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Using the Natural Rate Concept to Assess the Consistency of Projections Ten Years Ahead for Real Interest Rates and Inflation

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ABSTRACT

The concept of the "natural" long-term interest rate, a rate that is determined by the underlying production capability of the economy, provides a way to check the internal consistency of medium-term economic projections. Forecasters who use a neoclassical growth model to project the level of real Gross Domestic Product, as does the Congressional Budget Office, are implicitly projecting the natural rate, because the growth model simulates the production capability of the economy. The assumptions that are embedded in the growth model can be used to calculate the natural rate. In addition, economists have long noted that medium-term changes in inflation are related to the degree the natural rate is above or below people's expectations about the inflation-adjusted rate of return they will get on financial instruments. If the natural rate is above the expected real rate, inflation will tend to increase; if below, it will decrease. Therefore, the natural rate concept links the assumptions of the growth model, the projected change in the rate of inflation, and the projected change in the rate of interest.

Introduction

The concept of the "natural" long-term interest rate, a rate that is determined by the underlying production capability of the economy, provides a way to check the internal consistency of medium-term economic projections. Forecasters who use a neoclassical growth model to project the level of real Gross Domestic Product (GDP), as does the Congressional Budget Office (CBO), are implicitly projecting the natural rate, because the growth model simulates the production capability of the economy. The assumptions that are embedded in the growth model can be used to calculate the natural rate. In addition, economists have long noted that medium-term changes in inflation are related to the degree the natural rate is above or below people's expectations about the inflation-adjusted rate of return they will get on financial instruments. If the natural rate is above the expected real rate, inflation will tend to increase, if below, it will decrease. Therefore, the natural rate concept links the assumptions of the growth model, the projected change in the rate of inflation, and the projected change in the rate of interest.

The method for checking consistency described in this paper is only one of several of ways CBO evaluates projections of real long-term interest rates. No one procedure is uniformly superior, but this method is unique in that it provides insight into the relationship of several of different aspects of the projections.

Real and Natural Rates of Interest

The term "real long-term interest rate" can be used in two ways--retrospectively, or *ex post*, and prospectively, or *ex ante*. When used *ex post*, it means the rate of return after actual inflation is taken into account. Clearly, this notion of the real rate can be determined only after the fact–the real ten-year rate today cannot be measured because the performance of inflation over the next ten years is not known. When used *ex ante*, the term refers to an expectation, that is, it is what people think their rate of return on an asset will be after taking their expectation of inflation into account. The *ex ante* definition of the term is appealing because people act on their expectations, and therefore efforts to model their behavior should be based on an expectations concept. This concept is not clearly measurable insofar as expectations of inflation differ among people, but it can be estimated, both by surveying expectations directly and by using indirect methods such as one described later. This paper uses the *ex ante* sense of the term "real long-term interest rate."

The term can also differ according to the measure of inflation that is used. Some maintain that the consumer price index for all urban consumers (CPI-U) is the preferred measure, others the personal consumption price index, and still others the GDP price index. In addition, there are variants of each of these concepts (such as with or without energy and food prices, infrequently changing weights versus frequently changing, chain-type weights, etc.). In this paper, price change refers to the change in the CPI-U. As explained in the next section, using the CPI-U enables the construction of a series, with a sufficiently long-history, of a real long-term interest rate.

The real rate should not be confused with the concept of the natural rate of interest. The natural rate is not a financial market rate but instead a reflection of the underlying production capabilities of an economy–the availability of the factors and technologies of production in a broad sense–and is in essence a real return to capital. As described by its originator, Knut Wicksell:

> In general, we may say, it [the natural rate] depends on the efficiency of production, on the available amount of fixed and liquid capital, on the supply of labor and land, in short on all the thousand and one things which determine the current economic position of a community; and with them it constantly fluctuates.[1]

This natural rate, which varies with the changes in the underlying ability of the economy to produce, is unobservable, but as will be described later, it can be related to the assumptions of neoclassical growth models that try to characterize the determinants of long-term trends in production.

