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Putty-Clay Capital and an Index of Capital per Hour

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Abstract

This paper presents an index of capital per hour constructed using the assumption of putty-clay capital. That index is based on an aggregate putty-clay production function that both preserves the desirable empirical properties of putty-clay models and yields a closed-form solution for aggregate investment as a function of aggregate variables. The key to the production function is an index of capital per hour that is a logarithmic average of the ratios of capital to labor embodied in each unit of capital. That index separates out the capital-deepening portion of investment (more capital per worker) from the capital-broadening portion (more capital to employ more workers). The capital-deepening portion is identified using the cost of capital. The putty-clay index of capital per worker shows stronger growth during 2001-2004 than a putty-putty index, making the unusually strong growth of labor productivity during that period somewhat less of a puzzle.

Unless otherwise noted, the figures and tables in this paper were created by the author.

1 Introduction

Productivity has grown rapidly in the United States in recent years. From the first quarter of 2001 to the fourth quarter of 2004, output per hour in the nonfarm business sector grew at an annual rate of 4.1 percent, roughly double its 2.0 percent average rate of growth during the preceding 40 years. Although productivity often grows at above-average rates early in business-cycle expansions, its robust increase in recent years has been unusual (Congressional Budget Office, 2004, pp. 26-27). Researchers have suggested various explanations, including a delayed effect as firms learned how to use new technologies first purchased in the late 1990s (Basu, Fernald, and Shapiro, 2001).

Such rapid productivity growth is even more unusual because it has coincided with unusually weak investment in new plant and equipment. Nonresidential fixed investment averaged 10.4 percent of gross domestic product (GDP) from the second quarter of 2001 to the fourth quarter of 2004, below the 11.1 percent of GDP it averaged during the preceding 40 years despite an upward trend over time.

The impact of investment on productivity depends on whether the investment goes toward pure replacement demand, i.e., replacing depreciating capital with identical units; capital deepening, i.e., increasing the amount of capital used by an individual worker; or toward capital broadening, i.e., increasing the stock of capital to accommodate additional workers. Although both capital deepening and capital broadening augment the economy's capacity to produce output, capital deepening raises productivity while capital broadening does not.

The putty-putty model of capital traditionally used in growth accounting (e.g., Solow, 1956) offers little guidance on how to split investment into its capital-deepening and capital-broadening components. Under the putty-putty assumption, also called ex post variable proportions, businesses can vary the ratio of capital to labor both before and after capital is put in place. Implicitly, all capital is always fully utilized. If one uses actual labor hours as the labor input to production, as does the Bureau of Labor Statistics in its measure of multifactor productivity (U.S. Department of Labor, 1997), then one may misinterpret a cyclical rise in labor hours that reduces the ratio of capital to labor hours as a reduction in capital deepening.[1] On the other hand, if one corrects for this problem by using cyclically adjusted labor hours as the input to production, as the Congressional Budget Office (CBO) does in its projections (2001), one may misinterpret a cyclical rise in investment that raises capital per cyclically adjusted labor hour as an increase in capital deepening.

[1] Firms can adjust labor hours more rapidly than they can adjust the capital stock, so an increase in demand initially raises the ratio of labor hours to capital. In the putty-putty model, this reduces capital per labor hour, implying less capital deepening. The economic consulting firm Macroeconomic Advisers (2001) offsets the implied reduction in productivity by enhancing the normal procyclicality of total factor productivity, which is equivalent to abandoning the putty-putty assumption.

The putty-clay model of capital, originally developed by Johansen (1959), Solow (1962), and Phelps (1963), allows us to separately identify capital broadening and capital deepening in a way that is not distorted by the business cycle. Under the putty-clay assumption, also called ex ante fixed proportions, a business can choose among a wide variety of possible ratios of capital to labor before capital is ordered (putty) but cannot alter that ratio once capital is put in place (clay). The dependence of the capital-labor ratio on the cost of capital allows us to use the cost of capital to identify capital deepening, leaving capital broadening as a residual. As an added virtue, empirical estimates of the impact of the cost of capital on investment, such as those surveyed by Chirinko (1993), are more consistent with putty-clay capital than with putty-putty capital.

Past research has generally assumed that the production of individual units of capital, or "machines," is independent. That assumption has limited the usefulness of putty-clay models. Instead, this paper relies on an aggregate putty-clay production function. The key to that production function is an aggregate index of capital per worker equal to the logarithmic average of the ratios of capital to labor across all machines. Holding the ratio of capital to labor embodied in each machine constant over the machine's service life guarantees that investment retains the useful empirical properties of other putty-clay models. At the same time, the use of an aggregate index of capital per worker makes total factor productivity (TFP) of new capital equal to that for all capital, eliminating the problem in many putty-clay models that the TFP of new capital is independent of that of old capital and thus unobservable from aggregate data.

The production function has other important implications for investment not examined empirically in this paper. The accelerator depends on growth in output minus growth in productivity, rather than on growth in output alone. Replacement demand depends on changes in the cost of capital since the depreciating capital was installed. Investment depends on the expected future growth rate of productivity.

This paper focuses on the construction of an index of capital per labor hour. That index allows us to distinguish capital deepening from capital broadening in a way not distorted by the business cycle and thus to obtain a better measure of the impact of investment on labor productivity. I find that capital deepening has remained strong since the recession in 2001 despite the sharp slowdown in investment. That helps make the rapid growth of productivity during 2001-2004 somewhat less of a puzzle.

Section 2 presents a basic version of the putty-clay model to illustrate its basic workings and its implications for investment. Section 3 makes several modifications to the basic model to make it more realistic, allowing for varying utilization, multiple types of capital, changes in the capital intensity of the production function, time to build, taxes, a distribution of service lives, and a different treatment for inventories than for fixed capital. Section 4 discusses two other potential modifications not included in the model: permanent changes in machines used per labor hour and endogenous service lives of capital. Section 5 discusses the data used to construct the index of capital per hour. Section 6 discusses the resulting index and its implications for productivity. Section 7 presents conclusions.

2 The Basic Model

This section presents a basic version of the putty-clay model to illustrate how it works. That version uses several simplifying assumptions that are relaxed later in the paper. There are no taxes. The rate of return is expected to remain at its current level. There is a single type of capital. There is no time to build capital, and there are no costs of adjustment, so the number of machines always equals the number of employed workers.

The model employs several assumptions that are not unique to putty-clay models. I assume that production can be represented by a single production function producing one intermediate good. However, in order for the price of capital goods to differ from the average price of all output produced, I assume that the single intermediate good can then be turned into a range of final goods and services whose average price is equal to the price of the intermediate good. The amount of each of those goods and services that can be produced from the intermediate good is inversely proportional to its price. I assume that all workers are equally productive.

Firms are competitive and thus price-takers. All firms are identical, so that the profit-maximization decision can be analyzed using the economywide production function. The production function exhibits constant returns to scale.

Several additional assumptions are specific to the putty-clay model. I assume that the capital stock is composed of individual units I will call "machines." Ultimately, the labor input into production is proportional to aggregate hours worked, so the index described in this paper is referred to as an index of capital per hour. However, for expositional purposes, it is easier to describe the labor input as being composed of individual workers. Hence, the actual labor input to production is full-time-equivalent workers, or hours worked per year divided by 2000. In the basic model, each such worker uses one machine. As discussed in the next section, short-run changes in hours have the same effect on production whether they come from changes in employment or changes in the workweek.

Firms choose the quality of new machines on the basis of prices, output per worker, and expectations at the time they are purchased. Although the productivity of a machine can change over time with overall technology and with the quality of other machines being used, the quality of each machine and its associated labor input remain unchanged during its service life.

Because the machines are integral units, investment is assumed to take place in discrete periods. However, to make the math easier, production is assumed to be continuous.

2.1 The Production Function

At time t, the production function is

$$Y_t = A_t L_t \bar{k}_t^{\alpha}, \tag{1}$$

where Y is total output, A is total factor productivity, L is the number of workers, and $\bar{k}$ is the index of capital per hour. The workweek is constant over time, so each machine is used a fixed number of hours. Technical progress is Hicks-neutral, affecting the productivity of labor and capital equally. The production function in equation 1 is no different from the standard putty-putty production function.[2] The key difference between putty-clay and putty-putty lies in the specification of the capital input.

Let the workers at time t and the machines they use be indexed 1 to L_t. I assume that the aggregate index of capital per worker is a logarithmic average of the quality, or size, of each machine k_i used by worker i at time t. Thus,

$$\log\left(\bar{k}_t\right) = \sum_{i=1}^{L_t} \frac{\log\left(k_i\right)}{L_t}, \tag{2}$$

or

$$\bar{k}_t = k_1^{1/L_t} k_2^{1/L_t} \ \ldots \ k_{L_t}^{1/L_t}.$$

Using this latter expression, we can rewrite the production function as

$$Y_t = A_t L_t k_1^{\alpha/L_t} k_2^{\alpha/L_t} \ \ldots \ k_{L_t}^{\alpha/L_t}.$$

From that equation, one can see that the putty-clay production function is actually a huge Cobb-Douglas production function with each machine as a separate input to production.

Since the determinants of the quality of each machine of a single vintage are the same, we will instead differentiate machines by the time at which they were installed. In that case, the index of capital per worker can be expressed as

$$\log\left(\bar{k}_t\right) = \sum_{i=0}^{S-1} \frac{N_{t-i} \log\left(k_{t-i}\right)}{\sum_{i=0}^{S-1} N_{t-i}}, \tag{3}$$

where S is the service life of capital and N_{t-i} is the number of machines put in service at time $t-i$. With full employment, the total number of machines in service at time t $\sum_{i=0}^{S-1} N_{t-i}$ equals the number of workers L_t.

[2] If we define K_t as $\bar{k}_t L_t$, then $A_t L_t \bar{k}_t^{\alpha} = A_t L_t^{1-\alpha} K_t^{\alpha}$.

Note that the production function exhibits constant returns to scale. If the number of workers doubled, then a doubling of the number of each type of machine would double the amount of capital being used, but $\bar{k}$ would remain constant. From equation 1, it is obvious that output would also double.

2.2 Quality of New Machines

To determine the quality of machines of different vintages, we must examine a business's investment decision. The present discounted value of the portion of a firm's profits that depends on the quality of new machines over their service life, multiplied by the number of firms, is

$$\pi n_t = \int_{i=0}^{S} p_{t+i}\ Y_{t+i}\ e^{-({}_t r +\ {}_t \dot{p}) i}\ di - N_t\ q_t\ k_t, \tag{4}$$

where p is the price of output, ${}_t r$ is the expected real rate of return over the service life of capital put in place at time t, ${}_t \dot{p}$ is the expected rate of increase of output prices over the service life of capital put in place at time t, and q is the price index of capital, or price per unit of quality. The price of a new machine is thus the price index of capital times the quality of new capital $q_t k_t$. Each firm chooses the quality of new machines k_t to maximize profits.

