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THE STUDENT LOAN CONSOLIDATION OPTION: AN ANALYSIS OF AN EXOTIC FINANCIAL DERIVATIVE

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Abstract

The federal government makes subsidized federal financing for higher education widely available. The extent of the subsidy varies over time with interest rate and credit market conditions. A loan provision that adds considerably to the size and volatility of the subsidy is the consolidation option, which allows students to convert floating-rate federal loans to a fixed rate equal to the average floating rate on their outstanding loans. We develop a model to estimate the option's cost and to evaluate its sensitivity to changes in program rules, economic conditions, and borrower behavior. We model borrower behavior using data from the National Student Loan Data System, which provides new insights on the responsiveness of consumers to financial incentives.

1. Introduction

The federal student loan program makes low-cost federal financing for higher education widely available. Because legislation rather than market forces determines federal student loan terms, the extent to which these loans are subsidized varies over time with interest rate and credit market conditions. Comparison of rates charged on private student loans with rates charged on federal loans suggests a typical differential of several percentage points annually. From an economic perspective, imperfections in private credit markets and legal limitations on selling human capital forward might justify subsidizing credit for education, although the effectiveness of such policies is controversial.[1] In any case, one would expect an efficient subsidy to either reduce credit rationing or lower the cost of educational investment for target populations. Although the federal student loan program has these aspects,[2] it also contains costly provisions that are hard to justify on efficiency grounds.

One apparently inefficient provision is the consolidation option, an exotic financial derivative created by a few paragraphs in the Higher Education Act. It allows students to convert their floating-rate loans to a fixed rate equal to the average floating rate on their outstanding loans. For some borrowers, it also allows maturity extension.

The consolidation option is of interest for several reasons. For one, it represents a multi-billion-dollar public expenditure that has gone largely unmeasured and therefore has received little scrutiny. In addition, the benefits are distributed randomly across different cohorts of students, depending on interest rate conditions, rather than being distributed to help a target population. The consolidation option also demonstrates how the combination of programmatic complexity and the rules of federal budgeting can make it difficult to infer the full economic cost

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1 See, for example, De Fraja (2002), Dynarski (2002), Edlin (1993), Hanushek (1989), and Keane (2002). Gale (1991) points out that many federal credit programs probably have a small real effect on the allocation of credit, in many cases simply crowding out private borrowing and lending.

2 Further, legislation exempting student loans from dismissal in bankruptcy may alleviate market imperfections caused by restrictions on forward contracting of human capital.

of federal credit programs. Further, it provides a new setting in which to study how consumers respond to financial incentives and how their responses vary with the size of an incentive.

Recent legislation fixed the rate on new Stafford loans at 6.8 percent starting in July 2006 and made other changes that significantly lowered the value of the consolidation option for loans originated in July 2006 or later. The option is still valuable, however, on outstanding loans originated before that date. Consolidation may also reemerge as an issue if future legislation reinstates floating rates.

In this study, we develop an options pricing model to estimate the cost of the consolidation option and to evaluate the sensitivity of that cost to changes in rules and economic conditions. The model takes into account borrower behavior, program rules, the stochastic properties of interest rates, default, and the market price of risk. More broadly, the analysis demonstrates how modern options pricing methods can better inform public policy.[3] As far as we know, this is the first academic study to address consolidation and these surrounding issues.[4]

Data from the National Student Loan Data System (NSLDS) from the Department of Education is used to study borrower behavior and to calibrate the behavioral assumptions embedded in the valuation model. The data suggest that students respond to the time-varying incentives to consolidate with much higher rates of consolidation in years when the option is most valuable.[5] Further, interest rate sensitivity increases with the amount of debt outstanding. There is

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[3] For example, Falkenheim and Pennacchi (2003), Lucas and McDonald (2006), and Pennacchi and Lewis (1994) also use option pricing methods to determine the value of government liabilities.

[4] A study by the Congressional Budget Office (CBO, 2006) also examines the cost of the consolidation option. This paper differs from that study in that this paper uses a more complex interest rate model to better account for historic interest rate conditions and dynamics; uses previously unavailable student loan consolidation data to improve estimates of consolidation choice and to integrate that choice more formally into the valuation model; and explicitly formalizes the valuation model. This paper also provides specific estimates of consolidation costs for program years 1998 and 2005 and compares those estimates with a simulated distribution of consolidation costs based on information available in 1998.

[5] Although many recent studies have provided evidence of suboptimal financial behavior by individuals, few have reported on whether inefficiency decreases with the amount at stake. An exception is Calvet, Campbell, and Sodini (2006), who find more efficient portfolio allocations on the part of wealthier households in Swedish data.

some evidence of learning, because take-up rates have increased steadily over time even after controlling for interest rate incentives.

The pricing-model estimates reveal that the consolidation option has at times been extremely valuable. For instance, options exercised in 2004 had an intrinsic value of about $8.8 billion. Although the cost in some years was much lower than in the recent past, the analysis suggests that the possibility of high costs might have been foreseen.

From a policy perspective, the subsidy is likely to be an inefficient mechanism to encourage investment in education. It has little effect ex ante on the perceived cost of borrowing, confers benefits randomly across different cohorts of borrowers ex post, and provides the largest benefits to professional students graduating from high-tuition schools—those likely to have the best access to unsubsidized capital markets. The lack of transparency surrounding the cost of the option, however, has reduced its visibility to policymakers.

The remainder of the paper is organized as follows: Section 2 briefly describes the federal student loan program and the consolidation option. Section 3 outlines the options pricing model and critical modeling assumptions. Probit analysis of data from the NSLDS summarizes consolidation take-up rates and their sensitivity to borrower characteristics and market conditions. Section 4 discusses both historical and forward-looking cost estimates. Section 5 discusses policy implications and conclusions. Appendices A and B contain more-detailed descriptions of the options pricing model, supporting assumptions, and the statistical analysis of borrower behavior.

2. The Federal Student Loan Program

The federal student loan program is one of the largest credit programs operated by the U.S. government. In 2004, about 7.7 million students borrowed $55 billion in new federally backed loans, adding to the $432 billion in outstanding federal student loans.[6] This was up from $126 billion in outstanding loans in 1998, reflecting the rapid growth of education costs and strong appetite for consumer debt during that period.

The government makes credit available through two similar but competing student loan programs—the Federal Direct Student Loan (DL, or direct loan) program, and the older Federal Family Education Loan (FFEL, or guaranteed loan) program. In the former, the government lends funds directly to qualifying students. In the latter, it guarantees loans originated by private lenders against losses from default and pays lenders any shortfall between the rate charged to students and a promised minimum.[7] Schools have a choice of which program to adopt. Loan terms vary with the purpose of the loan; there are loans for undergraduates, graduate students, various types of professional students, and parents. Some students qualify for more highly subsidized loans on the basis of income. The terms offered to a given borrower are generally quite similar under the direct and guaranteed programs. Those loan terms—interest rates, maturity, loan limits, and so forth—are set by statute under the Higher Education Act.

Most loans made under the direct and guaranteed programs are Stafford loans, and these are the focus of our analysis. On Stafford loans originated between October 1998 and July 2006, students pay a variable interest rate based on a three-month Treasury rate that resets annually, plus a spread. The spread varies with the repayment status of the loan: It equals 1.7 percentage points when the student is in school, in the six-month grace period after leaving school, or in

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[6] About 10 percent of the total federal loan portfolio is loans to parents made under the Parent Loans for Undergraduate Students (PLUS) program.

[7] Currently, the student rate in repayment is the three-month Treasury rate plus 2.3 percentage points, whereas the rate guaranteed to lenders is the three-month commercial paper rate plus 2.34 percentage points.

periods of deferment and equals 2.3 percentage points otherwise.[8] The interest rate is capped at 8.25 percent.

2.1 The Consolidation Option

The consolidation option has three main features. First, it allows borrowers to combine one or more outstanding loans into a new loan. Second, borrowers pay a fixed interest rate on the consolidation loan equal to a weighted average of the rates prevailing on the loans they consolidate. This option to switch a loan from a floating rate to a fixed rate is termed a "swaption" in financial markets. Third, some borrowers have an extension option, which allows them to extend the maturity of their loans beyond what is otherwise permitted.[9] Borrowers have the opportunity to consolidate loans for as long as their original Stafford loans remain outstanding. They can also reconsolidate their previously consolidated loans with new Stafford loans, but the new fixed interest rate that the borrower obtains is a weighted average of the fixed rates on the previously consolidated loans and the floating rate on the original loans. Thus, borrowers have the valuable right to lock in the floating rate on each original loan only once.[10]

Consolidation also affects government costs through the program rules that govern payments to guaranteed lenders, because consolidation lowers the guaranteed rate of return to

8 For "subsidized" Stafford loans, which account for approximately 50 percent of the value of outstanding Stafford loans, borrowers pay no interest while they are in school, in grace or in other periods of payment deferment. The subsidized share of new loans is declining and is expected to continue to decline.