Estimating the Real Long-Term Rate

Two market-based measures of the real ten-year Treasury note rate are readily available. Since 1997, Treasury inflation-index securities (TIIS) have been issued, which give a direct measure of the real rate of interest.[2] The TIIS rate shown in Figure 1 is the "on-the-run" ten-year rate, which means that it strings together yields on only the newest of TIIS outstanding with an original maturity of ten years. This rate tended to stay above 3 percent until the onset of recession drove it below 2 percent. It averaged about 3.4 percent over the period of data on TIIS shown in Figure 1.[3]

The second measure is indirect in that it is based on surveys of expected inflation. An expected inflation measure that is drawn from surveys of economists and other participants in financial markets is subtracted from approximately concurrent yields on nominal Treasury securities of ten-year maturity. This provides a longer series than that of the TIIS data because such surveys are available back to 1979 for expected inflation ten years ahead and back to 1946 for expected inflation one-year ahead. These measures are also shown in Figure 1. It appears from the data that, although the newer, TIIS measure of real interest rates is higher than the measures using surveys of inflation expectations, it tends to move roughly in line with them. With some early notable exceptions, it also appears from the closeness of the real rate using either one-year or ten-year inflation forecasts that short-term and long-term inflation expectations have differed surprisingly little in surveys of market

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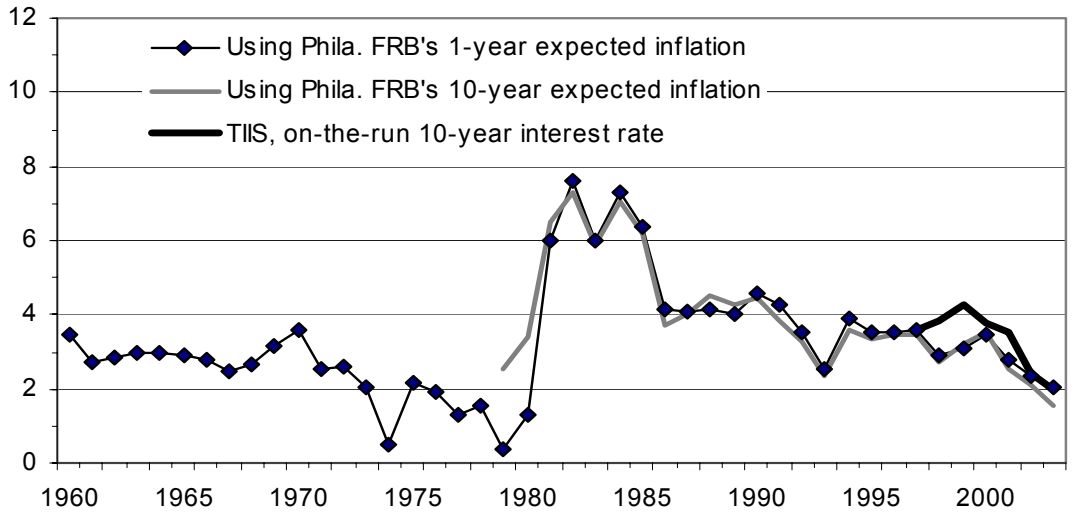
[1]Wicksell, *Interest and Prices* (1936), page 106. Wicksell's original, less detailed exposition appeared earlier, in an 1898 published lecture, "The Influence of the Rate of Interest on Commodity Prices."

[2]For additional discussion on using Treasury's inflation-indexed securities to estimate real interest rates and inflation expectations, see Mascaro (2003).

[3]For a discussion of other measures derived from TIIS rates (such as duration-adjusted, which attempts to minimize cyclical influences on real rates), see Bomfim (2001).

professionals.[4] The exceptions were in 1979 and 1980, when Federal Reserve monetary policy turned decisively to break the trend of rising inflation, a point that is returned to later on. Since then, however, the differences have been mostly minor. As a result, the remainder of this paper will use a merged series for the real ten-year Treasury rate that uses one-year forecasts of inflation until ten-year forecasts become available; when the series extends into 2004 and beyond, the composite series is augmented with the latest CBO projections. This composite real rate averaged 3.3 percent over 1960 to 2003.

Figure 1. Market-Based Measures of Real Ten-Year Treasury Rate, 1960 to 2003.



