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The Role of Saving in Economic Growth When the Cost of New Human Capital Depends on the Cost of Labor

Mark Lasky

Congressional Budget Office
Washington, DC
E-mail: mark.lasky@cbo.gov

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Abstract

Mankiw, Romer and Weil (1992) found that modifying the Solow growth model to include human capital substantially increases the impact on output of a change in the saving rate for physical capital because increased output induces greater investment in human capital. However, that conclusion rests on the assumption that the cost of new human capital is proportional to the price of output. If, instead, the cost of new human capital is proportional to the cost of labor, and thus proportional to labor productivity, the effect on output of a change in the saving rate for physical capital is the same as in the Solow growth model.

Introduction

In his classic 1956 article on economic growth, Robert Solow developed a model in which rates of saving and population growth help to determine the steady-state level of income per capita.[1] In that model, an increase in the saving rate boosts steady-state output by more than its direct impact on investment because the induced rise in income raises saving, leading to a further rise in investment. All investment is in physical capital.

In 1992, Mankiw, Romer and Weil (hereafter MRW) modified the Solow growth model to account for human capital as well as physical capital.[2] In their model, the steady-state level of income per capita depends on the rates of saving and investment for both physical capital and human capital. When the saving rate for physical capital rises, the increase in output stemming from greater investment in physical capital leads to additional investment in both physical and human capital. As a result, the impact of an increase in the saving rate for physical capital on the steady-state level of income per capita is much larger than in the Solow model. For reasonable parameter values, a given increase in the saving rate for physical capital has twice the impact on steady-state income per capita in the MRW model as in the basic Solow model.

The Solow and MRW models assume that the costs of both new physical capital and new human capital are proportional to the price of output. Thus, an increase in real output leads to a proportionate increase in real investment in human capital for any given rate of saving for human capital. However, the cost of new human capital appears more closely tied to labor compensation per hour than to the price of GDP.

This paper assumes that the cost of investing in human capital is proportional to the cost of labor, rather than to the price of GDP. That assumption breaks the chain linking investment in physical capital to investment in human capital. While an increase in physical capital boosts output, it also boosts the real wage, and thus the real cost of new human capital, by the same amount, leaving real investment in human capital unchanged. Essentially, the substitution effect offsets the income effect. An increase in the saving rate for physical capital has the same effect on output as in the basic Solow model.

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1. Robert M. Solow, "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, vol. 70, no. 1 (1956), pp. 65-94.

2. N. Gregory Mankiw, David Romer and David N. Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, vol. 107, no. 2 (1992), pp. 407-437.

Economic Growth in the Solow and MRW Models

The following descriptions of both the Solow and MRW models are taken from the paper by Mankiw, Romer and Weil:

The Solow Growth Model. In the Solow model, output is produced using two inputs, physical capital and labor, which are paid their marginal products. Output is produced according to a Cobb-Douglas production function with constant returns to scale, in which output at time t is:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \tag{1}$$

where $0 < \alpha < 1$. Y is output, K is the stock of physical capital, L is labor hours, and A is the level of technology. L and A grow at rates *n* and *g* respectively:

$$L(t) = L(0)e^{nt} \quad \text{and}$$
$$A(t) = A(0)e^{gt}.$$

Thus, the number of effective units of labor, A(t)L(t), grows at rate n+g. Rates of saving (s), population growth (n), technological progress (g) and depreciation (δ) are all assumed to be exogenous.

If *k* is the stock of capital per effective unit of labor (k=K/(AL)) and *y* is output per effective unit of labor (y=Y/(AL)), then equation (1) can be rearranged to form:

$$y(t) = k(t)^{\alpha}. \tag{2}$$

Capital per effective unit of labor *k* grows according to:

$$\begin{aligned} \dot{k}(t) &= sy(t) - (n + g + \delta)k(t) \\ &= sk(t)^{\alpha} - (n + g + \delta)k(t). \end{aligned} \tag{3}$$

Since income equals output, the saving rate *s* can be expressed as a fraction of output. By setting the left-hand side of (3) equal to zero, one can see that, in the long run steady state, the stock of capital per effective unit of labor *k* converges to:

$$k* = \left[s/(n + g + \delta)\right]^{1/(1-\alpha)}. \tag{4}$$

Steady-state income per worker can then be calculated by substituting the expression for k* from (4) into (2), substituting Y/AL for *y*, and rearranging:

$$\log\left[\frac{Y(t)}{L(t)}\right] = \log\left[A(0)\right] + gt + \frac{\alpha}{1-\alpha}\log(s) - \frac{\alpha}{1-\alpha}\log(n + g + \delta). \tag{5}$$

where log(x) denotes the natural logarithm of *x*. Capital's share in income (α) is roughly one third, so a one percent increase in the saving rate *s* raises income per worker by 0.5 percent. Counting only the direct impact of higher saving on output and ignoring the additional boost given to investment because of higher output, the coefficient on log(s) is just α, or one-third of one percent. The extra impact of saving on output comes from the effect of higher output on investment.

