VALUING FEDERAL LOANS AND LOAN
GUARANTEES USING OPTIONS-PRICING METHODS

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ABSTRACT

This paper provides more detail about the methods used in the Congressional Budget Office (CBO) study, "Estimating Subsidies for Federal Loans and Guarantees," (August 2004). That study estimates the market value of loans and loan guarantees under both credit reform and risk-adjusted approaches. The purpose of this paper is to increase the transparency of those methods for analysts who may wish to adopt them. Although this paper includes a brief overview of the basics, it is not a substitute for a textbook on options pricing. Rather, the discussion focuses on instances where the standard models were modified to better reflect the terms of the AWA and Chrysler guarantees.

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1. Introduction[1]

The CBO study, "Estimating Subsidies for Federal Loans and Guarantees," (August 2004) finds that including the market price of risk in cost estimates of federal loans and loan guarantees is necessary to measure cost comprehensively and, thereby, improves the comparability of those estimates with the cost of other federal programs in the budget. That study also estimates the costs of loan guarantees made to Chrysler and to America West Airlines with and without risk adjustment, and reports considerably higher costs with market risk.

Incorporating the market price of risk into subsidy cost estimates for loans and loan guarantees is accomplished most efficiently using options pricing techniques, which is the approach followed in "Estimating Subsidies for Federal Loans and Guarantees." That approach is useful because guarantees are a type of option and because the exposure of options to market risk varies over time. The latter feature of guarantees makes options pricing more accurate than traditional discounted present value estimates.

Interested readers of the CBO study may find the overview provided here of options-pricing models and the additional detail about the use of binomial models useful. This paper also explains the use of options pricing models to derive two key

parameters for valuing corporate loan guarantees–the value of the firm's underlying assets, and the volatility of the value of those assets. Since many government guarantees involve the receipt of warrants, the paper also explains how standard options-pricing methods are used to value warrants, and how their valuation differs under Credit Reform. Appendix I is an extended example of the effect of market risk on the price of guarantees that includes an explanation of why market risk leads to a discount rate that is different than the risk-free rate. Appendix II describes an alternative approach to estimating expected losses based on credit ratings.

2. Basics of Valuing Loan Guarantees Using Options-Pricing Methods

The pricing methods described in this paper rely on the observation that the payments on a loan guarantee are equivalent to those on a put option written by the government and held by the borrower.[2] Thus, the cost of the loan guarantee provided by the government may be estimated using either the Black-Scholes formula for put options or a binomial option-pricing model. The former is easier to use when it is applicable, but the latter can be applied much more generally. The Black-Scholes formula is appropriate when the government has first lien over all the assets of the borrower and the borrower cannot prepay the loan. In more complicated situations, for instance, if the loan can be prepaid or the government does not have a senior claim over all the assets of the borrower, the binomial option-pricing model is more appropriate. In addition, the default point, (i.e., the asset value at which the borrower chooses to default on its obligations or the lenders force the borrower to default), tends to change over the life of the loan guarantee, which also precludes using the standard Black-Scholes formula. Consequently, the binomial option pricing approach, which offers more flexibility, is used in computing the Chrysler and AWA loan guarantee values. The main model is constructed using an Excel spreadsheet and Visual Basic.[3]

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2.   As explained in the CBO study, the credit subsidy cost of a loan or a guarantee on a loan with the same terms is equivalent. Hence, the discussion refers only to loan guarantees, but applies equally well to direct loans.

3.   For a more complete description of binomial option pricing methods see McDonald, Robert, *Derivatives Markets*, Addison-Wesley, 2003.

When the binomial option-pricing model is used, the information required to compute the cost of the loan guarantee includes the following:

1) Current market value of assets (A)

2) Current book/market value of liabilities (L)

3) Market value of assets that triggers bankruptcy (Default Point, D, may vary with time)

4) Market value of assets that triggers prepayment (Prepayment Point, P, may vary with time)

5) Original time to expiration of loan guarantee (T)

6) Volatility of return on assets (σ_A)

7) Risk-free interest rate (r_f)

8) Dividend yield on assets (δ_A)

3. Estimating the Value of Guarantees and Guarantee Fees Using Binomial Asset Pricing Models

The value of loan guarantees and of associated guarantee fees were found using similar binomial asset pricing models. The binomial model approximates the future distribution of asset value over time, based on the initial asset value, asset volatility, and expected returns. The results also depend on the conditions that trigger default and prepayment.

