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Output, Investment, and Growth in a World of Putty-Clay

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Abstract

This paper presents a putty-clay model of capital that proceeds from profit maximization to an analytic solution for aggregate investment that fits the data well. The key innovation is a production function in which a geometric average of the capital-labor ratio embedded in each unit of capital replaces the aggregate capital-labor ratio in the standard Cobb-Douglas production function. The accelerator in this model depends on the growth rate of output in excess of labor productivity at full employment, rather than on the growth rate of output alone as in previous work. Growth of labor hours at full employment is an important driver of investment. Expectations of faster productivity growth boost current investment. The effect of productivity growth on the real rate of return in steady state is smaller than in the traditional neoclassical model. Cash flow and restrictions implied by the neoclassical model are not significant when added to the estimating equations. The putty-clay assumption allows growth of the capital stock to be split into its capital-widening and capital-deepening components. The model explains the sharp rise in the gap between capital income and business fixed investment in the early 2000s.

Economists have long had difficulty constructing a model of business investment that both fits the data well and is grounded in the theory of the firm. In the traditional neoclassical model first developed by Jorgenson (1963), the demand for capital is derived from the profit-maximizing decisions of firms, and so is consistent with theory. However, that model's assumption that capital is putty-putty, i.e., that firms are free to vary the number of workers required to operate any machine after it is put in place, implies that the short-run response of investment to a change in the ratio of output price to the cost of capital should be the same as the short-run response to the same percentage change in output. That implication is not borne out in empirical work (see Chirinko's (1993) survey). In addition, the dynamics needed to translate demand for capital into investment do not follow explicitly from firm's profit-maximizing decisions.

The q theory of investment, proposed by Keynes (1936) and developed by Tobin (1969), in which investment depends positively on the ratio of the market value of a firm to the replacement cost of its capital stock, does a better job of fitting the data but requires the ad hoc addition of variables like cash flow and implies unrealistically large costs to firms of adjusting the amount of capital they use. Cooper and Haltiwanger (2006) find adjustment costs of a more realistic magnitude using plant-level data from manufacturing, but the properties of their data set, especially the degree of serial correlation, are quite different from those of aggregate investment.

Past work using the putty-clay approach to capital, originally developed by Johansen (1959), Solow (1962), and Phelps (1963), has not solved that problem. Under the assumption of putty-clay capital, firms can choose among a wide variety of possible ratios of capital to labor when purchasing capital, but cannot alter that ratio once capital has been delivered. Thus, each unit of capital, or "machine," is used by a fixed amount of labor throughout its service life. The theoretical literature has generally assumed that new machines function independently of old machines. In that case, the productivity of new machines cannot be inferred from aggregate productivity—the productivity of existing capital—making empirical work difficult. The existing emprical work (e.g., Bischoff (1971)), while providing a good fit for the data, has required assumptions that strain the link with theory.[1]

This paper presents a putty-clay model of investment that is derived from the profit-maximizing decisions of firms and that provides a good fit for the aggregate data. The key innovation in the model is an aggregate production function in which a geometric average of the capital-labor ratio embedded in each existing machine replaces the aggregate capital-labor ratio in the standard Cobb-Douglas production function. The productivity of new machines is then a function of aggregate productivity, allowing an analytic solution for investment as a function of aggregate variables. Investment is the number of new machines multiplied by the capital-labor ratio embodied in those machines. With the addition of a variable capturing trends in the number of

[1] For example, Bischoff assumes that "entrepreneurs choose their factor proportions by a simple suboptimal rule of thumb" (p. 70).

machines per worker, estimated errors are small. Even without that variable, the model captures turning points well. The results are also consistent with previous empirical work finding that the short-run response of investment to variations in output is much larger than the short-run response to variations in the cost of capital.

The model also has several unique implications. Because investment depends on the number of new machines, which in turn is tied to labor hours, the accelerator—the response of investment to changes in demand—uses the growth rate of output less the growth rate of labor productivity at full employment. That differs from previous work, in which the accelerator depends on the growth rate of output alone. Higher productivity does increase the capital-labor ratio embodied in new machines, but does not lead to an accelerator effect. Because higher productivity leaves the number of new machines unchanged, a productivity shock cannot generate changes in employment through its effect on investment. In addition, the impact of a change in trend growth of productivity on the real rate of return in steady state is smaller than in the neoclassical model.

At the same time, expectations of faster productivity growth in the future directly raise investment, in contrast to the neoclassical model where expectations are irrelevant. Greater productivity in the future raises the marginal effect on revenue of buying a machine of higher quality.

Businesses use labor hours at full employment (the portion of labor hours independent of the business cycle) to help forecast the need for machines in the future, so growth of labor hours at full employment is a key driver of investment. That helps mitigate one of the problems of the neoclassical model: the dependence of investment on output, which in turn depends on the capital stock.

Putty-clay capital provides a way of distinguishing capital widening—an increase in capital that accommodates additional workers—from capital deepening—an increase in capital that boosts the output of existing workers. While both increase the economy's capacity to produce output, only the latter raises productivity. The model also allows for gradual changes in capital's coefficient in production through a variable measuring trends in machines per worker. Unusually strong investment in the late 1970s and early 1980s and an increase in capital's share of income at about the same time appear to reflect an increase in that variable. Finally, because the model has implications for both capital's share of income and business fixed investment, it explains most past variations of capital income less investment as a share of total output.

Section 1 presents the putty-clay production function and derives investment starting from the profit maximization decision of the firm. Section 2 examines the theoretical properties of the model, comparing the short-run behavior of investment and the rate of return in steady state to their properties in the standard neoclassical model. Section 3 describes the data used in this paper. Section 4 presents the results from estimating the model and tests its robustness against restrictions implied by the neoclassical model and a simple version of the q theory with cash flow. Section 5 calculates estimates of capital per labor hour and the implied capital stock. Section 6 shows the implications of the model for capital income minus investment.

1 The Model

I assume that capital is produced and used in the form of "machines," be they drill presses, aircraft, computers, office buildings, oil wells, or software packages.[2] Each machine is associated with a fixed amount of labor when it is in use. To take an obvious example, the number of workers it takes to drive a truck does not change over its service life. A firm is free to choose the amount of capital in a new machine (its "quality") at the time of investment, but once the machine is put in place, that amount is fixed. Ex ante, capital is putty; ex post, it is clay.

That assumption matters because the amounts of capital embodied in each existing machine are combined into an aggregate index of the quality of existing capital using a geometric average. The aggregate index of machine quality replaces the ratio of capital to labor in the standard Cobb-Douglas production function. Firms maximize profits, choosing the quality of new machines so that the present discounted value of the after-tax revenues from an extra dollar of quality equals the after-tax cost of that quality. In addition, I assume quadratic costs of utilization, so firms try to align the total number of machines with labor hours. Real investment is the number of new machines times the quality of new machines. Inventories, another form of capital necessary to calculate the total stock of capital, are discussed in Appendix A.

1.1 The Production Function and Capital per Labor Hour

At time t, output is determined by the production function

$$Y_t = A_t \, L_t \left(\bar{k}_t\right)^{\alpha}. \tag{1}$$

Y is total output, A is total factor productivity (TFP), L is labor hours, $\bar{k}$ is an index of capital per labor hour, and α is the coefficient of that index in production. Labor is homogeneous over time, so TFP includes both average labor quality and the technology of production. The only difference between that production function and the neoclassical production function is the treatment of capital per labor hour.

The index of capital per labor hour depends on the quality of each existing machine (k) and on the equilibrium number of machines per labor hour (x).[3] The quality of

[2] Although labor hours are the measure of labor used in this paper, one can think of a machine as the amount of a specific type of capital used by one worker. In the case of an office building, an office would be a structures "machine." One section of a retail establishment would also be a machine.

[3] The actual number of machines per labor hour may differ from x because of time to build. It takes longer to adjust the capital stock than it does to adjust the amount of labor.

each existing machine is fixed over its service life, so the index of capital per labor hour changes only as new machines are added to the stock, existing machines are scrapped, or the number of machines per labor hour changes. The first two factors encompass most of what economists normally think of as capital deepening. Examples include increased real investment in computers as the price of new computers declines, higher investment in response to tax incentives or a decline in the cost of funds, and the replacement of depreciating capital with higher-quality new capital due to a lower ratio of the cost of capital to the cost of labor than when the depreciating capital was installed. (That latter factor may explain at least part of the increase in labor productivity observed by Foster, Haltiwanger, and Krizan (2002) when a new retail establishment replaces an old one.)

Examples of a change in the number of machines per labor hour are a shift in demand toward sectors with a higher ratio of capital to labor, such as mining, and the shifting of tasks from people to computers as businesses adopted the personal computer. In practice, changes in x account for a small fraction of changes in $\bar{k}$; for example, $\bar{k}$ nearly quadrupled between 1955 and 2005, while x rose just 4 percent. The variable x is added to the model to control for changes in the importance of mining and for trends in capital per labor hour that cannot be explained by the determinants of k. If we allowed for unlimited variation of x, the equation for investment would become an identity.

To formalize an expression for $\bar{k}$, I assume there are M kinds of capital, denoted by the suffix m. The quality of machines of type m installed at time t, i.e., real dollars of investment per new machine, is km_t. Let $Rm_{t-i,i}$ be the number of type-m machines installed at time $t-i$ and remaining in service i periods later. Then the index of capital per labor hour for machines of type m at time t is

$$\log\left(\bar{k}m_t\right) = \sum_{i=0}^{\infty} \frac{Rm_{t-i,i} \log\left(km_{t-i}\right)}{Nm^K{}_t},$$

where $Nm^K{}_t = \sum_{i=0}^{\infty} Rm_{t-i,i}$, the total number of machines of type m in service at time t. Thus, $\bar{k}m$ is a geometric average of the quality of each type-m machine still in service. (In this paper, N denotes the number of new machines, while N^K denotes the number of existing machines.) Each machine is a one-horse shay, remaining fully operational until it is removed from the capital stock in its entirety.

To prevent the growth of the index of capital per labor hour from depending on the base year chosen for the price indexes, we must express it using first differences (Δ):

$$\Delta \log\left(\bar{k}_t\right) = x_{t-1} \left(\frac{\sum_{m=1}^{M} Nm^K_{t-1} \Delta \log\left(\bar{k}m_t\right)}{N^K_{t-1}} + \Delta \log\left(x_t\right) \right), \tag{2}$$

where $N^K_{t-1} = \sum_{m=1}^{M} Nm^K_{t-1}$. Logarithmic growth of capital per labor hour depends on a weighted average of logarithmic growth of capital per labor hour for each type of machine plus logarithmic growth of x. That sum is premultiplied by x_{t-1} so that a given percentage change in machine quality has a larger impact on output the more machines there are per labor hour. (For example, we want a 1.04 percent increase in all the $\bar{k}m$ when $x = 1$ to have the same impact on output as a 1 percent increase in all the $\bar{k}m$ when $x = 1.04$.) In addition, the parallel treatment of $\Delta \log\left(\bar{k}m\right)$ and $\Delta \log(x)$ in equation 2 implies that a permanent 10 percent increase in real investment would have the same long-run effect on capital per labor hour whether it were used to increase x or the quality of new machines.

The levels of quality of different types of machines are scaled so that machines of different types have the same importance in production. Thus, there are far fewer light truck "machines" in a light truck than there are aircraft "machines" in a commercial aircraft.

To solve the firm's profit maximization problem, we need to express $\log\left(\bar{k}_t\right)$ in levels. Equation 2 becomes

$$\log\left(\bar{k}_t\right) = \sum_{i=1}^{t} \log\left(X_{t-i}\right) + x_t \left(\frac{\sum_{m=1}^{M} Nm_t^K \log\left(\bar{k}m_t\right)}{N_t^K} + \log\left(x_t\right) \right), \tag{3}$$

where X_t is defined with

$$\begin{aligned} \log\left(X_t\right) &= -\Delta x_t \left(\frac{\sum_{m=1}^{M} Nm_t^K \log\left(\bar{k}m_t\right)}{N_t^K} + \log\left(x_t\right) \right) \\ &\quad - x_{t-1} \sum_{m=1}^{M} \Delta \frac{Nm_t^K}{N_t^K} \log\left(\bar{k}m_t\right). \end{aligned}$$

I assume that, when choosing the quality of new machines (km_t), firms expect $\log\left(X_{t+i}\right) = 0$ for all positive i.

To grasp the intuition behind $\bar{k}$, consider the special case in which $X_t = x_t = 1$. Index the existing machines from 1 to N_t^K. Then we can rewrite equation 3 as

$$\bar{k}_t = k_1^{1/N_t^K} k_2^{1/N_t^K} ... k_{N_t^K}^{1/N_t^K}. \tag{4}$$

Because each k_i is measured using real dollars of capital per labor hour, $\bar{k}$ is a geometric average of the capital-labor ratio embodied in each existing machine.

In this special case, $N^K = L$, so equations 4 and 1 can be combined to express production as

$$Y_t = A_t \, L_t \, k_1^{\alpha/L_t} \, k_2^{\alpha/L_t} ... k_{L_t}^{\alpha/L_t}. \tag{5}$$

Now rewrite the standard Cobb-Douglas production function as

$$Y_t = A_t \, L_t \left(\frac{K_t}{L_t} \right)^{\alpha}, \tag{6}$$

where K is the aggregate putty-putty stock of capital. When $x = 1$, the putty-clay production function is just a Cobb-Douglas production function in which the aggregate capital-labor ratio has been replaced by a geometric average of the capital-labor ratio of each machine.

Indeed, one can consider the Cobb-Douglas production function as a special case of the putty-clay production function in which firms are free to vary the quality of each machine throughout its service life. Given aggregate capital K_t, the firm maximizes output by distributing that capital equally across all machines, i.e., $k_1 = k_2 = ... = k_{L_t} = K_t/L_t$. In that case, equation 5 collapses to equation 6.

In the putty-clay model, machines are complements: an increase in k_1 raises the marginal product of an extra dollar of k_2. For example, a software upgrade is generally more valuable the more powerful is the personal computer using it. All else equal, the greater the quality of existing machines, the greater will be the quality of new machines. This is not true in the neoclassical model. The greater is K, the lower is the marginal product of an additional dollar of K.

Returning to the putty-clay production function in equation 1, output varies proportionately with labor hours in the short run. Unanticipated short-run variations of labor hours lead to more or less intensive use of the capital stock. Output per labor hour Y/L can even be procyclical if TFP depends on aggregate demand. A rising cost of utilization, discussed below, prevents firms from permanently setting N^K below xL.

The assumption that output varies proportionately with labor hours in the short run implicitly assumes that utilization rates for each machine are equal. That assumption departs from much of the putty-clay literature. Solow (1962), Phelps (1963), Solow et al. (1966), Calvo (1976), and Gilchrist and Williams (1998) assume that all machines above a market-clearing quality are in use while all others are idle. In that case, a short-run rise in output and labor hours increases the utilization of less-efficient capital, reducing average labor productivity. While that is certainly a plausible assumption, it is unhelpful in empirical work, given that economists since at least Fabricant (1942) have found short-run increasing returns to labor. The assumption that a machine's utilization does not depend on its quality relative to other machines also greatly simplifies the firm's investment decision.

The treatment of TFP in this paper also differs from that in much of the putty-clay literature. Johansen (1959), Phelps (1963), Solow et al. (1966), Calvo (1976), and Gilchrist and Williams (1998) all assume that the output of a given unit of capital is constant over its service life, i.e., that all technical progress is embodied in capital.[4] To determine the contribution of a machine to output, one must know not only its

[4] While Ando et al. (1974) do not assume that the output of a given unit of capital is constant over its service life, their assumption that the required labor input per unit of existing capital rises at a constant rate m per year leaves the productivity of existing capital independent of current technical progress.

quality but also its vintage. That assumption prevents easy aggregation of capital. In this paper, once one knows the quality of a machine, its vintage provides no additional information about its contribution to output.

Note that the production function exhibits constant returns to scale when the utilization rate is held constant. If the number of hours doubles, then an unchanged $\bar{k}$ implies a doubling of the number of machines and thus a doubling of the total amount of capital. From equation 1, output would also double.