9 Borrowers in the direct program with loans originated between 1998 and 2006 receive little incremental benefit from the extension option, as they can extend loan maturity on their direct Stafford loans with or without consolidating. Many borrowers in the guaranteed program benefit from the extension option, because only borrowers with balances above $30,000 can extend their loans without also consolidating. Consolidation allows all borrowers to extend their repayment period according to the rules governing the direct program. (After June 2006, the extension rules for the direct program are the same as for the guaranteed program.)

10 Students who have not exhausted their loan limits can lower their borrowing rate further by taking new student loans to pay off older consolidation loans in favorable interest rate conditions.

lenders by 0.75 percentage points annually,[11] and because lenders pay a small fee to the government at the time of consolidation. The cost saving to the government from lower fees paid is partially offset by the longer maturity of some consolidation loans and, historically, by the presence of a rate floor on lender payments.

2.2 Benefits to Students

The consolidation option provides students with several distinct benefits—an interest rate option, an extension option, and a liquidity benefit of lower monthly payments for some borrowers. Guaranteed lenders also advertise the convenience of a single bill each month.

Perhaps the greatest benefit to students comes from the interest rate option. For a given maturity, the market rate on a floating-rate loan is generally lower than the corresponding fixed rate because yield curves tend to slope upward. By being allowed to convert a floating rate into a long-term fixed rate, students often can lock in a favorable spread. Conversely, when floating rates are high, students can choose to defer consolidation. Hence, the floating-to-fixed-rate conversion option has significant value, but that value diminishes as the loan amortizes.

The term extension is also of value for qualifying borrowers in the guaranteed program. Stafford loans allow students to borrow at a below-market rate of interest.[12] Market rates on private student loans, a market that has grown rapidly and has become increasingly competitive in recent years, provide an indication of the fair market rate for government-sponsored student loans.[13] The annual interest subsidy is then the difference between the estimated fair market rate

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[11] Lenders receive a special allowance payment equal to the rate on three-month commercial paper (CP) plus 2.34 percentage points for Stafford loans, plus any floor income (a payment when rates fall below a floor on loans issued prior to June 2006). Consolidation lenders get CP + 2.64 - 1.05 = CP + 1.59. The difference is at least 0.75 percentage points and may be higher at times due to a rate floor.

[12] This is true even of "unsubsidized" Stafford loans. The government's measure of subsidy cost for budgeting does not include administrative costs or a charge for market risk, causing reported costs to be systematically lower than a market-value-based measure of opportunity cost (CBO, 2004).

[13] Private lenders offer rates that vary with a borrower's credit score and educational institution. To the extent that private loans are taken disproportionately by professional students (e.g., law, medicine, business), they may not be representative of the average Stafford loan credit quality.

and the government rate. By extending the maturity of the loan, the subsidy is received over a longer period, increasing its present value. We refer to this component of cost, from the perspective of the government, as the “extension cost.” The extension cost varies with the credit risk of the borrower, providing the greatest benefit to students with poor credit scores. Thus, all else equal, borrowers with low credit quality have a greater incentive to consolidate than do students with stronger credit records.

Extending loan maturity also lowers monthly payments. For students facing liquidity constraints, a lower monthly payment might be preferred even if it entails a higher interest rate.[14] Guaranteed lenders marketing consolidation loans emphasize this feature, which lowers monthly payments by about 40 percent. Apart from the previously discussed extension costs, providing liquidity to students by extending maturity is relatively inexpensive. In fact, extension may save money for the government for two reasons: First, lower payments may help some borrowers avoid default, and second, the desire for lower payments may cause some borrowers to consolidate even when interest rate conditions are unfavorable. These considerations underscore that the cost to the government, which is based only on financial considerations, need not equal the benefit to an individual student.

[14] Gross and Souleles (2002) document the propensity of liquidity-constrained consumers to borrow at high interest rates when credit limits on credit cards are increased.

3. Modeling Consolidation Value

The value of the consolidation option and its sensitivity to program rules, economic conditions, and behavioral assumptions are evaluated by the options pricing model described briefly in this section (and are explained in more technical detail in Appendix B). We begin with a discussion of the critical factors affecting the option value.

3.1 Factors Affecting Costs

3.1.1 Level, Persistence, and Volatility of Interest Rates

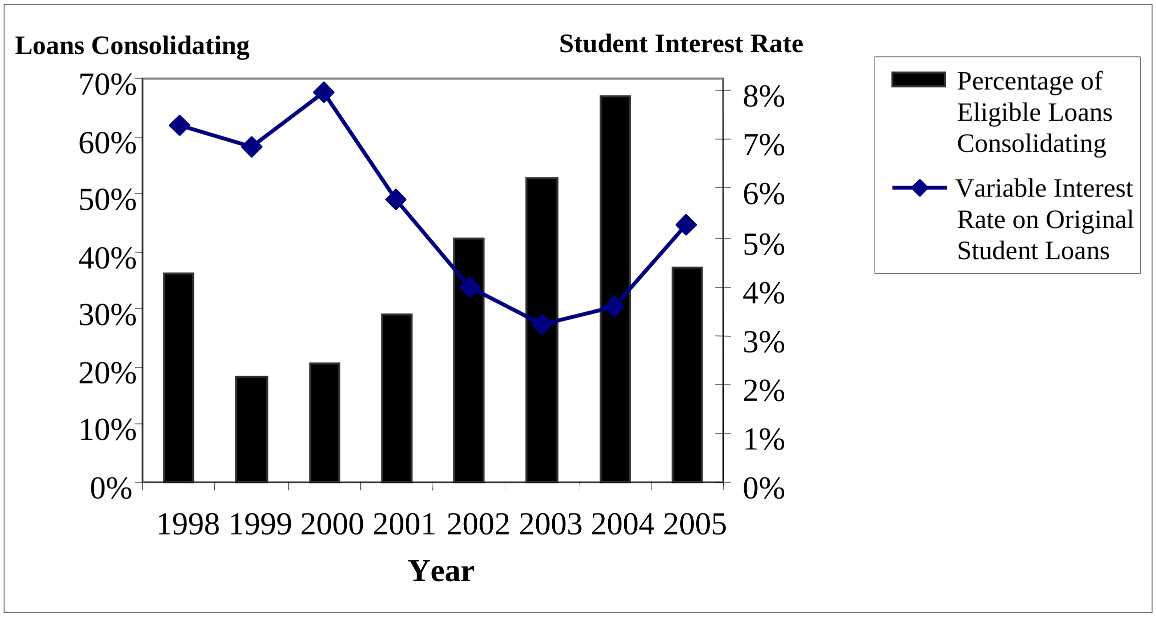
The value of the consolidation option depends on the term structure of interest rates, the persistence of interest rates over time, and volatility. Consolidation is most valuable to students and most costly to the government when short-term rates are lower than long-term rates, because then students can lock in a long-term borrowing rate that is below fair value. Because the term structure is upward-sloping most of the time, consolidation usually benefits students. Because interest rates exhibit a high degree of persistence, the value of the option varies with interest rate conditions at origination. As is generally true for options, higher interest rate volatility increases option value.

The well-known Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985) is used to generate a stochastic term structure of interest rates. We employ a two-factor version of the model, following Jagannathan, Kaplin, and Sun (2001), among others. The interest rate model is a critical input into the valuation exercise, because interest rates affect both cash flows and discount rates. The model posits a stochastic mean-reverting factor process, with volatility proportional to the square root of each factor. Conveniently, a closed-form solution is available to calculate the implied discount rates for arbitrary maturities. As described in Appendix B, the interest rate model is calibrated to match the historical volatility of the short-term Treasury rate and also, depending on the question at hand, to match the historical or current shape of the yield curve.

