaling parameters in use.

"Rosetta Stone" Table for Translating Among Parameter Schemes

Separable form of the Unified Scaling Relation (USL) F =		$g(\varepsilon)$ ×	$h(t)$ ×	$f(b)$	$B_{c2}^*(t,\varepsilon)/B_{c2}^*(0,0)$
Ekin (1980)	= C	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0]^s$	$[B_{c2}^*(t,0)/B_{c2}^*(0,0)]^{\eta}$	$f(b)$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ [B_{c2}^*(t,0)/B_{c2}^*(0,0)]$
	C	$(1-a\lvert\varepsilon_0\rvert^u)^s$	$(1-t^2)^{\eta}$	$b^p(1\text{-}b)^q$	$(1-a\lvert\varepsilon_0\rvert^u)\quad(1-t^2)$
(Book 2006)	C	$(1-a\lvert\varepsilon_0\rvert^u)^s+f(g_1,g_2)$	$(1-t^{\nu})^{\eta}$	$b^p(1\text{-}b)^q$	$(1-a\lvert\varepsilon_0\rvert^u)+f(a_1,a_2)\quad(1-t^{\nu})$
Summers (1991)	= C	$(1-900\lvert\varepsilon_0\rvert^{1.7})^1$	$(1-t^2)^{1.5}[1-0.31\,t^2(1-1.77\ln t)]^{-1/2}$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^2)[1-0.31\,t^2(1-1.77\ln t)]$
Cheggour & Hamp-	=	$A(\varepsilon)[B_{c2}^*(0,\varepsilon)]^n$	$(1-t^{\nu})^n$	$b^p(1\text{-}b)^q$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{\nu})$
-shire (1999, 2001)		$A(\varepsilon)[B_{c2}^*(0,\varepsilon)]^n$	$(1-t^{1.5})^n$	$b^p(1\text{-}b)^q$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.5})$
Keys & Hampshire	= C	$\alpha(\varepsilon)[T_c^*(\varepsilon)]^2\ [B_{c2}^*(0,\varepsilon)]^{n-2}$	$(1-t^2)^2\ (1-t^{\nu})^{n-2}$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{\nu})$
(2003)	C	$\alpha(\varepsilon)[T_c^*(\varepsilon)]^2\ [B_{c2}^*(0,\varepsilon)]^{n-2}$	$(1-t^2)^2\ (1-t^{1.28})^{n-2}$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.5})$
Godeke et al.	= C	$[B_{c2}^*(0,\varepsilon)]^n$	$(1-t^2)^{\nu}\ [1-0.31\,t^2(1-1.77\ln t)]^{\nu-\gamma}$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^2)[1-0.31\,t^2(1-1.77\ln t)]$
(1999)	C	$[B_{c2}^*(0,\varepsilon)]$	$(1-t^2)^2\ [1-0.31\,t^2(1-1.77\ln t)]$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^2)[1-0.31\,t^2(1-1.77\ln t)]$
Godeke et al.	= C	$[B_{c2}^*(0,\varepsilon)]^{\nu-\alpha\gamma}$	$(1-t^2)^{\gamma}\ (1-t^{1.52})^{\nu-\gamma}$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.52})$
(2006)	C	$[B_{c2}^*(0,\varepsilon)]$	$(1-t^2)\ (1-t^{1.52})$	$b^{0.5}(1\text{-}b)^2$	$[B_{c2}^*(0,\varepsilon)/B_{c2}^*(0,0)]\ (1-t^{1.52})$

By comparing terms, the table also gives equivalences among the various scaling parameters in use.

Example:

	$g(\varepsilon)$
Ekin (1980):	$[B_{c2}{}^{*}(0,\varepsilon)/B_{c2}{}^{*}(0,0]^{s}$
Keys & Hampshire (2003):	$\alpha(\varepsilon)\ [T_c{}^{*}(\varepsilon)]^{2}\ [B_{c2}{}^{*}(0,\varepsilon)]^{n-2}$

K & H also use: $\alpha(\varepsilon)/\alpha(0) = [T_c{}^{*}(\varepsilon)/T_c{}^{*}(0)]^{u}$ $T_c{}^{*}(\varepsilon)/T_c{}^{*}(0) = [B_{c2}{}^{*}(0,\varepsilon)/B_{c2}{}^{*}(0,\varepsilon)]^{1/w}$