Maximizing profits with respect to quality of new machine k_t yields first-order conditions:

$$0 = \alpha \int_{i=0}^{S} p_{t+i}\ \frac{Y_{t+i}}{\bar{k}_{t+i}} \frac{N_t}{\sum_{j=0}^{S-1} N_{t+i-j}} \frac{\bar{k}_{t+i}}{k_t}\ e^{-(r +\ {}_t \dot{p}) i}\ di - N_t\ q_t. \text{[3]} \tag{5}$$

Let y denote output per worker, and let ${}_t \dot{y}$ denote the expected rate of growth of output per worker over the service life of capital put in place at time t. Then, because the number of machines in service equals the number of workers, $y_{t+i} = Y_{t+i} / \sum_{j=0}^{S-1} N_{t+i-j}$. After solving out the integral and rearranging terms, equation 5 can be rewritten as

$$k_t = \frac{\alpha\ p_t\ y_t}{v_t}, \tag{6}$$

where

$$v_t = q_t\ \frac{r -\ {}_t \dot{y}}{1 - e^{-(r -\ {}_t \dot{y}) S}}. \tag{7}$$

The quality of new machines is proportional to output per worker y, to capital's coefficient in the production function α, and to the ratio of the price of output to the

[3] The quality of new machines purchased today influences the quality of new machines purchased tomorrow and so has a second-order effect on future output. However, by the envelope theorem, that effect does not affect profits and so can be ignored.

cost of capital v. Quality is negatively related to the real rate of return and positively related to the expected rate of growth of output per worker and to the service life. A higher nominal rate of return increases the discounting of future revenues from a machine, while higher rates of growth of output prices and output per worker raise those revenues. The more rapid the expected rate of growth of output per worker, the more a marginal unit of quality contributes to future output, and thus the greater is the quality of new machines. The expression for the cost of capital in equation 7 is similar to that in Ando, Modigliani, Rasche, and Turnovsky (1974), except for the inclusion of expected growth in productivity.

2.3 Investment

Real investment I equals the quality of new machines times the number of new machines:

$$I_t = N_t \; k_t. \tag{8}$$

In this basic model, the number of new machines equals growth in the number of workers plus the number of machines discarded, or

$$N_t = \Delta L_t + N_{t-S}. \tag{9}$$

We can also invert equation 8 to solve for the number of depreciating machines N_{t-S} in terms of investment and machine quality in period $t - S$

$$N_{t-S} = \frac{I_{t-S}}{k_{t-S}} = \frac{v_{t-S} \; I_{t-S}}{\alpha \; p_{t-S} \; y_{t-S}} \tag{10}$$

Substituting equations 9 and 10 into equation 8, real investment is

$$I_t = \left(\Delta L_t + \frac{I_{t-S}}{k_{t-S}} \right) \; k_t. \tag{11}$$

Using the definition of y, we can also express real investment as

$$I_t = \left(\Delta \left(\frac{Y_t}{y_t} \right) + \frac{I_{t-S}}{k_{t-S}} \right) \; k_t. \tag{12}$$

Substituting for k from equation 6, real investment can also be expressed as

$$I_t = \Delta \left(\frac{Y_t}{y_t} \right) \frac{\alpha \; p_t \; y_t}{v_t} + I_{t-S} \frac{p_t \; y_t \; / v_t}{p_{t-S} \; y_{t-S} \; / v_{t-S}},$$

where v_t is defined by equation 7.

This basic model illustrates several properties of investment in a putty-clay world:

- Investment depends positively on growth of the number of workers employed (from equation 11), or equivalently on growth of output in excess of productivity (from equation 12).
- Nonetheless, investment depends positively on the level of productivity, which is a determinant of the size of new machines.
- Replacement demand is not simply equal to the original value of the machines currently depreciating (I_{t-S}), but also takes account of growth in the desired size of new machines since the machines currently depreciating were put in place (k_t/k_{t-S}).
- If investment has varied in the past, the replacement component of current investment will exhibit replacement cycles, peaking S periods after past peaks of investment.
- As in the standard investment model, investment is inversely related to the real rate of return, and real investment is inversely proportional to the ratio of the price of new capital to the price of output.
- Investment depends positively on the expected rate of future productivity growth.

3 Enhancements to the Basic Model

Estimating an index of capital per worker for the U.S. economy requires several modifications of the basic model just presented. The number of hours a machine is used per year may vary in the short run, and the number of machines utilized may differ from the total number of machines. There are many different kinds of capital, and their relative importance in production varies over time. The capital intensity of production can vary over time with the importance of capital-intensive sectors. Because of adjustment costs and the time it takes to plan and build new capacity, the actual capital stock rarely equals the desired capital stock. Retirements of a given vintage of capital do not all occur simultaneously at a fixed amount of time after installation. The cost of capital must take account of taxes. Finally, inventories must be handled differently than other capital.

3.1 Utilization of Machines

In the real world, the number of machines utilized is generally smaller than the total number of machines. To address that issue, I normalize the quality of machines so that, on average, the number of machines equals the number of full-time-equivalent workers. Alternatively, I could normalize the quality of machines so that, on average, the number of utilized machines equals the number of full-time-equivalent workers.

Neither desired investment nor the index of capital per hour would be affected by that change.

Given that firms rarely use all their capital, which machines are idle? I assume that the distribution of machines used is identical to the distribution of all machines, and thus that capital per worker for utilized machines is identical to capital per worker for all machines. That assumption differs from the more common assumption in putty-clay models, e.g., Solow (1962) and Gilchrist and Williams (1998), that firms use only the most productive capital available. Although that assumption is perfectly reasonable, it implies that, all else equal, capital per worker should be countercyclical. In fact, however, labor productivity is procyclical. Incorporating the assumption that firms only use their most productive capital just makes it harder to explain the procyclicality of TFP.

The amount of output produced by a given stock of capital may vary in the short run for reasons other than changes in TFP. Because capital adjusts more slowly to demand shocks than does employment, demand shocks will cause procyclical variations in the ratio of workers to machines and thus in output per machine. For a given number of workers, an increase in the average workweek will also increase output per machine. Firms may also raise the output of a given stock of capital by increasing the number of shifts.

In each of those cases, output varies proportionately with hours worked in the short run. So, I assume that the labor input into production L_t is the number of full-time-equivalent workers and that variations in the workweek and the number of shifts have the same impact on production as variations in the number of workers. I also assume that firms expect all variations in the workweek and in the number of shifts to be temporary. The next section explores the potential impact of relaxing that assumption below.

3.2 More Than One Type of Capital

In the real world, there are many broad types of tangible capital—equipment and software, nonresidential structures, residential structures, land, and inventories—as well as intangible capital. Within most of those broader categories, in turn, there are many different kinds of capital. This paper focuses on the nonresidential tangible capital stock used by the nonfarm business and nonprofit institution sectors. Thus, the paper analyzes four broad categories of capital: equipment and software, nonresidential structures, (nonresidential) land, and inventories, indexed below as J. Equipment and software and nonresidential structures are further broken into numerous subcategories, indexed below as Mj for broader category j.[4]

[4] The Bureau of Economic Analysis also publishes data on several different kinds of inventory. This paper looks at the overall stock of real inventories and does not examine the effect of changes in the composition of those inventories on capital per hour.

For simplicity of exposition, I assume that each worker uses one "machine" of each of the broad categories: one piece of equipment or software, one structure, one plot of land, and one unit of inventory. In reality, at a point in time, several workers operate one airplane, while a worker who uses software also uses the computer running that software. However, the model would behave identically if each worker used some fraction of several different machines, as long as those fractions stayed constant during the service life of that machine.

For machines of type m with service life Sm, the logarithm of capital per worker is

$$\log\left(\bar{k}m_t\right) = \sum_{i=0}^{Sm-1} \frac{Nm_{t-i}\log\left(km_{t-i}\right)}{\sum_{i=0}^{Sm-1} Nm_{t-i}}, \tag{13}$$

where Nm_{t-i} is the number of machines of type m put in place in period t and km_{t-i} is the quality of those machines. We can then express the overall index of capital per worker as

$$\log\left(\bar{k}_t\right) = \sum_{j\in J}\left[\frac{\alpha j_t}{\alpha_t}\sum_{m\in Mj}\frac{Nkm_t}{\sum_{m\in Mj} Nkm_t}\log\left(\bar{k}m_t\right)\right], \tag{14}$$

where Nkm_t is the total number of machines of type m at time t, $\sum_{i=0}^{Sm-1} Nm_{t-i}$.

Equation 14 only holds at a point in time. If the relative importance of different types of capital changes, i.e., if the αj or the $Nkm_t/\sum_{m\in Mj} Nkm_t$ vary over time, equation 14 cannot be used to calculate a time series for $\bar{k}$. The reason is obvious if we consider shifts in the relative importance of two types of machine, one for which $\bar{k}m$ grows more rapidly than for the other because of different rates of growth in the prices of the two types of machine. Depending on the year used to index the prices used to calculate $\bar{k}m$, a shift in relative importances in the two types of machines could add or subtract from the growth rate of the overall index of capital per worker.

The relative usage of different types of machines changes over time because firms have the choice of many different production technologies given the same stock of capital. At the margin, firms switch from one technology to another when the output from the two technologies, given the existing stock of capital, is the same. Thus, in calculating capital per worker, I assume that changes in the distribution of machines and in the importance of capital in production (changes in α_t) have no effect on output at the time they occur.

To keep changes in the mix of machines and the importance of capital in production from affecting output, we must add an extra "transition" term to equations 1 and 14. Equation 1 becomes

$$\log\left(Y_t\right) = \log\left(A_t\right) + \log\left(L_t\right) + \alpha_t\log\left(\bar{k}_t\right) + tranY_t. \tag{15}$$

If the change in output is

$$\Delta\log\left(Y_t\right) = \Delta\log\left(A_t\right) + \Delta\log\left(L_t\right) + \alpha_{t-1}\Delta\log\left(\bar{k}_t\right),$$

then

$$\Delta tranY_t = -\Delta\alpha_t \log\left(\bar{k}_t\right).$$

Likewise, equation 14 becomes

$$\log\left(\bar{k}_t\right) = \sum_{j\in J}\left[\frac{\alpha j_t}{\alpha_t}\sum_{m\in Mj}\frac{Nkm_t}{\sum_{m\in Mj} Nkm_t}\log\left(\bar{k}m_t\right)\right] + tranK_t. \tag{16}$$

If the change in the index of capital is

$$\Delta\log\left(\bar{k}_t\right) = \sum_{j\in J}\left[\frac{\alpha j_{t-1}}{\alpha_{t-1}}\sum_{m\in Mj}\frac{Nkm_{t-1}}{\sum_{m\in Mj} Nkm_{t-1}}\Delta\log\left(\bar{k}m_t\right)\right],$$

then

$$\begin{aligned}\Delta tranK_t \quad = \quad & -\sum_{j\in J}\left[\Delta\left(\frac{\alpha j_t}{\alpha_t}\right)\sum_{m\in Mj}\frac{Nkm_t}{\sum_{m\in Mj} Nkm_t}\log\left(\bar{k}m_t\right)\right] \\ & -\sum_{j\in J}\left[\frac{\alpha j_{t-1}}{\alpha_{t-1}}\sum_{m\in Mj}\Delta\left(\frac{Nkm_t}{\sum_{m\in Mj} Nkm_t}\right)\log\left(\bar{k}m_t\right)\right].\end{aligned}$$

The change in the overall logarithmic index of capital per worker is a weighted average of the change in the logarithmic indexes of capital per worker for each type of capital.