3.1.2 Borrower Consolidation Behavior

The cost of consolidation to the government depends on when and whether borrowers choose to consolidate. Theoretically, interest rate conditions are the most important determinant of option value, and observed consolidation rates are in fact correlated with interest rate conditions, as shown in Figure 1.[15] Although the consolidation option has been available since 1994, it attracted little attention for many years because interest rates were relatively high and often close to the rate cap. In the mid 2000s, as short-term Treasury rates fell to historical lows, the rate of consolidation exceeded 70 percent. The intense marketing of consolidation loans by guaranteed Stafford lenders at that time may have contributed to rising consolidation rates. It is doubtful, however, that similarly high rates of consolidation would continue with a return to less favorable interest rate conditions.

Figure 1: Student Interest Rate versus Share of Eligible Loans Consolidating



15 Defining a consolidation rate is complicated by the restriction on in-school consolidations prior to 2005. The restriction implies that a loan originated in a particular year could be ineligible for consolidation for many years while the student completes his or her education. We approximate the loans eligible for consolidation in each year before 2005 as a weighted average of loans from past years, with a linearly increasing weight from 0 percent in the current year to 100 percent in loans at least five years old. Figure 1 includes reconsolidations of previously consolidated loans, increasing the reported rates relative to the new consolidations that are the basis of the cost estimates.

Despite the importance of interest rate conditions, the cost to the government is not as large as it would be under a theoretically optimal exercise policy by borrowers. As is the case for home mortgages, behavior is also affected by other factors that must be taken into account. Hence, we use a behavioral model of consolidation behavior, as described below, rather than an optimizing model.

Program rules influence consolidation behavior in several ways. Until 2005, borrowers in the guaranteed program could not consolidate loans during the in-school period, creating long and variable lags between consolidation and origination. Historically, the delay could be as short as a year or could be well over eight years (in the case of a loan taken by a freshman who goes on to pursue graduate studies). Relaxation of this rule caused a wave of in-school consolidation activity in 2005.

Two factors encourage borrowers to consolidate their loans earlier rather than later in the life of their loans. For loans issued prior to July 2006, an idiosyncrasy in the law allows loans that are consolidated in school or during a grace period to bear rates that are permanently 0.6 percentage points lower than if consolidation were postponed to a normal repayment period.[16] Further, early consolidation maximizes the principal balance upon which the subsidy is based, because the balance declines as the loan amortizes.

Whether it is optimal to consolidate to take advantage of the extension option is a more difficult question, the answer to which varies with borrower characteristics such as credit score and liquidity constraints. And, even if the theoretically optimal policy were apparent, actual exercise behavior would be likely to deviate from it. A further complication in estimating the cost of the extension option is that the portion of extensions to attribute to consolidation is uncertain. All students under the direct loan program, and students with high loan balances in the guaranteed program, have the extension option independent of whether they consolidate. Yet, empirically,

[16] The grace period applies to students while in school and in the first six months after leaving school. Allowing extension of the grace-period discount has proved to be costly, as shown in the sensitivity analysis.

most extensions occur with a consolidation. An unobservable but critical quantity is the percentage of loans that would have been extended in the absence of the consolidation option. The base case counterfactual assumption is that a constant fraction of eligible borrowers extend (see Appendix B.3). When consolidation is available, we assume, consistent with observed behavior, that all consolidating borrowers extend to the maximum allowable maturity. In the sensitivity analysis, we consider the possibility that the propensity to extend is the same with or without consolidation.

We estimate the effect of interest rate conditions and loan term extension on consolidation rates using a sample of loan data taken from the National Student Loan Data System administered by the Department of Education. The database comprises multiple linked files containing loan characteristics, borrower characteristics, and time-series information recording changes in the status of each loan since its origination. Using the data, we constructed a time series for each borrower in the sample that includes consolidation events, total loan amount outstanding (which determines eligibility for term extension), the consolidation interest rate, and other borrower and loan characteristics.

A Probit regression model is used to estimate the probability that a borrower consolidates in any year. This probability varies with the total loan balance outstanding, the variable interest rate on the original loans, the length of time the loans have been in repayment, and interactions between these variables. Consistent with optimizing behavior in the presence of fixed costs, interest rates interact with balance size to produce larger consolidation probabilities among borrowers with bigger balances when rates are lower. For instance, at a variable interest rate of 8 percent, a borrower with a balance of $60,000 or more has a 4 percent probability of consolidating in the year he or she enters repayment, whereas a borrower with a balance under $20,000 has a 2 percent probability of consolidating in that year. At a 4 percent interest rate, those probabilities increase to 39 percent and 4 percent, respectively. Appendix A contains a more detailed description of the data and estimation.

In the prospective cost estimates, consolidation behavior is modeled as stochastic but is sensitive to the considerations just described and is consistent with the Probit model estimates. In repayment, the probability of consolidation is positively related to the current interest rate advantage. Even when interest rate conditions are unfavorable, we assume a non-zero base rate of consolidation, consistent with historical experience. The sensitivity analysis explores several alternative behavioral parameterizations, which are described in detail in Appendix B.

3.1.3 Federal Borrower Interest Rates versus Market Interest Rates

As discussed above, the greater the difference between the interest rate on federal loans and the fair private interest rate, the greater the extension cost. The extension cost estimate is based on the typical spread between private and government loan rates and on default and recovery rates estimated from information from the Department of Education (see Appendix B).

The rate on all federal floating-rate student loans is capped at 8.25 percent. The cap affects option value and is reflected in the valuation model. The cap effectively increases the value of the original variable-rate loans, particularly in high interest rate periods. At the same time, it reduces the interest rate benefit of a fixed-rate consolidation, because it limits the worst-case interest rate a borrower would pay without consolidating.

3.2 An Options Pricing Model

The cost of the consolidation option is estimated by using an options pricing model, implemented numerically by using Monte Carlo simulation. Projected cash flows are discounted along each sample path at risk-adjusted rates. Cost is measured at the time of loan origination, or for some measurements, as the intrinsic value at the time of exercise. The marginal cost of the option is measured by taking the difference between the present value of net government cash flows on loans with and without the consolidation option.

Cash flows depend on the stochastic path of future interest rates, the program rules, and the behavioral decision rules. Each month, a random draw from a normal distribution determines the innovation in the short-term interest rate, and the corresponding term structure is derived from the CIR model described earlier. For the direct program, monthly cash flows to the government depend on whether the student is in school, the current short-term rate if the loan has not yet been consolidated or a fixed rate based on the short-term rate at the time of conversion if the loan has been consolidated, whether the interest rate cap on the student loan is binding, the average rates of default and prepayment, and an administrative charge.[17] For the guaranteed program, the relevant cash flows are between the government and guaranteed lenders. These cash flows consist of special allowance payments (SAP), which vary with consolidation status, and compensation to lenders for credit losses.

The discount rates reflect the opportunity cost of providing loan capital to students. There are at least two potential sources of priced risk that affect the discount rate—interest rate risk and credit risk. Estimating a credit risk premium for student loans directly is problematic, because there are no traded companies that specialize exclusively in private student loans. The approach taken here is to capture the price of interest rate risk by using a risk-neutral representation of the CIR model. The credit risk premium is represented by a spread over the risk-neutral rates and is inferred from the spread between private and federal student loans, adjusting for expected losses and administrative charges, as discussed in Appendix B.

[17] The administrative charge on direct loans cancels out when pricing the option as long as the old and new loans have similar maturity.

4. Results

4.1 Historical Experience

The realized cost of the consolidation option from 1998 to 2005, in total current dollars and as a cost per $100 of loans consolidated, is shown in Table 1. The calculation is based on the intrinsic value of realized consolidation volume in a given year. The intrinsic value is the difference between the present value of cash flows for a consolidated loan and the present value of cash flows for an unconsolidated loan from the date that the consolidation occurs.[18] In each case, we assume that borrowers take whatever term extension is available to them. For each program year, we take as a starting point the interest rate conditions in that year and simulate the loan cash flows and discount rates over the life of the representative set of loans.