Source: *Author's calculations, based on data from the Federal Reserve Bank of Philadelphia and Haver Analytics*

Estimating the Natural Rate from a Standard Textbook Growth Model

The standard model of economic growth implies a real return to capital that represents the natural rate of interest and that, after an adjustment described below, can serve as a reference level for the real interest rate on the ten-year Treasury note. When the economy converges to a steady state in which output grows at the rate equal to the growth of the labor force plus labor-augmenting technology, the real return to capital converges to a "natural" rate that depends upon parameters representing the share of income going to owners of capital, the rate of saving out of GDP, the rate

[4]Over 1981to2003, the average difference between ten- and one-year forecasts was 15 basis points and the standard deviation was 25 basis points. Data and documentation on expected inflation from the Philadelphia Fed are available at http://www.phil.frb.org/files/spf/cpie10.txt, for ten-year expectations and at http://www.phil.frb.org/files/liv/reqm1.txt,for one-year expectations.

of growth of the labor force, growth of total-factor productivity (TFP), and the rate at which capital depreciates.[5] In the model's stochastic form, assuming stochastic growth of labor and TFP, additional parameters of the natural rate include the variances of labor-force and TFP growth and their covariance.[6]

Let α represent the capital share in an aggregate production function, s be the overall saving rate to include saving from abroad, τ be the growth in total factor productivity, n be the growth rate of the labor force, δ be the rate of capital depreciation, V be the variance of the sum of labor-force growth and growth in labor-augmenting technical change, and ρ be the return to capital (its marginal product net of depreciation).

Using the result that the net return to capital is its marginal product less its rate of depreciation and assuming Cobb-Douglas technology yield the following expression for the steady-state return to capital net of depreciation:[7]

$$\rho = \frac{\alpha}{s}\left[\frac{\tau}{(1-\alpha)} + n + \delta - \frac{1}{2}V\right] - \delta \qquad (1)$$

The steady-state return to capital shown in Equation 1 varies inversely with the rate of overall saving, because higher saving raises the level of capital per worker, lowering capital's return. If capital's contribution to output (α) exceeds the overall saving rate (s), as is typically judged to be the case, higher depreciation rates (δ) will raise the return to capital.[8] Faster depreciating capital shortens the period to recover past investment outlays and prepare for new outlays to maintain capital per worker. Finally, the steady-state return also varies positively with increases in growth of TFP (τ) and the labor force (n), both of which raise the effective number of workers per unit of physical capital.

The variance term in Equation 1, V, indicates that stochastic labor and technology growth reduce

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[5]The result from growth theory used here is that the growth in labor-augmenting technical change is equal the growth in total-factor productivity divided by labor's share of income.

[6]Merton (1975) derives the case where only labor grows stochastically, but inclusion of stochastic TFP growth is a straightforward extension.

[7]As shown by Merton (1975), the expression in Equation 1 is the expected value of the net marginal product of capital. As such, it is the first moment of the steady-state distribution of the marginal product. If labor and labor-augmenting technical change each follow a log normal process, then the steady-state marginal product of capital will have a gamma distribution with the above first moment.

[8]Since 1960, overall saving (national plus inflows from abroad) as a share of GDP is estimated to have averaged about 16 percent, compared to a capital share of about 30 percent. For additional discussion, see Congressional Budget Office (August 2001), page 9.

the return to capital as firms increase capital per worker to compensate for the effects of random fluctuations in labor and technology on the level of output. To approximate the variance, CBO's series for growth rates of actual and "full-employment" labor and TFP are used as indicated here (where, *fe,* denotes full-employment counterparts of the actual series and *E* denotes expected value):

$$V = E(n - n_{fe})^2 + E\left(\frac{\tau - \tau_{fe}}{1 - \alpha}\right)^2 + 2E(n - n_{fe})\left(\frac{\tau - \tau_{fe}}{1 - \alpha}\right) \tag{2}$$

The return to capital is likely to embody an equity risk premium, q. The equity premium is defined on a per period basis as the real return to equity (equal to the dividend yield plus real capital gain) less the real, holding-period return on a riskless security (typically taken to be the one-period real return on Treasury bills). Using data supplied by Shiller, an estimate of this premium over 1960 to 2002 is about 5.7 percent.[9] With that adjustment the riskless natural rate, *r,* is computed as:

$$r = \rho - q \tag{3}$$

A summary of the estimates for the natural rate of interest and averages for the parameters used in the estimates is shown in Table 1, including data projected for the medium term which I have averaged over 2010 to 2014. The evolution of the natural rate is shown in Figure 2.

To construct the natural rate, annual values of the parameters were smoothed by using moving averages before computing Equation 1, and constant values were used for the variance term and the equity premium. The moving average was arbitrarily set at ten years in length, presumably long enough to approximate steady-state expected values. An alternative smoothing rule, such as using a centered nine-year moving average, would have a negligible effect on long-period averages, but would matter for the medium term projection because it would require extending data beyond the interval in current projections. The use of rules, such as statistical filters to extract underlying, time-varying trends from noisy series, could also be examined in subsequent research. The assumption of a constant equity-risk premium also could be relaxed as a separate undertaking.