3.3 Changes in the Mix of Goods Produced

The importance of capital in production can change either because of general changes in the importance of capital in production, as discussed above, or because the mix of output shifts toward goods and services whose production is more or less capital-intensive than average.[5] An increase in the importance of capital in production produces an increase in the desired quality of new machines, while a shift in the mix of output changes the number of machines used per worker. The latter option is particularly useful for modeling the mining sector, which uses much more capital per worker than other sectors. A desire to raise mining output, for example when oil prices are high, results in an increase in the number of mining "machines" rather than a generalized increase in investment in all capital.

[5] Strictly speaking, shifts in the mix of output are impossible in the one-good model presented in this paper. However, the actual modification introduced is not inconsistent with the model, and it helps capture a genuine feature of the economy.

Let xj denote the number of excess machines per worker for broad category of capital j due to output of the capital-intensive sector. For example, if mining uses 100 percent more nonresidential structures per dollar of output than other sectors and mining accounts for 5 percent of output, then xj for nonresidential structures is 1.00*0.05, or 0.05. Equation 16 becomes

$$\log\left(\bar{k}_t\right) = \sum_{j \in J} \left[\frac{\alpha j_t}{\alpha_t} \left(\log\left(1 + xj_t\right) + \frac{\sum_{m \in Mj} Nkm_t \log\left(\bar{k}m_t\right)}{\sum_{m \in Mjx} Nkm_t} \right) \right] + tranK_t, \quad (17)$$

where Mjx excludes mining. The $(1 + xj_t)$ term causes capital per worker $\bar{k}_t$ to increase as xj_t rises, i.e., as the number of machines per worker rises. The exclusion of mining machines from the denominator of the second term on the right-hand side magnifies the impact of an increase in the individual $\bar{k}m_t$ on overall $\bar{k}_t$ because an increase in the number of machines of type j per worker raises proportionately the impact of an increase in the quality of all machines of type j. The change in the index of capital is

$$\Delta \log\left(\bar{k}_t\right) = \sum_{j \in J} \left[\frac{\alpha j_{t-1}}{\alpha_{t-1}} \left(\Delta \log\left(1 + xj_t\right) + \frac{\sum_{m \in Mj} Nkm_{t-1} \Delta \log\left(\bar{k}m_t\right)}{\sum_{m \in Mjx} Nkm_{t-1}} \right) \right], \quad (18)$$

and the equation for $tranK_t$ changes correspondingly.

3.4 Time to Build

The basic model assumes that investment depends on changes in the desired number of machines and their cost at the time that capital is put in place. In reality, important lags exist between the time at which investment is determined and the time it takes place, collectively called "time to build," following Kydland and Prescott (1982). It takes time for businesses to determine the need for more capital and to plan for how to meet that need. The existence of unfilled orders for capital goods shows a gap between the time that such plans are made and the time equipment is delivered. For structures, the time between ground-breaking and final completion is considerable. Investment will also lag behind the changes in demand driving it if capital is “lumpy,” i.e., if businesses do not adjust their capital stock until some threshold level of adjustment is required.[6] Lags also may be added to the investment process if integrating several new factories into production during a short period of time is more costly than integrating those same factories into production over a longer period.

As a first step, assume that the interval between the beginning of the planning stage and the final delivery of capital—the time to build—is T periods and that the economy

[6] For evidence that capital is lumpy, see Cooper, Haltiwanger, and Power (1999) and Doms and Dunne (1998).

is always at full employment. Then the right-hand-side variables in equation 6 for the quality of capital are expectations at time $t-T$.

The number of new machines is also determined by expectations at time $t-T$. The total number of machines at time t, Nk_t will equal the time $t-T$ expectation of the number of workers at time t

$$Nk_t = {}_{t-T}L_t.$$

Let η represent expected growth in workers per period. Then

$${}_{t-T}L_t = \eta^T L_{t-T}.$$

Substituting the equation for ${}_{t-T}L_t$ into the equation for Nk_t and taking first differences, we have

$$\Delta Nk_t = \eta^T \Delta L_{t-T}.$$

The number of new machines at time t, N_t, then equals the desired change in machines plus retirements at time t (which I assume are known at time $t-T$), or

$$N_t = \eta^T \Delta L_{t-T} + N_{t-S}. \tag{19}$$

Now assume that the economy is not always at full employment. In addition, assume that the expected gap between actual employment and full employment $\bar{L}$ is a linear function of the current gap between actual employment and full employment, i.e.,

$${}_{t-T}L_t - {}_{t-T}\bar{L}_t = \theta\ \eta^T \left(L_{t-T} - \bar{L}_{t-T}\right),$$

where $\theta < 1$. Also, let

$${}_{t-T}\bar{L}_t = \eta^T \bar{L}_{t-T}.$$

Then equation 19 becomes

$$N_t = \eta^T \left(\theta \Delta L_{t-T} + (1-\theta)\Delta \bar{L}_{t-T}\right) + N_{t-S}.$$

Finally, assume that the number of machines put in place at time t is determined by expectations formed during periods $t-T+1$ through t. Then, as a rough approximation, we can say that

$$N_t = \bar{\eta} \left(\theta \frac{L_t - L_{t-T}}{T} + (1-\theta)\frac{\bar{L}_t - \bar{L}_{t-T}}{T}\right) + N_{t-S}, \tag{20}$$

where $\bar{\eta}$ is close to an average of the η^{t+i}.[7]

[7] In equation 20, N is expressed at the same frequency as T. So if investment is measured at an annual rate but the data are quarterly, the denominators of the fractions become $T/4$.

3.5 Retirements of Capital

The basic model assumes that all capital of a given vintage is retired at the same time, a fixed period after it is purchased. In the real world, service lives vary by type of capital and are spread over a long period of time.

I follow the treatment used by the Bureau of Labor Statistics (BLS). BLS assumes that retirements of a given vintage of fixed capital follow a truncated normal distribution centered on the expected service life and ranging from 0.02 to 1.98 times that service life (U.S. Department of Labor, 1983). BLS chose the value 0.98 because Hulten and Wykoff (1981a and 1981b) found that some assets are considerably older than the service lives estimated by the Bureau of Economic Analysis (BEA), and also to take account of assets that are accidentally destroyed when new. In BLS's formula, the standard deviation of the density function is 0.49 times the service life.

3.6 Taxes and the Cost of Capital

Corporate taxation affects the costs of all categories of fixed capital. In addition, state and local property taxes are levied on land and structures.[8] Each broad type of capital has distinct features that require separate treatment.

3.6.1 Equipment and Software

Firms choose the quality of new equipment machines of type m, km, that maximizes expected after-tax profits. The present discounted value of the portion of after-tax profits that depends on the quality of new machines of type m purchased at time t is

$$\pi m_t = \int_{i=0}^{Sm_t} (1 - u_{t+i})\, p_{t+i}\, Y_{t+i}\, e^{-({}_t rm + {}_t\dot{p}m)i}\, di - Nm_t\, qm_t\, km_t\, (1 - Cm_t - Zm_t^u)\,, \tag{21}$$

where Sm_t is the average service life of capital of type m put in place at time t, u is the combined federal and state and local tax rate on corporate income, ${}_trm$ is the expected real rate of return over the service life of capital of type m installed at time t, ${}_t\dot{p}m$ is the expected rate of output price inflation over the service life of capital of type m installed at time t, and Cm is the investment tax credit for new type-m capital. The present discounted value of depreciation allowances per dollar of capital

[8] Indirect taxes levied on output fall on both capital and labor, and thus I do not consider them as part of the cost of capital.

of type m installed at time t, Zm^u_t, is

$$Zm^u_t \equiv \int\limits_{i=0}^{Sm_t} u_{t+i}\ Dm_{t,i}\,(1 - Bm_t\ Cm_t)\,e^{-({}_t rm +\ {}_t\dot{p}m) i}\ di, \tag{22}$$

where $Dm_{t,i}$ is the share of depreciation allowances taken i years after time of purchase t, and Bm is the share of the investment tax credit that is deducted from the allowable base for depreciation.

To simplify the first-order conditions for maximization of 21, I assume that the expected rate of corporate taxation equals the current rate, so that u_t replaces u_{t+i}. Rearranging the first-order conditions as in the basic model, we find that quality of new machines of type m is

$$km_t = \frac{\alpha j_t\ p_t\ \bar{y}_t}{vm_t}, \tag{23}$$

where

$$vm_t \equiv qm_t \frac{{}_t rm - {}_t\dot{y}m}{1 - e^{-({}_t rm - {}_t\dot{y}m) Sm_t}} \frac{1 - Cm_t - u_t\ Zm_t}{(1 - u_t)}, \tag{24}$$

$$Zm_t \equiv \int\limits_{i=0}^{Sm_t} Dm_{t,i}\,(1 - Bm_t\ Cm_t)\,e^{-({}_t rm +\ {}_t\dot{p}m) i}\ di,$$

$\bar{y}_t$ is cyclically adjusted output per worker, αj_t is the coefficient of equipment and software in the production function, and ${}_t\dot{y}m$ is the expected rate of growth of output per worker y during the service life of type-m capital installed at time t.[9] I use cyclically adjusted output per worker to remove variations in output per worker that firms know are temporary and thus ignore in making their investment decisions. Adding time to build turns the right-hand side of equation 23 into a moving average.

For equipment, equation 24 must be modified to account for scrappage value, which BEA counts as a subtraction from investment in equipment. Assuming that the present discounted scrappage value as a fraction of the purchase price is Sc, equation 24 becomes

$$vm_t \equiv qm_t \frac{{}_t rm - {}_t\dot{y}m}{1 - e^{-({}_t rm - {}_t\dot{y}m) Sm_t}} \left(\frac{1 - Cm_t - u_t\ Zm_t}{(1 - u_t)} - Sc \right). \tag{25}$$

I use equation 24 for software, which has no scrappage value.

[9] I assume firms expect the production function coefficient αj_t to remain constant in the future. If one relaxed that assumption, the $-{}_t\dot{y}m$ in equation 24 would become $-{}_t\dot{y}m -_t \dot{\alpha} j$.

3.6.2 Land

The fact that land does not depreciate but the structures on it do means that land per worker is not permanently set at the rate at which the land is first put into service but can change whenever an old structure depreciates and a new structure replaces it. When land is first put into service, the expected profit function for that land during the service life of the structure with which it is put into service is

$$\begin{aligned} \pi l_t &= \int_{i=0}^{Sl} (1-u_{t+i})\, p_{t+i}\; Y_{t+i}\; e^{-({}_trl+\;{}_t\dot{p}l)i}\; di - Nl_t\; ql_t\; kl_t\, (1-Cl_t - Zl_t^u) \\ &\quad - \int_{i=0}^{Sl} ux_{t+i}\, (1-u_{t+i})\, Nl_t\; ql_t\; kl_t\; e^{\Phi l*i}\; e^{-({}_trl+\;{}_t\dot{p}l)i}\; di, \end{aligned} \tag{26}$$

where the suffix l denotes land, ux is the property tax rate (multiplied by $1-u_{t+i}$ because property taxes are deductible against corporate income taxes), and Φl is the rate of price appreciation of land over its service life.