1.2 The Firm and Profit Maximization

1.2.1 The Profit Function

I assume an arbitrary number of monopolistically competitive firms. Demand for the output of the jth firm (Yj_t) is given by

$$Yj_t = fj_t Y_t \left(\frac{pj_t}{p_t}\right)^{\eta},$$

where fj is the jth firm's share of the market when all firms charge the same price ($\sum fj = 1$), p is the aggregate price of output, pj is the price charged by the jth firm, and η is the price elasticity of demand for each firm's output. For simplicity, I assume that the fj are constant over time. The implications of that assumption are discussed below.

Businesses seek to maximize the expected present discounted value (PDV) of current and future profits, $PDV(\pi)$. For simplicity, I assume no taxation and only one type of capital. The effect of the corporate income tax on investment, which is identical to its effect in the traditional neoclassical model, and the effect of property taxes on investment are developed in Appendix B and included in the empirical section below. The impact of multiple types of capital on investment is explored later in this section.

The PDV of current and future profits of the jth firm at time t is given by

$$\begin{aligned} PDV(\pi j)_t \quad = \quad & \int_{i=0}^{\infty} pj_{t+i} Yj_{t+i} e^{-ri} di - \int_{i=0}^{\infty} \bar{w}_{t+i} Lj_{t+i} e^{-ri} di - \sum_{i=0}^{\infty} q_{t+i} kj_{t+i} Nj_{t+i} e^{-ri} \\ & - \int_{i=0}^{\infty} \frac{1}{2} \xi_{t+i} \bar{w}_{t+i} Lj_{t+i} \left(1 - \frac{Nj^K_{t+i}}{x_{t+i} Lj_{t+i}} \psi\right)^2 e^{-ri} di, \end{aligned} \qquad (7)$$

where r is the nominal cost of corporate funds (or nominal rate of return) expected at time t, $\bar{w}$ is straight-time compensation per hour, regarded by each firm as exogenous, q is the price index for capital goods, N_t is the number of machines installed at time

Table 1: Sample Values of the Cost of High Utilization

α	ψ	x	ξ
0.25	0.6	1.05	1.68
0.24	0.6	1.05	1.58
0.25	0.5	1.05	1.73
0.25	0.6	1.16	1.97

t, ξ_{t+i} and ψ govern the cost of high utilization rates ($\psi < 1$), and the suffix j denotes a variable for the jth firm.[5] I assume that

$$\xi_{t+i} = \left(\frac{1-\psi}{\alpha} \frac{\psi}{x_t} - \frac{1}{2} \left(1 - \psi^2\right) \right)^{-1}. \tag{8}$$

That assumption is motivated by convenience rather than by any deep economic rationale: it generates $N^K = xL$ in equilibrium. Table 1 shows values of ξ for possible combinations of α, ψ, and x.[6]

For simplicity, I assume that A, $\bar{k}_{t-1}$, and the distribution of existing vintages are identical for each firm, so that each firm faces the same production function. Only the number of machines (N^K) and market share (f) differ among firms.

The first two terms on the right-hand side of equation 7 are familiar, except that $\bar{w}L$ only includes straight-time labor compensation. The third term is the PDV of investment: q is the price index of new capital goods, while $kjNj$ is real investment. Because N is a discrete number of machines purchased, rather than a rate of investment, I assume that investment occurs at the beginning of each period, meaning the PDV is calculated with a summation rather than an integral.

The final term in equation 7 is the cost of high utilization rates (xL/N^K is the utilization rate, indexed to 1), needed because the production function contains no penalty to a firm for buying few machines and then operating them at a high rate. One obvious component of that cost is higher hourly compensation for the overtime and extra shifts needed to obtain extra production from a limited amount of capital. (The marginal value of an extra hour of leisure rises as weekly hours worked increase, and so overall hourly compensation—including any deferred portion—should increase with hours per week.) In addition, part of the cost of higher utilization must fall on capital, since the share of labor compensation in total income is not procyclical, as it would have to be if the entire cost of increased utilization accrued to labor. One example would be extra wear and tear on capital that results in additional maintenance in later periods or earlier retirement. The cost of high utilization could also proxy

[5] Throughout the paper, rates of growth and rates of return in the equations correspond to the unit in which time is measured. Thus, if time is measured in quarters, rates of growth and rates of return are quarterly rates. However, in any discussion of data, I cite annual rates.

[6] The values for α assume that $\alpha\,(1 + 1/\eta)$ is 0.26 and η is -26 (for $\alpha = 0.25$) or -13 (for $\alpha = 0.24$). The values for x are the miminum and maximum between 1960 and 2006.

for an absolute limit on the amount of production possible from a given capital stock coupled with a permanent reduction in market share fj when a firm cannot meet the demands of its customers, so that future demand Yj depends inversely on current xL/N^K. Whatever the costs of high utilization rates, they cannot include a contemporaneous reduction in the efficiency of production, since that would violate the production function. Utilization costs are expressed as a function of labor hours rather than output so that the choice of kj, which affects output but not labor hours, does not depend on expected future rates of utilization.

1.2.2 First-Order Conditions

Each firm maximizes profits over three variables: the quality of new machines (kj), the number of new machines purchased (Nj), and labor hours (Lj). Given aggregate output (or demand) Y and aggregate price p, the firm's choice of labor hours determines its output (Yj) and the price charged to customers to sell that output (pj). To simplify the mathematics of the firm's investment decision, I assume identical service life S for all machines, so that firms expect machines installed at time t to depreciate at time $t + S$.[7]

After a few substitutions and cancellations, the three first-order conditions for maximization of profits become

$$\alpha\left(1+\frac{1}{\eta}\right)\int_{i=0}^{S} pj_{t+i}\frac{Yj_{t+i}}{Nj^K_{t+i}}\frac{1}{kj_t}x_{t+i}e^{-ri}di = q_t, \tag{9}$$

$$\int_{i=0}^{S}\xi_{t+i}\frac{\bar{w}_{t+i}}{x_{t+i}}\left(1-\frac{Nj^K_{t+i}}{x_{t+i}Lj_{t+i}}\psi\right)\psi e^{-ri}di = q_t kj_t, \tag{10}$$

and

$$\left(1+\frac{1}{\eta}\right)pj_t\frac{Yj_t}{Lj_t} = \bar{w}_t\left[1+\frac{1}{2}\xi_t\left(1-\left[\frac{Nj^K_t}{x_tLj_t}\psi\right]^2\right)\right]. \tag{11}$$

Equation 9 states that profits are maximized when the PDV of the additional revenues produced by an extra dollar of machine quality equals the cost of an extra dollar of machine quality. According to equation 10, the reduction in the costs of high utilization from adding another machine equals the cost of that machine.[8] Equation

[7] In the empirical section, I calculate depreciation following the treatment used by the U.S. Department of Labor (1983). Retirements of type-m capital installed at time t follow a truncated normal distribution centered around the end of the expected service life $t + Sm_t$.

[8] The first-order conditions for Nj_t do not include the effect of Nj_t on $\bar{k}j_{t+i}$. If the quality of new machines grows over time, then an extra machine purchased today will raise the average productivity of the capital stock in the near term, when the quality of that machine is greater than the average quality of existing machines, but reduce it when the machine nears retirement and is less productive than most existing capital. The PDV of the net impact is small, roughly 1 percent of the cost of capital for equipment and 2 percent for structures, and complicates the algebra considerably, so I ignore it.

11 states that the marginal revenues from an additional hour of labor equal the costs of that hour—straight-time pay plus the costs caused by a higher utilization rate.[9]

1.3 Investment: The Quantity and Quality of New Machines

1.3.1 The Number of New Machines with No Time to Build

To solve this system of equations, we must make some further assumptions. I assume that firms expect $Nj^K/(xL)$ to remain constant from time t to time $t+S$, so that we can replace $Nj^K_{t+i}/(x_{t+i}L_{t+i})$ with $Nj^K_t/(x_tL_t)$. Since Nj^K_{t+i} is fixed from $t+i$ to $t+i+1$ but Lj can vary during that interval, the assumption that $Nj^K/(xL)$ remains constant cannot hold in continuous time. Consequently, L_{t+i} denotes average labor hours from $t+i$ to $t+i+1$ and Y_{t+i} denotes average total output from $t+i$ to $t+i+1$ in the remainder of the paper. With time to build, discussed below, the firm makes its investment decisions for time t long enough before time t that it can assume that $Nj^K/(xL)$ will be at its equilibrium level at time t when making those decisions, and thus can remain constant from time t to time $t+S$. In addition, I assume that firms expect x to remain constant from time t to time $t+S$. Equation 8 then implies we can replace ξ_{t+i} with ξ_t.

To solve for Nj^K_t, we first use equation 11 to substitute for $\bar{w}_{t+i}$ and equation 8 to substitute for ξ in equation 10. After setting the left-hand side of the resulting equation equal to the left-hand side of equation 9 (multiplied through by kj_t) and considerable algebra, we find that $Nj^K_t = x_tLj_t$. Summing over all firms,

$$N^K_t = x_tL_t. \tag{12}$$

At the time a firm plans its capital stock for time t, it desires that the number of machines equal machines per labor hour times expected labor hours. To achieve the desired stock, the number of new machines (N) installed by all firms at time t must equal the desired change in the total number of machines plus the number of machines depreciating at time t (dep):

$$N_t = N^K_t - N^K_{t-1} + dep_t.$$

What happens if desired N is negative? In practice, depreciation is large enough that this does not occur at the aggregate level. However, if we allow for variations in market share (the fj), individual firms may face a situation in which they desire to reduce Nj but cannot. Caballero (1999) shows that irreversibility of investment leads firms to invest less than they otherwise would. Adding irreversibility of the number

[9] In practice, firms can vary the intensity of effort over the business cycle (see Appendix C). High intensity of effort must have some cost, or firms would keep intensity high all the time. In that case, labor productivity at full employment should replace actual labor productivity (Yj_t/Lj_t) in equation 11.

of machines to the putty-clay model would raise $N^K/(xL)$ above 1 during periods when irreversibility is binding, thereby reducing it below 1 when irreversibility is not binding. While that may have an important adverse effect on the equilibrium level of investment, I do not consider irreversibility of the number of machines in this paper. Rather, the putty-clay model assumes the irreversibility of machine quality.

1.3.2 The Quality of New Machines

To derive the quality of new machines (k), first observe from equation 10 that if $Nj^K_{t+i}/(x_{t+i}L_{t+i}) = 1$, then current and future kj is the same for all firms. The assumption that $\bar{k}j_{t-1}$ and A_t are equal for all firms, coupled with equation 1, implies that Yj/Lj, and thus pj, are the same for all firms now and in the future.[10] Using that equality across firms and setting x_{t+i}/Nj^K_{t+i} equal to $1/Lj_{t+i}$, we can rewrite equation 9 as

$$\alpha\left(1+\frac{1}{\eta}\right)\int_{i=0}^{S} p_{t+i}\frac{Y_{t+i}}{L_{t+i}}e^{-ri}di = q_t k_t. \tag{13}$$

Let y represent labor productivity (Y/L) and $\dot{y}$ represent its expected trend rate of growth.[11] I assume that time to build is long enough that firms assume that any procyclical component of Y/L present when they pick k_t will be gone by time t, so that Y/L can be expected to grow at its trend rate thereafter. To simplify the algebra, I assume that firms solve their maximization problem using the average expected growth rate of output prices ($\dot{p}$) and nominal rate of return expected to prevail over a machine's service life.

Inserting those expectations into equation 13, solving the integral, and rearranging terms yields

$$k_t = \alpha\left(1+\frac{1}{\eta}\right)\frac{p_t\, y_t}{v_t}, \tag{14}$$

where

$$v_t = q_t\frac{{}_tr - {}_t\dot{p} - \dot{y}}{1-e^{-\ ({}_tr-{}_t\dot{p}-\dot{y})S}} \tag{15}$$

and ${}_tr$ and ${}_t\dot{p}$ are, respectively, the nominal cost of funds and rate of growth of output prices expected at time t. Equation 14 states that the quality of new machines is proportional to output per hour, y, and to the ratio of the price of output (p) to the cost of capital (v). Workers are given more capital to work with the more productive they are and the cheaper capital is. Productivity in turn depends both on TFP and on the quality of existing machines ($\bar{k}$), since machines are complementary. The

[10] To be exact, that conclusion also requires that Nj^K grow at the same rate across firms.

[11] In this paper, $\dot{y}$ equals its sample average in all periods, so there is no need for a time subscript.

expression for the cost of capital v is similar to that found by Ando et al. (1974), except that it substitutes $r - \dot{p} - \dot{y}$ for $r - \dot{p}$ and excludes taxes.[12]

The cost of capital in the standard putty-putty model in the absence of taxes, v^{PP}, is

$$v_t^{PP} = q_t \left({}_t r - {}_t \dot{q} + \frac{1}{S} \right),$$

where ${}_t\dot{q}$ is the expected growth rate of q at time t. In both models, a higher price of capital goods and a higher nominal rate of return raise the cost of capital, while a longer service life (or lower rate of depreciation) reduces the cost of capital. The sizes of the effects of higher rates of return and a longer service life on the cost of capital differ between the models due to different functional forms. There are two other important differences.

First, the real rate of return is the nominal rate minus the rate of overall price inflation in the putty-clay model, but the nominal rate minus the rate of growth of capital prices in the putty-putty model. In the putty-putty model, the marginal product of existing computers falls as they are puttied together with new computers, raising the cost of capital required to equate marginal revenue with marginal cost. In the putty-clay model, however, a fall in the price of computers drives down the price of existing computers but does not make them less productive. Instead, the marginal revenue of an additional dollar of machine quality grows with output prices. That rate of growth is subtracted from the nominal cost of funds.

Second, in the putty-clay model, existing machines become more productive as labor productivity rises. All else equal, the more rapidly labor productivity will rise in the future, the more valuable is each dollar of quality added to a new machine today. Hence, the expected rate of growth of labor productivity is subtracted from the expected real rate of return. In the putty-putty model, the increased productivity of existing machines disappears as they are puttied together with new capital. Consequently, productivity growth has no effect on the cost of capital in that model.

When there is more than one type of capital, the cost of type-m capital (vm) depends on the price (qm) and service life (Sm) of such capital. In addition, if $r - \dot{p}$ is expected to change over time, ${}_t r - {}_t \dot{p}$ may differ at time t for types of capital with different expected service life. In the absence of time to build, real investment in type-m capital at time t is $km_t Nm_t$.

[12] Equation 15 is also similar to the formula for determining the payment on a home mortgage with a fixed rate. In that case, the term of the mortgage replaces S and the mortgage rate replaces $r - \dot{p} - \dot{y}$.

1.3.3 The Number and Quality of New Machines with Time to Build

The change in the number machines in a period depends on information from previous periods because of various factors collectively called "time to build," following Kydland and Prescott (1982). It takes time for businesses to gather information and to translate that information into an investment plan. The existence of unfilled orders for capital goods gives evidence of a further time lag between the time those plans are made and the arrival of new capital. For structures, considerable time can elapse between groundbreaking and final completion of a project. Investment will also lag behind changes in the desired stock if capital is "lumpy," i.e., if businesses do not adjust their capital stock until some threshold level of adjustment is required.[13]

Time to build for the quality of new machines need not be the same length as time to build for the number of new machines. Lumpy investment, adjustment costs, and the information-gathering interval between changes in demand and the development of a plan to meet them all suggest that the time to build for N exceeds the time to build for k.

Time to build is usually modeled by assuming costs of adjustment, either of the capital stock or of investment. However, adding a cost of adjusting the number of new or existing machines greatly complicates the model. Consider the addition of a cost term containing $(Nj_{t+i} - Nj_{t+i-1})^2$ to equation 7. Equation 10 would then include $Nj_t - Nj_{t-1}$ and $Nj_t - Nj_{t+1}$, making the number of new machines depend on past and expected rates of investment. That in turn would cause the utilization rate, xL/N^k, to deviate from 1. From equation 9, we can see that kj varies positively with expected utilization. Because of the cost of adjusting the rate of purchase of new machines, firms would satisfy part of the need for greater capacity by purchasing machines of greater quality. To calculate the quality of new machines, we would not only need to know labor productivity and the cost of capital, but also past and expected N. Instead of adjustment costs, I assume a fixed schedule of planning and deliveries.