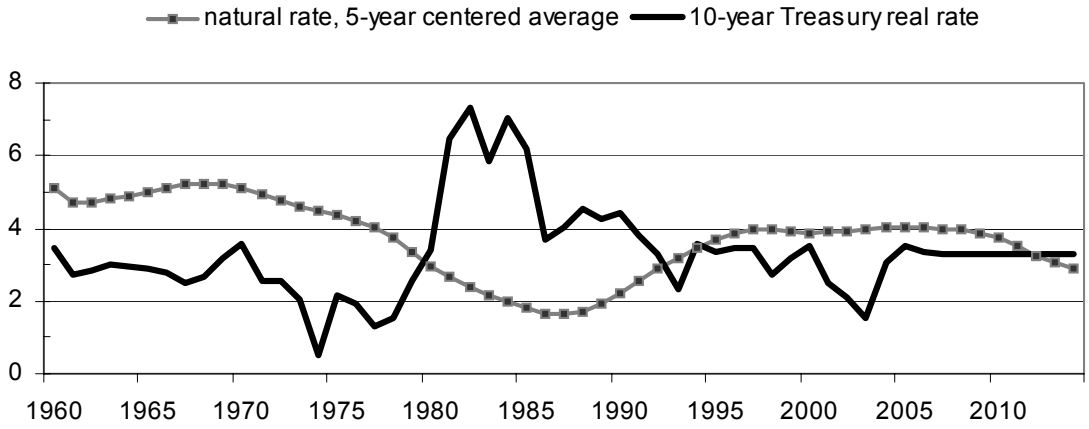
[9]Shiller's data contain some differences with the discussion in the text that enable him to extend his series back to 1887. For interest rates he variously uses inflation-adjusted rates on four- to six- month commercial paper and rates on six-month certificates of deposit, compounded at the start and middle of each year to obtain an annual nominal holding period return. For inflation he uses the all-commodities producer price index. I have not rigorously addressed these differences, but rough calculations suggest the natural rate could be higher, possibly by about 90 basis points. I plan to address this issue more fully in subsequent research. Additional information on Shiller's data is at: http://www.econ.yale.edu/~shiller/data/chapt26.html.

Table 1: Natural Rate: Parameter Inputs and Estimates from the Growth Model

	1970	1980	1990	2000	Medium-Term Projection[6] 2010-2014
Capital Share [1] (α)	0.300	0.300	0.300	0.300	0.300
Saving Rate[2,3] (s):					
National	0.160	0.172	0.149	.142	0.115
National + Foreign	0.153	0.170	0.166	.158	0.164
TFP [3,4] (τ)	0.020	0.010	0.008	.013	0.013
Labor-Force Growth[3,4] (n)	0.013	0.021	0.016	.014	0.010
Variance, [1,5] (V)	0.011	0.011	0.011	.011	0.011
Depreciation[3] (δ)	0.039	0.043	0.046	.050	0.054
Equity Risk Premium [1] (q)	0.057	0.057	0.057	.057	0.057
Implied Natural Rate (r)	0.052	0.029	0.021	.038	0.033

1. It is assumed throughout that the capital share is 0.3, the equity risk premium is kept at its 1960-2002 value of 5.7 percent, and the variance parameter equals the value computed over 1951-2002.
2. National saving as a ratio to GDP and gross private domestic investment as a ratio to GDP, respectively, with data from Department of Commerce, Bureau of Economic Analysis and Congressional Budget Office (2004) projections..
3. Data shown for these series are ten-year averages.
4. Growth in TFP and the labor force are computed from derived series designed to approximate "full-employment" levels. They are discussed in detail in Congressional Budget Office (2001) and are available from CBO.
5. The variance of growth in productivity and labor is computed from "error" series for each component. The error series for each component consists of the difference between actual growth and growth in a "full-employment" series mentioned in note 4.
6. Data used to compute 2010-2014 averages are from Congressional Budget Office (2004).

Figure 2. Natural Rate and Ten-Year Treasury Real Rate, 1960 to 2014.