Equation 26 must be modified to take account of land's infinite service life. I assume that land is depreciated for tax purposes only when it is first put into service. After its first use, firms expect the marginal contribution of land to nominal revenues to rise by the expected increase in the price of land, as in a putty-putty model, rather than by the expected increase in nominal output per worker. Thus, the variable ${}_t\dot{y}l$ is a function of expected growth in productivity during the service life of the first structure and expected growth in the real price of land thereafter. To simplify the expression for the cost of land, I assume that ${}_t\Phi l$ equals ${}_t\dot{y}l +_t \dot{p}l$.

Solving the first-order conditions, we find that the cost of capital for land is

$$vl_t \equiv ql_t\;\; ({}_trl - {}_t\dot{y}l + \bar{u}x_t * (1-u_t))\, \frac{1-Cl_t - u_t\; Zl_t}{(1-u_t)}, \tag{27}$$

where $\bar{u}x_t$ is the expected property tax rate. Without the depreciation term, the cost of land would be the real rate of return less real growth of the land's productivity plus property taxes.[10]

3.6.3 Structures

In calculating the expected value of property taxes, I assume that firms expect the assessed value of a structure to rise with nominal output per worker but fall proportionately with the number of years remaining in its service life. Thus, in equation 26, the term $e^{\Phi m*i}$ is replaced by $(1-i/Sm_t)\, e^{({}_t\dot{y}+{}_t\dot{p})i}$. In addition, to account for brokers' commissions, I assume that purchasers of new structures pay brokers a

[10] The cost of capital for land is only used to calculate the importance of land in production αj. I calculate the index of nonresidential land per worker using data from BLS.

commission at rate b, which is the same for all structures. The cost of capital for structures is then

$$\begin{aligned} vm_t &= (1+b)\, qm_t \frac{{}_t rm - {}_t\dot{y}m}{1-e^{-({}_t rm - {}_t\dot{y}m)Sm_t}} \frac{1 - Cm_t - u_t\ Zm_t}{(1-u_t)} \\ &\quad + (1+b)\, qm_t \left(\frac{1}{1-e^{-({}_t rm - {}_t\dot{y}m)Sm_t}} - \frac{1}{Sm\,({}_t rm - {}_t\dot{y}m)} \right) \bar{u}x_t. \end{aligned} \tag{28}$$

3.7 Inventories

Because inventories are held only for a short period of time, it is tempting to assume that firms can adjust the "quality" of inventories, i.e., the desired level of inventories held per unit of output, very quickly. However, a look at the inventory-sales ratio suggests that, once one removes the effects of the business cycle on sales, the ratio responds very slowly, if at all, to changes in interest rates, which are the cost of holding inventories. Instead, the desired level of inventories per unit of output is a function of existing technology and capital and does not respond quickly to changes in the rate of return.

Assume that each year, a fraction $1/Sj$ firms set the inventory to output ratio $\tilde{k}y$ that they will use for Sj years. Then those firms choose $\tilde{k}y$ to maximize the profit function

$$\begin{aligned} \pi j_t &= \int_{i=0}^{Sj} (1-u_{t+i})\, p_{t+i}\ Y_{t+i}\ e^{-({}_t rj +\ {}_t\dot{p}j)i}\ di \\ &\quad - \int_{i=0}^{Sj} (1-u_{t+i})\, qj_{t+i}\ \tilde{k}y_t\ \frac{Y_{t+i}}{Sj}\ r_{t+i}\ e^{-({}_t rj +\ {}_t\dot{p}j)i}\ di. \end{aligned}$$

where the suffix j denotes inventories.[11] I assume that the value of inventories rises with the price of output during the time they are held, implying that the pre-tax cost of inventories is the real interest rate times the value of inventories held. Similarly, I assume that firms expect the price of inventories qj to rise at the same rate as the price of output p over the period for which the inventory technology is being chosen.

Rearranging the first-order conditions, we find that

$$\tilde{k}y_t = \frac{\alpha j_t\ p_t}{{}_t rj\ qj_t}. \tag{29}$$

The desired ratio of inventories to output is proportional to the importance of inventories in production and to the ratio of the price of output to the price of inventories,

[11] As an approximation, I assume that firms expense inventories. Also, I use $1/Sj$ as an approximation for the share of workers using the inventory technology chosen at time t. Assuming the labor force grows over time, the actual share would be somewhat larger at first and somewhat smaller just before the inventory technology is replaced.

and inversely proportional to the expected real rate of return. The index of inventories per worker $\bar{k}j$ is found by multiplying an Sj-year moving average of the desired ratio of inventories to output $\tilde{k}y_t$ by output per worker y_t.

3.8 Summary

Equation 18 is used to calculate the index of capital per worker. For equipment, software, and structures, indexes of type-m capital per worker $\bar{k}m$ are calculated using equation 13, modified so that retirements of machines installed at time t are determined by a truncated normal distribution centered on $t + S$. The indexes of machine quality km used to construct those $\bar{k}m$ are calculated using

$$km_t = \frac{1}{Tm} \sum_{i=0}^{Tm-1} \frac{\alpha \dot{j}_{t-i}\ p_{t-i}\ \bar{y}_{t-i}}{vm_{t-i}}.$$[12]

The number of new machines Nm is found by modifying equation 8 for multiple types of capital and inverting it:

$$Nm_t = Im_t / km_t,$$

where Im is real gross investment in type-m capital. The variable xj equals the number of mining-specific machines divided by the total number of nonmining-specific machines for equipment and software and for structures.[13]

The cost of capital vm_t is defined using equation 25 for equipment, equation 24 for software, and equation 28 for structures. I use BLS data to calculate land per worker, as discussed below. Inventories per worker equal cyclically adjusted output per worker $\bar{y}_t$ times a 15-year moving average of the desired ratio of inventories to output $\tilde{k}y_t$ defined using equation 29.

4 Other Potential Modifications

Two other potential modifications to the basic model were not incorporated in the index of capital per hour: permanent changes in machines used per full-time-equivalent worker and endogenous service lives of capital. This section discusses the theoretical impacts of those modifications.

[12] I assume that Tm is 4 for equipment and software and 5 for structures.

[13] Using investment data for 1997 from Meade, Rzeznik, and Robinson-Smith (2003), the mining industries use about the same amount of nonmining-specific capital per dollar of output as other industries. Thus, mining-specific investment, defined as mining and oilfield machinery and mining exploration, shafts, and wells, captures the extra capital due to the greater capital intensity of mining.

4.1 Machines Used per Full-Time-Equivalent Worker

In the short run, a 1 percent rise in the average workweek, a 1 percent rise in the number of shifts worked per machine, and a 1 percent rise in the number of workers each have the same impact on output. However, once a firm expects those changes to be permanent, they have different effects on investment. Changes in the workweek and the number of shifts per machine affect the desired quality of new machines but not the desired number, whereas changes in the number of workers affect the desired number of new machines but not their quality.

For this section, we return to the assumptions of the basic model except for the assumption that the number of machines equals the number of full-time-equivalent workers. Let W_t be the number of workers, h_t be hours per worker per year divided by 2000, and γ_t be the number of shifts, or the number of workers per machine. Then $L_t = h_t W_t$ and the number of machines in use is W_t/γ_t.

The labor input to production is the number of machines in use, or W_t/γ_t, times annual hours of usage per machine (divided by 2000), or $h_t\gamma_t$. But that is just L_t, the number of full-time-equivalent workers. So the production function is identical to that in equation 1. The index of capital per hour is identical to that in equation 3. The only difference is that now the total number of machines need not equal the number of full-time-equivalent workers.

As in the basic model, the firm desires to maximize the present discounted value of profits as given by equation 4. The first-order conditions are identical to those in the basic model, except that the desired number of machines is now $L_t/\left(h_t\gamma_t\right)$ rather than L_t. Solving for the optimal quality of new machines, we have

$$k_t = \frac{\alpha\ p_t\ y_t\ h_t\ \gamma_t}{v_t},$$

where

$$v_t = q_t\ \frac{r -\ {}_t\dot{y} -\ {}_t\dot{h} -\ {}_t\dot{\gamma}}{1 - e^{-\left(r-\ {}_t\dot{y}-\ {}_t\dot{h}-\ {}_t\dot{\gamma}\right)S}},$$

and ${}_t\dot{h}$ and ${}_t\dot{\gamma}$ are the expected exponential rates of growth of the workweek and of the number of shifts. The optimal quality of new machines is equal to that in the basic model multiplied by the intensity of usage of machines as given by $h_t\gamma_t$.

The equation for real investment becomes

$$I_t = N_t\ k_t = \left[\Delta\left(L_t/\left(h_t\gamma_t\right)\right) + N_{t-S}\right]\ k_t.$$

Note that if current and expected h and γ equal 1, then investment collapses to that in the basic model.

To illustrate the impact of those changes to the basic model, consider the impact of a permanent reduction in the workweek h_t that leaves total hours worked L_t and the number of shifts γ_t unchanged and thus raises employment W_t. (Those assumptions were presumably the intent of France's introduction of a 35-hour workweek in 1998.) The optimal quality of new machines k_t falls proportionately to the change in h_t. The shorter the period of time a machine will be used, the less the incremental revenue from using machines of higher quality. At the same time, the desired increase in the number of machines $\Delta \left(L_t / \left(h_t \gamma_t \right) \right)$ is sharply higher in period t. The net result is higher investment in period t but lower investment in subsequent periods. (In the real world, time to build would spread the initial positive impact on investment over a few years.)

In a steady state, however, the reduction in h reduces output and investment. To simplify the steady-state analysis, assume that relative prices, TFP, the real rate of return, labor hours, the workweek, and the number of shifts are all expected to remain constant. Then $k_t = \bar{k}_t$. The number of new machines each period equals the number of replacement machines, or $L_t / \left(h_t \gamma_t \right) / S$. Real investment is

$$I_t = \frac{L_t}{\left(h_t \gamma_t \right) \ S} \ k_t = \frac{\alpha \ p_t \ Y_t}{v_t \ S}.$$

If output Y was unaffected by the reduction in hours, real investment would also be unaffected. The increased number of new machines would exactly offset the reduction in the quality of new machines. But the reduction in the workweek would reduce $\bar{k}$, which would in turn reduce output and thus investment. A given dollar of investment is less effective in producing output as it is spread over more workers.

The results would be the same if we considered a reduction in the number of shifts γ instead of a reduction in the workweek. The number of machines would rise, giving a short-run boost to investment, but their average quality would decline more than proportionately in the new steady state.

I do not incorporate hours per week or the number of machines per worker in the model because the two variables are probably inversely correlated, and I have no data on the latter. An inverse correlation would occur if the downward trend in weekly hours is related to the increase in employment at retail establishments, including restaurants. Such establishments frequently use part-time workers and employ multiple shifts. Taken alone, the downward trend in weekly hours implies that the rate of growth in the number of machines is more rapid than estimated and the rate of growth in capital per hour is slower than estimated.