(for interpolative purposes)

Subsitituting gives: $[B_{c2}{}^{*}(0,\varepsilon)/B_{c2}{}^{*}(0,0]^{(1/w)\,[u+2+w(n\text{-}2)]}$

Therefore we have the equivalence: $s = (1/w)\,[u+2+w(n\text{-}2)]$

Illustrate using scaling parameters from Taylor and Hampshire (2006):

Separable par. *s*

	n	w	u	s = (1/w) [u+2+w(n-2)]	
MJR (OST) wire	3.069	2.545	-0.912	1.496505	
Bronze-route (Vac) wire	2.457	2.216	0.051	1.382542	
Internal-tin (EM-LMI) wire	2.338	1.936	-0.056	1.342132	

N.B. Note that the parameters *n*, *w*, *u* are commingled with respect to temperature and strain.
e.g., new temperature data → update value of *n* ,
but then *u* and *w* change to keep the strain part the same (no update in strain data).
With the separable strain parameter set, *s* would automatically remain unchanged.
Bottom line: Easy to update separable set as additional data become available.

For engineering, how well does the simple separable set (i.e. first line in table) represent all the other model expressions (i.e. all the other lines in the table)?

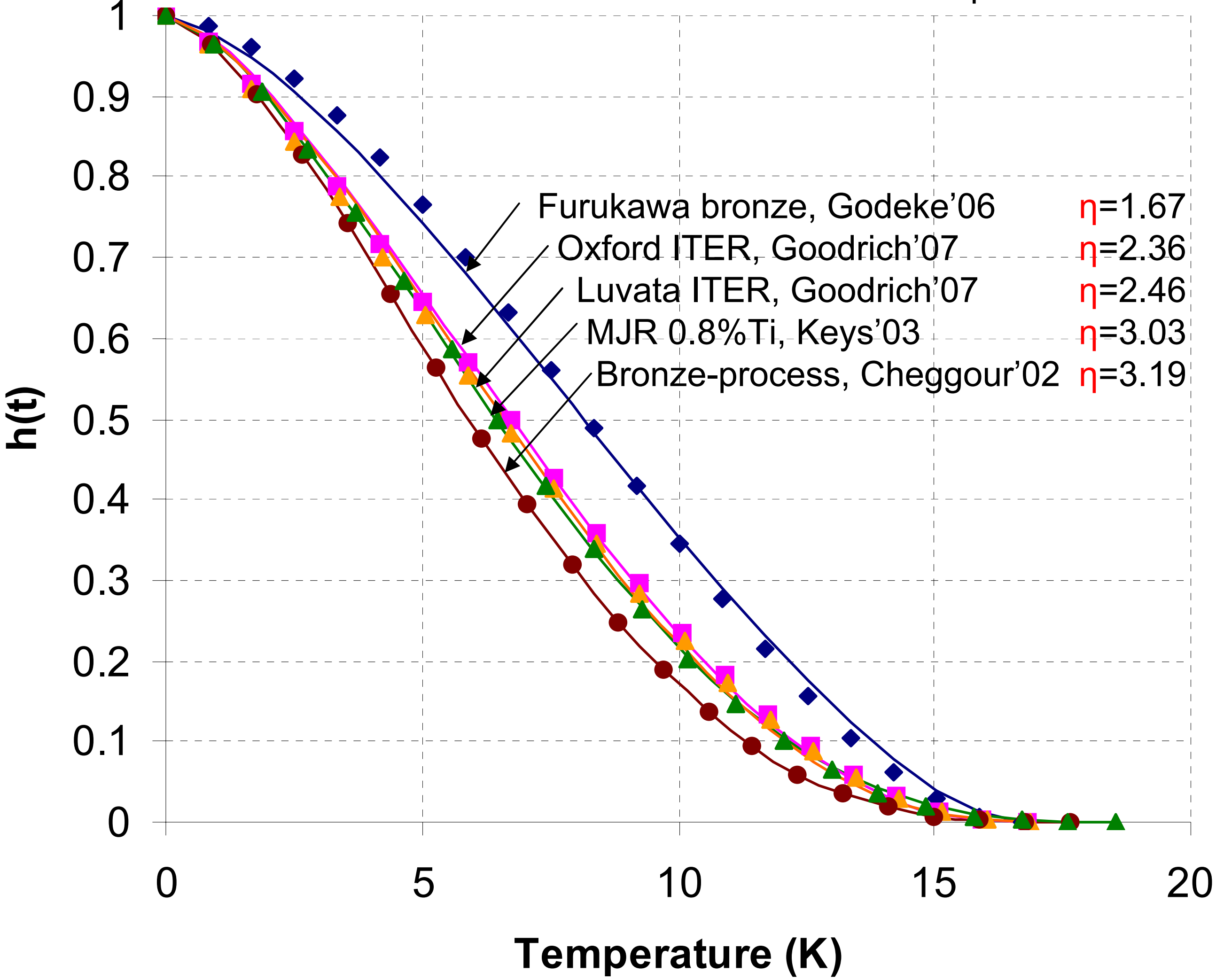
Temperature part *h*(*t*):