Start by assuming that firms choose N_t^K based on information available at time $t-T$. Throughout this section, I assume that firms know x_t with certainty at every stage of time to build. Then, from equation 12, N_t^K depends on firms' expectations of L_t at time $t-T$. Let $E_{t-T}(J_t)$ denote time $t-T$ expectations of variable J at time t. Output equals labor hours times labor productivity, so

$$E_{t-T}(L_t) = \frac{E_{t-T}(Y_t)}{E_{t-T}(y_t)} \tag{16}$$

for each firm and for the economy as a whole.

Let $\bar{Y}$ and $\bar{L}$ denote output and labor hours at full employment, i.e., output and labor

[13] For evidence that capital is lumpy, see Doms and Dunne (1998) and Cooper, Haltiwanger, and Power (1999).

hours consistent with an unemployment rate stable at the full employment level.[14] Assume that firms expect the difference between actual output and output at full employment in the future to be a linear function of today's difference between actual output and output at full employment:

$$E_{t-T}(Y_t) - E_{t-T}(\bar{Y}_t) = \theta\left(Y_{t-T} - \bar{Y}_{t-T}\right). \tag{17}$$

I assume firms expect the gap between output and output at full employment to narrow over the time-to-build horizon, so that $\theta < 1$.

The expected growth rate of output at full employment equals the expected growth rate of hours at full employment ($\dot{n}$) plus $\dot{y}$, so

$$E_{t-T}(\bar{Y}_t) = \bar{Y}_{t-T}\; e^{(\dot{n}+\dot{y})T}. \tag{18}$$

Inserting this into equation 17 and rearranging, we have

$$E_{t-T}(Y_t) = \left(e^{(\dot{n}+\dot{y})T} - \theta\right)\bar{Y}_{t-T} + \theta Y_{t-T}. \tag{19}$$

Output expected at time t is a linear combination of actual output and output at full employment at time $t-T$.

Assume that firms expect the procyclical component of labor productivity to disappear by the end of the time-to-build horizon and labor productivity at full employment to grow at rate $\dot{y}$, so $E_{t-T}(y_t) = \bar{y}_{t-T}e^{\dot{y}T}$. Substituting this expression and equation 19 into equation 16, substituting $\bar{L}_{t-T}$ for $\bar{Y}_{t-T}/\bar{y}_{t-T}$, and rearranging, we have

$$E_{t-T}(L_t) = \left(e^{\dot{n}T} - \frac{\theta}{e^{\dot{y}T}}\right)\bar{L}_{t-T} + \frac{\theta}{e^{\dot{y}T}}\frac{Y_{t-T}}{\bar{y}_{t-T}}. \tag{20}$$

Expected future labor hours, and thus the desired number of machines, are a linear combination of labor hours at full employment and the ratio of actual output to labor productivity at full employment. Actual output has a smaller weight the greater the expected narrowing of the gap between actual output and output at full employment, i.e., the lower the θ.

Assume that at time $t-T$, each firm places an order for $\frac{1}{T}Oj_{t-T}$ new machines to be delivered in each period from $t-T+1$ to t, a total order of Oj_{t-T}. The standard deviation of ΔN^K is smaller than the standard deviations of $\Delta\bar{L}$ and $\Delta Yj/\bar{y}$, implying that firms' response to movements in $\bar{L}$ and $Yj/\bar{y}$ is spread out over time. For structures, which take several quarters to complete, the filling of an order over several quarters is a natural assumption. Assume that firms place orders at time $t-T$ that

[14] Because labor hours are slow to adjust to output, they may not be at their full employment level if the unemployment rate is currently at its full employment rate but was higher or lower than that rate in previous quarters.

optimize expected Nj_t^K, i.e., $E_{t-T}\left(Nj_t^K\right) = x_t * E_{t-T}\left(Lj_t\right)$.[15] The desired number of machines is a linear combination of the $\bar{L}j$ and $Yj/\bar{y}$, which can be summed over firms, so we can analyze aggregate orders O_{t-T} directly.

At time $t-T-1$, the number of machines expected at time $t-1$, $E_{t-T-1}\left(N_{t-1}^K\right)$, equals orders placed at or before time $t-T-1$ but not yet delivered plus the expected volume of orders placed after $t-T-1$ but delivered before or at time $t-1$. As an approximation, I assume that the expected volume of orders placed after $t-T-1$ but delivered before or at time $t-1$ is the same at time t as at time $t-1$.[16] Then

$$E_{t-T}\left(N_t^K\right) = E_{t-T-1}\left(N_{t-1}^K\right) + O_{t-T} - E_{t-T-1}\left(O_{t-T}\right) + \frac{1}{T}\sum_{i=0}^{T-1} E_{t-T}\left(O_{t-T+i}\right).$$

Rearranging terms, we have

$$\begin{aligned} O_{t-T} &= E_{t-T}\left(N_t^K\right) - E_{t-T-1}\left(N_{t-1}^K\right) + \frac{T-1}{T} E_{t-T-1}\left(O_{t-T}\right) \\ &\quad -\frac{1}{T}\sum_{i=1}^{T-1} E_{t-T}\left(O_{t-T+i}\right). \end{aligned} \tag{21}$$

To solve that expression, we must know how firms form expectations of future orders. I will assume that they expect the same level of orders that they would in steady state, i.e., if $\bar{L}$ and $Yj/\bar{y}$ grew at rate $\dot{n}$. If O grows at rate $\dot{n}$, then

$$\begin{aligned} \frac{T-1}{T} E_{t-T-1}\left(O_{t-T}\right) - \frac{1}{T}\sum_{i=1}^{T-1} E_{t-T}\left(O_{t-T+i}\right) &= \frac{T-1}{T} O_{t-T}\left[1 - \sum_{i=1}^{T-1}\left(1+\dot{n}\right)^i\right] \\ &\approx O_{t-T}\left[1 - e^{\dot{n}T/2}\right]. \end{aligned} \tag{22}$$

Substituting from equation 22 into equation 21 and rearranging, we have

$$O_{t-T} = e^{-\dot{n}T/2}\left[E_{t-T}\left(N_t^K\right) - E_{t-T-1}\left(N_{t-1}^K\right)\right]. \tag{23}$$

The volume of orders made at time $t-T$ is slightly smaller than the change in the desired number of machines at time t because the expected volume of orders to be made after $t-T$ but delivered before or at t is somewhat greater than the volume of orders made after $t-T-1$ but delivered before or at $t-1$.

[15] That prevents a positive shock between periods $t-T$ and t from producing a permanent oscillation in orders. For example, a positive shock in period $t-1$ would lead firms to raise orders in that period to meet the new higher target for N_t^K. However, that would leave deliveries too high at time $t+1$, leading to a reduction in orders made in period t. That zigzag pattern of orders would continue ad infinitum.

[16] The only reason that is an approximation is the assumption that firms always expect labor hours at full employment to grow at rate $\dot{n}$. Consequently, if the *level* of $\bar{L}_{t-T}$ is 1 percent greater than expected at time $t-T-1$, the *growth* of $\bar{L}$ in subsequent periods will rise by 1 percent.

To obtain an expression for total investment, we must add in orders for replacement machines. Replacement demand is smooth enough that we can assume that deliveries of replacement machines equal the number of machines depreciating in every period. Once replacement demand is factored in, O_{t-T} can be negative. The number of new machines is then replacement demand plus a moving average of past orders:

$$N_t = \frac{1}{T}\sum_{i=1}^{T} O_{t-i} + dep_t.$$

Using equations 12, 20, and 23 to replace O_{t-i}, we have

$$N_t = \frac{1}{T}\sum_{i=1}^{T}\left[\left(e^{\dot{n}T/2} - \frac{\theta}{e^{(\dot{y}+\dot{n}/2)T}}\right)\Delta\left(x_t\bar{L}_{t-i}\right) + \frac{\theta}{e^{(\dot{y}+\dot{n}/2)T}}\;\Delta\left(x_t\frac{Y_{t-i}}{\bar{y}_{t-i}}\right)\right] + dep_t$$

The number of new machines is a function of lagged growth of labor hours at full employment and lagged growth of output in excess of labor productivity plus replacement demand.

With more than one type of capital, the number of new machines of type m must also take account of changes in the desired mix of capital and differences in time to build between different types of capital. Ideally, separate orders Om_{t-Tm} would be placed for each type of capital at time $t - Tm$, with Tm being the time to build type-m capital. However, if the desired ratio of type-m machines to all machines (Nm^K/N^K) is changing, the portion of orders Om_{t-Tm} delivered before $t-T$ would cause the actual ratio to differ from the desired ratio in periods between $t-T$ and t. That makes it difficult to observe the desired ratio. Instead, I assume that various aggregate orders O_{t-Tm} are placed, with the share of the order going to different types of machines able to vary with desired Nm^K/N^K. The number of new machines of type m is then given by

$$\begin{aligned} Nm_t \;=\; & \left(\frac{Nm_t^K}{N_t^K}\right)^* \sum_i \left\{\beta m_i \Delta\left[x_t\bar{L}_{t-i}\right] + \gamma m_i \Delta\left[x_t\frac{Y_{t-i}}{\bar{y}_{t-i}}\right]\right\} \\ & + \Delta\left(\frac{Nm_t^K}{N_t^K}\right)^* x_{t-1}\bar{L}_{t-1} + depm_t + \epsilon m_t, \end{aligned} \tag{24}$$

where $\left(Nm_t^K/N_t^K\right)^*$ is desired Nm^K/N^K at time t, $\sum_i(\beta m_i + \gamma m_i) > 1$, and ϵm_t is an error term.

Machine quality must also be chosen at the time an order is made. The quality of machines delivered at time t will roughly equal the average quality ordered from periods $t-T$ to $t-1$. I assume that firms expect the nominal rate of return and tax law will be the same at time t as at time $t-T$. In that case, the quality of new orders will grow at rate ${}_t\dot{p} + \dot{y} - {}_t\dot{q}$. To attain the desired average quality of new machines delivered at time t, firms order quality

$$\alpha\left(1+\frac{1}{\eta}\right)\frac{p_{t-T}\,\bar{y}_{t-T}}{v_{t-T}}e^{({}_t\dot{p}+\dot{y}-{}_t\dot{q})T/2}$$

at time $t-T$. Consequently, the quality of delivered machines of type m is given by

$$km_t = \alpha\left(1+\frac{1}{\eta}\right)\sum_i Gm_i \frac{p_{t-i}\,\bar{y}_{t-i}}{vm_{t-i}} e^{({}_t\dot{p}+\dot{y}-{}_t\dot{q})i/2}, \tag{25}$$

where Gm_i is the fraction of machines delivered at time t that are ordered at time $t-i$.[17] Total real investment in type-m capital at time t is $Nm_t km_t$.

The maximum lag length for the number of machines exceeds the maximum lag length for the quality of machines. Gathering information on the state of demand, necessary for determining the number of new machines but not their quality, precedes the placing of an order for machinery or the groundbreaking for a structure. Perhaps more important, capital is lumpy, so new orders depend partly on changes in labor hours and output occurring in earlier periods.

2 Properties of the Model

Many of the properties of the putty-clay model can most easily be seen in comparison to the properties of the more familiar neoclassical model. This section discusses the behavior of investment to illustrate the short-run properties of the model and the behavior of the rate of return in steady state to illustrate the long-run properties of the model. I ignore taxes, since the corporate income tax has the same effect on the cost of capital in the putty-clay and putty-putty models. In both models, a corporate income tax raises the cost of capital unless there is expensing, while an investment tax credit reduces the cost of capital.[18] Property taxes have different effects in the two models, but the difference is not large.

2.1 Properties of Investment

In the absence of taxes and with $\eta = -\infty$ (perfect competition), the desired stock of putty-putty capital given a Cobb-Douglas production function in Jorgenson's (1963) neoclassical model is $K_t^* = \alpha p_t Y_t / v_t^{PP}$, where

$$v_t^{PP} = q\left({}_tr - {}_t\dot{q} + \delta\right)$$

and δ is the rate of geometric depreciation of the capital stock corresponding to $1/S$. (For simplicity, I assume a single type of capital in both models, so the m suffixes

[17] Since structures are used with land, a structure "machine" is composed of both structure and land. I assume that the cost of the land portion of such a "machine" is 32 percent as large as the cost of the structure portion, so km_t from equation 25 is multiplied by $1/1.32$ for structures and $0.32/1.32$ for land. (See Appendix C for further details.) In practice, that assumption affects only x and the estimated growth rate of overall $\bar{k}$.

[18] That result assumes a given cost of corporate funds. If the cost of corporate funds is endogenous, then a corporate income tax can affect the cost of capital even if new investment can be expensed.

are eliminated.) Net investment is assumed to be a weighted average of current and past changes in desired capital (ΔK_t^*). Gross investment I^{PP} is then given by

$$I_t^{PP} = \alpha \sum_i \chi_i \Delta \left(\frac{p_{t-i} Y_{t-i}}{v_{t-i}^{pp}} \right) + \delta K_{t-1}. \tag{26}$$

In the absence of taxes, with $\eta = -\infty$, $x = 1$, and one type of capital, gross investment in the putty-clay model is

$$I_t^{PC} = \alpha \left\{ \sum_i \left[\beta_i \Delta \bar{L}_{t-i} + \gamma_i \Delta \left(\frac{Y_{t-i}}{\bar{y}_{t-i}} \right) \right] + dep_t \right\} \sum_j G_j \frac{p_{t-j}\, \bar{y}_{t-j}}{v_{t-j}^{PC}}, \tag{27}$$

where[19]

$$v_t^{PC} = q \frac{{}_t r - {}_t \dot{p} - \dot{y}}{1 - e^{-\ ({}_t r - {}_t \dot{p} - \dot{y}) S}}.$$

Several important differences between the putty-clay and putty-putty models can be seen by comparing the two investment equations. Perhaps the most characteristic difference between the models is that investment depends on the level of the real cost of capital in the putty-clay model but on growth of the real cost of capital in the putty-putty model. In a putty-clay world, the quality of existing machines is fixed once they are in place, so firms cannot go back and remold existing machines when interest rates, stock prices, or tax rates change. In a putty-putty world, however, they can.

In the putty-clay model developed in this paper, the accelerator term depends on both the growth of labor hours at full employment and the growth of output in excess of labor productivity at full employment.[20] That is, firms invest in both capital and labor when they cannot meet the growth of demand solely by increasing the productivity of their existing capital and labor. In the putty-putty model, however, the accelerator term depends on the growth of real output, regardless of the rate of growth of labor productivity. In that model, if real output is growing at a 3 percent rate, firms feel the same need to increase physical capacity whether productivity is also growing at a 3 percent rate or not growing at all. Another way of putting this is that the impact of faster growth of output on investment in the putty-clay model depends on whether that extra growth comes from faster growth of hours or from faster growth of productivity, while in the putty-putty model the source of that extra growth is irrelevant. Previous empirical work based on putty-clay capital has defined the accelerator using the growth of output, the same as in the putty-putty model.

However, it would be incorrect to say that investment is independent of productivity in the putty-clay model. The presence of $\bar{y}$ in the second summation in equation 27

[19] For simplicity, I leave the $\exp\left[(\dot{p}_t + \dot{y} - \dot{q}_t)\, i/2\right]$ term out of equation 27. That means that G_j is actually the product of the G_j from equation 25 and $\exp\left[(\dot{p}_t + \dot{y} - \dot{q}_t)\, i/2\right]$.

[20] The growth of output in excess of labor productivity at full employment is similar to the growth of labor hours, which is the growth of output in excess of actual labor productivity.

means that investment is proportional to the level of productivity as long as higher productivity is reflected in higher output. If demand keeps pace with technology, investment depends on the level of productivity in the putty-clay model and on the growth of productivity in the putty-putty model. If demand growth fails to keep pace with productivity, improved technology can actually slow putty-clay investment in the short run, a result found empirically by Basu, Fernald, and Kimball (2004).

The presence of the growth of labor hours at full employment in the putty-clay investment equation reflects the assumption that firms expect actual labor hours to gradually converge to labor hours at full employment. Hence, changes in labor hours at full employment provide information about future employment and output, and thus influence investment.

The models' treatment of replacement demand differs in two ways. First, replacement demand in the putty-putty model is a constant fraction of the existing stock while in the putty-clay model it depends on the mix of vintages in the existing stock. Geometric depreciation of the capital stock, as in the putty-putty model, is impossible in the putty-clay model if productivity rises over time. Eventually, the productivity of new machines becomes so much larger than that of existing machines that firms willingly discard existing machines.