Estimates of guarantee value using a binomial tree are based on the following steps:

1. A binomial tree is created, where each node in the tree represents the market value of the guaranteed firm's assets on a future date, and in a particular "state of the world" that reflects one possible history of asset returns. Going forward any one step in the binomial tree, the market value of assets takes two possible values (hence the name "binomial"). The two outcomes are selected to capture the assumed mean and variance of asset returns over the next short time period. A probability also is associated with an up or a down move (in such a way that the target mean return and variance are matched, and so that the tree structure is tractable). These probabilities are constant throughout the tree.

2. The assumed rules for prepayment and default are applied at each node to determine whether a default or prepayment will occur. Given these rules and the value of assets, the size of any loss to the government at each node is

calculated.

3. The insight exploited by options pricing theory is that the cost to the government at each node is equivalent to the payoff on a portfolio consisting of a fraction of the firm's assets and risk-free borrowing or lending. This "replicating portfolio" is different at different nodes, reflecting that the amount of market risk implicit in the guarantee changes over time. The replicating portfolio is easily identified and calculated, using standard methods. Since the model provides the value of the replicating portfolio at each node, and since its value is equal to the value of the guarantee payments, the calculation yields the value of the guarantee. The value of guarantee fees received is found similarly.

While the procedure is computationally straightforward, it is not immediately obvious how risk adjustment is being incorporated. The answer is that the risk-adjusted market rate enters the calculation of market value via the portion of the replicating portfolio invested in the firm's assets. The larger the effective position in the firm's assets, the higher the risk. The market risk is also magnified by the fact that the guarantee is equivalent to a highly levered position in the assets. This effect on the position of the guarantor means that the guarantee is like financing a purchase of the firm's risky assets with risk-free debt.

Credit reform cost estimates and cost estimates using market prices may both be obtained from the same binomial tree. However, to obtain credit reform cost

estimates, the present value of the cash flows from the government is determined by

discounting back to the present at a risk-free rate rather than determining the value of

a replicating portfolio. This method is used in the CBO study.

4. Estimating Asset Values and Volatilities

To value the cost of government loan guarantees using option-pricing models, it is necessary first to estimate the market value and volatilities of the assets of the firm receiving the guarantee. The market value of assets of a firm is equal to the sum of the market value of its liabilities, including its long and short-term debt and other fixed obligations, and the market value of its equity. (The market value of assets is not equal to the book value of assets, in general, since book value reflects historical costs and market values reflect investor expectations about future value.) The volatility of asset values depends on the volatilities of liabilities and equity, and on the correlations between them. In general, the market value and volatility of the equity of publicly traded firms can be calculated from recent share price data. If the market value and volatilities of all the liabilities of the firm and the correlation coefficient between the equity and debt returns is known, it is possible to calculate the market value and volatilities of the assets directly. Although the data required for estimating the market value and volatilities of the publicly traded debt of a firm may be available, the corresponding parameters for other liabilities ("non-trading liabilities") of the firm are difficult to estimate, but often significant. An alternative approach is thus needed for estimating the market value of the assets and the asset volatility.

For healthy firms, the simplest approach to estimating the market value of assets is to set it equal to the sum of the book value of liabilities and the market value of

equity. The inherent assumption in these calculations is that the market value of liabilities is closely approximated by their book value. This approximation can be expected to give good estimates of the market value of the assets of companies with strong balance sheets, e.g., for firms with negligible probability of default on their debt as evidenced by their very high debt ratings. However, this procedure can give incorrect results for companies in distress, as was the case for Chrysler and AWA, because the market value of their liabilities will be significantly lower than the book value of the liabilities. An alternative is to estimate these parameters using one of the two procedures described below. Both involve approximations with known limitations but they produce similar estimates for both Chrysler and America West Airlines.

4.1   Merton's Derivative Pricing Model

Over the past several years, some firms (most notably, KMV Corporation, a subsidiary of the rating agency, Moody's) have proposed using Merton's general derivative pricing model for understanding the link between the market value of the firm's assets and the market value of its equity. Equity has the residual claim on the assets after all other obligations have been met. It also has limited liability. A call option on the underlying assets has the same properties as equity. Thus, equity can be thought of as a call option on the assets of the firm. The holder of a call option on the assets has a claim on the assets after meeting the strike price of the option. When equity is valued as a call option, the strike price of the call option is equal to the book

value of the firm's liabilities. If the value of the assets is insufficient to meet the liabilities of the firm, then the shareholders, as holders of the call option, will not exercise their option and will leave the firm to its creditors.