Temperature Comparison

Symbols show various $h(t)$ model expressions from the literature.
Solid curves show the close fit of all these model expressions to the following simple function:

$$h(t) = (1 - t^{1.5})^{\eta}$$

in terms of one parameter: η.



Thus, the single temperature parameter η (Fietz and Webb parameter) represents all the model expressions surprisingly well.

Also, provides a single temperature-scaling parameter η for tabulating the temperature dependence of the various conductors.

Strain part $g(\varepsilon_0)$:

Two experimental results:

1. Over the moderate strain range ($-0.5\% < \varepsilon_0 < \varepsilon_{0,irr}$) , the simple function:

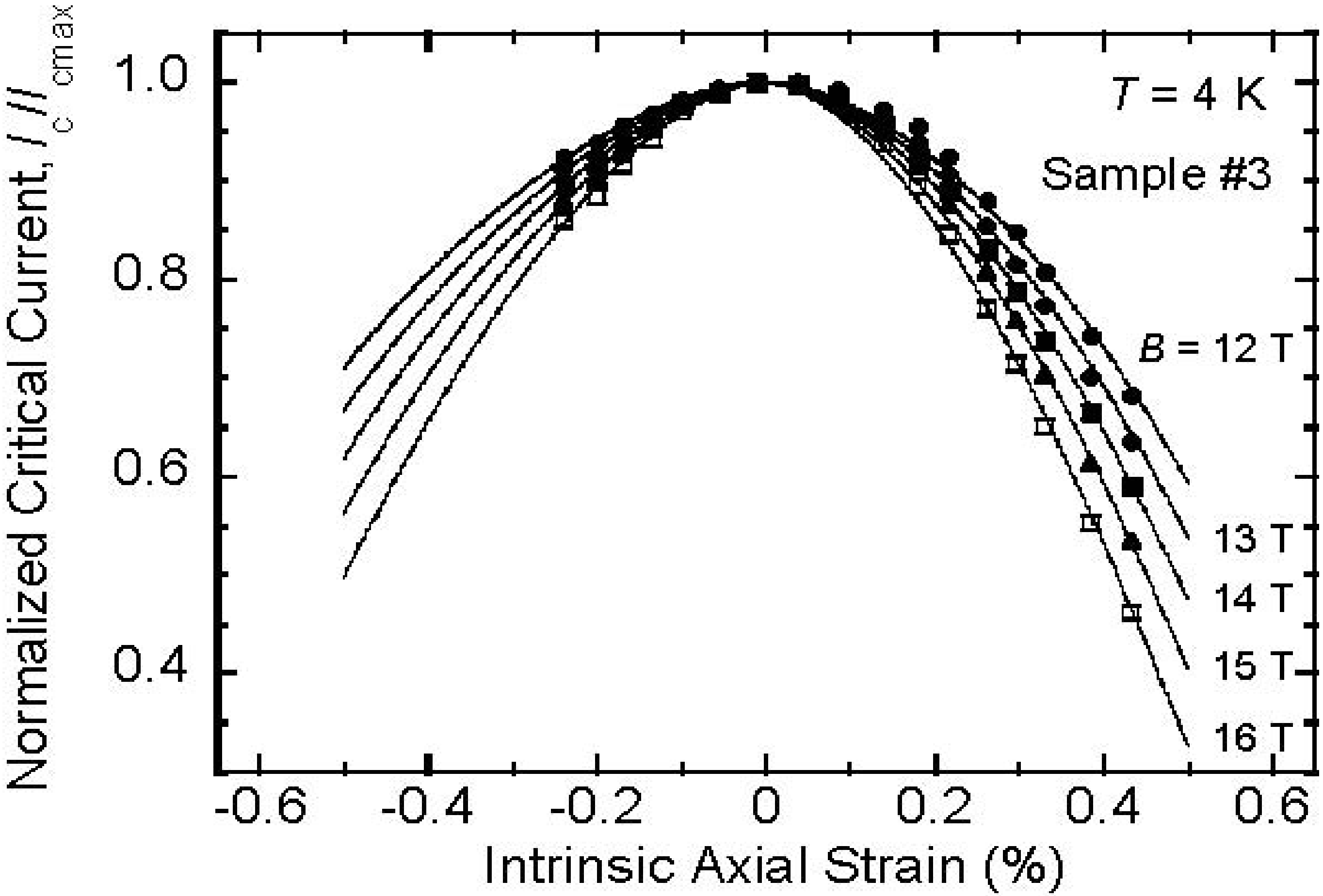
$$g(\varepsilon_0) = [B_{c2}^*(0,\varepsilon_0)/B_{c2}^*(0,0)]^s$$