4.2 Endogenous Service Lives

This paper assumes that service lives are exogenous. However, service lives could be endogenous in two ways. First, the service life of a machine may depend on the number of hours it has been used. A period of increased utilization would increase wear and tear on capital and thus force firms to accelerate replacements. A period of lower utilization would allow firms to stretch out the replacement schedule.

The assumption that service lives depend on utilization would not have large impacts on the index of capital per hour. Because firms expect that utilization after installation will equal its historical average, the quality of new machines would be unchanged by assuming that service lives depend on future utilization. However, the pattern of retirements and thus the average age of the existing stock would be affected somewhat. More important would be the implications for business-cycle theory. If service lives depend on wear and tear, then the marginal cost of production is higher than if service lives are exogenous.

Second, the service life could be determined by endogenous replacement, as suggested by Feldstein and Rothschild (1974). If the optimal quality of new capital rises enough relative to that of existing capital, the differential between the productivity of new and old machines can become so great that it becomes economical for a firm to discard existing machines and replace them with new ones. In economic terms, the rental value of the old machines (which is different from the cost of capital in the putty-clay model) becomes negative. A firm would actually have to be paid to continue to use old machines rather than replace them. That is probably the situation for computers and software.

One solution, not implemented in this paper, is to use a mix of those assumptions. One might assume endogenous replacement for computers and software, service lives that vary with usage for other equipment, and exogenous service lives for structures.

5 Data

This section describes the data used to construct an index of capital per worker. Land and inventories are handled somewhat differently than equipment, software, and nonresidential structures.

5.1 Fixed Investment

Constructing an index of capital per worker requires quarterly data for investment and the price index of investment for each type of fixed capital shown in the appendix. BEA publishes nominal investment and price indexes back to 1959 for most of the required series. Real investment is obtained by dividing nominal investment by the price index. BEA classifies investment in structures differently in the years after 1997 than in the years before. I use the pre-1997 classifications and splice them with the post-1997 classifications using the 1997 data, which are available for both systems of classification. For a few series that are not available separately all the way back to 1959, such as the two categories of trucks, I apportion nominal investment in the broader category (trucks, buses, and truck trailers for the two categories of trucks) using the shares in the last year for which the components are available separately. In those cases, I also assume the price indexes of the components move proportionately.

From 1947 to 1958, annual data are available for most of the detailed types of capital, but quarterly data are only available for the broader categories listed in the appendix. Within each year, the price index and real investment of each subcategory is assumed to move in the same way as the price index and real investment for the broader category. Detailed series that are only available summed with other series are disaggregated as described in the previous paragraph.

Annual data are available from BEA for 1929 to 1946. To create investment data for the period before 1929, which are not currently available from BEA, I determined the steady-state growth path of investment required to reproduce BEA estimates of the stock of each type of capital in 1929, assuming depreciation rates as in Bureau of Economic Analysis (2003) and annual real growth of 3.11 percent of output in the nonfarm business sector from Gordon (1999).[14]

5.2 Output

Output and the labor input are chosen to match the capital input and thus cover nonfarm business plus nonprofit institutions less value added for nonfarm tenant housing. Data for real and nominal output (used to construct the price index p_t) are from BEA. To calculate real value added in nonfarm tenant housing prior to 1990, I use the price index for consumption of nonfarm tenant housing.

5.3 Workers

BLS provides data on labor hours for the nonfarm business and nonprofit institution sectors back to 1947. I extend labor hours back to 1929 using data on full-time-equivalent employees from BEA, and before 1929 using growth rates from Gordon (1999). Labor hours for rental housing (a small number) are subtracted out using data on compensation and proprietors' income. The number of workers is the number of full-time-equivalent workers, found by dividing labor hours by 2000.

As indicated by equation 20, investment is determined by a combination of growth in workers and growth in the number of workers at full employment. The latter variable is not directly observable, so I estimate it using a Kalman filter exploiting the relationship between the unemployment rate and the demand for labor. Part of the drop in unemployment resulting from increased demand for labor occurs with a lag, because businesses initially obtain part of the desired increase in hours by extending the workweek, hiring more workers later.

[14] For the steady-state growth rate of investment to equal the steady-state growth rate of output requires that the cost of capital grow at the same rate as the price of output.

Table 1: Kalman Filter Estimate of Labor Hours at Full Employment

Parameter Estimated	Coefficient	z-Statistic
c_1	0.0052	1.5
c_2	0.77	3.5
c_3	-30.2	-19.2
c_4	-11.1	-6.2
c_5	0.370	-4.9
The sample is quarterly, from 1946:q2 to 2004:q3.		

I assume that labor hours at full employment are determined by

$$\ln\left(\bar{L}_t\right) = SV_t + c_1 t + c_2 \ln\left(NP_t\right), \tag{30}$$

where SV is the state variable, defined to be a random walk, t is a time trend equal to 1 in the first quarter of 1901 and rising by 0.25 in each subsequent quarter, and NP is the population aged 16 and older. The Kalman filter is estimated by assuming that the difference between the unemployment rate ru and the unemployment rate at full employment $\bar{r}u$ is determined by current and lagged ratios of actual labor hours L to labor hours at full employment $\bar{L}$

$$ru_t - \bar{r}u_t = c_3 \ln\left(\frac{L_t}{\bar{L}_t}\right) + c_4 \ln\left(\frac{L_{t-1}}{\bar{L}_{t-1}}\right) + c_5\left(SV_t - SV_{t-1}\right), \tag{31}$$

where $c_3>0$, $c_4>0$, and $c_5>0$. The final term captures the temporary rise in unemployment stemming from a jump in labor force participation. Data for $\bar{r}u$ is from the Congressional Budget Office (CBO). Coefficients obtained in estimating the Kalman filter defined by equations 30 and 31 are shown in Table 1.

5.4 Output per Worker

In making their investment decisions, firms ignore variations in output per worker that they know to be temporary. I assume that TFP A_t has a procyclical component U, intensity of effort, that depends on current and lagged values of the ratio of output to output at full employment:

$$U_t = \left(\frac{Y_t}{\bar{Y}_t}\right)^{b_1}\left(\frac{Y_{t-1}}{\bar{Y}_{t-1}}\right)^{b_2}\left(\frac{Y_{t-2}}{\bar{Y}_{t-2}}\right)^{b_3}\left(\frac{Y_{t-3}}{\bar{Y}_{t-3}}\right)^{1-b_1-b_2-b_3}, \tag{32}$$

where $\bar{Y}$ is output at full employment, $b_1>0$, $b_2<0$, and $b_3<0$. The assumption that demand affects productivity has a long history in the literature (e.g., Kuh, 1965). Initially, businesses meet a positive shock to demand by boosting both labor hours and worker effort. Over time, a greater share of the increase in output is met through additional labor hours. I assume that effort reverts back to pre-shock levels within a year.

Table 2: Kalman Filter Estimate of Productivity at Full Employment

Parameter Estimated	Coefficient	z-Statistic
b_1	0.432	25.0
b_2	-0.226	-14.9
b_4	0.0123	8.4
The sample is quarterly, from 1947:q1 to 2004:q3.		

Intensity of effort enables the ratio of output to output at full employment to deviate from the ratio of labor hours to labor hours at full employment:

$$\frac{Y_t}{\bar{Y}_t} = U_t \frac{L_t}{\bar{L}_t}.$$

Substituting for U_t from equation 32, replacing $\bar{Y}_{t-i}$ with $\bar{L}_{t-i}\bar{y}_{t-i}$, and rearranging, we find

$$\begin{aligned} \ln\left(\frac{L_t}{\bar{L}_t}\right) &= (1-b_1)\ln\left(\frac{Y_t}{\bar{L}_t\bar{y}_t}\right) - b_2\ln\left(\frac{Y_{t-1}}{\bar{L}_{t-1}\bar{y}_{t-1}}\right) \\ &\quad -b_3\ln\left(\frac{Y_{t-2}}{\bar{L}_{t-2}\bar{y}_{t-2}}\right) - (1-b_1-b_2-b_3)\ln\left(\frac{Y_{t-3}}{\bar{L}_{t-3}\bar{y}_{t-3}}\right). \end{aligned} \tag{33}$$

Using values of $\bar{L}_t$ obtained in the previous section, I estimated the unobservable $\bar{y}_{t-i}$ employing a Kalman filter, where

$$\ln(\bar{y}_t) = sv_t + b_4 t.$$

The state variable sv_t is assumed to follow a random walk. I assume that the coefficient on the final term on the right-hand side of equation 32 is half as large as the second-to-last term, i.e., $(1-b_1-b_2-b_3) = b_3/2$. Estimated coefficients are shown in Table 2.

5.5 Cost of Capital

The cost of capital vm has several determinants: the price index of new investment (discussed above), the rate of return, expected growth of productivity, the investment tax credit, the corporate tax rate, the treatment of depreciation allowances, the property tax rate, the cost of brokers' commissions, and the value of scrappage.

5.5.1 Rate of Return

Conceptually, the nominal rate of return is the rate of payments by business to capital (including retained earnings), net of corporate and property taxes, plus the rate of revaluation of assets due to inflation. The real rate of return is the nominal cost of funds less the rate of growth of output prices. In a putty-clay world, assets are revalued according to the change in the value of what they produce, i.e., the rate of growth of output prices. Thus, the revaluation of assets due to inflation and the rate of growth of output prices are the same, and the real rate or return is just net payments to capital.[15]

For corporations, the real rate of return is a weighted average of the after-tax yield on debt and the after-tax yield on equity. I assume that the after-tax rate of return for all legal forms of organization is the same as the after-tax rate of return for nonfarm nonfinancial corporations.

During the period for which it is available (1970 to 2002), Moody's average corporate bond yield roughly equals the average of the yields on Aaa-rated corporate debt and Baa-rated corporate debt, also from Moody's. So, the after-tax yield on debt is calculated as an average of the yields on Aaa-rated and Baa-rated corporate debt times $1 - u_t$. The overall corporate tax rate u_t is

$$u_t = uf_t + usl_t * (1 - uf_t),$$

where uf_t is the federal statutory corporate income tax rate and usl_t is the effective average state and local corporate tax rate, multiplied by $(1 - uf_t)$ because such taxes are deductible from federal corporate income taxes. Data for the federal statutory rate come from CBO and from Gravelle (1994). The state and local tax rate equals state and local corporate tax collections divided by profits before tax for corporate business.

The after-tax yield on equity is calculated using the dividend-discount model. According to that model, the after-tax yield on equity equals the dividend yield for the nonfarm nonfinancial business sector divided by the average historical ratio of dividends to after-tax profits in that sector from 1947 through the second quarter of 2004, 0.507.