Second, replacement demand in the putty-putty model is tied to the original value of the capital being replaced. If depreciating capital cost $1 billion in 2000 chained dollars when it was purchased, then replacement demand is $1 billion in 2000 chained dollars. In the putty-clay model, however, replacement demand is tied to the current quality of new machines, not to the quality of depreciating machines. If productivity has grown or the real cost of capital has fallen since the depreciating machines were purchased, replacement demand will exceed the original value of the depreciating machines.

The cost of capital also differs between the models. Geometric decay in the putty-putty model allows the rate of depreciation to be added linearly to the rate of return. Fixed service lives in the putty-clay model lead to a more complex functional form for the cost of capital. They also lead to a lower cost of capital for the same service life, all else equal. Although the expected service lives are the same, the present discounted value of the marginal output from an extra dollar of investment (in the putty-clay case, an extra dollar of machine quality) is greater in the case of fixed service lives, meaning a lower cost of capital is needed for the firm to break even.

In the putty-clay cost of capital, the expected rate of growth of nominal labor productivity ($\dot{p} + \dot{y}$) is subtracted from the nominal corporate cost of funds (r). The marginal revenue of an extra dollar of machine quality rises over time with the price of output and with labor productivity. Expectations of future growth have an important effect on investment. In a putty-putty world, however, the future value of an extra dollar of investment today depends on the future price of new investment. Thus, the marginal revenue of an extra dollar of investment grows at the rate of the price index for investment.

Changes in the desired mix of capital can have important effects on investment by type of machine in the putty-clay model. An increase in the desired share of type-m machines in overall capital produces a large transitory increase in investment in type-m capital (from a larger $\Delta\left(\frac{Nm_t^K}{N_t^K}\right)$ in equation 24) and a smaller permanent increase (from a larger $\frac{Nm_t^K}{N_t^K}$ and $depm_t$ in equation 24).

2.2 The Rate of Return in Steady State

To examine the properties of the putty-clay and putty-putty models along the steady-state growth path, assume exogenous saving rate s, $\dot{p} = \dot{q}$, no taxes, $\eta = -\infty$, and $x = 1$. In steady state, Y/L and $\bar{k}$ grow at the same rate $\dot{y}$ in both models. In the standard neoclassical model of Solow (1956), the real rate of return in steady state is

$$rr^{PP} = \frac{\alpha}{s}\left(\dot{y} + \delta + \dot{n}\right) - \delta.$$

The rate of return in steady state for the model of this paper differs from that result for two reasons: the putty-clay nature of capital and the use of a fixed service life instead of geometric depreciation. To illuminate the source of differences with the standard model, I solve the putty-clay model under the assumptions of both fixed service life and constant failure rate.[21]

2.2.1 Putty-Clay: Constant Failure Rate

With a constant failure rate, the fraction of machines purchased at t and still in use at time $t + i$ is $e^{-\delta i}$. Firms choose k_t such that

$$k_t = \frac{\alpha\,\bar{y}_t}{(rr + \delta - \dot{y})}. \tag{28}$$

Saving equals investment, so $sY_t = N_t k_t$ or

$$k_t = \frac{s\,Y_t}{N_t}. \tag{29}$$

With $x = 1$, labor hour equals the number of machines in operation, so

$$L_t = \int_{i=0}^{\infty} N_{t-i}\, e^{-\delta i} di = \frac{N_t}{\delta + \dot{n}}. \tag{30}$$

[21] As noted earlier, geometric depreciation is unrealistic in a putty-clay model with technological progress, whether the depreciation is from reduced efficiency of machines or a constant failure rate. Geometric depreciation is assumed only for illustrative purposes.

Combining equations 28 and 29, substituting for N_t using equation 30, replacing Y_t/L_t with $\bar{y}_t$, and rearranging, we have

$$rr^{CF} = \frac{\alpha}{s}(\delta + \dot{n}) - \delta + \dot{y}. \tag{31}$$

In the absence of technological progress, the real rate of return is identical to that in the putty-putty model. Firms have no desire to change the quality of existing machines, so putty-putty and putty-clay are the same in steady state. However, if there is technological progress and $\alpha/s > 1$, a higher rate of return is needed to equate investment with saving for putty-putty capital than for putty-clay capital. As $\dot{y}$ increases for a given r, the additional investment to increase the quality of old machines in the putty-putty framework exceeds the additional investment to build higher-quality new machines to take advantage of expected rising productivity in the putty-clay framework.

2.2.2 Putty-Clay: Fixed Service Life

With no taxes, fixed service life, and the price of output equal to the price of new capital, equations 14 and 15 can be combined as

$$k_t = \alpha\, \bar{y}_t \frac{1 - e^{-(rr-\dot{y})S}}{rr - \dot{y}}.$$

In steady state with $x = 1$,

$$L_t = \int_{i=0}^{S} N_{t-i}\, di = \int_{i=0}^{S} N_t\, e^{-\dot{n}i}\, di = N_t \frac{1 - e^{-\dot{n}S}}{\dot{n}}. \tag{32}$$

Following the same steps as in the case of a constant failure rate, we obtain the expression

$$\frac{rr^{PC} - \dot{y}}{1 - e^{-(rr^{PC}-\dot{y})S}} = \frac{\alpha}{s}\frac{\dot{n}}{1 - e^{-\dot{n}S}}. \tag{33}$$

2.2.3 A Numerical Example

Since an analytic solution for rr^{PC} is impossible, this section uses a numerical example to examine steady-state properties of the model. I look at five changes that each produce a 1.5 percentage point increase in rr^{PP}: an increase in $\dot{y}$, an increase in $\dot{n}$, a reduction in S (equivalent to an increase in δ), a reduction in s, and an increase in α.

In the base case, $\dot{y} = 0$, $S = 20$, $\dot{n} = 0.01$, $s = 0.2$, and $\alpha = 0.3$. With no technological progress, rr^{PP} is identical to rr^{CF} (see Table 2). A higher rate of return is needed

Table 2: The Rate of Return in Steady State

Parameters					Rate of Return (rr, in percent)		
						Putty-Clay	
						Constant	Fixed
$\dot{y}$	$\dot{n}$	S $(1/\delta)$	s	α	Putty-Putty	Failure Rate	Service Life
0	0.01	20	0.2	0.3	4.00	4.00	5.55
0.01	0.01	20	0.2	0.3	5.50	5.00	6.55
0	0.02	20	0.2	0.3	5.50	5.50	6.73
0	0.01	12.5	0.2	0.3	5.50	5.50	8.17
0	0.01	20	0.1714	0.3	5.50	5.50	7.50
0	0.01	20	0.2	0.35	5.50	5.50	7.50

in the putty-clay model with fixed service life to equate investment with saving. As long as rr is positive, the present discounted value of the additional revenues from an extra dollar of machine quality today is greater for fixed service life than for constant failure rate, i.e.,

$$\int_{i=0}^{S} e^{-rr*i} di > \int_{i=0}^{\infty} e^{-(rr+1/S)i} di \text{ for } rr > 0.$$

An increase in $\dot{y}$ from zero to 0.01 raises r by only 1.0 percentage point in the putty-clay models, no matter what type of depreciation is used. With α, s, $\dot{n}$, and S unchanged, $rr - \dot{y}$ is unchanged, so any increase in $\dot{y}$ produces a comparable increase in rr. Firms cannot change the quality of existing machines, so a larger expected increase in $\bar{y}$ increases only the quality of new machines. With putty-putty capital, however, firms want to increase the quality of every existing machine as $\bar{y}$ rises. A larger increase in rr is needed to crowd out the extra investment in that model.

The smaller positive impact of productivity growth on the real rate of return in the putty-clay model helps to explain why low real interest rates coincided with rapid productivity growth in the early 2000s. Kroszner (2006) sketches the standard rationale for effects of productivity growth on the real interest rate in a putty-putty framework and then notes that interest rates have been very low in the early 2000s given rapid productivity growth. That result is less surprising in a putty-clay framework, especially if above-average growth is not expected to persist over the service life of new capital.

In every other case, the results for the putty-putty model are identical to those for the putty-clay model with constant failure rate. In those cases, it is the assumption of fixed service life that causes differences between the putty-putty and putty-clay models. An increase in $\dot{n}$ raises rr by 1.18 percentage points with fixed service lives, less than with constant failure rate. With constant failure rate, the ratio of replacement machines to the total is always $1/S$. However, with fixed service lives, the ratio of replacement machines—machines put in place S years ago—to the total number of machines declines as $\dot{n}$ increases.[22] Because replacement demand is relatively smaller, so is rr.

Note that the effect of higher output growth in the putty-clay model depends on whether it comes from more rapid growth in labor hours or in productivity. In the putty-putty model, the source of faster growth is irrelevant.

A reduction in S and equivalent rise in δ raise rr significantly more with fixed service life than with constant failure rate. In either model, a more rapid rate of depreciation raises replacement demand, and thus raises rr. However, that initial increase in rr increases the disparity in the PDV of additional revenues from an extra dollar of machine quality between fixed service life and constant failure rate, requiring a larger increase in rr for fixed service life.

A reduction in s and an increase in α have about the same effect on rr in each model. While the absolute rise in rr is somewhat larger in the putty-clay model than in the putty-putty model, the percentage increase is slightly smaller.

3 Data

This section provides an overview of the data used in estimating the equations for investment and a more detailed discussion of the estimated time series for x. Appendix C discusses the other data in more depth.

I estimate equations for five categories of private nonresidential fixed investment: computers and peripheral equipment; communication equipment; software; other producers' durable equipment excluding agricultural equipment and mining and oilfield machinery; and nonresidential structures excluding farm and mining exploration, shafts, and wells. The model's assumption that the difference between actual output and output at full employment is correlated from one year to the next does not hold for farming, so investment in agricultural machinery and farm structures is not included. Excluding mining-specific capital (mining and oilfield machinery and mining exploration, shafts, and wells), the ratio of capital to output in mining is about the same as in the other industries using nonresidential fixed capital. Consequently, investment in mining-specific capital is not included in the investment equations and is instead treated as a change in x. Such investment fits the model poorly anyway, since the desired output of this industry is determined by the relative price of mining output rather than by aggregate demand.

The sectoral coverage of output and labor hours is defined to be consistent with investment. Labor hours at full employment ($\bar{L}$) and output per hour at full employment ($\bar{y}$) are not directly observable, and are estimated with Kalman filters, as

[22] Mathematically, equation 32 implies that the ratio of replacement machines to total machines, $N_t/L_t - n$, equals $\frac{n}{1-e^{-nS}} - n$ with fixed service lives. That expression is decreasing in n.

discussed in Appendix C.

The real rate of return is a weighted average of the cost of debt and the cost of equity for nonfinancial corporate business. The cost of equity is calculated using the dividend-discount model. That method is less than ideal but necessary, given that other measures rely on estimates of profits and thus of depreciation that are inconsistent with the putty-clay model. The expected growth rate of $\bar{y}$ ($\dot{y}$) at every time t is set equal to 1.91 percent, the coefficient of time in a regression of $\ln(y)$ on time during 1949-2005. Hence, this paper does not estimate the impact of variations in $\dot{y}$ on investment.

The pattern of retirements of capital follows the treatment used by the Bureau of Labor Statistics (BLS). In U.S. Department of Labor (1983), BLS assumes that retirements of a given vintage of fixed capital follow a truncated normal distribution centered on the expected service life and ranging from 0.02 to 1.98 times that service life. The standard deviation of the density function is 0.49 times the service life. In contrast to BLS, however, retirements are measured in numbers of machines, not constant dollars.

I assume that the observed values for xo and for each ratio Nm^K/No^K, where No^K denotes all machines except mining-specific capital and inventories, equal a desired value, used by firms to determine the number of new machines in equation 24, plus an error. Observed xo equals actual No^K divided by a rough estimate of businesses' forecast of labor hours when orders for those machines were placed.[23] I assume $\alpha\left(1+\frac{1}{\eta}\right)$ equals 0.26. (A larger value for α would result in a proportionately smaller estimate of xo.) The desired values of Nm^K/No^K are estimated by applying a Hodrick-Prescott filter to the raw data for 1954 to the end of the sample. To isolate long-run trends, the filter for xo uses more smoothing than is standard.[24]

Raw and filtered data for xo from 1929 through the third quarter of 2006 are shown in Figure 1. The most striking feature of the data is the large drop in xo during the Great Depression and World War II and the rebound of xo back near its 1929 level in the postwar period. An obvious explanation is that businesses extended the service lives of capital beyond their normal length from the early 1930s through the early 1950s. During the Great Depression, firms may have used hoarded labor to perform extra maintenance on existing plant and equipment. During World War II, shortages likely prevented businesses from purchasing as much new capital as they would have liked, forcing them to further extend the service lives of existing capital. With shortages at an end after World War II, businesses aggressively replaced the obsolete capital they had been unable to replace earlier. Because of the unusual

[23] That estimate is $mov13\left(\bar{L}^*\right)\left(0.6+0.4*mov13\left(L^*/\bar{L}^*\right)\right)$, where $mov13(y)$ denotes a 13-quarter moving average of y and L^* and $\bar{L}^*$ are two-quarter moving averages of L and $\bar{L}$, respectively. The coefficient of 0.4 on $mov13\left(L^*/\bar{L}^*\right)$ is broadly consistent with estimates of the γm_i.

[24] For xo, λ equals 6400, twice the normal amount of smoothing. Changing the amount of smoothing for the Nkm/Nko affects the individual investment equations, but has little impact on the overall fit.

character of business investment during the early postwar period, I begin the sample period for the estimation of investment in 1957.

One other important feature of xo is a rise from about 0.95 in the late 1970s to more than 1.0 by the late 1980s. The adoption of the personal computer, which occurred during that period, can explain much of the increase, but the amount spent on computers is too small to explain the entire change in xo.

One interesting possibility is that xo depends on the expected volatility of demand. The derivation of investment earlier in this paper assumes that, under uncertainty, the PDV of profits calculated using expected values of the variables equals expected PDV of profits. That assumption is reasonable for revenues, labor costs, and investment costs. However, the costs of utilization are highly nonlinear. An increase in expected volatility raises the expected cost of utilization, because the increase in the cost of high utilization is greater than the reduction in the cost of low utilization. That will cause firms to increase both the ratio of machines to labor hours (N^K/L), which the model interprets as an increase in x, and the ratio of p to $\bar{w}$.

Figure 2 shows total x excluding mining and computers. That measure of machines per hour rose steadily from the late 1960s until the late 1980s, perhaps reflecting an increase in expected volatility of output, and has fallen steadily since the early 1990s as expected volatility has declined. Under that interpretation, greater expected volatility caused firms to target a higher average level of excess capacity in order to avoid the costs of periods of high utilization. The cost of maintaining that excess capacity also led firms to raise the ratio of p to $\bar{w}$, putting upward pressure on inflation.

4 Estimation

The model fits the data well, and the estimated parameters generally conform to the theory. However, the handling of machines per labor hour has important effects on the fit of the model. We can reject the implications of the neoclassical model for the role of the cost of capital and the form of the accelerator. A basic q model of investment has little effect on the results.

4.1 Results of Estimating the Investment Equations

The model to be estimated is the system composed of equations 24 to 25 with xo substituted for x and No^K for N^K. The goal is to determine the βm_i, γm_i, and Gm_i that produce the best fit. Estimating those parameters jointly in a single equation for each type of capital proves troublesome. The sum of the Gm_i should be 1 for each type of capital, but is significantly less than 1 for each type of capital when freely estimated. Hence, the Gm_i must be treated separately from the βm_i and γm_i.

The estimation procedure for each type of capital is to first choose a set of Gm_i, then to solve equation 25 for km_t, then to divide real investment by km_t to get Nm_t, and finally to estimate equation 24 to obtain the βm_i and γm_i. The objective is to minimize the error in equation 24. I constrain the Gm_i to sum to 1 for each type of capital. To impose some structure on the choice of the Gm_i, I use moving averages. The fact that contemporaneous xo_t is multiplied by various lags of the $\bar{L}_{t-i}$ and $Y_{t-i}/\bar{y}_{t-i}$ rules out polynomial distributed lags for the βm_i and γm_i. Instead, I use moving averages. No constraint is imposed on the sums of the βm_i and γm_i.