fits the available data for Nb_3Sn quite well (errors < few percent) in terms of one strain-exponent parameter: *s*.

Example: Luvata ITER Nb_3Sn wire

$g(\varepsilon_0) = [B_{c2}^*(0,\varepsilon_0)/B_{c2}^*(0,0)]^s = (1 - a\,|\varepsilon_0|^{1.7})^s$

$s = 0.7$, $a^- = 1230$, $a^+ = 1670$



From Cheggour et al. ASC ’06, #3LW06

Two experimental results (continued):

1. Over the moderate strain range ($-0.5\% < \varepsilon_0 < \varepsilon_{0,irr}$), this fits available data for Nb_3Sn quite well (errors < few percent) in terms of one strain-exponent parameter: *s*.

2. Remarkable result for MgB_2:

In general, the strain part of prefactor K [i.e. $g(\varepsilon)$] depends on:

1) $B_{c2}^*(0,\varepsilon)$ Strain dependence of the upper critical field
2) $\kappa_1(0,\varepsilon)$ Strain dependence of Kappa
3) $A(\varepsilon)$ Other strain dependence of the pinning strength

Thus, assuming it can be modeled <u>only</u> in terms of $B_{c2}^*(0,\varepsilon)$ is a fairly drastic assumption:

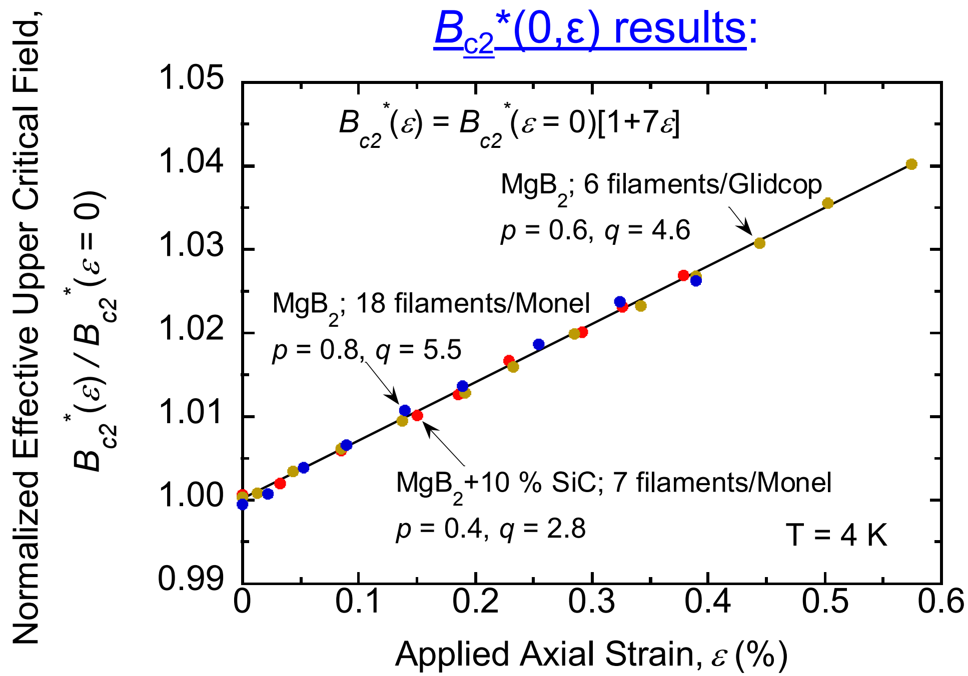
i.e., $g(\varepsilon) \; \alpha \; [B_{c2}^*(0,\varepsilon)]^{s}$

But consider the results for the MgB_2 system (which is intrinsically very different from the A15s):

MgB_2

linear dependence for $B_{c2}^*(0,\varepsilon)$ (hydrostatic-strain dominated?)