The weights on debt and equity are constructed from data in the flow-of-funds accounts of the Federal Reserve Board. Debt is the value of credit-market instruments of nonfarm, nonfinancial business, whereas equity is the market value of equities of nonfarm, nonfinancial business. I use two-quarter moving averages because the flow-of-funds data are for the end of each quarter. For years prior to 1952, when flow-of-funds data are unavailable, the market value of equity is calculated using the index for the S&P 500, while the market value of debt is calculated as a function of nominal GDP and the ratio of business interest payments to the corporate bond

[15] The equality of the real rate of return with expected payments to capital net of taxes is consistent with the Bureau of Labor Statistics' methodology for creating the cost of capital, as described in Appendix C of U.S. Department of Labor (1983). Similarly, Harper, Berndt, and Wood (1989) show that using an "internal own rate of return model" defined using expected payments to capital net of taxes "yields the same rental prices ... as would the nominal internal rate of return model provided average capital gains were employed." The putty-clay model meets that condition.

yield.

The rate of return as calculated above is probably not the correct rate to use for long-lived capital. In particular, the rate of return on stocks is the average rate expected to prevail over the average remaining service life of the existing stock of tangible and intangible capital. For new capital with an expected service life beyond that average, businesses could reasonably assume that the real cost of funds would return to its historical average. For capital with expected service life greater than 10 years, the expected real rate of return is

$$ {}_{t}rm = r_t + \left(1 - \frac{10}{Sm_t}\left[1 + \ln\left(Sm_t/10\right)\right]\right)\left(\bar{r} - r_t\right), $$

where r_t is the current real rate of return and $\bar{r}$ is the historical average real rate of return.[16] Given the sharp and apparently permanent drop in real rates of return in the early 1950s, I assume $\bar{r}$ is 7.65 percent (the 1921-1950 average) through 1950 and then falls linearly to 5.60 percent (the 1951-2004 average) from 1955 on. For land, I used the same real rate of return as for office buildings, including medical buildings. For inventories, the expected real rate of return (the real cost of holding inventories) over the 15-year lifetime of a new inventory technology is two-thirds of the current real rate of return plus one-third of the long-term real rate of return.

The expected nominal rate of return used to discount depreciation allowances equals the expected real rate plus the expected rate of growth of output prices. I assume that expected inflation is a function of current and past inflation and the difference between the unemployment rate and the full-employment unemployment rate. The more positive the latter, the more people expect inflation to decelerate from past rates. I first estimated a relationship with consumers' expectations of inflation from the University of Michigan survey of consumers as the dependent variable, and then adjusted that relationship for the average historical difference between output inflation and consumer price inflation:

$$ {}_{t}\dot{p} = 0.48 + 0.41\dot{p}_t + 0.21\dot{p}_{t-1} + 0.10\dot{p}_{t-2} + 0.09\dot{p}_{t-3} - 0.28(ru_t - \bar{r}u_t) $$

For long-lived capital, I made the same adjustment to expected inflation for reversion to historical rates as I made to the real rate of return.

5.5.2 Tax Treatment

The methods of depreciation (straight-line, accelerated, sum of digits, and expensing), tax lifetimes, and declining-balance parameters used to calculate the timing of depreciation allowances $Dm_{t,i}$ are taken from Gravelle (1994), as are the basis adjustment Bm and investment credit rates Cm. Data for the methods of depreciation used are adjusted to account for the temporary increases in the amount of expensing

[16] The same formula is used to determine ${}_{t}r$ in equation 29, setting Sm equal to 15.

allowed under the Job Creation and Worker Assistance Act of 2002 (JCWAA) and the Jobs and Growth Tax Relief Reconciliation Act of 2003 (JGTRRA). I assume the same tax treatment for land as for commercial property.

I assume that nonprofit institutions and tax-exempt cooperatives use expensing, which is mathematically equivalent to setting the corporate tax rate for those entities to zero. I assume that all new religious, educational, and vocational structures are purchased by nonprofits and that 75 percent of new hospitals and special-care structures and medical equipment and instruments are purchased by nonprofits.

The numerator of the property tax rate is total state and local property taxes less taxes on production and imports for farms and housing from BEA. The denominator, taken from the flow-of-funds accounts, is the value of nonresidential real estate held by nonfarm, noncorporate business plus the nonresidential share of real estate in nonfarm, nonfinancial corporate business. I assume that the expected property tax rate at time t $\bar{u}x_t$ is an average of the current property tax rate and its 20-year moving average.

5.5.3 Other

Service lives for most assets are taken from Bureau of Economic Analysis (2003). The service life for autos is taken from U.S. Department of Labor (2001). The service life for computers is derived by calculating the depreciation rate implied by BEA data on computer investment and the real net stock of computers, adjusted for an assumed declining balance parameter.

I assume that businesses expect output per worker to grow at the average over the sample period of 2.2 percent per year. For land, I use 1.5 percent, the average of expected productivity growth and the average real growth of the price of land. Brokers' commissions as a share of other investment in structures b is 0.7 percent, its average during 1959-1997. The present discounted value of sale of scrap as a fraction of the purchase price Sc is 0.6 percent.

5.6 Stock of Land

Annual data on the real input of land into nonfarm business production are available from BLS. Those data include land associated with tenant-occupied housing. I assume that the nonresidential portion of total land varies with the ratio of BLS's index of nonresidential structures to the sum of that index and 0.25 times the index of rental residential capital. That ratio is roughly consistent with both a regression of annual changes of the index for land on annual changes of the indexes for nonresidential and residential structures, and with the nominal share of the productive capital stock of nonresidential structures in the productive stock of all nonfarm business structures.

The price index for land is the price deflator for land from BLS.

5.7 Importance of Capital in Production (αj)

Although one can calculate time series for the various αj series using equation 23, those series are much too erratic to be realistic estimates of technologies that change slowly over time. Instead, for equipment and software, I developed a smooth series that kept the implied ratio of nonmining machines to workers close to 1.0 over most of the 1947-2004 period (see Figure 1 on page 31).[17]

Extending αj for equipment and software back to 1929, the estimated ratio of machines to workers falls sharply from the middle of the Great Depression until the end of World War II and then bounces back strongly in the late 1940s and early 1950s. That pattern suggests that businesses retired equipment more slowly than normal from 1935 to 1945, at first because they could shift labor hoarded during the Great Depression from production to maintenance and repair of equipment, and later because shortages prevented firms from purchasing as much new equipment as they would have liked during World War II. Then, during the late 1940s and early 1950s, firms increased the number of machines back to desired levels.

It proved impossible to develop a smooth αj for structures that kept the ratio of nonmining-structure "machines" to workers at realistic levels over the entire available period. A lower αj before the mid-1950s would raise the ratio of machines to workers closer to 1.0 in the 1940s and 1950s but would push the ratio in 1929 even further above 1.0. As a result, I assumed that the sum of the αjs for equipment and software and for structures was constant at a level that produced a ratio of structure machines to workers of about 1.0 in recent years (see Figure 2 on page 31). (The two αj series are discontinuous between 1929 and 1930 because the 1929 values are used for the entire period up to 1929.)

It is unclear how to compute an αj for land, since the method by which BLS constructs the amount of land in service is not necessarily consistent with the putty-clay model. Given that the ratio of the nominal value of land to nominal output appears to be reasonably stationary, I assume that αj for land is constant at 0.0287. That figure equals the sample average of the ratio of the nominal value of nonresidential land to output at full employment (0.349) times the sample average of the cost of capital for land (8.23 percent).

The relevant αj for an increase in inventories per worker is the average αj prevailing when all existing inventory technologies were put in place, i.e., a 15-year moving average. Because the αj for inventories is not necessary to construct the stock of

[17] Because of the effect of time to build, the ratio of machines to workers is procyclical. To account for that, Figure 1 is constructed using a weighted moving average of actual workers and workers at full employment. The moving average is 12 quarters for equipment and software and 16 quarters for structures. The weight on actual workers is 0.5 for equipment and software and 0.4 for structures.

inventories per worker, it can be calculated by inverting equation 29 and taking a 15-year moving average.

Variations in the estimated overall importance of capital in production stem both from variations in the importance of inventories and in the variation of mining-specific capital (see Figure 3 on page 32). The average over the 1947-2004 period is 0.253.

6 Results

The putty-clay index of capital per hour shows some important differences with measures of capital per labor hour based on the putty-putty model. The putty-clay index eliminates movements in capital per worker stemming from cyclical variations in employment and investment. Even after accounting for those cyclical differences, however, the putty-clay and putty-putty measures of capital per hour show some important differences, most notably faster growth in the putty-clay measure during 2002-2004. Those differences stem from different assessments of the portions of investment devoted to capital deepening and capital broadening. The more rapid growth in the putty-clay measure of capital per hour since 2001 helps explain continued rapid growth in productivity despite weak investment. Two important factors driving recent capital deepening are improvements in computers and software and a real rate of return that is still low relative to its historical average.

6.1 Differences Between Putty-Clay and Putty-Putty Measures of Capital per Hour

The differences between capital per labor hour in the putty-clay and putty-putty models can most easily be seen by comparing the growth rate of the putty-clay index of capital per hour with the BLS measure of capital per labor hour used to calculate multifactor productivity in the nonfarm business sector (see Figure 4 on page 33). Because labor hours are easier to adjust than the capital stock, the putty-putty measure of capital per worker is strongly countercyclical, rising sharply as employment is reduced in recessions and rising slowly or even falling as workers are rehired. The putty-clay measure of capital per hour depends on the cost of capital and so leaves out those cyclical movements. Growth in capital per hour is slightly procyclical because recessions are often triggered at least partly by a rise in the real rate of return.

Adjusting the putty-putty measure of capital per hour for cyclical movements in labor hours, as CBO does in preparing its estimates of potential GDP, eliminates a substantial portion of the difference between the putty-clay and putty-putty estimates of capital per hour (see Figure 5 on page 33). However, a new difference emerges. The putty-putty measure of capital per hour tends to grow more slowly early in economic expansions and more rapidly late in expansions than the putty-clay measure. The putty-clay model interprets the acceleration of investment during an expansion as an increase in the number of machines necessary to accommodate unemployed labor rejoining the workforce, i.e., as capital broadening. The putty-putty model instead interprets that acceleration as an increase in the growth rate of capital per worker, or capital deepening.

Beyond those cyclical differences, two other differences between putty-clay capital per hour and CBO's putty-putty capital per hour are worth noting. First, putty-clay capital per hour grows much more slowly during the early 1950s than putty-putty capital per hour. In the putty-clay model, the high cost of capital during the late 1940s and early 1950s meant that much of the investment during that period was capital broadening. Businesses were continuing to make up for the shortfall in investment during World War II. The putty-putty model assumes any growth in capital in excess of the growth in labor hours at full employment is capital deepening.

Second, putty-clay capital per hour has grown much more rapidly than putty-putty capital per hour since 2001. The putty-putty measure interprets the slowdown in investment, coming at a time when labor hours at full employment continued to grow, as a reduction in capital deepening. However, capital deepening in the putty-clay model is determined by the cost of capital, which has remained relatively low. Thus, the sharp slowdown in investment implies a sharp reduction in capital broadening.