The MRW Model. Mankiw, Romer and Weil added human capital to the Solow growth model. The production function then becomes:

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}, \tag{6}$$

or

$$y(t) = k(t)^{\alpha} h(t)^{\beta}, \tag{7}$$

where H is the stock of human capital, h=H/(AL), and other variables are defined as above. Workers contribute both labor hours and human capital to production.

Since there are two forms of capital, there are also two saving rates: the fraction of income invested in physical capital s_k and the fraction of income invested in human capital s_h. The MRW model assumes that human capital depreciates at the same rate δ as physical capital. Thus, the growth rates of physical and human capital per effective unit of labor are determined by:

$$\dot{k}(t) = s_k y(t) - (n + g + \delta)k(t) \tag{8}$$

and

$$\dot{h}(t) = s_h y(t) - (n + g + \delta)h(t). \tag{9}$$

Steady state values of k and h (k* and h*) can be found by setting the growth of *k* and *h* equal to zero, substituting equations (8) and (9) into (7), and rearranging:

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)} \text{, and}$$

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)} .$$

These steady-state values can then be used to obtain an expression for steady-state output per worker similar to (5):

$$\begin{aligned} \log\left[\frac{Y(t)}{L(t)}\right] = \log\left[A(0)\right] + gt + \frac{\alpha}{1-\alpha-\beta}\log(s_k) + \frac{\beta}{1-\alpha-\beta}\log(s_h) \\ - \frac{\alpha+\beta}{1-\alpha-\beta}\log(n + g + \delta). \end{aligned} \tag{10}$$

Adding human capital makes the impact of the saving rate on steady-state output much larger than in the standard Solow model. Assuming that the values of α and β are each roughly 1/3, then a one percent rise in the saving rate for physical capital boosts steady-state output by one percent, compared with a rise of just 0.5 percent in the basic Solow model. In the MRW model, the extra income resulting from higher investment in physical capital leads to additional investment in both physical and human capital. Note that output per labor hour in steady state grows at rate *g* in both the standard and modified Solow models.

Adding a Variable Relative Cost of Human Capital to the MRW Model

Both the Solow and MRW models assume that the cost of new human capital relative to the price of output is exogenous with respect to other variables in the model. This assumption guarantees that when a rise in the saving rate for physical capital pushes output higher, investment in human capital increases by the same percentage amount as output, further augmenting the rise in output.

However, the ratio of the cost of new human capital to the price of output is probably not exogenous with respect to the saving rate. Over time, the price index for consumption of education and research has moved much more closely with labor compensation per hour, and so with labor productivity times the GDP price index, than with the GDP price index alone (see Figure 1). In addition, the opportunity cost of human capital—foregone labor income—is tied directly to compensation per hour. Given that both the price of new human capital and the opportunity cost of obtaining human capital are roughly proportional to labor compensation per hour, the overall cost of new human capital is roughly proportional to compensation per hour.

Assume that output can be transformed into new human capital at a rate inversely proportional to the real price of new human capital. The equation governing growth in human capital per unit of effective labor then becomes:

$$\dot{h}(t) = s_h y(t) / p_h(t) - (n + g + \delta)h(t), \tag{11}$$

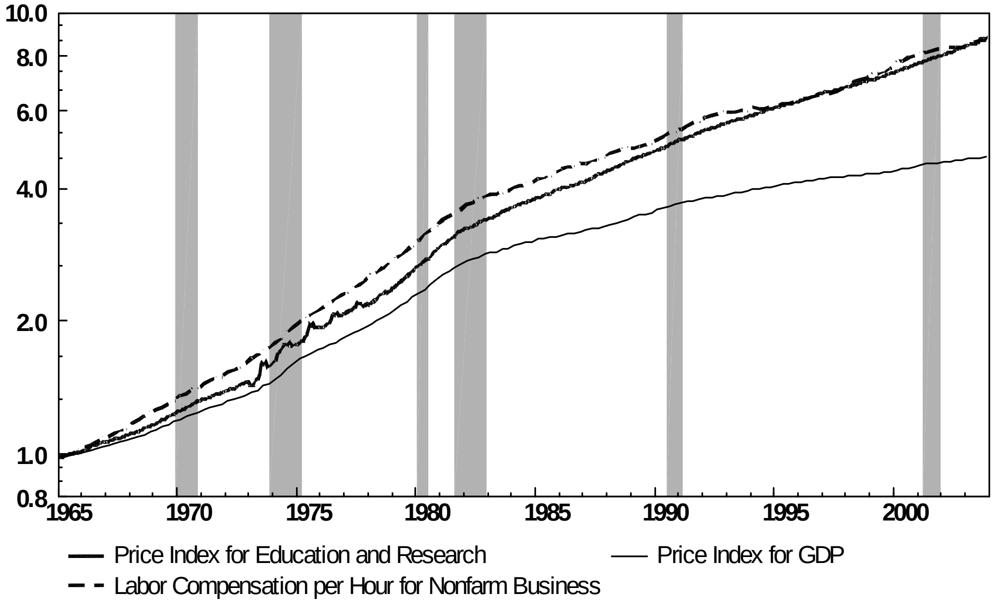
where p_h is the real cost of new human capital, or the cost of new human capital divided by the price of output. If the cost of human capital and the price of output are always equal, as in MRW, (11) simplifies to (9).

For real compensation per hour *w*, the first order conditions for profit maximization, given production function (6), yield:

$$w(t) = (1 - \alpha - \beta)Y(t) / L(t).$$

That is, real compensation per hour is proportional to labor productivity.

Figure 1. Price Indexes of Education, GDP, and Labor Compensation
(1965=1.0)



Sources: Dept. of Commerce, Bureau of Economic Analysis; Dept. of Labor, Bureau of Labor Statistics

Assume that p_h equals real compensation per hour w. Then:

$$p_h(t) = (1-\alpha-\beta)Y(t)/L(t) = (1-\alpha-\beta)y(t)A(t). \tag{12}$$

The price of new human capital is proportional to labor productivity (Y/L, or yA). As labor productivity rises, the cost of producing new human capital rises relative to the cost of producing other output. Substituting (12) into (11), growth in human capital per unit of effective labor is:

$$\dot{h}(t) = \frac{s_h}{A(t)(1-\alpha-\beta)} - (n+g+\delta)h(t).$$

Under these assumptions, human capital per unit of effective labor h is not constant in steady state. Instead, the economy moves toward a steady-state growth path in which human capital per worker H/L, or hA, is constant. On this path, human capital per unit of effective labor is inversely proportional to A:

$$h^*(t) = s_h \frac{1}{A(t)} \frac{1}{(1-\alpha-\beta)} \frac{1}{(n+\delta)}.$$

Along the steady-state growth path, income per worker is:

$$\log\left[\frac{Y(t)}{L(t)}\right] = \frac{1-\alpha-\beta}{1-\alpha}\log\left[A(0)\right] + g\frac{1-\alpha-\beta}{1-\alpha}t + \frac{\alpha}{1-\alpha}\log(s_k)$$
$$+\frac{\beta}{1-\alpha}\log(s_h) - \frac{\alpha}{1-\alpha}\log(n+g+\delta) \tag{13}$$
$$-\frac{\beta}{1-\alpha}\log(n+\delta) - \frac{\beta}{1-\alpha}\log(1-\alpha-\beta).$$

In this model, the impact of a change in the saving rate for physical capital is the same as in the basic Solow model. That is, the coefficient of log(s_k) is the same as the coefficient of log(s) in the basic Solow model. The link between the saving rate for physical capital and the stock of human capital in the MRW model is broken, because a given amount of output buys less real investment in human capital as the cost of that investment rises. The impact on output of a change in the saving rate for human capital is also smaller than in the MRW model. The higher output per worker produced by greater investment in human capital does not induce a further increase in human capital because the higher output per worker drives up the cost of investing in human capital.

In addition, output per worker grows at a rate less than *g* along the steady-state growth path. Increased technological progress raises the amount of output used to purchase new human capital, but also raises the cost of new human capital, leaving real investment in human capital unchanged. While both effective labor and physical capital grow at rate *g*, there is no growth in human capital per worker in steady state.

The conclusion that output per worker grows at a rate less than *g* along the steady-state growth path depends on the price of new physical capital being equal to the price of all output. In fact, however, the ratio of the price of new physical capital to the price of all output has declined over time. Thus, the ratio of real physical capital to real output has grown over time. If real prices of new physical capital decline rapidly enough, output per worker can grow as fast or faster than g on the steady-state growth path. However, the impact of changes in the saving rate are still the same as in the basic Solow model.