In Merton's approach, the option nature of equity is exploited to derive the asset value and asset volatility of the firm's underlying assets implied by the equity's market value and equity volatility. Both the equity value and equity volatility can be observed from share price data. In particular, the option price (equity value) and option price volatility (equity volatility) are used for determining the implied asset value and asset volatility.

Valuing equity as a call option on the firm's assets using the Black-Scholes option pricing formula for a non-dividend paying stock:

$$E = AN(d_1) - Xe^{-r_f T} N(d_2) \tag{1}$$

$$d_1 = \frac{\ln(A/X) + (r_f + 0.5\sigma^2_A)}{\sigma_A \sqrt{T}} \tag{2}$$

$$d_2 = d_1 - \sigma_A \sqrt{T} \tag{3}$$

In the Black-Scholes framework, the volatility of equity and the volatility of assets

are related by:

$$\sigma_E = \sigma_A N(d_1)\frac{A}{E} \qquad\qquad (4)$$

where E is the equity value, A is the asset value, X is the book value of total liabilities, T is time to expiration of call option, σ_E is the volatility of common stock, σ_A is the volatility assets, N(.) is the cumulative normal distribution, and r_f is the risk-free interest rate.

The Black-Scholes formula given by equation (1) and the relation between equity volatility and asset volatility provided by equation (4) provide two equations, which can be solved simultaneously to obtain the two unknowns, asset value (A) and asset volatility (σ_A).

As an example, consider a firm with a market capitalization (E) of $3 billion, an annual equity volatility (σ_E) of 40 percent and total liabilities (X) of $10 billion. Further assume that all the liabilities are due within one year (T=1 year) and that the risk free rate (r_f) is 5 percent. The asset value and asset volatility implied by the equity value, equity volatility and liabilities are calculated by solving the call price and volatility equations, (1) and (4), simultaneously. In this case, the implied market value of the firm's assets is $12.51 billion, and the implied asset volatility is 9.6%. (This result implies that the market value of debt is $9.51 billion, $.49 billion lower

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than its book value due to the possibility of default.)

The Black-Scholes model described above, while limited in certain dimensions, is widely understood and provides a useful framework to review the issues and to generate rough estimates. In practice, however, it can be important to use a more general option-pricing relationship that allows for a detailed specification of the firm's liabilities. In Merton's approach, the market value and volatilities of assets are calculated by simultaneously solving two equations (1 and 4). KMV extends this procedure to handle firms with more complex capital structures, and their approach is one that could be adopted by budget analysts.

The KMV approach is likely to give reasonable estimates of the market value and volatilities of assets of healthy firms. For firms in distressed conditions, the book value of the liabilities is likely to be above both the market value of the assets and the market value of the liabilities. In addition, the market value of equity will only be a small fraction of the book value of total liabilities for a distressed firm. (For instance, the market value of equity of AWA was about $138 million, while the book value of liabilities was more than $1,500 million, when the government guaranteed $420 million in new loans.) Consequently, the equity of distressed firms is similar to a deep out-of-money call option. It is well known that the implied volatility estimated from deep out-of-money options may not accurately reflect the actual volatility of the asset on which the option is written. Consequently, an alternative approach, described below, can be used instead. It also involves error because it

relies on statistical estimates of asset betas which are difficult to measure.

4.2   An Alternative Approach

An alternative model for estimating the volatility of assets is based on the observation that the option and the underlying asset on which the option is written will have same "Sharpe Ratios." The Sharpe ratio (S) is defined as: $S = \dfrac{r_e - r_f}{\sigma}$,

where r_e is the expected return, r_f is the risk free rate and σ is volatility. Using an asset pricing model known as the capital asset pricing model (the "CAPM") returns are given by: $r_e - r_f = \beta * (r_m - r_f)$ where (r_m - r_f) is the market risk premium.

In the case of loan guarantees, asset refers to the assets of the firm and the option refers to the equity of the firm. Therefore,

(Beta of Asset*Market Risk Premium)/Volatility of Asset =

(Beta of Equity*Market Risk Premium)/Volatility of Equity

It follows that:

Volatility of Asset = Volatility of Equity*(Beta of Assets/Beta of Equity) (5)

It is possible to estimate the volatility of equity, the beta of assets and the beta of equity for any publicly traded firm. These data can be used for calculating the

volatility of the assets using equation (5).