Table 1: Consolidation Volume and Estimated Government Consolidation Cost, 1998–2005

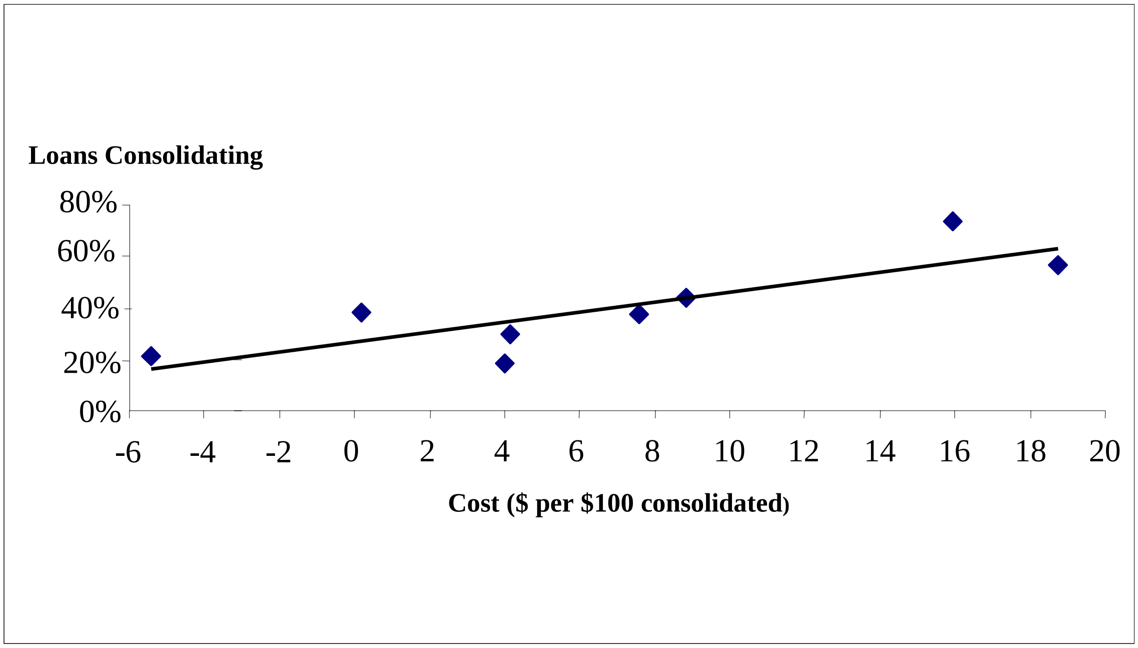
Consolidation Year	Consolidation Volume (millions of $)	Consolidation Cost (millions of $)	Consolidation Cost ($ per $100)
1998	12,312	21	0.17
1999	10,217	409	4.01
2000	15,466	-841	-5.44
2001	26,411	1,099	4.16
2002	39,283	3,480	8.86
2003	43,770	8,198	18.73
2004	55,272	8,800	15.92
2005	29,203	5,832	7.60

Table 1 shows that the costs vary considerably from year to year, ranging from a savings to the government of $5.44 per $100 of loans consolidated in 2000 to a cost of $18.73 per $100 of loans consolidated in 2003. That volatility reflects changes in interest rate conditions and borrower behavior. The scatter plot in Figure 2 shows that borrowers consolidate at higher rates

[18] The behavioral parameters are not relevant, however, because consolidation is assumed to have occurred in that year.

when it is in their interest to do so, although a considerable number consolidate even when it results in cost savings to the government.

Figure 2: Share of Eligible Loans Consolidating versus Government Consolidation Cost, 1998–2005



4.2 Were High Costs Predictable?

An interesting question is, to what extent could the recent high costs of this policy have been predicted? In other words, was there just a particularly unlucky draw of interest rates, or could the magnitude of realized costs have been anticipated by lawmakers? To evaluate this question, we used the CIR model, calibrated with initial conditions corresponding to interest rate conditions in 1998, to generate the distribution of the term structure of interest rates several years into the future. Figure 3 shows the distribution of cost per $100 of loans consolidated, at the time of consolidation, based on 2,000 draws of the forward distributions of term structures. The calculations suggest that, although the very high costs realized in 2003 and 2004 were unlikely ex ante, at the end of four years, there was a 23 percent chance of costs in excess of $8 per $100 of loans consolidated. Table 2 contains descriptive statistics for the distribution of intrinsic values at other terminal horizons, in each case starting from initial interest rate conditions in 1998. Clearly, this risk exposure could have been avoided, while still allowing conversion to a fixed rate, by indexing the consolidation rate to current long-term fixed rates rather than to the current floating rate.

Figure 3: Simulated Distribution of Intrinsic Value Simulated Four Years Forward from Interest Rate Conditions in 1998

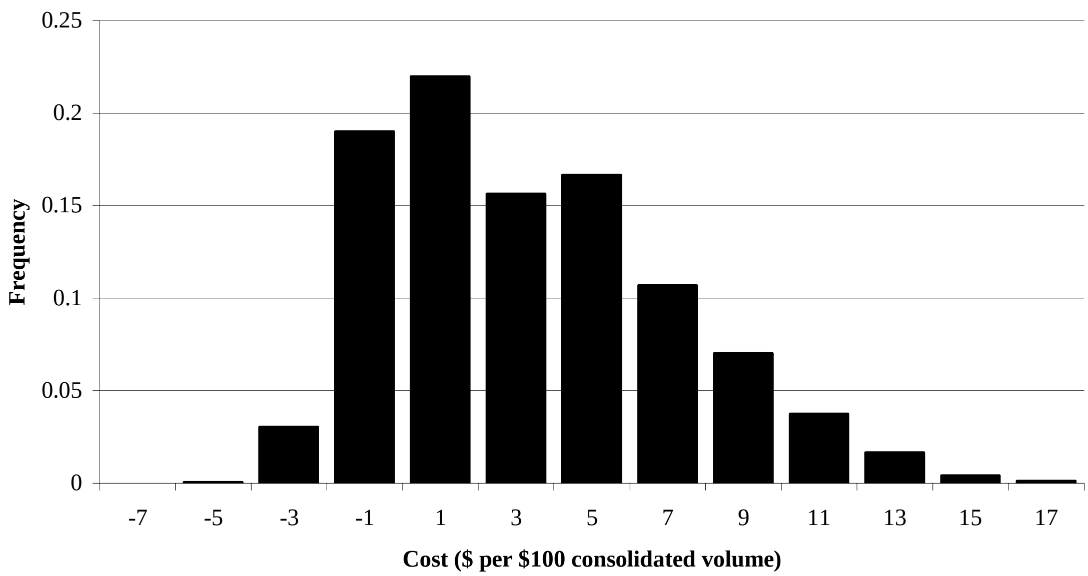


Table 2: The Distributional Properties of the Intrinsic Cost of Consolidation, Taking Initial Interest Rate Conditions as at 1998 and Simulating Interest Rates a Set Number of Years

	Number of Years				
	2	4	6	8	10
Mean	3.33	3.51	3.58	3.56	3.66
Median	3.01	2.95	2.90	3.01	3.10
Standard Deviation	3.40	3.77	3.97	3.92	3.96
Relative Skewness	0.61	0.70	0.71	0.68	0.65

4.3 Prospective Costs

The historical cost numbers reported above are conditional on a consolidation having occurred (the “intrinsic” option value). An alternative measure that is more relevant from an accrual perspective is the cost of consolidation at the time of origination of a Stafford loan. Compared with the historical cost calculations, this measure requires taking into account consolidation behavior and program rules that affect the time between origination and consolidation and whether a consolidation occurs at all.

Under the base-case assumptions, and under interest rate conditions prevailing in each year from 1998 to 2005, the cost of the consolidation option per $100 of Stafford loans originated is shown in Table 3. The table also shows the three-month bill and ten-year bond rates at those times, which explain much of the difference in cost across years. The cost varies less over time than in the historical calculations of intrinsic value (Table 1) because the delay between origination and consolidation provides time for interest rates to revert toward their historical average, making initial interest rate conditions less important. The cost is lower on average because it includes loans that never consolidate or that have a long delay until consolidation.

Table 3: Prospective Consolidation Costs and Interest Rates, 1998–2005

Origination Year	Three-Month T-Bill Rate	Ten-Year T-Bond Rate	Consolidation Cost ($ per $100)
1998	4.98%	5.50%	1.33
1999	4.57%	5.90%	1.22
2000	5.69%	6.10%	0.80
2001	3.49%	5.28%	2.05
2002	1.70%	4.93%	3.54
2003	0.92%	3.33%	6.44
2004	1.27%	4.73%	4.21
2005	2.97%	4.00%	3.70

Legislation reauthorizing student loans through 2012 was passed in early 2006. Interest rates on Stafford loans originated after July 2006 revert to a fixed 6.8 percent, with consolidation interest rates on those loans as high as 6.875 percent. This legislation also eliminates the possibility of consolidation while in school. Although there was little discussion of the effect on the value of the consolidation option, these changes effectively eliminate the option's value for loans originated after 2005. Even so, understanding the cost of this option is useful. Switching to a fixed rate for students, especially while maintaining floating-rate reimbursement to lenders, poses new risks to the government,[19] and it seems probable that lawmakers will contemplate a return to a floating rates in the future.