Equation 24 can be estimated using ordinary least squares (OLS) if the dependent variable, $Nm_t - \Delta\left(\frac{Nm_t^K}{No_t^K}\right)\bar{L}_{t-1}\,xo_{t-1} - depm_t$, is stationary. While one might expect this expression to drift upward over time, it has in fact been negative in recent years, so I test for a unit root using an augmented Dickey-Fuller test assuming a constant and no drift. Using one-sided p values, the probability of a unit root is 0.5 percent for software, 1.6 percent for computers and peripherals, 3.6 percent for communications equipment, 16.0 percent for other equipment, and 38.7 percent for structures. Table 3 shows the results using OLS. Corrections for autocorrelation are discussed below.

The top panel of Table 3 shows the Gm_i used in equation 25 to calculate the quality of new machines (km). The remainder of the table shows the results from estimating equation 24 for the number of new machines (Nm) using least squares. Consistent with the theory, those equations are estimated without a constant.[25] Because of their volatility, the variables $\bar{L}_t$ and $Y_t/\bar{y}_t$ are replaced by their two-quarter moving averages.

The results generally confirm two implications of the theory. First, the lag length for the number of new machines exceeds the lag length for the quality of new machines. Second, in general, the longer the time to build, the greater the weight on labor hours at full employment ($\sum_i \beta m_i$) and the less the weight on actual labor hours ($\sum_i \gamma m_i$). Thus, hours at full employment receive more than 70 percent of estimated $\sum_i \beta m_i + \gamma m_i$ for investment in structures, but have no role in the equation for computers. Software is an anomaly, with a long time to build but with a zero weight on labor hours at full employment. That equation has a low R^2, however, warranting some scepticism.

Based on the discussion above, we expect $\sum_i \beta m_i + \gamma m_i$ to be slightly greater than 1. The estimated coefficients for communications equipment, other equipment, and structures are consistent with that expectation. However, the sum of coefficients appears too large for computers and software.

Taken together, the equations fit the data well (see Figure 3).[26] The model generally

[25] Estimates of standard errors in the table are calculated assuming that xo_t and the Gm_i are given. Because they are instead estimated, the standard errors in the table may be biased.

[26] In Figure 3, the fitted value of nominal investment for each type of capital equals the fitted value for Nm times the actual value for km times the actual price index (qm). Those fitted values for nominal investment are then summed. For the categories of investment created by aggregating

Table 3: Least Squares Results for Investment Equations

	Computers	Software	Telecom. Equipment	Other Equipment	Structures
km (Eq. 25)					
Lags (i)	*0*	*0 to 3*	*0 to 3*	*0 to 8*	*1 to 4*
Sum of Gm_i	1	1	1	1	0.7
Lags (i)					*5 to 8*
Sum of Gm_i					0.3
Nm (Eq. 24)					
Lags (i)			*1 to 5*	*0 to 9*	*0 to 7*
Sum of βm_i			0.276	0.611	0.426
(t-statistic)			(5.3)	(25.6)	(19.5)
Lags (i)			*6 to 10*		*8 to 14*
Sum of βm_i			0.285		0.342
(t-statistic)			(5.5)		(17.2)
$\sum_i \beta m_i$	0	0	0.561	0.611	0.767
Lags (i)	*0 to 1*	*2 to 6*	*2 to 7*	*0 to 6*	*0 to 1*
Sum of γm_i	0.159	0.388	0.278	0.260	0.015
(t-statistic)	(2.6)	(7.3)	(12.6)	(20.5)	(3.1)
Lags (i)	*2 to 5*	*7 to 13*	*7 to 14*	*7 to 14*	*2 to 8*
Sum of γm_i	0.734	0.398	0.199	0.180	0.148
(t-statistic)	(9.6)	(6.6)	(7.8)	(12.6)	(18.1)
Lags (i)	*6 to 9*	*14 to 18*			*9 to 16*
Sum of γm_i	0.280	0.353			0.112
(t-statistic)	(4.4)	(7.2)			(12.6)
$\sum_i \gamma m_i$	1.173	1.139	0.477	0.440	0.276
$\sum_i \beta m_i + \gamma m_i$	1.173	1.139	1.038	1.051	1.043
R^2	0.646	0.612	0.814	0.911	0.959
D.W.	0.65	0.18	0.42	0.41	0.29
Sample	1965:q1-2006:q3	1965:q1-2006:q3	1957:q1-2006:q3	1957:q1-2006:q3	1957:q1-2006:q3

captures the timing and magnitude of changes in investment. One notable exception is a growing overestimate after 2004. In the model, the shortfall of investment after 2004 is the counterpart to the decline in raw xo during that period shown in Figure 1.

When the equations for other equipment and structures are freely estimated in first differences, the estimated $\sum_i \beta m_i + \gamma m_i$ are an unrealistically low 0.48 and 0.59, respectively. Hence, I constrain those sums to equal the sums of coefficients obtained using OLS. The resulting estimates of γm_i and βm_i are each within 0.04 of the OLS estimates for other equipment and within 0.02 of the OLS estimates for structures. When the equations are estimated with an AR1 correction, the estimated βm_i and γm_i are not significantly different than the OLS estimates for computers and communications equipment. However, estimated $\sum_i \beta m_i + \gamma m_i$ is less than 0.78 for the other three categories of investment. When I constrain $\sum_i \beta m_i + \gamma m_i$ for those categories to be the same as for OLS, most individual coefficients are not significantly different from the OLS estimates.

4.2 Effect of Holding Machines per Labor Hour Constant

Estimating the model while holding xo constant at its sample average worsens the model's performance. Estimates of $\sum_i \beta m_i + \gamma m_i$ are well above 1 for each category of investment, ranging from 1.114 for communications equipment to 1.259 for computers. In addition, the model does not fit the data as well (see Figure 4). Fitted values are well below actual investment during periods when xo is estimated to have risen—the late 1950s and 1980-1986—and are even further above actual investment during the period after 2002, when xo is estimated to have been falling. Even so, the model still picks up turning points fairly well.

4.3 Tests of Key Implications of the Neoclassical Model

The most characteristic difference between the putty-clay model and the standard neoclassical model is the treatment of $\Delta(p/vm)$, the change in the reciprocal of the real cost of capital. That term has no effect on investment in the putty-clay model, but a large impact on investment in the neoclassical model. We can test that implication of the neoclassical model by adding

$$\frac{Nm_t^K}{No_t^K} \sum_i \varphi_i \, \bar{L}_t \, xo_t \frac{\Delta(p_{t-i}/vm_{t-i})}{p_{t-i}/vm_{t-i}}$$

to equation 24. That is the additional term one gets after dividing a variant of equation 26 containing Nm^K/No^K and xo by equation 25. According to the neoclassical

finer detail—other equipment and structures—I use an aggregate km. That variable is created by inserting an aggregate vm into equation 25 and then multiplying the resulting km^* by the coefficient obtained from a regression of Im/Nm on km^*.

model, $\sum_i \varphi_i$ should be roughly 1. According to the putty-clay model, it should be zero.

The results are generally inconsistent with the neoclassical model. For computers and software, $\sum_i \varphi_i$ is negative no matter what specification is used. For communications equipment and other equipment, $\sum_i \varphi_i$ is small and statistically insignificant. For structures, $\sum_i \varphi_i$ is statistically significant, but quite small (0.019 for an eight-quarter moving average). That structures fall so far from the neoclassical model is perhaps surprising. Although the labor requirement of many types of equipment, for example motor vehicles, is obviously fixed during their service life, the labor requirement of a building can be changed through remodeling. However, the very small estimate of $\sum_i \varphi_i$ indicates that such remodelings in response to changes in the real cost of capital play little role in explaining investment in structures.

The neoclassical model also implies that the accelerator term should depend on changes in output alone rather than on changes in the ratio of output to labor productivity. We can test that implication by adding

$$\frac{Nm_t^K}{No_t^K} \sum_i \left\{ \beta m_i' \frac{\Delta\left[\bar{L}_{t-i}\, \bar{y}_{t-i}\, xo_t\right]}{\bar{y}_{t-i}} + \gamma m_i' \frac{\Delta\left[Y_{t-i}\, xo_t\right]}{\bar{y}_{t-i}} \right\}$$

to equation 24. If the neoclassical hypothesis about the form of the accelerator is correct, the quantity $\sum_i \beta m_i' + \gamma m_i'$ should roughly equal 1 and $\sum_i \beta m_i + \gamma m_i$ should equal zero. Instead, the estimated $\sum_i \beta m_i' + \gamma m_i'$ are negative for each type of capital. Consequently, we can reject the neoclassical hypothesis that the accelerator depends on changes in output and not on changes in the ratio of output to labor productivity.

4.4 Tests of a Basic q Model of Investment with Cash Flow

Fazzari, Hubbard, and Petersen (1988) found that cash flow is an important determinant of investment, especially for firms that face financing constraints.[27] They proposed adding cash flow to a model of investment based on Tobin's q:

$$I_t/K_t = \mu + \sum_i g_i\, Q_{t-i} + \sum_i h_i \left(CF_{t-i}/K_{t-i}\right), \tag{34}$$

where I is nonresidential fixed investment, K is the capital stock, Q is Tobin's q, and CF is cash flow gross of dividends. K is measured as tangible assets at current cost. Q is defined as $(V + B - MI)/K$, where V is the market value of equity, B is the market value of debt, and MI is the market value of inventories.

I estimated equation 34 using aggregate variables, with I defined to exclude capital specific to mining and farming.[28] Consistent with q theory, $\sum_i g_i$ is positive. In

[27] Although Kaplan and Zingales (1997) argue that cash flow is not a good measure of financing constraints, they still find it highly significant in determining investment.

[28] I estimate equation 34 using a lagged eight-quarter third-degree polynomial distributed lag for both Tobin's q and cash flow.

addition, $\sum_i h_i$ equals 0.69, meaning that each dollar of additional cash flow increases investment by 69 cents. One might expect some of the effects of cash flow to spill over to investment in mining capital and inventories, but those variables have an incorrect positive sign when added to the right-hand side of equation 34.

To account for the putty-clay model, I added I_t^{pc}/K_t to equation 34, where I_t^{pc} is the fitted value of nominal investment from the putty-clay model. (To be consistent, I, K, and CF are all nominal.) After that variable is added, $\sum_i g_i$ becomes negative so Q is removed from the regressions. With or without Q, cash flow fades in importance once fitted values for investment from the putty-clay model are added. In every specification, the estimated sum of coefficients on cash flow, $\sum_i h_i$, is less than 0.025. F-tests indicate that cash flow is significant at the 95 percent confidence level, but that effect is perverse: rising cash flow significantly reduces the level of investment. Thus, this simple version of the cash flow model adds little explanatory power to the putty-clay model of investment, and Q has the wrong sign. If a firm does not need more capacity, it does not build more capacity, no matter what cash flow or the market value of existing capacity is.

There are a few caveats to those results. First, recent papers examining the q model with cash flow have used firm-level data, not the aggregate data used in this section. Second, the fitted values for investment in the putty-clay model take x as exogenous. To the extent that cash flow may help to explain x, or to the extent that I have not incorporated a variable like x in the cash flow model, cash flow may still have a role in explaining investment. Finally, although I do not develop it, the model in this paper implies that changes in market share (fj) have important effects on which firms invest. (For an individual firm, a 1 percent increase in market share has the same effect as a 1 percent rise in aggregate demand.) Cash flow may thus be significant in investment equations using firm-level data as a proxy for changes in a firm's market share.

5 Estimates of Capital per Labor Hour and the Capital Stock

The ratio of capital to labor plays an important role in determining labor productivity in Solow's (1957) well-known framework for growth accounting. Traditional measures of the ratio of capital to labor hours based on homogeneous (putty-putty) capital cannot distinguish between capital deepening—an increase in the amount of capital used by a given worker that adds to labor productivity—and capital widening—an increase in the capital stock to accommodate an increased number of workers. In contrast, the index of capital per hour in the putty-clay model excludes capital widening, and thus can be used to isolate the true impact of investment on labor productivity.

The failure of traditional measures of the ratio of capital to labor to exclude changes unrelated to capital deepening leads to counterintuitive movements in that ratio. Consider the response to a positive shock to demand. In the very short run, labor hours rise, albeit less than proportionately to output because of procyclical productivity, while capital is nearly unaffected. Thus, the shock raises labor productivity (output per hour) but reduces standard measures of the ratio of capital to labor. After a year or two, labor hours have adjusted to the higher level of output for the most part, eliminating the procyclical gain in productivity. The capital stock is also larger than it would be without the shock, but it has not yet risen in proportion to the increase in output or hours, so the capital-output ratio remains lower than what it would have been in the absence of the shock to demand. Thus, a positive shock to demand reduces the growth rate of standard measures of the ratio of capital to labor. At the same time, dividing the capital stock by hours at full employment overestimates capital deepening because the shock boosts investment but not hours at full employment.

Those patterns are visible in the data on labor productivity and capital per labor hour published by the BLS for the nonfarm business sector (see Figure 5).[29] For 1949-2005, there is actually a slight negative correlation between the growth of labor productivity and the growth of what theory says should be one of its primary determinants, capital per labor hour. While growth of labor productivity slows during recessions, capital per labor hour grows at its most rapid pace during and immediately after recessions.

The putty-clay index of capital per labor hour instead isolates capital deepening. The primary component of that index, $\bar{k}$, depends only on the growth of the quality of machines. The other component, x, is constructed to focus on trends in the number of machines per labor hour at full employment. Consequently, growth in the index of capital per labor hour does not show any obvious cyclical movements (see Figure 6).[30] The strongest growth of capital per labor hour occurs during the late 1990s in response to rapid growth of TFP and a low cost of corporate funds due to the boom in stock prices.

The putty-clay model can be used to construct a capital stock consistent with labor hours at full employment. To convert the putty-clay index of capital per labor hour into a capital stock, I multiply by a two-year moving average of labor hours at full employment. For the most part, that measure of the capital eliminates the slowdown in growth after every recession in the BLS measure of the capital stock (see Figure 7). The more rapid growth of the putty-clay capital stock than of the putty-putty capital

[29] At present, BLS has published data for the nonfarm business sector on a NAICS basis only for 1987-2004. To this series, I splice data for the business sector in 2005 and data for the nonfarm business sector on an SIC basis for 1948-1986. BLS estimates include the output, capital, and labor hours of nonfarm tenant housing, while the putty-clay estimates do not. The impact of those differences in coverage on growth rates is small.

[30] As mentioned above, the figure excludes data from before 1957 because of the uncertainty of how to measure x during a period when businesses were still rebuilding their capital stocks after constraints on investment during World War II.

stock during 2002-2003 makes it somewhat less difficult to explain the unusually strong growth of labor productivity during those years.

6 Capital Income and Investment

Weak to moderate business investment in the presence of strong corporate profits since 2003 has puzzled many analysts. According to the q model of investment with cash flow, unusually high levels of profits and cash flow should translate into unusually strong business investment, limiting the rise in business saving. Private-sector economists and other researchers (e.g., Cardarelli and Ueda (2006), Hatzius et al. (2006), and Loeys et al. (2005)) have suggested that business investment instead remains moderate relative to GDP due to excessive investment in the late 1990s. However, Desai and Goolsbee (2004) argue that capital overhang was not a factor in the collapse of investment in the early 2000s.

An alternative view, based on the model in this paper, is that investment is driven primarily by the need to expand the stock of productive resources, not by the profitability of resources already in use. In that view, any factor that reduces the need to increase the number of machines, such as a slowdown in the growth of the labor force at full employment, will reduce investment relative to capital income.

6.1 Capital Income

Variations in x can affect capital's share of income. Modifying equation 14 for type-m capital and multiplying through by $Nm_t^K vm_t$ yields

$$\begin{aligned} Nm_t^K\, vm_t\, km_t &= \alpha\left(1+\frac{1}{\eta}\right) p_t\, y_t\, Nm_t^K \\ &= \alpha\left(1+\frac{1}{\eta}\right) p_t \frac{Y_t}{L_t} Nm_t^K. \end{aligned}$$

In the putty-putty model, the left-hand side of the equation exactly equals the income earned by type-m capital. In the putty-clay model, that result is only approximate, but it is empirically useful. Summing over all types of capital and replacing $\sum_m Nm_t^K/L_t$ with x_t, we have

$$\text{income of tangible capital} \approx \alpha\left(1+\frac{1}{\eta}\right) x_t\, p_t Y_t.$$

If total income is p_tY_t and income from intangible capital is $-\frac{1}{\eta}p_tY_t$, then

$$\text{labor income} \approx \left(1+\frac{1}{\eta}\right)[1-\alpha x_t]\; p_t\, Y_t.$$

Dividing through this equation by $p_t Y_t$ produces

$$\frac{w_t}{p_t y_t} \approx 1 + \frac{1}{\eta} - \alpha \left(1 + \frac{1}{\eta}\right) x_t, \tag{35}$$

where w is (labor income) $/L$, or compensation per hour. The left-hand side of the equation is labor's share of income.