6.2 Sources of Growth in Putty-Clay Capital per Hour

The putty-clay model allows us to divide investment into pure replacement demand—the cost of replacing depreciating machines with new machines of the same quality; capital deepening—the extra cost of replacing depreciating machines with new machines of optimal quality; and capital broadening—the cost of increasing the number of machines (see Figure 6 on page 33). Pure replacement demand as a percent of nominal output at full employment rises slowly over time as a result of the shift of the capital stock toward items with shorter service lives. Capital deepening varies with the increase in the quality of new machines compared with those they replace. Capital broadening is primarily a lagged function of growth in demand in excess of growth in labor productivity. In addition, as discussed above, capital broadening occurred during the late 1940s and early 1950s to make up for a shortfall in investment during World War II. (Alternatively, one might classify that as replacement demand.)

One important factor in capital deepening in recent years has been the rapid increase in the use of computers and software (see Figure 7 on page 34). The price indexes of computers and software have generally declined relative to the price indexes for other types of capital, so a shift of the capital stock toward computers and software tends to boost the rate at which the quality of new machines rises. Improvements in the quality of computers and software accounted for 44 percent of the total growth in capital per worker in 2003, even though those two types of capital accounted for just 23 percent of gross fixed investment and a smaller fraction of net investment.

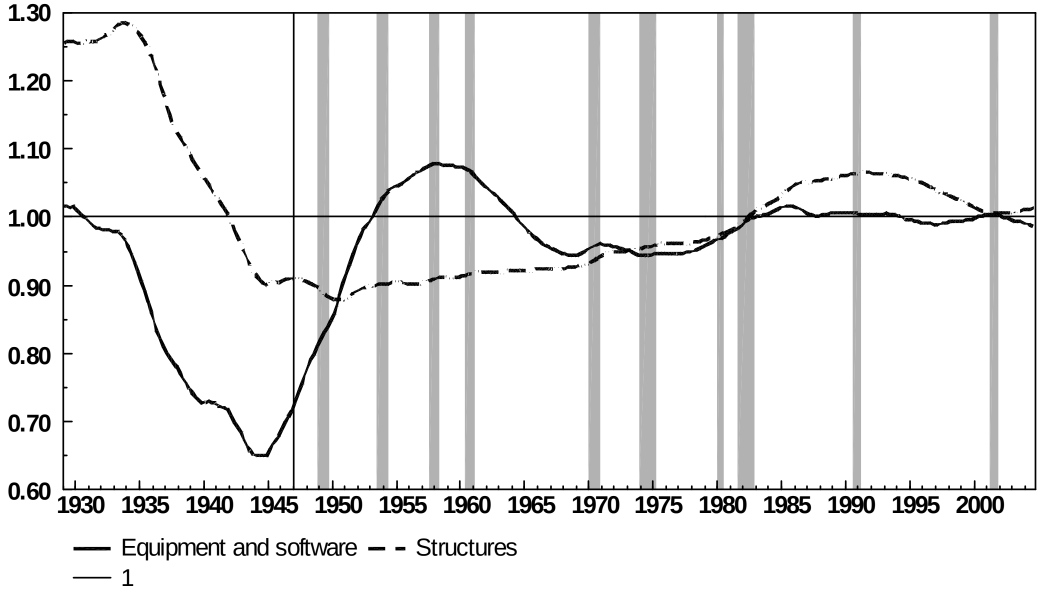
Another important determinant of capital deepening, or growth in capital per worker, is the real rate of return. The lower the real rate of return, the higher the quality of new machines and thus the more rapid the growth in capital per hour. Figure 8 shows this inverse correlation between growth in capital per hour and the real rate of return. Capital deepening has been strongest during periods when the real rate of return was low—the 1960s and early 1970s, and again in the period since the mid-1990s. On the other hand, high real rates of return during the late 1940s and early 1950s held growth in capital per hour to a minimum. High real rates during the late 1970s and 1980s also held down growth in capital per hour, but their adverse effect was partially offset by strong investment in petroleum structures and accelerated depreciation for structures.

7 Conclusions

The putty-clay model of capital offers a more useful way of thinking about the impact of investment on productivity than the more commonly used putty-putty model. It allows one to calculate a measure of capital per labor hour that is not distorted by cyclical fluctuations in employment and investment. The first-order conditions for optimal size of new machines allow one to identify capital deepening using the cost of capital and labor productivity adjusted for intensity of usage.

Although the putty-clay model cannot explain why productivity growth accelerated during 2001-2004, it at least removes the mystery of why the slowdown in investment did not cause productivity growth to slacken. That slowdown was the result of the near-disappearance of capital broadening, reflected in minimal employment growth over the same period. Businesses were investing only to replace depreciated capital, not to add new workers. Of the nearly 1.9 percentage-point acceleration in the growth of output per hour in private business less housing between the first quarter of 2001 and the third quarter of 2004 compared with the prior 10 years, I find that 0.2 percentage points was due to cyclical factors, between 0.1 percentage point and 0.2 percentage points was due to increased capital deepening, and the remaining 1.5 percentage points is unexplained.

Figure 1. Ratios of "Machines" to Full-Time-Equivalent Workers*
(Private nonfarm less housing)



* Workers are an average of actual workers and workers at full employment.

Figure 2. Production-Function Coefficients for Nonresidential Capital
(Private nonfarm less housing)

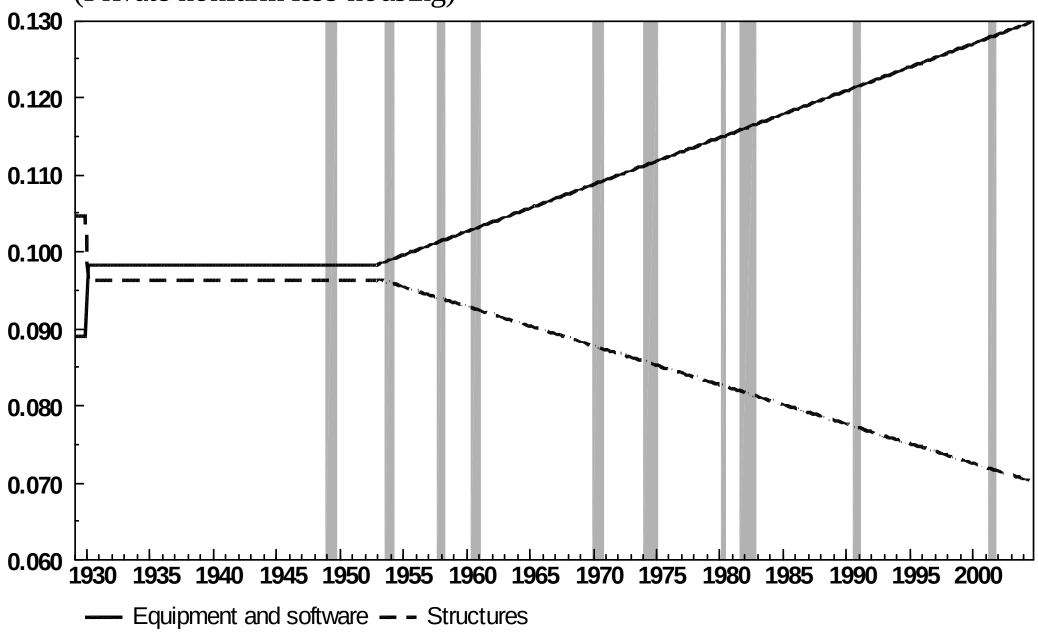
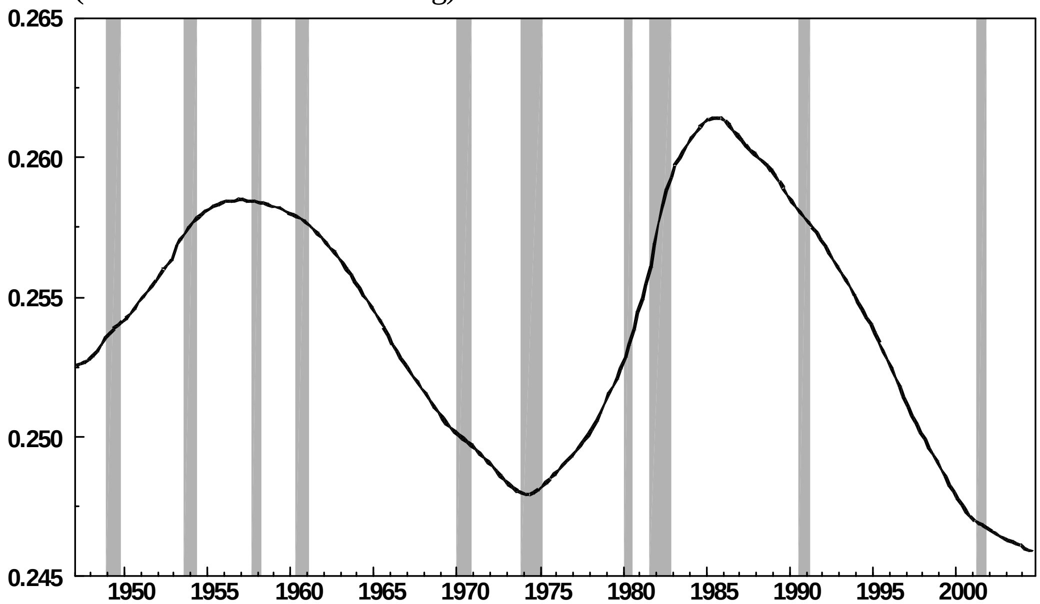


Figure 3. Importance of Nonresidential Capital in Production*

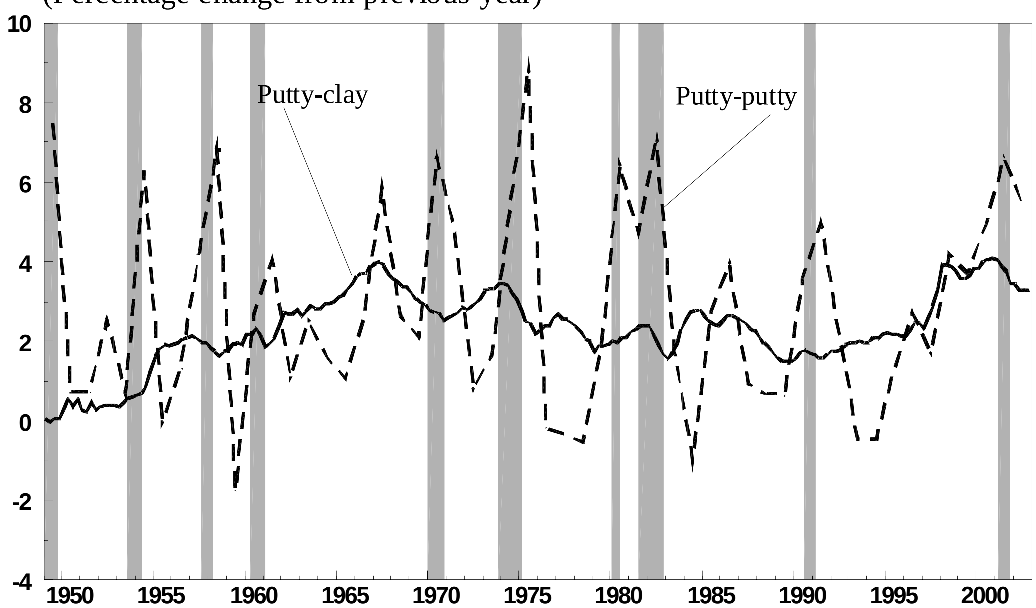
(Private nonfarm less housing)



* Equal to the coefficient of capital in production adjusted for mining.