This method may be more robust for firms in financial distress than the KMV approach because the beta of the assets of a firm should depend primarily on its business or the industry in which the firm operates. On the other hand, the difficulty of estimating betas precisely might lead some analysts to prefer the KMV approach, which does not require estimating industry betas.

After estimating the volatility of assets using equation (5), the market value of assets can be calculated using the Black-Scholes formula for call options. The market value of equity is the value of the call option on the firm's assets. The value of the call option depends on asset value, strike price, risk-free rate, asset volatility, time to maturity of the option and the dividend yield. The market value of the assets is the only unknown among these variables.

5.   Valuation of Warrants

The government sometimes accepts warrants as partial compensation for granting loan guarantees to distressed corporations. This was the case for both America West Airlines and Chrysler. To estimate the net cost to the government on a market value basis, the market value of the warrants also must be determined.

A warrant to purchase stock is analytically similar to a call option–the right but not the obligation to purchase shares at a specified price for a specified period. A call option on a stock allows the holder to buy the stock at a pre-specified price (strike price) on or before a pre-specified date, regardless of its current price. At expiration, the value of a stock option is the maximum of the difference between the market price and the strike price or zero and the holder will exercise the option only if it is to her advantage to do so. A warrant differs from a traded call option in that the number of shares outstanding increases when warrants are exercised (this effect is called dilution).

Warrants often have much longer maturities than traded call options. The value of either a warrant or a call option generally increases with maturity for two reasons. First, stocks tend to increase in value over time, and more time provides greater chance for movement in a stock price. At the time a warrant or call option expires, its value is worth the difference between the current stock price and the exercise price, or zero (when the current stock price is below the exercise price). Because of

the time value of a warrant, a rational investor would not exercise it prior to the expiration date. An investor who wanted to lock in gains if the stock price had risen above the exercise price would generally get a bigger return by selling the option on the open market.

The value of a warrant or call option also increases with volatility, because this increases the probability that the future stock price will exceed the exercise price. Finally, the value of a warrant or call option increases with the level of the risk-free interest rate because a call option permits the holder to defer purchase and thereby save the financing, or carrying, cost of the asset.

5.1   Dilution Adjusted Black-Scholes

Market participants commonly use the dilution adjusted Black-Scholes model to value warrants. This model assumes that stock returns are normally distributed and that stock prices follow a random walk (diffusion process). The Black-Scholes model values "European" options–those options that can be exercised only at maturity. American options can be exercised at any time, though it is seldom optimal to do so before maturity. (The exception is when a stock pays a dividend.) For this reason, the value of an American call option is same as the value of a European call option when the stock pays no dividend, as is expected to be the case for Chrysler and AWA. Thus, both Chrysler and AWA warrants may be treated as European warrants and valued using the Black-Scholes model.

Accordingly, the market value of a warrant share (W) is given by:

$$W = \left[S^* e^{-\delta T} N(d_1) - X e^{-r_f T} N(d_2) \right] * \left(\frac{N}{N + M} \right) \qquad (6)$$

$$d_1 = \frac{\ln\left(\frac{S^* e^{-\delta T}}{X e^{-r_f T}} \right)}{\sigma \sqrt{T}} + \frac{\sigma \sqrt{T}}{2} \qquad (7)$$

$$d_2 = d_1 - \sigma \sqrt{T} \qquad (8)$$

where $S^* = S + (M/N)W$, which is substituted for stock price, S, in the standard Black-Scholes formula to account for the effect of the increase in the number of shares on exercise of the warrant, or dilution. There are eight variables in equations (6)-(8). They are: share price at the time the warrant is issued (S), strike price or the price at which shares can be purchased using the warrant (X), the number of regular shares outstanding (N), the number of shares that the warrant holder can buy (M), time to expiration of warrant (T), volatility of common stock return (σ), risk-free interest rate (r_f) and dividend yield (δ). An iterative procedure is used for solving equation (6) since warrant price (W) is needed as an input to calculate S^* which is needed for calculating the warrant price (W) in turn.

The inputs into this formula are for the most part the same as for the binomial model used to value loan guarantees and associated fees. There are, however, a few considerations specific to warrant valuation.