4.4 Sensitivity Analysis

We examine the sensitivity of the prospective cost estimate in 2005 to several of the key economic, behavioral, and programmatic parameters: initial interest rate conditions, the volatility of interest rates, the risk spread in the discount rate, consolidation behavior, and extension behavior. We also estimate the cost of the grace-period loophole, which allows borrowers a permanent rate decrease of 0.6 percentage points if they consolidate during the grace period, and the cost of allowing immediate consolidation instead of forcing delay until leaving school. All costs are expressed as a present value per $100 of original Stafford loans at origination.

4.4.1 Interest Rates and Spreads

As discussed earlier, the cost of the consolidation option increases with a reduction in short-term interest rates and an increase in the volatility of interest rates. The cost also is affected by the credit spread, but there are two partially offsetting effects—a higher credit spread makes the extension option more valuable but raises the discount rate, which decreases the present value.

[19] Most important, guaranteed lenders will continue to be reimbursed based on a floating rate, introducing increased volatility into the federal payments to lenders.

Table 4 shows that moderate changes in the interest rate model, relative to a base-case discount rate of the Treasury rate plus 2 percentage points, have only small effects on prospective costs. The last column shows that a one-factor CIR model is more sensitive to interest rates, mostly because a single-factor model produces less-plausible term structure variation.[20]

Table 4: Prospective Cost Sensitivity to Interest Rate Model

	($ per $100 of Stafford originations)			
Origination Year	Discount Rate T+1%	Discount Rate T+3%	High Volatility	One-Factor Model
1998	1.13	1.46	2.17	0.87
1999	1.09	1.29	1.99	0.98
2000	0.66	0.89	1.49	0.29
2001	1.89	2.14	2.88	2.58
2002	3.45	3.56	4.17	7.48
2003	6.15	6.57	7.41	10.25
2004	4.13	4.22	4.88	8.89
2005	3.32	3.93	4.63	3.72

4.4.2 Borrower Behavior

The extent to which borrowers respond to the economic value of the option is estimated with considerable uncertainty. We compare the results of the base case, in which consolidation rates are quite sensitive to interest rates, with the alternative of fixing the consolidation rate at close to the historical average of 45 percent of eligible borrowers. Comparing the first two data columns of Table 5 shows that a fixed consolidation rate increases the cost to the government in some years and decreases it in others. The reason that a fixed consolidation rate is not uniformly less costly is that the benchmark rule was not chosen to be optimal but rather to mirror observed behavior.

[20] For both one- and two-factor interest rate models, we used the parameter estimates of Jagannathan, Kaplin, and Sun (2001).

Table 5: Prospective Cost Sensitivity to Behavioral Variations

Origination Year	Benchmark	No Sensitivity of Consolidation to Interest Rates	More Frequent Extensions in the Counterfactual Case
	($ per $100 of Stafford Originations)		
1998	1.33	2.06	0.89
1999	1.22	2.24	0.92
2000	0.80	1.96	0.71
2001	2.05	2.48	1.43
2002	3.54	2.97	2.52
2003	6.44	3.23	4.35
2004	4.21	3.09	3.07
2005	3.70	2.55	2.20

Another uncertainty is the extent to which the option to consolidate increases the propensity to extend loan maturity. As mentioned earlier, even without consolidation, all borrowers under the direct loan program, and borrowers with combined balances in excess of $30,000 in the guaranteed program, can request a term extension without consolidating. However, consolidation increases the incentive to extend maturity, because the rate reduction obtained with consolidation increases the value of extension. The empirical evidence does not offer much guidance on the size of this effect, because the legislation that enabled borrowers to extend loan maturity took effect at about the same time that the consolidation option became highly valuable. In the base case, we assume that in the counterfactual scenario where extension is available but consolidation is not, each year borrowers will extend the term of their loans with a constant probability.[21] An alternative set of cost estimates, shown in the third data column of Table 4, is more conservative in that we assume borrowers extend their loans in the counterfactual case as frequently as they consolidate them (where the rules governing term extension allow this).

[21] This is equal to the estimated base rate of consolidation in the Probit model when the borrower interest rate is at the 8.25 percent cap, so it might reflect the proportion of students who are anxious to extend maturity.

Comparison of the first and third data columns of Table 5 shows that raising the probability of extensions in the absence of consolidation lowers the cost by approximately 30 percent.

4.4.3 Effects of Related Policies on Consolidation

The cost of the grace-period rate reduction on Stafford loans is limited by the six-month length of the grace period after graduation.[22] The legislation that introduced the consolidation option, however, did not make the grace-period discount temporary on consolidation loans. Relative to a hypothetical consolidation loan with a discount that ends with the grace period, the incremental cost of this loophole has averaged $0.78 per $100 of Stafford loans originated.

In May 2005, the Department of Education appeared to change its position on in-school consolidations in the guaranteed program, writing that, contrary to popular opinion, in-school consolidation is permissible under current law.[23] Together with favorable interest rate conditions that were expected to be transitory, the decision triggered a wave of consolidation activity. Had in-school consolidation been a common practice over the entire period of 1998 to 2005, we estimate that it would have increased the prospective cost of the consolidation option by an average of $0.55 per $100 of Stafford loans originated.[24]

Guaranteed lenders can avoid dealing with some troubled loans by declining to consolidate them, causing those students to turn to the direct program for consolidation. We do not try to quantify the effect on cost, however, because it would require considerable additional modeling and assumptions.

[22] The grace period can be reactivated by returning to school for additional education, increasing the grace period's average value.

[23] The ruling was not a response to any legislative change but rather was a response to inquiries by guaranteed lenders. In-school consolidation was already available to direct-program borrowers at that time.

[24] We assume that in-school consolidation also is sensitive to current interest rate conditions but that, for any level of interest rates, consolidation is less likely in school than after graduation.

5. Discussion and Conclusions

In this study, we develop a model to value the student loan consolidation option and use it to estimate the cost to the government of this provision under a variety of assumptions about economic conditions and student behavior. The analysis implies that, between 1998 and 2005, the option had an ex post cumulative cost to the government of about $27 billion. Although this estimate is sensitive to model assumptions and particularly to the interest rate model, the estimate remains strikingly large for a wide range of modeling assumptions. The incidence of the subsidy has been largely random, conferring the greatest benefit on those cohorts who happened to graduate when interest rate conditions were most favorable. It also differentially benefited professional students with the largest loan balances and those with the sophistication to manage their loans efficiently. Because most entering students are unlikely to be aware of the option's true value, it is unlikely to affect educational outcomes significantly.

Why, then, has this subsidy persisted? We believe that several factors make its cost relatively easy to overlook or, at least, to ignore. As with many other subsidies involving an opportunity cost but not a direct transfer of funds, there is no cash trail. The current budget treatment of student loans obscures the cost.[25] The Office of Management and Budget treats consolidation loans as new loans, resulting in a zero budget cost for future consolidations associated with newly originated Stafford loans. CBO does incorporate expected future consolidation costs into its cost estimates for new Stafford loans, but neither budget agency breaks out the cost of anticipated future consolidations associated with current Stafford loan originations.

From a technical perspective, an options pricing approach is arguably the best way to measure the value of this type of provision. It is not, however, the only way to see that the option is extremely valuable. For instance, a back-of-the-envelope calculation based on locking in a

[25] Student loans, like other federal credit programs, are accounted for under the rules of the Federal Credit Reform Act. See CBO (2004) for a summary of those rules.

2 percent annual interest rate advantage on a 20-year amortizing loan produces a cost estimate that is in line with the predictions of our more complicated model. In fact, the high rate of consolidation in recent years is evidence that the option is widely recognized as valuable by student borrowers.