Figure 4. Growth Rates of Capital per Hour

(Percentage change from previous year)

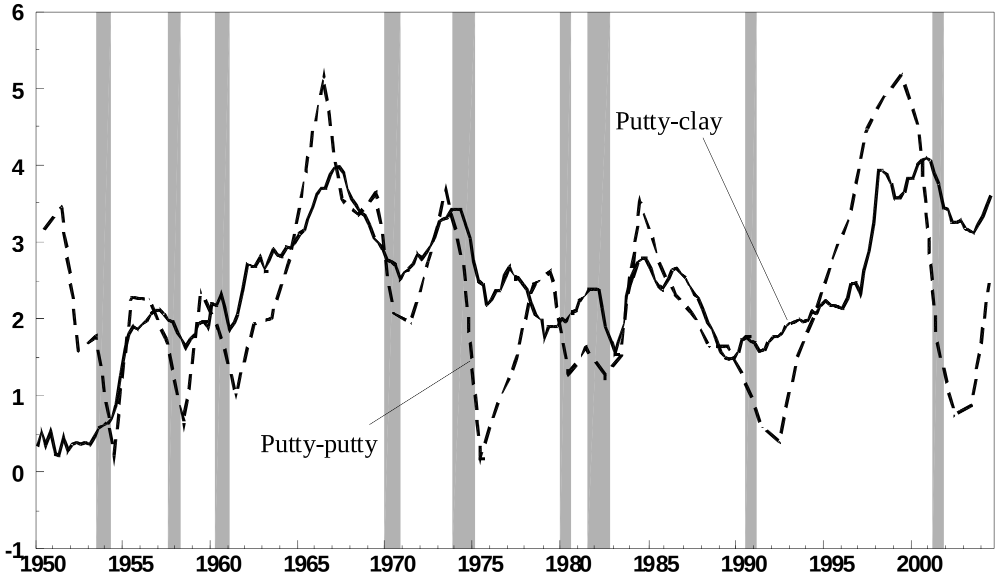


Sources: Author; Bureau of Labor Statistics.

Notes: Putty-clay capital per hour is for private nonfarm less housing.
Putty-putty capital services per hour of all persons, from BLS, is for nonfarm business.

Figure 5. Growth of Capital per Hour

(Percentage change from previous year)



Sources: Author; Congressional Budget Office.

Notes: Putty-clay capital per hour is for private nonfarm less housing.
Putty-putty capital per hour, from CBO, is for private nonfarm.

Figure 6. Components of Business Fixed Investment

(Excludes farming; percentage of nominal output at full employment)

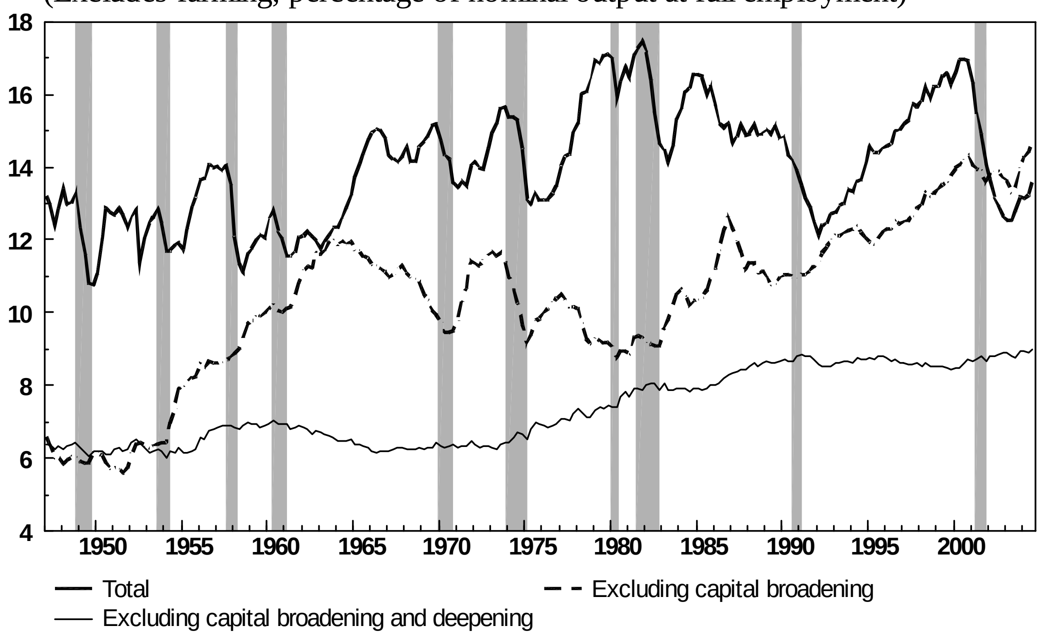


Figure 7. The Contribution of Computers and Software to Capital Deepening (Percentage change from previous year)

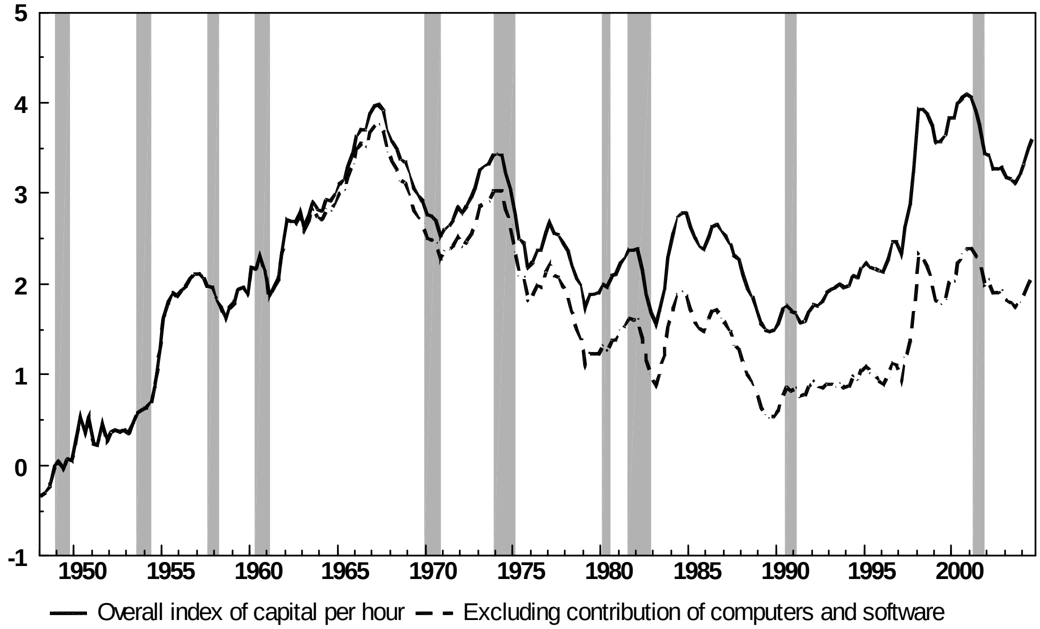
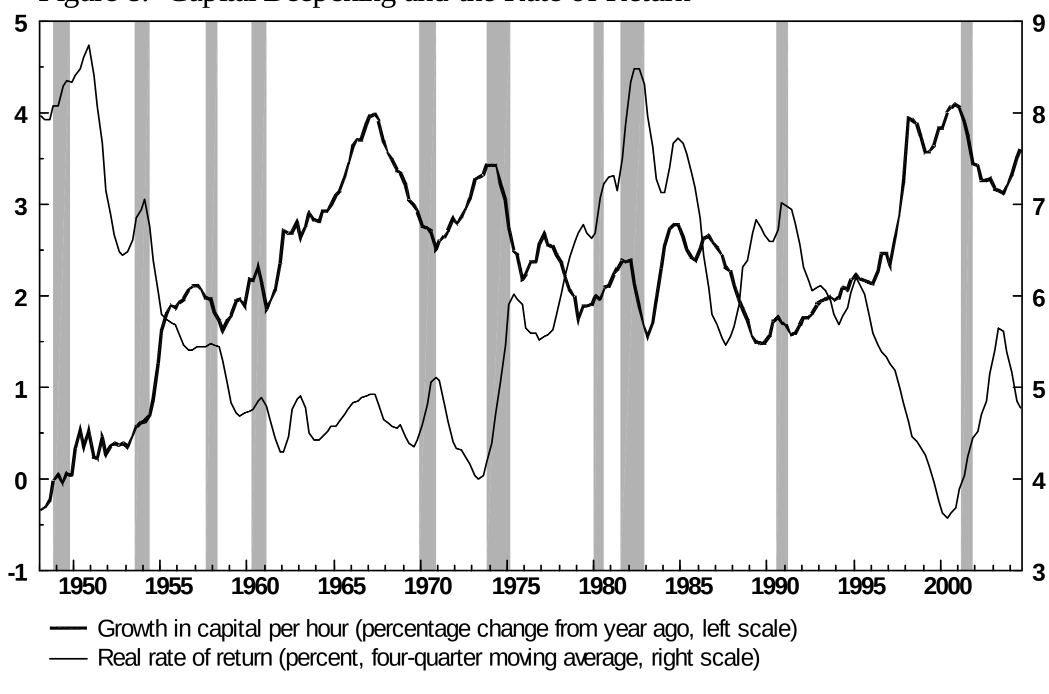


Figure 8. Capital Deepening and the Rate of Return



Appendix: Types of Capital

Nonfarm, nonresidential equipment and software comprises the following types of capital:

Information-processing equipment and software

- Computers and peripheral equipment
- Software
- Communication equipment
- Medical equipment and instruments
- Nonmedical instruments
- Photocopy and related equipment
- Office and accounting equipment

Industrial equipment

- Fabricated metal products
- Steam engines
- Internal combustion engines
- Metalworking machinery
- Special-industry machinery, not elsewhere classified
- General industrial, including materials-handling, equipment
- Electrical transmission, distribution, and industrial apparatus

Transportation equipment

- Light trucks (including utility vehicles)
- Other trucks, buses, and trailers
- Autos
- Aircraft
- Ships and boats
- Railroad equipment

Other equipment

- Household furniture
- Other furniture
- Construction tractors
- Other construction machinery
- Mining and oilfield machinery
- Service-industry machinery
- Household appliances
- Miscellaneous electrical
- Other

Nonfarm, nonresidential structures comprises the following types of capital:

Commercial and health care

- Office, including medical buildings
- Other commercial structures
- Hospitals and special-care facilities

Manufacturing

Power and communication

- Electric power
- Other power
- Communication

Mining exploration, shafts, and wells

- Petroleum and natural gas
- Mining

Other structures

- Religious
- Educational

Other buildings

Railroads

Other

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