The share price (S) used in the warrant price calculations should be the share price immediately after the government loan guarantee has been publicly announced. Taking an example from the CBO study, AWA's share price jumped from $2.75 to $4 following the announcement of the loan guarantee. If the share price is set equal to $2.75 instead of $4, the warrant share price is in the range of $1.49 to $1.82, which implies that the total market value of the warrant for 18.8 million common shares is $28.01 million to $34.22 million. However, warrants are to buy stock in the post-guarantee AWA. Therefore, the correct stock price to use in warrant calculations is the post-guarantee price. Chrysler's share price immediately after the government agreed to guarantee the loan was $7.50.

The value of a warrant or stock option is very sensitive to the assumed volatility of the share price, as measured by the standard deviation of the average percentage change in stock prices. Other things equal, the price of a warrant will increase with the expected variability of a firm's share price. Warrants to purchase stock in a firm whose share price shows little variability over time will be priced below those for a firm subject to large swings in the market value of its equity.

For a distressed company issuing a long-lived warrant, the question is how to

estimate the average volatility over the life of the warrant, taking into account that

the current volatility is likely to be higher than long-run volatility. When publicly

traded option prices are available, an estimate of volatility can be backed out from

the option prices. This is known as the "implied volatility," and it can be used as an

input to value other options or warrants. For instance, when Chrysler issued the

warrant, the volatility of the common stock was extremely high--101 percent. The

estimated value of the warrants using a volatility of 101 percent is $6.67 a share.

Alternatively, the implied volatility of the 5-year warrants that were publicly traded

was 70.5 percent on May 12, 1980. Substituting a volatility of 70.5 percent for 101

percent in the Black-Scholes options pricing model generates a price of $5.60 for the

warrants. Thus, the total value of the warrants for 14.4 million shares ranged from

$80.6 million to $96.5 million, depending on which volatility assumption is chosen.

The number reported in the CBO study reflects the lower implied volatility, which

seems more appropriate for the government's long-lived warrants.

5.2   Valuation Under Credit Reform

The difference between the estimated market value of warrants and the credit-reform

estimates is that credit reform uses a risk-free rate for discounting cash flows, rather

than a risk-adjusted rate.[4] As with market valuation, it is assumed that the warrants

can be exercised only at the end of the maturity period, so their terminal value

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4.   Options-pricing methods, including the Black-Scholes model use an analytical technique known as
       "risk-neutral pricing" to estimate the market value of risky options. The use of risk-free rates with this
       technique does not imply that investors are risk-neutral or that markets use risk-free rates to discount
       risky future cash flows. Risk-neutral pricing is simply a computationally efficient means of replicating
       risk-adjusted market prices. For details, see McDonald, *Derivatives Markets*, pp. 336-40 and 358-63.

depends on the stock price at that time relative to the strike price. The distribution of possible stock prices at the end of the maturity period is simulated and the warrant payoffs corresponding to these simulated stock prices are calculated. The expected value of warrant payoff at maturity is discounted using a risk-free interest rate to obtain the warrant value at the time it was issued.

For the AWA and Chrysler estimates under Credit Reform, and consistent with the assumptions underlying the Black-Scholes formula, a log normal distribution of stock prices at the end of the warrant maturity period is assumed. With a log normal distribution, the stock prices can be represented by:

$$S_T = S_0 e^{(\alpha - \delta - 0.5\sigma^2)T + Z\sigma\sqrt{T}} \tag{9}$$

where S_T is the share price at the end of the warrant maturity period, S_0 is the initial share price, α is the CAPM-based expected return on common stock, δ is the dividend yield, T is the time to maturity, Z is the standard normal random variable and σ is the volatility of the common stock.

For any share price at time T, denoted by S_T, the payoff for each warrant share that can be exercised only at warrant maturity is given by:

Warrant Payoff at time T = max (0.0, (NS$_T$ + MX)/(N+M) - X) (10)

where N is the number of common shares, M is the number of warrant shares and X

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is the strike price for warrants. By averaging warrant share payoffs at several share prices (i.e., at different values of the standard normal random variable Z), the expected payoff to the warrant share at maturity is calculated. The expected payoff of each warrant share at its maturity is discounted at a risk-free interest rate to determine the warrant share price when the warrant is originally issued. This procedure effectively replicates the assumptions of the market-based warrant valuation, except for the use a risk-free discount rate.

Appendix I: The Effect of Market Risk on the Price of Loans and Loan Guarantees

This appendix consists of an example illustrating how an aversion to market risk translates into an effect on discount rates and asset prices.