Because equation 35 is only approximate and because we do not know $1/\eta$, the equation must be estimated. If firms set prices using $\bar{y}$ rather than y, the left-hand side of equation 35 should be $w/(p\bar{y})$. (The ratio $y/\bar{y}$ is then added as an independent variable.) If real compensation per hour is slow to react to changes in productivity, w_t will also depend on current and lagged y_t/y_{t-1}. In addition, because a large share of mining income goes to capital, high energy prices reduce real compensation per hour. I find that mining investment as a share of nominal output is a better proxy for the impact of energy prices on w than are energy prices, having a strongly significant negative coefficient in the estimated equation.

The fitted value for capital's share of income equals 1 minus the fitted value of equation 35.[31] The equation captures most of the important movements in capital's share of income but underpredicts in the early 1970s, possibly due to price controls, and overpredicts in the mid 1990s and the early 2000s (see Figure 8).

6.2 Capital Income Minus Investment

Subtracting the model's prediction for investment from its prediction for capital income (the nonlabor income of private nonfarm business plus nonprofits less tenant housing) yields the model's prediction for capital income minus investment (see Figure 9).[32] I focus on capital income less investment rather than on retained earnings less net investment, a more common measure of the financial surplus of business, because the putty-clay model implies nothing about profits' share of total capital income or about the share of profits paid out as dividends.

The model explains high levels of capital income less investment in the 1960s, low levels in the late 1970s and early 1980s, and the sharp rise in the level during the early 2000s. Most of that rise during the early 2000s stemmed from a sharp slowdown in the growth of labor supply, measured as labor hours at full employment ($\bar{L}$). Figure 10 shows the impact on capital income less investment of letting $\bar{L}$ grow at its average

[31] Labor income is measured as compensation of employees less that of general government, government enterprises, farming, and housing, plus 65 percent of nonfarm proprietors' income. That sum is multiplied by the ratio of GDP to gross domestic income in order to make income consistent with output used to estimate $\bar{y}$. The independent variables are a constant, $y_t/\bar{y}_t$, nominal mining investment divided by $p_t \bar{y}_t L_t$, and a 12-quarter fourth degree PDL of y_t/y_{t-1}.

[32] Values for mining investment are actuals rather than fitted values from a model. However, since mining investment is also the proxy for the effect of high energy prices on capital income, the net effect of mining investment on capital income less investment is not large.

growth rate over 1954-2006.[33] Rapid growth of $\bar{L}$ added to investment in the late 1970s and early 1980s, reducing the financial surplus of business, while slow growth of $\bar{L}$ raised that surplus in the early 2000s. In fact, differing rates of growth of labor hours at full employment between 1978-1982 and 2003-2004 explain almost the entire 5.5 percentage point difference in capital income less investment as a share of output between those two periods.

Why has the uninvested portion of capital income remained so high since 2004 when the model indicates it should have declined? Part of the explanation lies in a widening gap between actual investment and the level of investment predicted by the model. More important, however, has been a sharp rise in capital's share of income. The putty-clay model can explain why that increase in income has not directly aided investment, but cannot explain the rise in capital income itself.

While it is difficult to explain the abrupt rise in capital's share of income between 2003 and 2006, the general rise in capital's share of income since the early 1990s may be due to an increasing importance of intangible capital, such as patents and firm-specific know-how. The model would capture this as a reduction in the absolute value of η. Intangible capital may have grown in importance with the increased usage of information technology. For example, in its fiscal 2006, Microsoft Corporation accounted for more than 0.6 percent of capital income (pretax profits plus depreciation) but less than 0.1 percent of services from tangible capital.[34] Other explanations for a rise in capital's share of income are also possible.

[33] The interval for calculating the growth rate begins in 1954 because lagged growth of $\bar{L}$ affects capital income less investment. I also change the growth rate of $Y/\bar{y}$ by the same percentage amount as the growth rate of $\bar{L}$.

[34] Microsoft's depreciation of tangible capital was 0.08 percent of total depreciation of nonfarm nonresidential capital.

Appendix A: Inventories

Although inventories have a very short service life and their cost is closely tied to the cost of funds, the real stock of inventories has a much larger short-term response to output than to the cost of funds, just like fixed capital. Inventories seem to have a putty-clay quality that lasts much longer than the service life of an inventory.

Even so, inventories are modelled somewhat differently than fixed capital. In contrast to a "machine" of fixed capital, the quality of which is fixed over its service life, an inventory "machine" varies in quality with labor productivity at full employment ($\bar{y}$) during its service life. Conceptually, a warehouse and the shelves in the warehouse are of a fixed size during their service life, but the value of each item on those shelves rises with labor productivity. Thus, an inventory "machine," or unit of inventory technology, consists of a certain amount of inventories held per real dollar of output per labor hour, denoted $k@y$, rather than an amount of inventories per labor hour. The amount of inventories corresponding to that "machine" is $k@y \cdot \bar{y}$.

If the cost of holding inventories is the after-tax real cost of funds times the value of inventories, then the PDV of the portion of current and future profits that depends on the type of inventory technology adopted at time t ($k@y_t$) is

$$\begin{aligned} PDV(\pi)_t &= \int_{i=0}^{Sm} (1-u_{t+i})\, p_{t+i}\, Y_{t+i} e^{-ri} di \\ &\quad - \int_{i=0}^{Sm} (1-u_{t+i})\, Nm_t\, qm_{t+i}\, k@y_t\, \bar{y}_{t+i}\, r_{t+i}\, e^{-ri} di, \end{aligned}$$

where Sm is the expected service life of new inventory technology, Nm_t is the number of new units of inventory technology adopted at time t, and qm_{t+i} is the price index for inventories at time $t+i$. We can solve for $k@y_t$ by: maximizing $PDV(\pi)_t$ over $k@y_t$; making the same assumptions about u, Y, and r as for fixed capital; assuming that the expected rate of growth of qm is the same as that for p; substituting a constant r for r_{t+i}; and solving the integrals. After rearranging terms,

$$k@y_t = \frac{\alpha\, p_t}{r\ qm_t}.$$

Inventory technology is proportional to the ratio of the price of output to the price of inventories and inversely proportional to the real cost of funds expected over the life of the technology.

Appendix B: The Cost of Capital with Taxes

With taxes, the present discounted value of future after-tax profits for the jth firm is

$$
\begin{aligned}
PDV(\pi j)_t = & \int_{i=0}^{\infty} (1-u_{t+i})\, pj_{t+i} Yj_{t+i} e^{-ri} di - \int_{i=0}^{\infty} (1-u_{t+i})\, \bar{w}_{t+i} Lj_{t+i} e^{-ri} di \\
& - \sum_{i=0}^{\infty} q_{t+i} kj_{t+i} Nj_{t+i} \left(1 - C_{t+i} - Z^u_{t+i}\right) e^{-ri} \\
& - \sum_{i=0}^{\infty} (1-u_{t+i})\, propj_{t+i} e^{-ri} di \\
& - \int_{i=0}^{\infty} \frac{1}{2} (1-u_{t+i})\, \xi_{t+i} \bar{w}_{t+i} Lj_{t+i} \left(1 - \frac{Nj^K_{t+i}}{x_{t+i} Lj_{t+i}} \psi\right)^2 e^{-ri} di, \qquad (36)
\end{aligned}
$$

where u is the tax rate on corporate income, C is the investment tax credit for new capital, and *prop* is the PDV of property taxes to be levied on new structures.[35] The price of output pj excludes all taxes on production and imports except property taxes, since those other indirect taxes are not part of the after-tax revenue of the firm and do not affect the relative costs of capital and labor. Z^u_t, the PDV of the tax value of depreciation allowances per dollar of capital installed at time t, is given by

$$
Z^u_t \equiv \int_{i=0}^{S_t} u_{t+i}\, D_{t,i} \left(1 - B_t\, C_t\right) e^{-ri} di,
$$

where $D_{t,i}$ is the share of depreciation allowances taken i years after time of purchase t, and B_t is the share of the investment tax credit that is deducted from the allowable base for depreciation. (The service lives of some types of capital have changed over time, so the service life of capital installed at time t is denoted S_t.)

The PDV of property taxes levied on new structures installed at time t, $propj_t$, is given by

$$
propj_t = \int_{i=0}^{S_t} ux_{t+i} Nj_t\, q_t\, kj_t \left(1 - \frac{i}{S_t}\right) e^{(\dot{p}+\dot{y})\,i} e^{-ri} di,
$$

where ux is the property tax rate. The expression assumes that the property tax base of a structure declines linearly over its service life subject to a rising trend as nominal output per labor hour rises. Property taxes are deductible from corporate income taxes, and so are multiplied by $1 - u_{t+i}$ in the equation for after-tax profits.

[35] The equations in this appendix assume a single type of capital. If there are many types of capital, the formula for the cost of capital remains the same except that the variables C, Z^u, D, and S are specific to the type of capital.

The treatment of property taxes in this paper differs from that in the literature. Empirical studies of investment—e.g., Hall and Jorgenson (1967), Bischoff (1971), and Tevlin and Whelan (2003)—generally ignore taxes on production and imports altogether. On the other hand, in U.S. Department of Labor (1983), the Bureau of Labor Statistics includes all indirect taxes in constructing the cost of capital.

For structures, the first-order conditions for kj and Nj become

$$\begin{aligned}
&\alpha\left(1+\frac{1}{\eta}\right)\int_{i=0}^{S_t}(1-u_{t+i})\,pj_{t+i}\frac{Yj_{t+i}}{Nj^K_{t+i}}\frac{1}{kj_t}x_{t+i}e^{-ri}di \\
&= q_t\left(1-C_t-Z^u_t\right) \\
&+\int_{i=0}^{S_t}(1-u_{t+i})\,ux_{t+i}q_t\left(1-\frac{i}{S_t}\right)e^{(\dot{p}+\dot{y})\,i}e^{-ri}di
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
&\int_{i=0}^{S_t}(1-u_{t+i})\,\xi_{t+i}\frac{\bar{w}_{t+i}}{x_{t+i}}\left(1-\frac{Nj^K_{t+i}}{x_{t+i}Lj_{t+i}}\psi\right)\psi e^{-ri}di \\
&= q_t kj_t\left(1-C_t-Z^u_t\right) \\
&+\int_{i=0}^{S_t}(1-u_{t+i})\,ux_{t+i}q_t kj_t\left(1-\frac{i}{S_t}\right)e^{(\dot{p}+\dot{y})\,i}e^{-ri}di.
\end{aligned} \tag{38}$$

Combining equations 37 and 38 gives the same result as combining equations 9 and 10, because the added tax terms cancel out, so it is still true that $N^K_t = x_t L_t$.

To obtain the cost of capital, we solve equation 37 for kj_t (equal to k_t) using the same assumptions as above. (The equation for machine quality is unchanged.) The cost of capital is

$$v_t = q_t\frac{{}_tr-{}_t\dot{p}-\dot{y}}{1-e^{-({}_tr-{}_t\dot{p}-\dot{y})S_t}}\frac{1-C_t-u_tZ_t}{1-u_t}$$

for equipment and software (on which property taxes are not assessed), where

$$Z_t \equiv \int_{i=0}^{S_t} D_{t,i}\left(1-B_t\,C_t\right)e^{-\,{}_tri}\;di,$$

and for structures is

$$\begin{aligned}
v_t &= q_t\frac{{}_tr-\,{}_t\dot{p}-\dot{y}}{1-e^{-({}_tr-\,{}_t\dot{p}-\dot{y})S_t}}\frac{1-C_t-u_tZ_t}{1-u_t} \\
&\quad+q_t\,ux_t\left(\frac{1}{1-e^{-({}_tr-\,{}_t\dot{p}-\dot{y})S_t}}-\frac{1}{\left({}_tr-\,{}_t\dot{p}-\dot{y}\right)S_t}\right).
\end{aligned}$$

For land, the present discounted value of profits includes the discounted value of land when it is sold. (If the firm reuses the land, that notional sales value cancels out the future purchase price of land.) Assuming that the sales price of land is expected to grow at the rate ${}_t\dot{p}+\dot{y}$, equation 36 has an added term:

$$+\sum_{i=0}^{\infty}(1-u_{t+i})\,q_{t+i}kj_{t+i}Nj_{t+i}e^{-(r-\dot{p}-\dot{y})S_t}e^{-ri}.$$

In addition, the base for property taxes does not decline during the service life of land, so

$$propj_t=\int_{i=0}^{S_t}ux_{t+i}Nj_t\,q_t\,kj_t\,e^{(\dot{p}+\dot{y})\,i}e^{-ri}di.$$

Solving the first-order conditions for maximization of profits with respect to kj_t, we find

$$\begin{aligned} v_t \;&=\; q_t\frac{{}_tr-{}_t\dot{p}-\dot{y}}{1-e^{-({}_tr-{}_t\dot{p}-\dot{y})S_t}}\frac{1-C_t-u_tZ_t}{1-u_t}+q_t\,ux_t \\ &\quad -q_t\,e^{-({}_tr-{}_t\dot{p}-\dot{y})S_t}\frac{{}_tr-{}_t\dot{p}-\dot{y}}{1-e^{-({}_tr-{}_t\dot{p}-\dot{y})S_t}}. \end{aligned}$$

The final right-hand-side term comes from the present discounted value of the sales price of the land.

Appendix C: Data

Price indexes and real and nominal quantities for 49 detailed categories of private nonresidential fixed investment are taken from the National Income and Product Accounts of the Bureau of Economic Analysis (BEA) for 1929 on, and from BEA's fixed asset tables for earlier years. (The latter data are necessary for calculating the number of machines depreciating during the sample period.) For years before 1901, necessary for calculating retirements of certain long-lived assets, I use 1901 stocks and assume a steady-state growth path before 1901 using growth rates from Gordon (1999). Price indexes and real investment for the two aggregate investment categories are obtained using the formula for chain weighting.

Nominal output is gross value added for the nonfarm business and nonprofit institution sectors less gross value added of nonfarm tenant housing, output of government enterprises, and nonproperty taxes on production and imports, all from BEA. Nonfarm tenant housing and government enterprises are excluded because their investment is not part of nonresidential fixed investment. Taxes on production and imports other than property taxes are not part of the after-tax revenue of the firm and do not affect the relative costs of capital and labor. Property taxes are excluded from the equation used to chain real output, meaning firms treat higher property taxes as a reduction in the net price they receive for output.

Table 4: Kalman Filter Estimate of Labor Hours at Full Employment

Parameter	Coefficient	z-Statistic
c_1	-31.0	-19.5
c_2	-11.6	-7.0
c_3	11.6	7.0
c_4	0.0086	2.8
c_5	0.48	2.4
The sample is quarterly, from 1947:q2 to 2006:q1.		

The sectoral coverage of labor hours is the same as that for output. Labor hours for nonfarm business and nonprofit institutions come from the Bureau of Labor Statistics. Hours for government enterprises are from BEA. I use compensation and proprietors' income in housing sector output to calculate labor hours for nonfarm tenant housing.

Labor hours at full employment, an unobservable variable, are estimated using a Kalman filter combining two equations. First, the difference between the unemployment rate (ru) and the unemployment rate at full employment ($\bar{r}u$) depends negatively on the current and lagged ratios of actual labor hours to $\bar{L}$ (increases in labor demand) and positively on growth in $\bar{L}$ (increases in labor supply):

$$ru_t - \bar{r}u_t = c_1 \ln\left(\frac{L_t}{\bar{L}_t}\right) + c_2 \ln\left(\frac{L_{t-1}}{\bar{L}_{t-1}}\right) + c_3 \ln\left(\frac{\bar{L}_t}{\bar{L}_{t-1}}\right), \tag{39}$$

where $c_1<0$, $c_2<0$, and $c_3>0$. Second, labor hours at full employment follow

$$\ln\left(\bar{L}_t\right) = SV_t + c_4\, t + c_5 \ln\left(NP_t\right), \tag{40}$$

where the state variable SV is a random walk, t is a time trend rising by 0.25 per quarter, and NP is the population aged 16 and older. Data for $\bar{r}u$ are from the Congressional Budget Office (CBO). Coefficients obtained in estimating the Kalman filter defined by equations 39 and 40 are shown in Table 4.