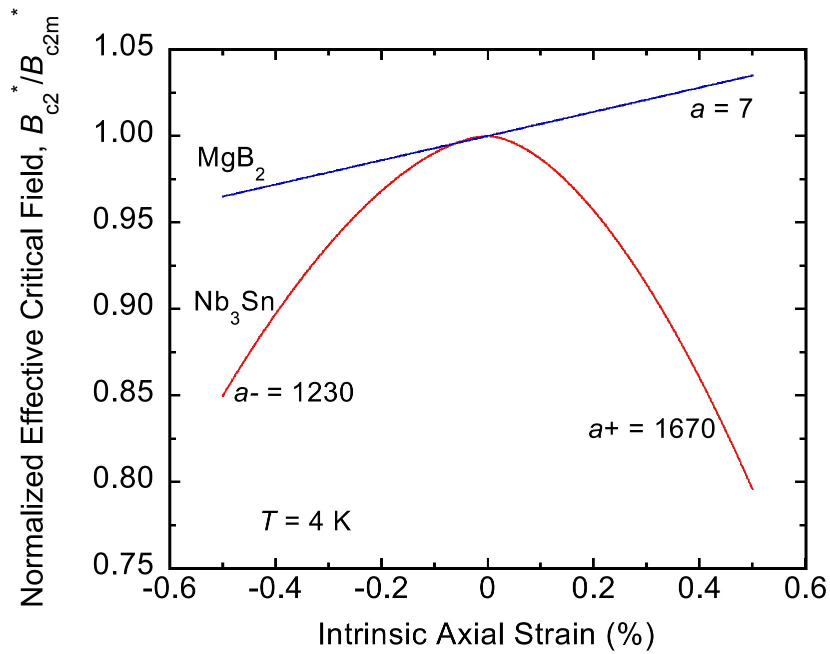
$B_{c2}^*(0,\varepsilon)$ results:



versus

A15s

power-law or parabolic dependence for $B_{c2}^*(0,\varepsilon)$ (deviatoric-strain dominated)



From Cheggour & Ekin, HEP workshop 2006

Fundamentally different $B_{c2}^*(0,\varepsilon)$ for the two systems.

So, what is ε dependence of ***K*(0,ε)?**

The data are described by the following equations:

$$B_{c2}^{*}(\varepsilon,4K) = B_{c2}^{*}(0,4K)\left[1 + a\varepsilon\right]$$

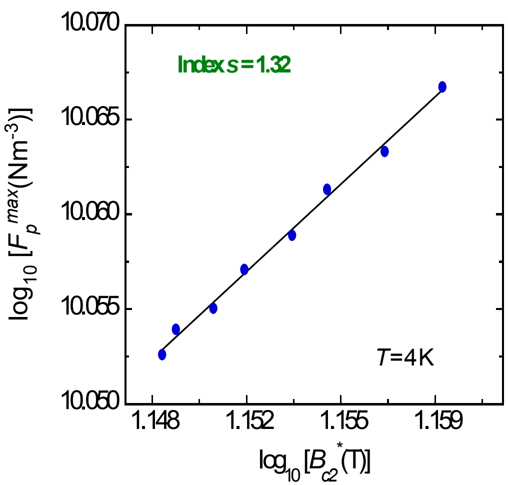
$$F_p = K(T,\varepsilon)\, b^{p}(1-b)^{q}$$