Source: Author's calculations, based on data from the Federal Reserve Bank of Philadelphia, Haver Analytics, and the Congressional Budget Office

As shown in Table 1 and Figure 2, the natural rate is estimated to have been above 4 percent until the middle 1970s and well above the real rate on the ten-year Treasury note until the end of the 1970s. The declining phase of the natural rate from the early 1970s to the middle 1980s mostly reflects a slowdown in productivity growth and a rising depreciation rate, reinforced by a slowdown in labor-force growth in the first half of the 1980s. Its recovery since then reflects a further rise in depreciation, a pickup in TFP growth, and a slightly lower average rate of national saving plus inflows of saving from abroad.

The Relationship between the Natural Rate, the Real Rate, and Inflation

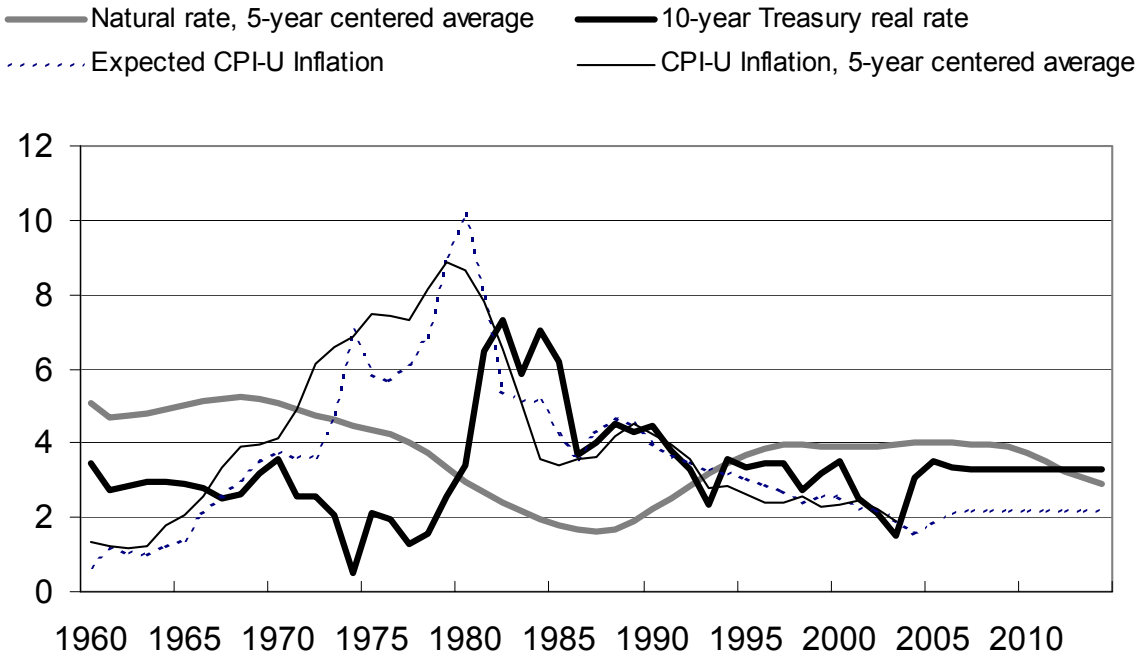
According to a long-held view, the gap between the natural rate and the real rate shown in Figure 2 is related to the trend in inflation in an important way. Wicksell apparently was the first economist to argue that the price level is related to the relationship between the real rate and the natural rate.[10] If the real rate, which is determined by credit market conditions and people's inflation expectations, is below the natural rate, which is determined by production capabilities, a boom in investment-type spending ensues, eventually driving prices higher as resource use tightens. Conversely, a real rate above the natural rate dampens investment activity and eventually reduces the price level. Only when the real and natural rates are aligned will conditions in credit markets be consistent with conditions of full utilization in production and eventual price stability. Wicksell referred to changes in the price level, whereas today this theory may be used to describe changes in the inflation rate.[11]

Figure 3 shows the relationship between inflation and the estimated values of the natural rate and the real rate. During the two decades of rising inflation in the 1960s and 1970s, the real rate was below the natural rate, as Wicksell would have predicted. In surprising conformity with Wicksell's view, inflation and expectations of inflation peaked at about the time the real interest rate began to rise and move above the natural rate as a result of the Federal Reserve's anti-inflation policy. The subsequent decline in inflation coincided with a natural rate below the real rate until about the middle of the 1990s as the Federal Reserve steered inflation to low levels. For the second half of the 1990s, the two rates were roughly aligned, and inflation stabilized at a level considered to be effective price stability. Finally, the current period of low real interest rates in the vicinity of 2 percent, coincides, according to the estimates, with a much higher natural-rate estimate of about 4 percent.