To estimate the unobservable variable $\bar{y}$, I start by assuming that

$$\frac{Y_t}{\bar{Y}_t} = \frac{L_t}{\bar{L}_t} U_t, \tag{41}$$

where U is cyclical intensity of effort and $\bar{Y} \equiv \bar{y}\bar{L}$. Intensity of effort depends on current and lagged values of $Y/\bar{Y}$:

$$U_t = \left(\frac{Y_t}{\bar{Y}_t}\right)^{b_1} \left(\frac{Y_{t-1}}{\bar{Y}_{t-1}}\right)^{b_2} \left(\frac{Y_{t-2}}{\bar{Y}_{t-2}}\right)^{b_3} \left(\frac{Y_{t-3}}{\bar{Y}_{t-3}}\right)^{1-b_1-b_2-b_3}, \tag{42}$$

where $b_1>0$, $b_2<0$, and $b_3<0$. Intensity of effort reverts to unity within a year after a shock to $Y/\bar{Y}$ absent additional shocks. Combining equations 41 and 42, substituting $\bar{y}\bar{L}$ for $\bar{Y}$, and rearranging, we can express $\ln\left(L_t/\bar{L}_t\right)$ as a function of current and lagged $\ln\left(Y_t/\left(\bar{L}_t\,\bar{y}_t\right)\right)$.

Table 5: Kalman Filter Estimate of Productivity at Full Employment

Parameter Estimated	Coefficient	z-Statistic
b_1	0.448	26.2
b_2	-0.230	-15.2
b_3	-0.127	-7.5
b_4	0.021	8.9
The sample is quarterly, from 1948:q1 to 2006:q2.		

A signal equation for the Kalman filter is created by substituting for $\bar{y}_t$ in that expression using

$$\ln(\bar{y}_t) = sv_t + b_4 t,$$

where the state variable sv_t follows a random walk. Estimated coefficients are shown in Table 5.

The cost of capital is calculated as if all investment is done by nonfinancial corporate business. The methods of depreciation (straight-line, accelerated, sum of digits, and expensing), tax lifetimes, and declining-balance parameters used to calculate the $Dm_{t,i}$ are taken from Gravelle (1994), as are the basis adjustment B and investment credit rates Cm. The methods of depreciation used also account for the temporary increases in the amount of expensing allowed under the Job Creation and Worker Assistance Act of 2002 (JCWAA) and the Jobs and Growth Tax Relief Reconciliation Act of 2003 (JGTRRA).

I assume that nonprofit institutions use expensing, which is equivalent to setting the corporate tax rate for those entities to zero. To implement that assumption, I treat 65 percent of medical equipment and instruments, all new religious, educational, and vocational structures, and 75 percent of new hospitals and special-care structures as purchased by nonprofits and thus expensed. For mining, depreciation depends on the value of depletion, which is based on the output of the mine independent of its tax lifetime. Consequently, I substitute the service life for the tax lifetime. Also, since the real output of a well declines over time, I assume accelerated depreciation with a 150 percent declining balance.

The numerator of the property tax rate is total state and local property taxes less taxes on production and imports for farms and housing, from BEA. The denominator, taken from the flow-of-funds accounts, is the value of nonresidential real estate held by nonfarm, noncorporate business plus the nonresidential share of real estate in nonfarm, nonfinancial corporate business.

Conceptually, the numerator of the nominal rate of return is payments by business to owners of debt and equity (including retained earnings) after taxes plus the revaluation of assets due to inflation, while the denominator is the market value of debt and equity.[36] The real rate of return is the nominal rate of return less the rate of growth of output prices. In a putty-clay world, assets are revalued according to the change in the value of what they produce, i.e., the rate of growth of output prices. Thus, the revaluation of assets due to inflation and the rate of growth of output prices are the same, and the numerator of the real rate of return is payments to debt and equity after taxes, including retained earnings.[37] Consequently, the real rate of return is a weighted average of the after-tax bond yield and the yield on equities.

The pretax bond yield is an average of the Moody's seasoned corporate bond yields on Aaa- and on Baa-rated corporate debt. Interest payments are deductible from corporate taxes, so the after-tax bond yield is the pretax yield times $1 - u_t$, where

$$u_t = uf_t + usl_t * (1 - uf_t),$$

uf_t is the federal statutory corporate income tax rate, and usl_t is the effective average state and local corporate tax rate, multiplied by $(1 - uf_t)$ because such taxes are deductible from federal corporate income taxes. Data for the federal statutory rate come from Gravelle (1994) and CBO. The state and local tax rate equals state and local corporate tax collections divided by corporate profits before tax for domestic industries. The same u_t is used in calculating the cost of capital v_t.

The after-tax yield on equity is calculated using the dividend-discount model. That yield equals the dividend yield for the nonfarm, nonfinancial business sector divided by the average historical ratio of dividends to after-tax profits in that sector from 1947 through 2005, 0.514. To adjust for a surge in earnings repatriated from abroad in 2005, which count as negative dividends, I splice nonfarm, nonfinancial dividends with net corporate dividends.

The weights on debt and equity are constructed from data in the flow-of-funds accounts of the Federal Reserve Board. Debt is the value of credit-market instruments of nonfarm, nonfinancial business, whereas equity is the market value of equities of nonfarm, nonfinancial business. I use two-quarter moving averages because the flow-of-funds data are for the end of the quarter.

I assume that businesses do not discount the entire future stream of revenues from a long-lived asset using only the current rate of return. Instead, the expected real rate of return for type-m capital (${}_trrm$) is a weighted average of the current rate of return (rr_t) and the historical average rate of return $\bar{r}r_t$, with the weights depending on the service life of type-m capital. For capital with service life of five years or less,

[36] The expression "after taxes" is necessary because interest payments are deductible from the tax base for corporate income.

[37] The equality of the real rate of return with expected payments to capital net of taxes is consistent with the Bureau of Labor Statistics' methodology for creating the cost of capital, as described in Appendix C of U.S. Department of Labor (1983). Similarly, Harper, Berndt, and Wood (1989) show that using an "internal own rate of return model" defined using expected payments to capital net of taxes "yields the same rental prices ... as would the nominal internal rate of return model provided average capital gains were employed." The putty-clay model satisfies that condition.

${}_t rrm = rr_t$. For longer-lived capital,

$$ {}_t rrm = rr_t + \left(1 - \frac{5}{Sm_t}\left[1 + \ln\left(\frac{Sm_t}{5}\right)\right]\right)\ (\bar{r}r_t - rr_t)\,. $$

The weight on rr_t is 0.85 when $Sm = 10$, 0.60 when $Sm = 20$, and 0.33 when $Sm = 50$. Given the sharp and apparently permanent drop in rr_t in the early 1950s, I assume $\bar{r}r_t$ is 7.35 percent (the average for 1921-1950) through 1950 and then falls linearly to 5.56 percent (the average for 1951-2005) by the end of 1954, where it remains from 1955 on.

This paper does not estimate an equation for inventory investment, but the number of units of inventory technology and their quality are necessary to calculate the index of capital per labor hour and the capital stock. I assume that the current inventory technology is a geometric average of technologies chosen over the previous 15 years.[38] Then $\bar{k}m$ for inventories is found by multiplying that expression by $\bar{y}$, so that

$$ \log\left(\bar{k}m_t\right) = \log\left(\bar{y}_t\right) + \sum_{i=0}^{59} \frac{\log\left(k@y_{t-i}\right)}{60}. $$

The expected real cost of funds used to determine $k@y_t$ is a weighted average of the current and average real cost of funds, $\frac{1}{3}rr_t + \frac{2}{3}\bar{r}r_t$.

From 1978 on, ${}_t\dot{p}$ equals the expected inflation rate on consumer purchases one year ahead from the University of Michigan's surveys of consumers less the average difference over that period between that rate and the growth rate of p. That expected inflation rate is regressed on current and lagged inflation and on $ru - \bar{r}u$. (Rational consumers expect inflation to accelerate when unemployment is low, giving $ru - \bar{r}u$ a negative coefficient.) That equation is used to calculate ${}_t\dot{p}$ before 1978. The variable ${}_t\dot{p}m$ equals a weighted average of ${}_t\dot{p}$ and the historical average growth of p using the same weights as for ${}_t rrm$. The relationship between ${}_t\dot{p}$ and unemployment seems different before 1952, so $ru - \bar{r}u$ has a coefficient of zero during that period.

Service lives for most assets are from the Bureau of Economic Analysis (2003). The service life for autos is taken from the U.S. Department of Labor (2001). The service life for computers is derived by adjusting the depreciation rate implied by BEA data for an assumed declining balance parameter.

The number of new machines, necessary for calculating retirements of machines, is estimated by using equation 25 to solve for km_t and then dividing real investment by that km_t. The lag structure in equation 25 is the same as that ultimately chosen for each category of capital in the investment equations.

[38] The service life for inventory technology is likely related to the service life for fixed capital. I choose 15 years as a round number that is somewhat larger than the service life of equipment but well below the service life of structures. Ideally, the timing of the introduction of new inventory technology should correspond to the timing of fixed investment, rather than be a simple moving average. However, given the uncertainty of the service life of inventory technology, further refinement will not necessarily lead to a more accurate answer.

Each structure "machine" includes both a structure and the land used with it. The putty-clay model requires knowledge of the value of land associated with each structure when it was put in place, data we do not have. I assume that λ, the ratio of the cost of a new structure to the cost of the accompanying land (abstracting from expectations, the ratio of $vm * km$ for structures to $vl * kl$ for land), is constant over time.[39] $Nm = Nl$ implies that $Il/kl = Im/km$, and so $Il = Im\lambda\frac{vm}{vl}$. Given historical data for Im, vm, and vl, we can calculate the implied value of nonresidential land in use for a hypothetical λ and compare it with estimates from BLS. The data support a value for λ of roughly 0.32.

To calculate costs of capital for the two aggregate investment categories, we have the choice of combining the individual vm or of using aggregated rm, Sm, etc. to solve for an aggregate v. To more easily isolate the impacts of changes in the real rate of return and tax law on investment, I choose the latter. I use contemporaneous weights (i.e., chain weighting) to calculate qm. For other variables, however, the use of contemporaneous weights can introduce large quarter-to-quarter movements, especially for the tax variables. Instead, each weight is calculated by inserting the H-P-filtered Nm^K/No^K and trend values for $\bar{L}$ and $Y/\bar{y}$ into equation 24, solving to obtain a raw weight, and dividing by the sum of the raw weights. For tax lives and service lives, weights are exponential rather than arithmetic.[40] The weights on $\bar{r}r_t$ and rr_t do not vary over time, but are set to the sample average. The desired number of inventory "machines" per labor hour, used in the calculation of the index of capital per hour, equals the filtered values of the ratio of the number of such machines to $\frac{1}{5}\sum_{i=0}^{4} L_{t-i/4}\, e^{(\dot{n}+\dot{y})/2}$ for quarterly data, where $\dot{n}$ and $\dot{y}$ are annual rates.

[39] I assume that the price index for land is the same as that for new structures. Given that assumption, the cost of capital for land is the same as the cost of capital for structures, with two exceptions. First, because land does not depreciate, firms expect the basis for property taxes to grow at a rate of ${}_t\dot{p}m + \dot{y}$. Second, the cost of land is reduced by the PDV of the after-tax sales value of land when the structure on it is retired.

[40] For tax lives, each weight is multiplied by one minus the share of investment expensed, since the tax life is irrelevant when there is expensing.

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Figure 1. Machines per Labor Hour, Excluding Mining and Inventories, 1929-2006