Appendix A: Estimating the Frequency of Borrower Consolidation from the National Student Loan Database

Approximately 700,000 borrower records were randomly selected from the National Student Loan Database System (NSLDS) in January 2006. For each borrower, the database contains a record of all current and previous loans. Each loan record contains the loan amount disbursed, the amount outstanding in January 2006, the type of loan, and the current status of the loan. In addition, each loan record contains a full history of loan status changes and associated dates. We simplified the complicated loan status variable into seven categories, four of them open and three closed. The open status categories, in which the loan is still open and collectible, are:

- G: In school or in the six-month grace period. Usually, the borrower does not make payments on a loan in this status.
- F: In forbearance or deferment. The borrower does not make payments in this status.
- R: In repayment.
- d: In default. A borrower who fails to make the prescribed minimum repayments for more than 270 days is in default.

The closed status categories are:

- D: Collected, negotiated, or written off.
- P: Paid in full.
- C: Consolidated. Note that some loans that were coded as paid in full in the data were actually consolidated. (We have corrected this error where possible by linking paid-in-full records to the corresponding origination of a consolidated loan for that borrower.)

We analyzed the transition of loans from the repayment status (R) to consolidation status (C) at the borrower level by constructing a time series of consolidation events and associated controls for each borrower. This gives us approximately 3 million pooled observations on the following variables:

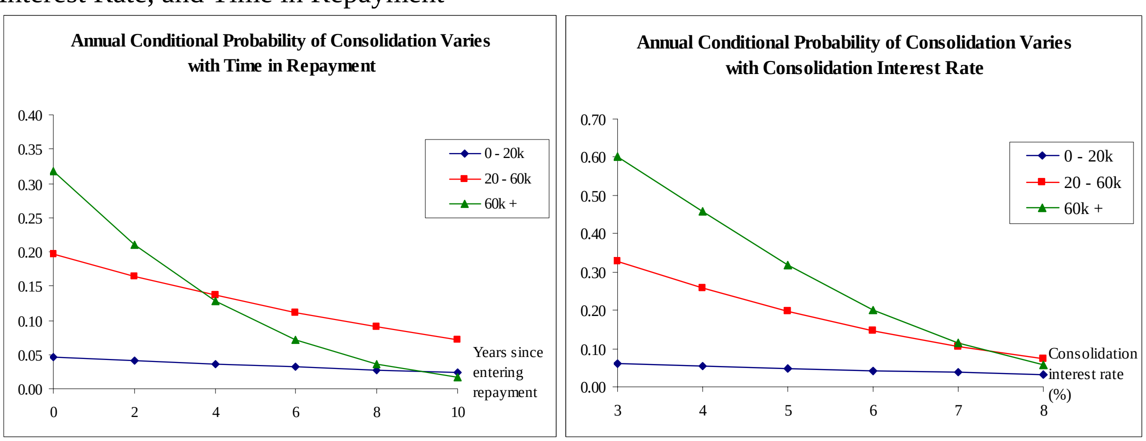
1. CONSOL: Consolidation indicator variable taking the value 1 if the borrower consolidates currently outstanding loans at a particular point in time, 0 otherwise. This is the endogenous variable in the Probit regression model.
2. BALCATx: Dummy variable indicating the total value of the borrower's outstanding original loans. The categories are as follows: x = 0, balances of $0 to $20,000; x = 1, balances of $20,001 to $60,000; and x = 2, balances exceeding $60,000. Because we do not observe the principal outstanding for every loan in every year, we estimated it from the disbursed amount of each loan under assumed interest rates and an assumed amortization schedule.
3. RATE: The weighted average interest rate on the borrower's outstanding original loans. This is the rate a borrower would lock in if he or she consolidated (ignoring any rate incentives he or she may have received from a guaranteed lender).
4. YRS IN REPAY: Weighted average time the loans have been in repayment.
5. Dummy variables for the program the loans originated in. Loan consolidation rates appear to be higher in the guaranteed than in the direct program. Moreover, borrowers with loans from both programs are more likely to consolidate than those with loans from a single program.
6. Dummy variables for the year of the decision, proxying for the changes over time in the information borrowers receive about consolidation.

Table A-1 below reports the results from the Probit regression. We have not attempted to adjust the standard errors for the possibility of individual specific random effects in the consolidation results. However, parameter estimates are consistent despite this form of mis-specification (see Maddala, 1987), and the large sample size ensures that, even after such a correction, parameter estimates would retain statistical significance. More important, parameter estimates are of the expected sign and magnitude. Notably, interest rates interact with balance size to produce larger consolidation probabilities among borrowers with bigger balances when rates are lower. Figure A-1 reports the estimated consolidation probability for borrowers with particular characteristics for various interest rates, balance sizes, and time in repayment.

Table A-1: Parameter Estimates and Standard Errors from a Probit Regression of Consolidation Decisions on Covariates

Parameter	Estimate	Standard Error	Chi-Square	Pr > ChiSq
Intercept	-1.0458	0.0292	1,279.66	<.0001
HAS DIRECT	-0.0846	0.0035	570.02	<.0001
HAS BOTH	0.3734	0.0051	5,380.53	<.0001
BALCAT1 (20-60k)	1.1457	0.0624	337.10	<.0001
BALCAT2 (60k+)	2.8545	0.3533	65.29	<.0001
RATE	-0.1227	0.0032	1,449.70	<.0001
RATE*BALCAT1	-0.0426	0.0075	32.30	<.0001
RATE*BALCAT2	-0.2104	0.0427	24.32	<.0001
YRS IN REPAY	-0.0276	0.0006	1,943.63	<.0001
YRS IN REPAY*BALCAT1	-0.0324	0.0013	636.90	<.0001
YRS IN REPAY*BALCAT2	-0.0317	0.0044	51.40	<.0001
YRDUM (POST 2002)	0.6949	0.0048	21,103.70	<.0001
YRDUM (1998-2002)	0.3148	0.0044	5,066.74	<.0001

Figure A-1: Estimated Consolidation Probabilities by Outstanding Balance, the Consolidation Interest Rate, and Time in Repayment



Appendix B: Simulation Methodology

To estimate the cost of the consolidation option, we employ a five-step process:

1. Use the Cox-Ingersoll-Ross (CIR) model to simulate a risk-neutral path of interest rates for various maturities. It is the basis for discounting credit-risk-free nominal cash flows in the model.
2. Compute cash flows: In the prospective cost estimates, the government's stream of cash flows are computed across each interest rate path, the default state of the loan, and stochastic consolidation and extension behavior. In the intrinsic value calculations, cash flows across each interest rate path and the default state are computed as in the prospective case. However, the date of consolidation, and hence current interest rate conditions, are taken as known and given. Consolidation occurs at the beginning of repayment, and the maximum allowable extension is assumed.
3. Discount the government's default-contingent cash flows using a state price deflator to account for default risk and the simulated risk-free interest rate to account for time value of money.
4. Compute the cost of consolidation to the government for each loan type (direct or guaranteed) by taking the difference between the present value of cash flows with and without the option to consolidate, averaging across the simulated risk-neutral interest rates and stochastic consolidation and extension behavior.
5. Aggregate cash flows for a representative set of loans from the two lending programs.

B.1 Interest Rates

We adopt the CIR model to simulate future paths of Treasury rates, and we adjust rates upward for the systematic component of default risk, as described in Sections B.4 and B.5. In the

CIR model, the instantaneous interest rate, $R(t)$, is the sum of a constant and n factors, $z_i(t)$, for i = 1,…,n, the state variables in the model:

$$R(t) = \overline{R} + \sum_{i=1}^{n} z_i(t) \tag{1}$$

Each factor obeys a mean reverting square root process:

$$dz_i(t) = \kappa_i\left[\theta_i - z_i(t)\right]dt + \sigma_i\sqrt{z_i(t)}dZ_i(t) \tag{2}$$

where κ_i is the speed of mean reversion, θ_i is the mean reverting rate, σ_i is the volatility, and $dZ_i(t)$ is a standard Weiner process independent across factors.