$$K(T,\varepsilon) = C\left[1 + a\varepsilon\right]^{s}$$

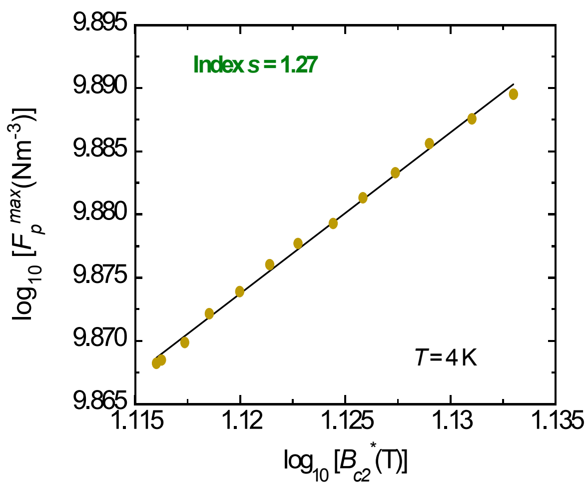
a = 7, s = 1.2 - 1.3;
C, p, and q constants;
$b = B / B_{c2}^{*}$

$K(0,\varepsilon)$ results (proportional to F_p^{max}) for three conductors are:

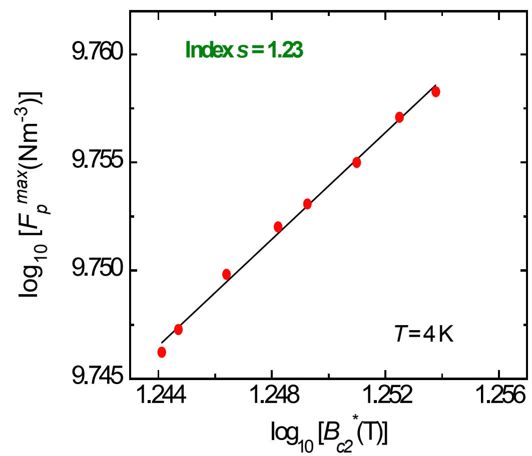
Undoped MgB_2 wire (#1)



Undoped MgB_2 wire (#2)



10 % SiC-doped MgB_2 wire (#3)



Cheggour & Ekin, HEP workshop 2006

Thus, the ε dependence of K [i.e. $g(\varepsilon)$] is also linear(!) and therefore can still be correlated to the ε dependence of B_{c2}^{*}, even though B_{c2}^{*} is fundamentally different in the MgB_2 system.

i.e., $g(\varepsilon)\ \alpha\ [B_{c2}^{*}(0,\varepsilon)]^{s}$ still holds!

Conclusion – Key Points

- Thus, strain dependence of B_{c2}^* appears to dominate $g(\varepsilon)$, even for intrinsically different superconductor systems.
- Separable parameter set robust across the A15s and MgB_2 (also Chevrel)
  Some fundamental justification of separability provided by the pinning-model expressions.
- For engineering, a single temperature parameter η and single strain parameter s represent the different pinning-model expressions surprisingly well.
  They can also be used to translate among the various model expressions.
- This provides a user-friendly means of building up scaling parameter set from separate strain and temperature measurements.
  A complete data matrix $J_c(B,T,\varepsilon)$ is not necessary to directly measure individual scaling parameter values.
  The set can be built one parameter at a time, as data become available for a particular conductor.
- For engineering purposes, s and η also offer a simple means of comparing ε and t behavior of different conductors.
- Separable parameter values are consistent and therefore useful for predictive purposes when needed.

(More details and examples in book.)

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End

Request

When analyzing large matrixes of $J_c(B,T,\varepsilon)$ data, please determine $K(T,\varepsilon)$ and $B_{c2}^*(T,\varepsilon)$ at each T and ε point, and then work with these K and B_{c2}^* data.

If, on the other hand, a global simultaneous fit is made to the raw data:
--simpler procedure
--but fitting function may not be justified (difficult to see without the K and B_{c2}^* data)
--parameter values become inconsistent with simultaneously fitted large arrays of parameters.

Two figures from the book follow:

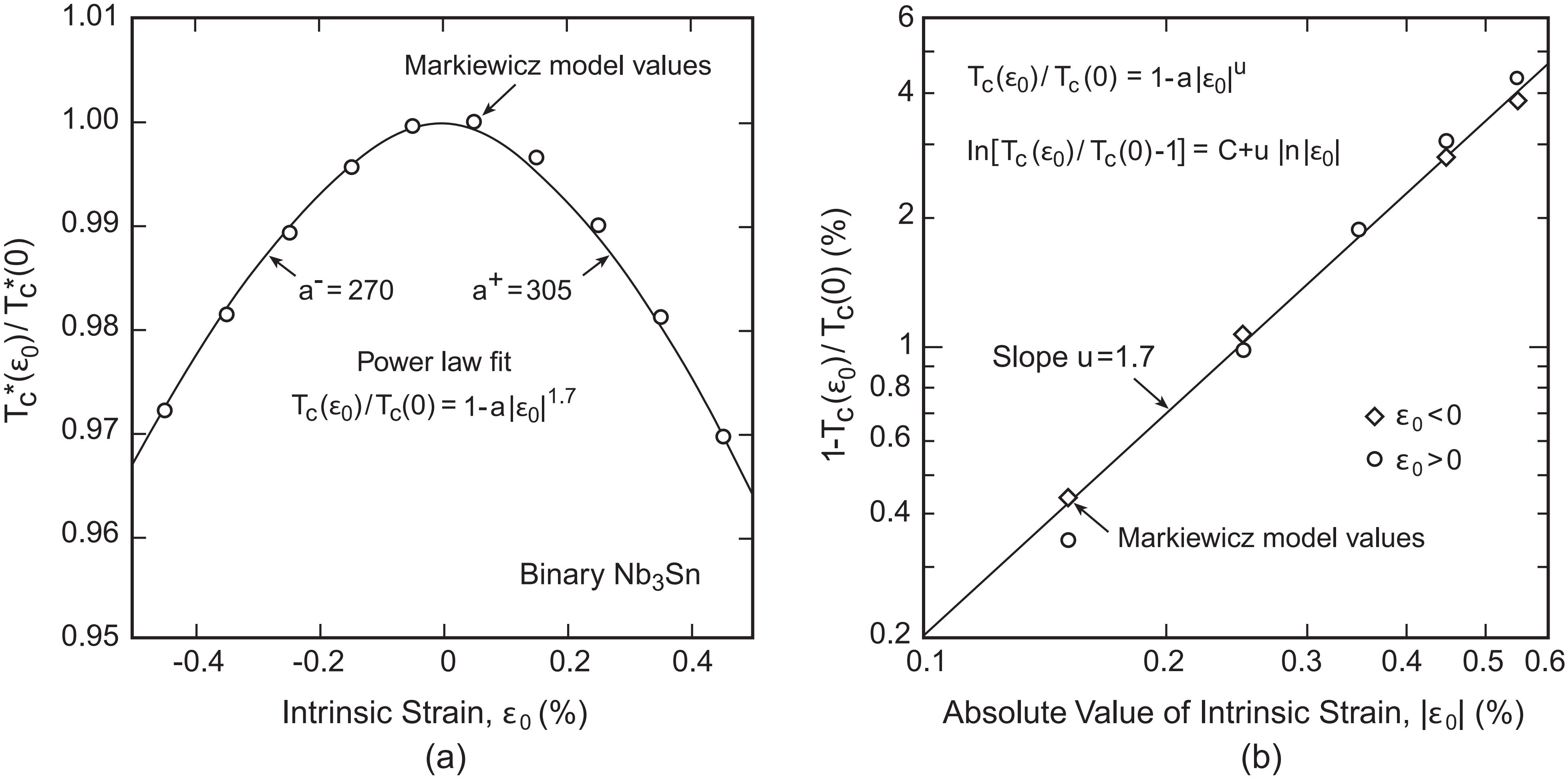

FIG. 10.32 Fundamental basis of the power law at moderate intrinsic strains ($-0.5\ \% < \varepsilon_0 < \varepsilon_{0irr}$): (a) Strain dependence of the critical temperature of binary Nb_3Sn calculated by introducing phonon anharmonicity into the McMillan/Kresin equation (from Markiewicz 2004). The model shows that the power-law dependence given by Eq. (10.21) arises mainly through the principal part of the second invariant of the deviatoric strain tensor. (b) Calculated results replotted as $\log\{1-[T_c(\varepsilon_0)/T_c(0)]\}$ vs. $\log|\varepsilon_0|$, showing that the anharmonicity model gives the canonical power-law exponent $u = 1.7 \pm 0.1$ without any adjustable parameters for both tensile and compressive intrinsic strains.