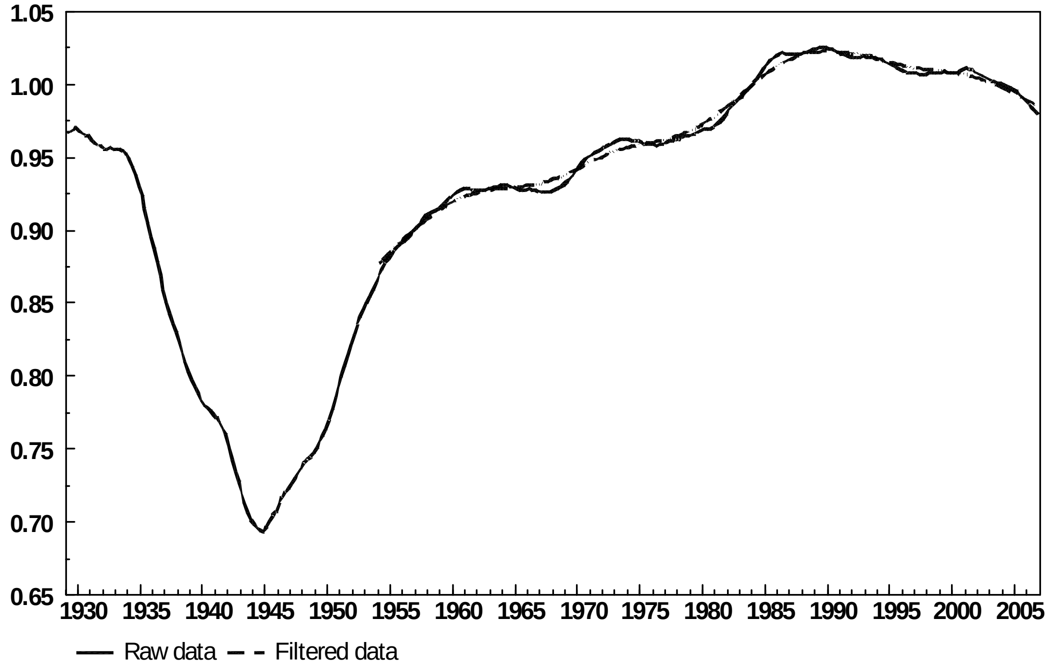


Figure 2. Machines per Labor Hour, Excluding Mining and Computers, 1960-2000

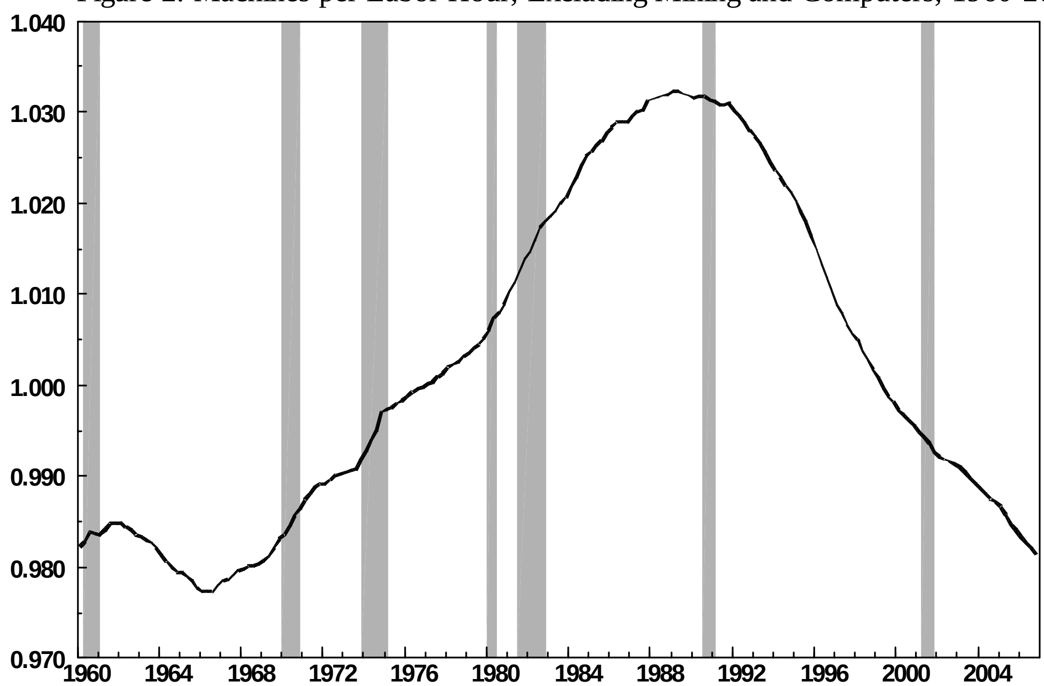
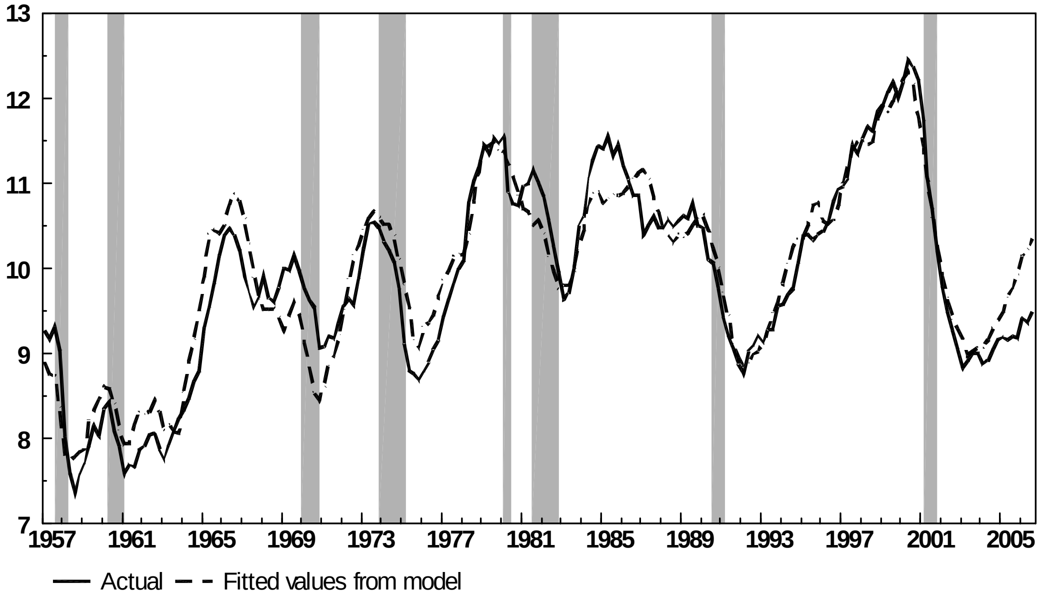
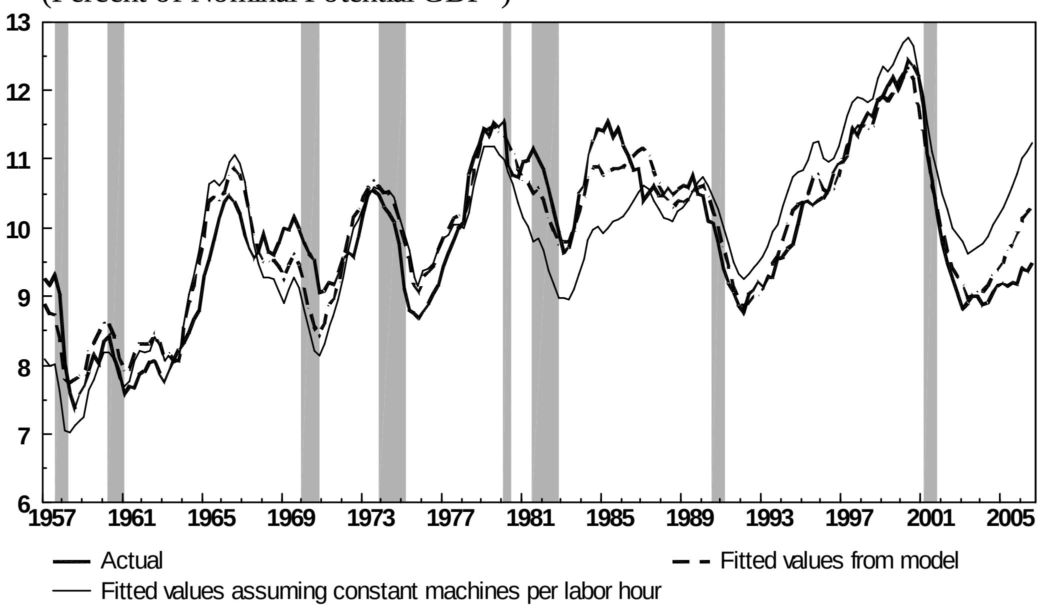


Figure 3. Business Fixed Investment, Excluding Mining and Farming, 1957-2006 (Percent of Nominal Potential GDP*)



* Potential GDP is taken from unpublished data used for Congressional Budget Office (2007).

Figure 4. Business Fixed Investment, Excluding Mining and Farming, 1957-2006 (Percent of Nominal Potential GDP*)



* Potential GDP is taken from unpublished data used for Congressional Budget Office (2007).

Figure 5. Putty-Putty Labor Productivity and Capital per Hour, 1949-2006
(Percent growth, nonfarm business)

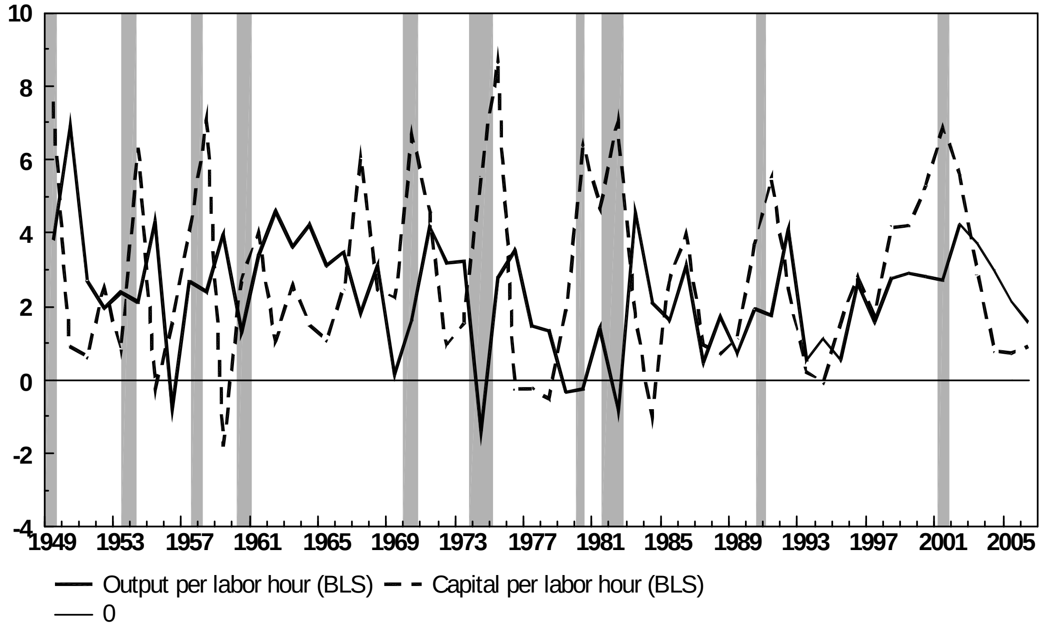


Figure 6. Two Measures of Growth of Capital per Labor Hour, 1957-2006
(Percent change)

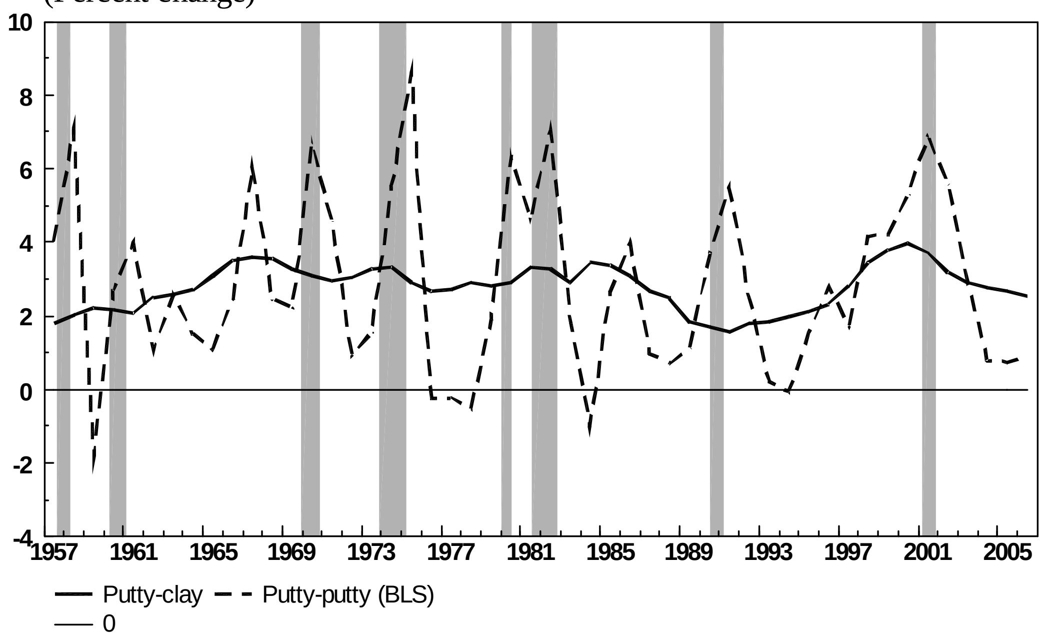


Figure 7. Two Measures of Growth of the Capital Stock
(Percent change)

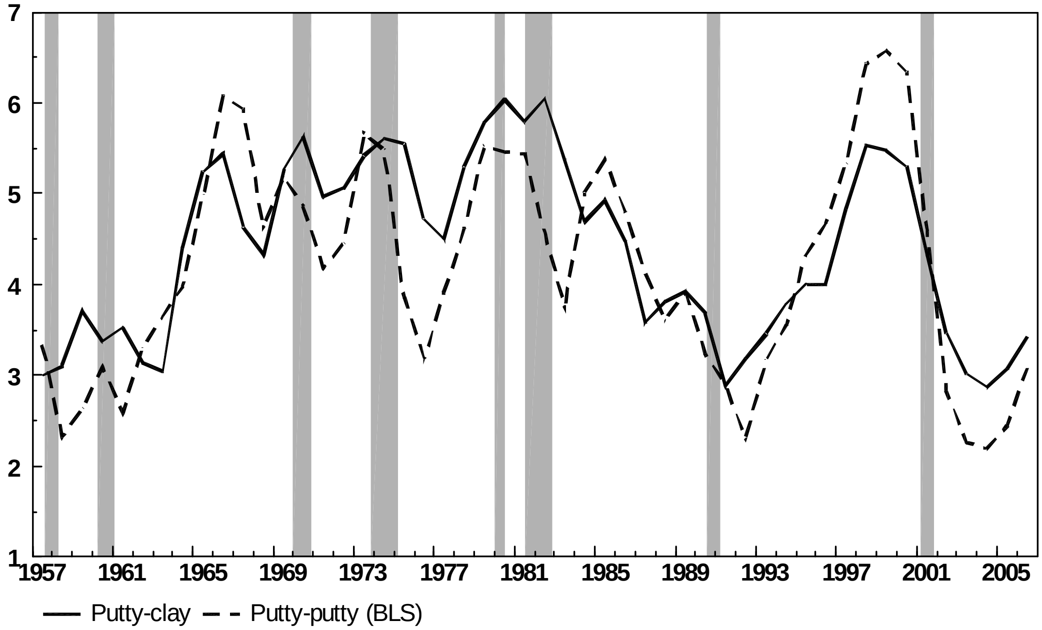


Figure 8. Capital's Share of Income
(Percent, private nonfarm business plus nonprofits less tenant housing)

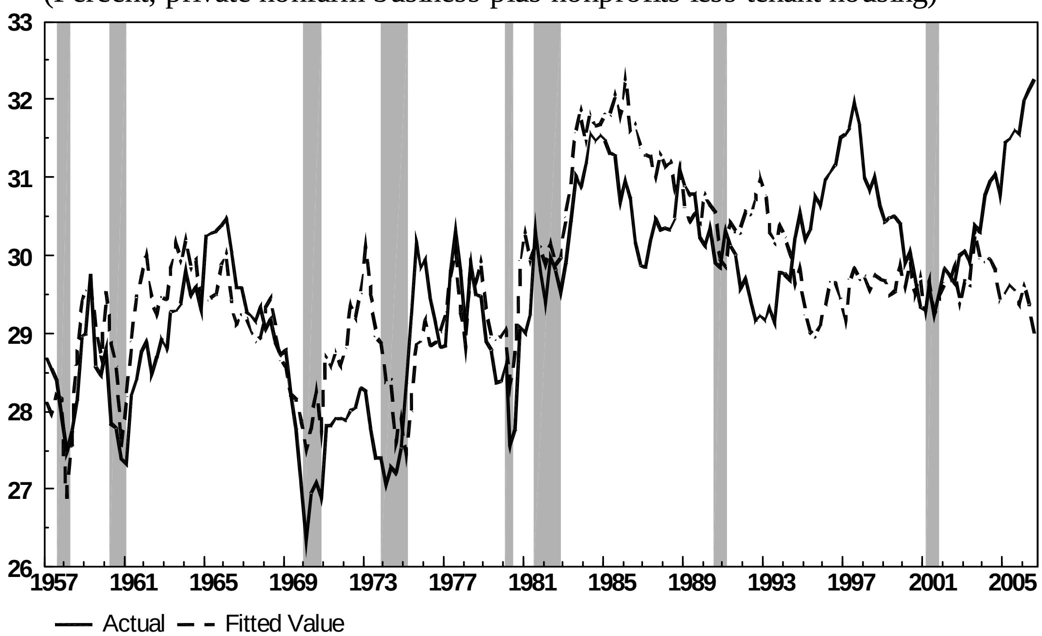


Figure 9. Capital Income Minus Investment
(Percent of private nonfarm business output plus nonprofits less tenant housing)

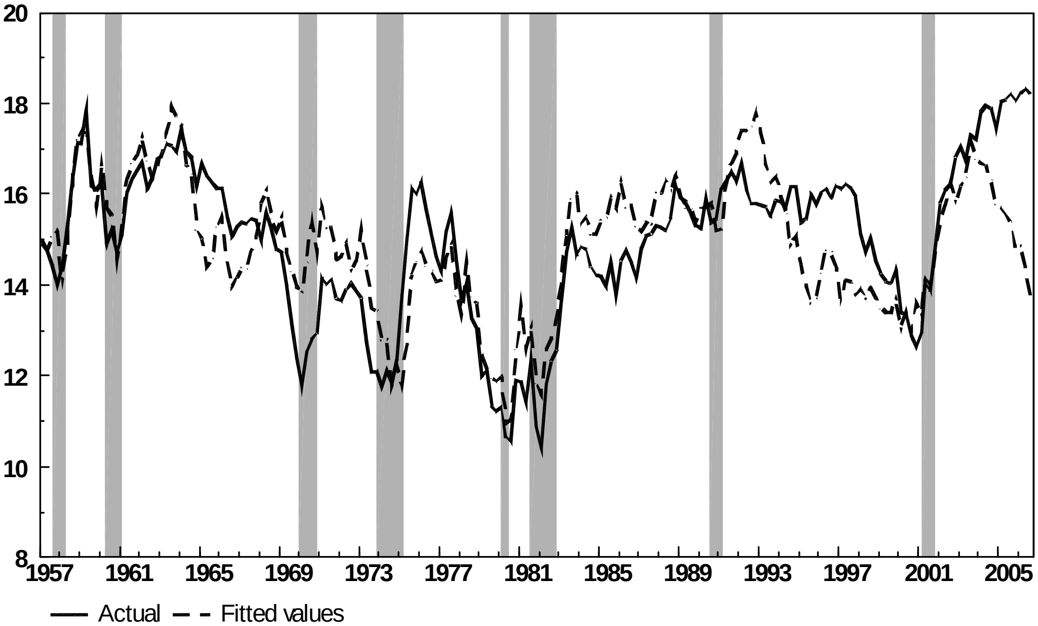


Figure 10. Effect of Labor Supply on Capital Income Minus Investment
(Percent of private nonfarm business output plus nonprofits less tenant housing)