Market risk is costly because people would pay more today to receive of $100 next year if the economy is in a downturn, than they would to receive $100 next year if the economy is in a boom. This is equivalent to saying that people are willing to pay for insurance against bad economic times.[5]

To make this reasoning precise, assume that the booms and busts have equal probabilities of 50 percent. Also suppose that investors are willing to pay $45 today to receive a safe promise of $100 next year if the economy is in a boom, and $49 today to receive $100 next year if the economy is in a downturn (so-called "state payoffs"). Then the price today of $100 for sure next year is $94.00 ($49 + $45), the price of buying a claim for $100 in both the good and bad states. This implies a risk-free rate of 6.38%, since $100/(1.068) = $94.

Consider a risky government loan with an expected return of 6.38%, the risk-free rate. For instance, assume that the government lends out $100 at a rate of 12.76%, and that the government anticipates the loan will be repaid in full if the economy is

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5.   This higher value of payment when income is low is also an implication of diminishing marginal utility of wealth.

in a boom, but that the loan will only repay principal in a downturn. Using credit

reform assumptions, the loan would appear to be unsubsidized, because its value is

$\frac{.5(112.76)+.5(100)}{1.0638} = 100$. Private investors, however, would assign a value to the loan

of less than $100, because payments are relatively low when they are most valuable.

Investors would value the loan at $100\left(\frac{49}{100}\right)+112.76\left(\frac{45}{100}\right) = \$99.74$, which implicitly

discounts the loan at the higher rate of 6.66% since $\frac{.5(112.76)+.5(100)}{1.066} = \$99.74$. The

market would assign a subsidy value (or loss) of $0.26 per $100 of funds loaned. If

the government attempted to re-insure the guarantee with private investors, it would

appear to lose $0.13 per $100 guaranteed.

Alternatively, suppose the government guarantees a similarly risky loan. With the

government guarantee, the borrower could obtain a rate only slightly above the risk-

free rate of 6.38%, since the private lender would set a rate on the loan taking into

account the government guarantee. Assume again that the loan is repaid in full in a

boom, including interest of 6.45%, but that it only repays principal in a downturn.

In a boom there is no cost, but in a downturn the cost is $6.45. Under credit reform,

the budget would report a present value cost of $\frac{.5(0)+.5(6.45)}{1.0638} = \$3.03$. Private

guarantors would assign the higher cost of $6.45\left(\dfrac{49}{100}\right)+0\left(\dfrac{45}{100}\right)=\$316$ to the

guarantee, reflecting that they anticipate paying out money when it is especially

valuable.

Appendix II: Credit Rating-Based Alternatives for Estimating Expected Losses

The rules of credit reform permit a variety of approaches to estimating the probability and severity of losses on loan guarantees, and the value of expected fees paid. For comparability, and because it is arguably the most reliable approach, the credit-reform estimates reported in the CBO study are based on the same model of future asset values, defaults, and prepayments as the market-based estimates. As a robustness check, and to demonstrate an alternative that may be easier to use in some circumstances, the credit reform value of guarantee fees is also estimated using default probabilities based on credit ratings. This approach relies on historical default probabilities and recovery rates associated with debt of a particular rating. The weakness of this approach is that within a rating class, the probability and especially the severity of default can vary widely, making the estimates imprecise.

The expected cash flows from guarantee fee payments in i^{th} year are given by:

$$CF_i = (1 - CPD_i)*(1 - P_p)^i*F_i$$

where CPD_i = Cumulative probability of default by year "i", P_p = annual prepayment probability, F_i = Promised guarantee fee payments in the ith year. The default probability is based on the historical default performance of the debt with the same credit rating as that of the firm whose debt the government is guaranteeing. The prepayment probability is based on the annual probability of credit upgrade by two

notches. For example, consider the prepayment probability of a CCC rated debt. Historical statistics available from CreditMetrics indicate that a CCC rated debt has 41.53% probability of getting upgraded one category to B and 6.8% probability of an upgrade to BB or higher within the first year of issuance. In this situation, the prepayment probability is set to 6.8%. The expected guarantee fee payments are discounted at a risk-free rate to determine the value of guarantee fees at the time government extends the guarantee. Following this approach, the expected value of loan guarantee fee payments to the government is $69.49 million in AWA case and $38.55 million in Chrysler case. Both numbers are somewhat higher than those estimated using the model of cash flows corresponding to the options calculations ($52.5 million for AWA and $28.9 million for Chrysler).