Under the risk-neutral (or equivalent martingale) measure:

$$dz_i(t) = \overline{\kappa}_i\left[\overline{\theta}_i - z_i(t)\right]dt + \sigma_i\sqrt{z_i(t)}dZ_i(t) \tag{3}$$

where

$$\overline{\kappa}_i = \kappa_i + \lambda_i \tag{4}$$

and

$$\overline{\theta}_i = \frac{\kappa_i\theta}{\kappa_i + \lambda_i} \tag{5}$$

λ_i is the constant market price of risk for factor, i. The price at time t of a zero-coupon bond with unit coupon and expiring at T is:

$$p(t,T) = e^{-\overline{R}(T-t)}\prod_{i=1}^{n} A_i(t,T)\,e^{-B_i(t,T)y_i(t)} \tag{6}$$

where

$$A_i(t,T) = \left[\frac{2\gamma_i \exp\left[(\gamma_i + \overline{\kappa}_i)(T-t)/2\right]}{(\gamma_i + \overline{\kappa}_i)\left[\exp\left[\gamma_i(T-t)\right]-1\right]+2\gamma_i}\right]^{2\overline{\kappa}_i\overline{\theta}_i/\sigma_i^2} \tag{7}$$

$$B_i(t,T) = \frac{2\exp\left[\gamma_i(T-t)-1\right]}{(\gamma_i + \overline{\kappa}_i)\left[\exp\left[\gamma_i(T-t)\right]-1\right]+2\gamma_i} \tag{8}$$

and

$$\gamma_i = \sqrt{\bar{\kappa}_i^2 + \sigma_i^2} \tag{9}$$

The yield to maturity, y, of a zero-coupon bond maturing at T is

$$y(t,T) = \frac{-\ln p(t,T)}{T-t} \tag{10}$$

Jagannathan, Kaplin, and Sun (2001) estimate the factors from the two-factor model, using weekly LIBOR rates of various maturities from 1995 through 1999, as shown in Table B-1.

Table B1: Parameters for the Cox-Ingersoll-Ross Two-Factor Interest Rate Model

$\bar{R}$ = -0.229

Factor	κ	θ	σ	λ
1	0.392	0.272	0.0153	-0.00038
2	0.0532	0.0162	0.0430	-0.0592

Under these parameters, factor 1, with the stronger degree of mean reversion, drives the gap between long- and short-term rates, and factor 2 determines long-term rates. We subtract 20 basis points from $\bar{R}$ to reflect the average spread between three-month Treasury and LIBOR yields.

For each Monte Carlo run, initial levels of the state variables are calibrated to fairly price an initial three-month Treasury bill rate and ten-year Treasury bond rate. For each simulation, the instantaneous rate is sampled monthly for as many months as the maximum maturity of the student loan, using a discrete approximation of the risk-neutral process in equation (3).

B.2 Government Cash Flows in the Direct and Guaranteed Programs

The cost of consolidation for a particular loan is the difference in the present value of government cash flows when a student has access to the consolidation option and when the student does not have access. The cash flows themselves depend on the evolution of the principal balance and interest charged on the student loans, which we outline now.

Loans originate at time 0, begin repayment at time T^R, and have a maturity of T^M, so the loan is repaid in T^R+T^M months. T^M depends on whether the consolidation option is exercised or, in the counterfactual case, the loan term is extended. The original maturity of Stafford loans is 10 years. Section B.3 describes the stochastic rules governing consolidation and extension.

Interest accrues on outstanding principal every month. The interest rate prior to consolidation is linked to the yield on the three-month Treasury bill on May 30 each year and is fixed for a year. Thus, the reference rate, $\tilde{R}^j_{12k+i-1}$, in month i of year k is:

$$\tilde{R}^j_{12k+i-1} = \exp\left[4y^j\left(12k, 12k+3/12\right)\right]-1, \forall i = 0,...,11, k = 0,1,2,... \tag{11}$$

The borrower pays an interest rate that depends on whether he or she is in repayment and whether he or she has consolidated. Extension of loan term without consolidation does not affect the interest rate. Before consolidation and before entering repayment, the student rate, r_s, is the lower of the reference rate plus 1.7 percentage points or the interest rate cap of 8.25 percent,

$$R^j_{S,t} = \min\left[\tilde{R}^j_t + 1.7\%, 8.25\%\right], \ \forall t < T^R \ and \ t \le t^c \tag{12}$$

After entering repayment but prior to consolidation, the student rate is the lower of the reference rate plus 2.3 percentage points or the interest rate cap:

$$R^j_{S,t} = \min\left[\tilde{R}^j_t + 2.3\%, 8.25\%\right], \ \forall t \ge T^R \ and \ t \le t^c \tag{13}$$

After consolidation, the student rate is fixed at the rate prevailing at the time of consolidation, t^c:

$$R^j_{S,t^c+k} = R^j_{S,t^c} \ \ \forall k > 0 \tag{14}$$

The variable P^j_t denotes the evolution of principal (prior to default) over time in each simulation j. Given an initial principal of $P^j_0 = P_0$, principal evolves according to:

$$P^j_{t+1} = P^j_t\left[1 + r^j_{S,t}\right] - A^j_{t+1} \tag{15}$$

where

$$r_{S,t}^{j} = \left(1 + R_{S,t}^{j}\right)^{\frac{1}{12}} - 1 \qquad (16)$$

is the monthly compounding student rate. The prescribed monthly payment, A_t^j, depends on the loan status and is based on amortizing the principal at the current interest rate over the remaining life of the loan:

$$A_{t+1}^{j} = \begin{cases} \dfrac{P_t^j r_{S,t}^j}{1-\left(1+r_{S,t}^j\right)^{-k}}, & t \geq T^R \\ 0, & t < T^R \end{cases} \qquad (17)$$

In the direct program, the government's cash flows on performing loans are the student loan payments less any administrative fees:

$$A_t^j - fP_t^j \qquad (18)$$

where *f* is the proportional administrative fee. The fee is 0.75 percent per annum in the benchmark calibration, reflecting typical servicing and other administrative costs. In default, the government recovers in proportion to the present value of remaining payments.

In the guaranteed lending program, the government's cash flows are the quarterly payments to lenders—the special allowance payment less any consolidation fee paid by lenders to the government—while the loan is in good standing and the lump-sum payment of outstanding principal and accrued interest in the event of default. We ignore administrative costs because they are largely borne by the guaranteed lender.

The quarterly SAP is the difference between the student rate and the three-month commercial paper rate plus a spread, but has a floor of zero. We assume that the annualized three-month commercial paper rate, R_C, tracks the T-bill rate with a 20 basis point spread:

$$R_{C,t}^{j} = \exp\left[4y^{j}\left(3k, 3k+3/12\right)\right] + .002 - 1, \forall t = 1,2,...,T \qquad (19)$$

Absent default, the government's cash flow in each month is the SAP less any consolidation fee paid from lenders to the government, or 1.05 percent of principal. We denote the net guarantee payment from the government while the loan is in good standing by G:

$$G_t^j = \begin{cases} -P_{3k}^j \max\left[R_{C,3k}^j + 1.74\% - R_{S,3k}^j, 0\right]/4, & 3k < T^R \text{ and } 3k < T^C \;\; \forall\, k = 0,1,2,... \\ -P_{3k}^j \max\left[R_{C,3k}^j + 2.34\% - R_{S,3k}^j, 0\right]/4, & 3k \ge T^R \text{ and } 3k < t^C \;\; \forall\, k = 0,1,2,... \\ -P_{3k}^j \left(\max\left[R_{C,3k}^j + 2.64\% - R_{S,3k}^j, 0\right] - 1.05\%\right)/4, & 3k \ge t^C \;\; \forall\, k = 0,1,2,... \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

In default, the government pays the outstanding principal, P_t^j, to the lender, assumes the loan, and recovers in proportion to the present value of the remaining outstanding payments. The default and recovery rates used in the calibration are described in Section B.5.

Several additional factors affect the timing and magnitude of student loan cash flows, but we omit them from the analysis because we expect their impact on consolidation cost to be small. Voluntary prepayment has the effect of reducing the cost to the government by shortening effective loan maturity, whereas deferment and forbearance, by delaying payment, increase that cost. Subsidized Stafford loans offer a higher subsidy than so-called unsubsidized Stafford loans, because interest does not accrue while the student is in school. In the prospective cost estimates, this affects the principal balance at the time repayment begins, making it smaller than it otherwise would be. In the analysis, all loans are unsubsidized loans.