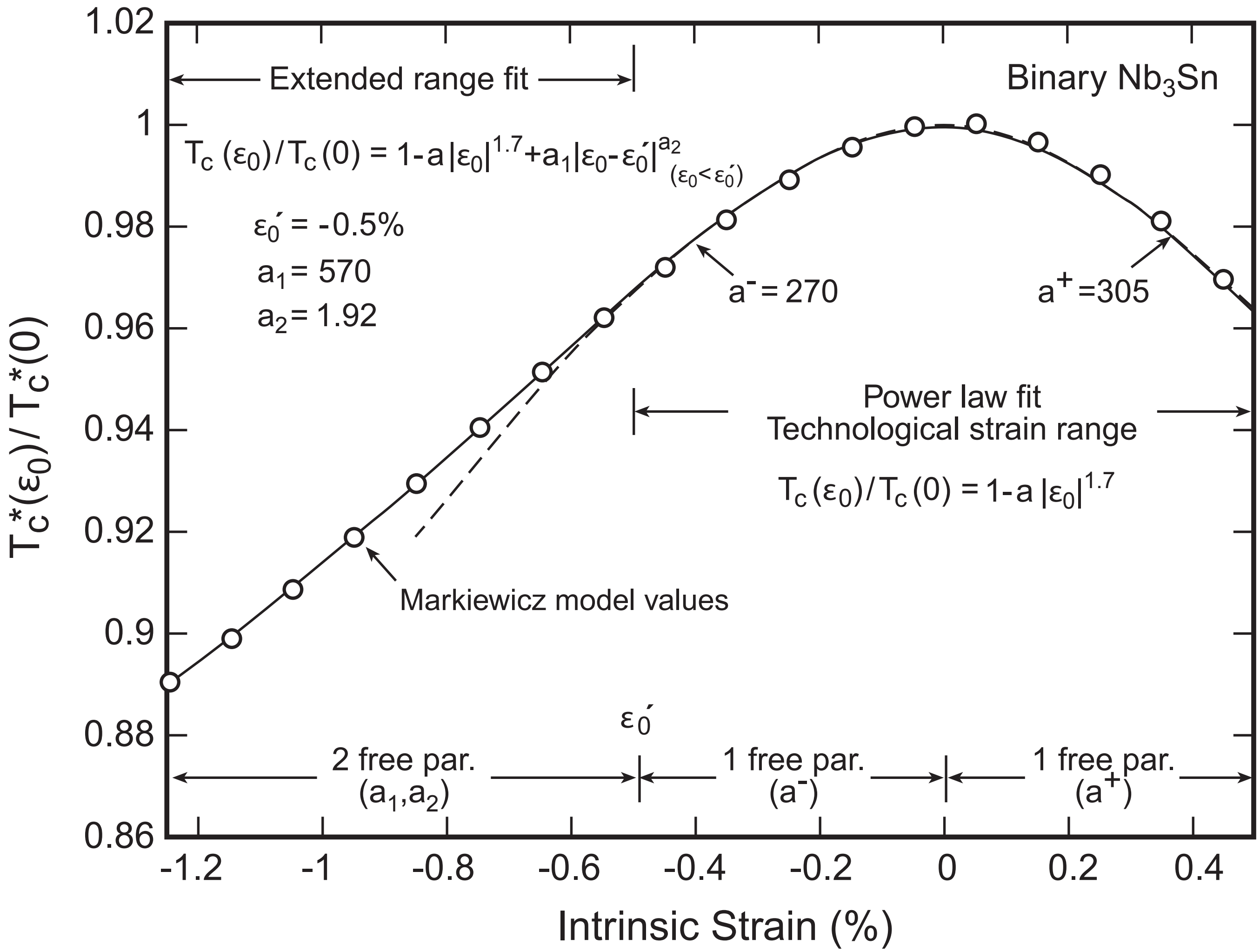

FIG. 10.33 Critical temperature of binary Nb_3Sn calculated over an extended strain range from a three-dimensional deviatoric strain model by Markiewicz (2004, 2005). The solid curve shows an extended-range fit of the model results with an expression of the same form as Eq. (10.23) for the effective upper critical field. Fitting forms of this type are useful for analytically representing the strain dependence at high compression while preserving a consistent value of the parameter *a* for characterizing the intrinsic peak region ($-0.5\ \% < \varepsilon_0 < 0.4\ \%$) where most magnets are designed. In the Markiewicz model, the transition to a positive second derivative at high compressive strains mainly arises from the third invariant of the deviatoric strain tensor. Extrinsic factors, such as copper and bronze yielding or conductor damage at very high compressive strains, may also contribute to a positive curvature.