[10]Wicksell (1898).

[11]As Milton Friedman (1968) noted in his presidential address to the American Economic Association, Wicksell did not distinguish between nominal and real interest rates. In that same address, Friedman also introduced the term "natural rate of unemployment" to describe the theory of an expected inflation-augmented Phillips curve as a counterpart to Wicksell's natural rate of interest.

Figure 3. Real Ten-Year Rate, Natural Rate from Growth Model, Expected and Actual CPI-U Inflation, 1960 to 2014[1,2]



Notes to Figure 3:

1. Expected inflation uses Philadelphia Federal Reserve data to 2003 and Congressional Budget Office (2004) forecast thereafter.
2. The version of the actual CPI-U used here is the research series published by the Bureau of Labor Statistics from December 1977 to the present. The CPI-U series is extended back by CBO to 1960 using all of the official categories of the CPI-U except the series for shelter, for which CBO approximated the conceptual treatment of the shelter index that is now in both the research series and the current official CPI-U.

Source: Author's calculations, based on data from the Federal Reserve Bank of Philadelphia, Haver Analytics, and Congressional Budget Office.

If the estimate of the current gap between the natural rate and the real rate shown in Figure 3 is approximately correct, and if it continues like the one of the early 1960s, it could be a precursor of rising inflation unless the gap is offset by a neutral, rather than the current, unusually accommodative stance of monetary policy.[12] That prospect may be what the Chairman of the Federal Reserve had in mind in his recent, semiannual testimony on monetary policy:

[12]Interest in the concept of the natural rate apparently has emerged among central banks. See, for example, Laubach and Williams (2001), Orphanides and Williams (2002), Meyer (2003), Humphrey (2004), and Pianalto (2004). In the language of monetary-policy discussions, however, some central banks, such as the Reserve Bank of Australia (2001), refer to the level of the policy rate that would be consistent with the natural rate as the "neutral rate of interest." Keynes (1936, page 243) also used "neutral" to describe the level of the natural interest rate at full employment, distinguishing it from rates prevailing when the economy was in states of unemployment equilibria.

> To be sure, the Federal Open Market Committee's current judgment is that its accommodative posture is appropriate to foster sustainable expansion of economic activity. But the evidence indicates clearly that such a policy stance will not be compatible indefinitely with price stability and sustainable growth; the real federal funds rate will eventually need to rise toward a more neutral level. However, with inflation very low and substantial slack in the economy, the Federal Reserve can be patient in removing its current policy accommodation.[13]

Checking the Consistency of CBO's January 2004 Projections

The CBO projections for 2004 to 2014 can be checked against the theoretical relationship between the natural rate, the real rate, and inflation postulated by Wicksell. As noted above, the assumptions of the growth model for growth in TFP, the labor force, and depreciation, plus an assumption about the size of the equity premium, imply a CBO projection of the natural rate during the latter years of the projection period on the order of 3.3 percent (see Table 1, rightmost column). Because the CBO projection also assumes only a quite mild increase in the rate of inflation on average, from 1.6 percent in 2004 to an average of 2.2 percent in the medium term (see Figure 3), CBO, by the reasoning in this paper, should forecast the real rate to converge to the natural rate. That is, the two rates should be similar on average over the projection period if inflation does not have a strong upward or downward trend. In fact, as seen in Figure 3, CBO does project an increase in the real ten-year rate that is roughly in line with the estimate of the natural rate from 2005 through the balance of the projection period to 2014. As also can be seen from Figure 3, CBO also projects a stable inflation rate of about 2.2 percent-a level that, taking issues of measurement into account, might easily be regarded as within the range of approximate price stability.[14] If, instead, the real rate remained below the natural rate for an extended period, the theory implies that inflation would accelerate more than indicated by the CBO forecast. In effect, CBO's forecast assumes the Federal Reserve will achieve an inflation rate that approximates price stability by returning the stance of monetary policy away from being unusually accommodative and towards being neutral.

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[13]Alan Greenspan (2004), page 4.

[14]Most discussions of price stability tend to focus on core measures that exclude the more volatile food and energy prices. Recently, Greenspan (2004, page 3) suggested that the inflation rate of 0.9 percent for the core personal-consumption deflator in 2003 was consistent with price stability. Also in 2003, the corresponding inflation rate for the core, fixed-weight CPI-U was 1.2 percent, and the inflation rate for the CPI-U forecasted by CBO was 1.9 percent.

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