B.3 Timing of Consolidation and Extension

We posit a rule for the intensity of consolidation for a given loan that is consistent with the Probit model described in Appendix A. Specifically, consolidation is decreasing in the student interest rate and decreasing in the time since repayment begins. We assume borrowers consolidate loans during the grace period, consistent with the rule for consolidation at other times, but cannot consolidate at all while they are in school.[26] Thus, the annualized probability of consolidation, $q_{C,t}$, at month t is

$$q_{C,t}^{j} = \begin{cases} 0, & t < T^{R} - 6 \\ \Phi\left(\beta_1 + \beta_2 r_{S,t}^{j}\right), & T^{R} - 6 \le t < T^{R} \\ \Phi\left(\beta_1 + \beta_2 r_{S,t}^{j} + \beta_3 \max\left(\left[t - T^{R}\right]/12, 0\right)\right), & t \ge T^{R} \end{cases} \qquad (21)$$

where Φ is the cumulative standard normal distribution function and β_1 through β_3 are the loan type specific parameters reported in Table B-2 below and are consistent with Probit estimates in Appendix A. We also apply the rule for consolidation to capture extension behavior in the counterfactual case with no consolidation. In the benchmark simulation without consolidation, we assume a constant rate of extension equal to the rate of consolidation when the interest rate is at the 8.25 percent cap. This implies that the rate of extension is invariant to the level of the student rate. If anything, standard theory would predict that the sensitivity of extension behavior to short-term interest rates should be positive, because, relative to market rates, the present value of an extension increases when interest rates are closer to the cap.

Table B-2: Parameters Determining the Annual Frequency of Consolidation

	β_1	β_2	β_3
Consolidation: Eligible for 10-year term	-1.56	-0.06	-0.03
Consolidation: Eligible for 20-year term	0.00	-0.20	-0.06
Consolidation: Eligible for 30-year term	1.20	-0.37	-0.17
No Consolidation: Eligible for 10-year term	-1.79	0.00	-0.03
No Consolidation: Eligible for 20-year term	-0.82	0.00	-0.06
No Consolidation: Eligible for 25- or 30-year term	-0.27	0.00	-0.17

Consolidation allows students to extend their original 10-year loans to as long as 30 years. To estimate the cost of the option to the government, several factors relating to the maturity change are accounted for. One such factor is that annual administrative expenses, such as servicing and collection, will be incurred for a longer period. Losses from defaults also must be

[26] For the 2006 academic year, borrowers were allowed to consolidate during their in-school period.

treated consistently. The performance history of consolidated and unconsolidated loans suggests a higher cumulative default rate for consolidation loans. On a per annum basis, the default rates are similar, so we assume the same 2 percent per annum default rate for all loans.

B.4 Adjusting Discount Rates for Default Risk

Under the CIR model, the risk-neutral monthly compounded discount rate, d_t, for default-free but possibly interest-rate-contingent monthly cash flows is

$$d_t = \frac{1}{p\left(t, t+1/12\right)} - 1 \;\forall t = 0,1,2,... \tag{22}$$

In both the direct and guaranteed lending programs, the underlying payments between parties are contingent on default. We adopt a parsimonious approach to default that treats guaranteed and direct lending cash flows consistently.

We assume that default occurs with equal probability in each month until the borrower completely repays the loan. Implicitly, we assume that the priced component of default risk is orthogonal to the priced component of interest rate risk. Suppose there is a pair of simple one-period securities traded in every period. The first is risk-free, offering a certain payoff of $1 in one period's time. The second is a risky claim that pays $1 if the borrower does not default and α if the borrower does default. The fair price of the default-free claim is:

$$\frac{1}{1+d_t} \tag{23}$$

With a constant risk premium of π and a default probability of q, the fair price of the risky claim is:

$$\frac{1-q(1-\alpha)}{\left(1+d_t\right)\left(1+\pi\right)} \tag{24}$$

More conveniently, we can define state price deflators to value cash flows in t+1 paid if the borrower defaults:

$$\frac{h}{1+d_t} \tag{25}$$

and if the borrower does not default:

$$\frac{1-h}{1+d_t} \tag{26}$$

where *h* is the risk-neutral probability of default:

$$h = \frac{\pi + q(1-\alpha)}{(1+\pi)(1-\alpha)} \tag{27}$$

B.5 Calibrating State Prices for Default-Contingent Prices

Parameters consistent with the state-price representation for default risk can be inferred from market rates on student loans and historical default and recovery experience. Many financial institutions (e.g., Sallie Mae, Citibank) compete to offer private student loans to qualifying students. Interest on private student loans is most commonly quoted as a spread over LIBOR or Prime. The spread varies widely with a student's credit score and educational institution. Based on lender Web sites and on data from one large private lender, the Treasury rate plus 4 percentage points appears to be a typical private rate spread (adjusting for the difference between LIBOR and Prime, and Treasury).

In a competitive market, components of the spread over Treasury charged by private lenders include: (1) compensation for expected losses net of recoveries; (2) servicing and other ongoing administrative costs, including expected collection costs and taxes; (3) fixed origination and marketing costs, amortized over the expected life of the loan; and (4) a market risk premium.

Data from the NSLDS suggest a cumulative default rate over the life of a typical Stafford loan of 15 percent. Default rates vary over the life of a loan, with the rate decreasing as the loan ages. Abstracting from the time pattern, an annual default rate of 2 percent is consistent with this cumulative experience. We do not have data to ascertain whether private lenders experience higher or lower default rates than the government, so we assume the default rates are similar.[27] Estimates from the NSLDS suggest a recovery rate of 40 to 60 percent on defaulted loans. We assume the midpoint of 50 percent in the computation of consolidation cost.

Discussions with private lenders suggest that typical costs for servicing and other ongoing administrative costs total about 0.5 percent per annum. We attribute another 0.5 percent to other expenses, such as taxes and amortized origination and marketing costs.

We can now infer the market risk premium. Starting with the 4 percent spread and subtracting 1 percent for default losses (a 2 percent default rate net of the 1 percent recovery rate) and 1 percent for various administrative costs leaves 2 percent as an estimate of the annual risk premium. Using a binomial representation of the various components of cash flows, and using the parameterization described here, the discount rate is adjusted for the price of market risk. (See Lucas and Moore, 2006, for a more detailed description of the risk adjustment.)

B.6 Aggregate Cost of Consolidation

The forward-looking measure of consolidation cost is the sum of the costs of consolidation at origination for a representative cohort of loans issued in each year of the program. The cost of consolidation at origination of any individual loan in a cohort is:

- The present value of government cash flows under a loan originated when the borrower is eligible to consolidate; less
- The present value of cash flows under a loan originated where the borrower is not eligible to consolidate but may be eligible for term extension.

Each year's cohort of originated loans comprises direct and guaranteed loans, loans from borrowers with different total balances (determining their eligibility for various loan maturities

[27] Students with private loans have more debt, because they usually exhaust the limit on government borrowing before turning to the more expensive private market. This may also lower recovery rates for private lenders. Conversely, the students most likely to exceed federal limits—those at costly private schools or pursuing professional degrees—are more likely to obtain well-paying jobs, lowering the expected default rate.

under consolidation and term extension), and loans with different repayment start times based on how close to graduation the borrower is. In each year, and roughly consistent with data, we assume that the fraction of volume originated is 75 percent in the guaranteed program and 25 percent in the direct loan program. For guaranteed loans, the assumed distribution of loan balances is such that:

- 30 percent of originated volume is not eligible for any term extension. The borrower must pay off the loan over the original 10-year term.
- 30 percent of originated volume is eligible for 20-year term extension if consolidation is available. In the counterfactual where consolidation is not available, borrowers are not eligible for term extension.
- 40 percent of originated volume is eligible for 30-year term extension if consolidation is available. In the counterfactual, a term extension of 25 years is available.

For the direct program, borrowers have a symmetric opportunity to extend the maturity of their loans with and without consolidation:

- 30 percent of originated volume is not eligible for any term extension.
- 30 percent of originated volume is eligible for 20-year term extension.
- 40 percent of originated volume is eligible for 30-year term extension.

In a given year, repayment of each loan originated begins only after the borrower finishes school. We assume that the loans originated have repayment times uniformly distributed between one and five years. The aggregate cost of consolidation is the sum of the individual costs of consolidation across both direct and guaranteed programs, categories of maturity extension eligibility, and repayment start times.

For the intrinsic value calculations, we compute the cost of consolidation at the time the loans are consolidated. For simplicity, we assume the distribution of loan amounts and eligibility for term extension is the same as for the forward-looking estimates. However, we no longer treat term extension in the absence of consolidation as stochastic; instead, we simply assume that borrowers take advantage of any term extension available, both in fact and in the counterfactual.

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