Expected Multi-Hop Power Consumption in Mobile Ad-Hoc Networks

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Abstract — **Mobile Ad-Hoc Networks provide the means to reduce significantly the power required for routing from source to destination through multi-hops between other nodes in the network. In the presence of mobility, only continuous updating can guarantee routes which minimize this power. This paper performs a theoretical study, establishing closed-form lower and upper bounds on the expected power corresponding respectively to continuous updating and to routes never updated. We introduce a mobility model which allows examining the behavior of the expected power between the two extrema as a function of the update period. The derived expressions vary according to the number of nodes in the network, their roaming area, the mobility of the nodes, the power exponent which governs their consumption, and time. This analysis of a one-dimension network in addition provides the basic equations which enable its extension to two dimensions, currently under investigation.**

Index Terms — **Energy, MANET, Mobility, Update Period.**

I. Introduction

Mobile Ad-Hoc Networks provide the means to reduce significantly the power consumption for routing from source to destination through multi-hops between other nodes in the network. Given the power required to transmit between neighboring nodes as a function of the distance between them, the distributed Bellmann-Ford algorithm [1] can compute the minimum-power route between a source and destination in the network. In the case of mobile nodes, this route remains valid for only a short period of time [2] and necessitates frequent updating to maintain it.

This work develops analytical expressions for the upper and lower bounds on the expected power to route in a one-dimensional network for a number of routing scenarios. The expressions vary according to the number of nodes in the network, their roaming area, the mobility of the nodes, the power exponent which governs their consumption, and time. Frequent updating maintains minimum-power routes, but introduces many overhead messages throughout the network [3, 4]. Conversely, updating preserves the battery life of the nodes, and so the connectivity of the network. We study the effect of the update period on the expected power and energy. The basic equations presented here also serve in the extension to a two-dimensional network, currently under investigation.

This paper is organized as follows. Section II develops a lower bound on the expected power required to transmit between source and destination nodes with known positions in a network. The extended result in the subsequent section covers the more general scenario of the pair randomly positioned, and cases with multiple source and/or destination nodes. Section IV derives analogous results for the upper bound on the expected power. The mobility model introduced in Section V enables studying the expected power as a function of the update period and computing the associated expected energy, or battery life. Lastly Section VI describes some areas of work in progress and extensions of the results here for more general applicability.

II. Minimum Power between Fixed Source and Destination

Consider the scenario of routing from a *source* node S to a *destination* node D, separated by a distance d, through n intermediate nodes indexed through i. Let the position $x_S = 0, x_D = d$, and the *independent* positions of the intermediate nodes have the density function

$$f_x(x_i) = \begin{cases} \frac{1}{d} & , \ 0 \le x_i \le d \\ 0 & , \ otherwise \end{cases} . \tag{1}$$

Assuming $P_{ij} = |x_j - x_i|^r$ the power necessary to route from node i to j according to some exponent r which relates increasing power consumption to increasing distance for $r \ge 1$, $P_{ij \cdots k}$ below describes the power to transmit from S to D through the n intermediate nodes and $n+1$ hops, if S routes to i, i routes to j, ..., and k routes to D.

$$P_{ij \cdots k} = |x_i - x_S|^r + |x_j - x_i|^r + \cdots + |x_D - x_k|^r \tag{2}$$

The *minimum power* required to route from S to D includes all nodes in between the two, dividing the total transmission distance into the largest number of hops. Each node in the route transmits to the immediate node with a greater position, in other words according to their position order. It is trivial to show through the Minkowski inequality [5] that this route is minimal for $r \ge 1$.

$\mathcal{E}(P_F^m)$ denotes the minimum power (m) for the scenario of fixed (F) source and destination. Equation 3 expresses this value through the sum of the expected values of the minimum power given a particular ordering of the intermediate nodes, each weighed by the probability of that ordering. Hence the sum spans the n permutation of the n elements of the index set of the intermediate nodes, denoted by ${}_n\mathcal{P}_{\{i,j,\cdots,k\}}$.

$$\mathcal{E}(P_F^m) = \sum_{{}_n\mathcal{P}_{\{i,j,\cdots,k\}}} p(x_i \le x_j \le \cdots \le x_k)\mathcal{E}(P_{ij\cdots k}|x_i \le x_j \le \cdots \le x_k) \quad (3)$$

$$= n!\, p(x_1 \le x_2 \le \cdots \le x_n)\mathcal{E}(P_{12\cdots n}|x_1 \le x_2 \le \cdots \le x_n) \quad (4)$$

As the intermediate nodes have the same position density, all terms in the sum of (3) are equal to each other, and since there are $n!$ permutations, the sum can be written in terms of the $P_{12\cdots n}$ component alone, chosen arbitrarily.

The position independence of the intermediate nodes allows expressing the probability of the condition in (4) as a product of the probabilities $p(x_{i-1} \le x_i) = \frac{d-x_{i-1}}{d}$ (see eq. 1) with x_i as the random variable:

$$p(x_1 \le x_2 \le \cdots \le x_n) = p(x_1 \le x_2) \cdots p(x_{n-1} \le x_n) \quad (5)$$

Now enforcing this condition of dependence amongst the nodes by choosing without loss of generality x_1 as the independent variable, x_2 as a dependent variable of x_1, x_3 as a dependent variable of x_2, and so on, gives the following conditional position densities (see eq. 1):

$$f_x(x_i|x_{i-1} \le x_i) = \begin{cases} \frac{1}{d-x_{i-1}} & , x_{i-1} \le x_i \le d \\ 0 & , otherwise \end{cases}$$

Now we can write the expected value in (4) as

$$\mathcal{E}(P_{12\cdots n}|x_1 \le x_2 \le \cdots \le x_n) = \quad (6)$$

$$\int_0^d \int_0^d \cdots \int_0^d f_x(x_1) f_x(x_2|x_1 \le x_2) \cdots f_x(x_n|x_{n-1} \le x_n) P'_{12\cdots n} dx_n \cdots dx_2 dx_1,$$

where $P'_{12\cdots n}$ denotes the value in (2) without the absolute signs. The absolute signs can be discarded here since the position ordering ensures that all quantities of $P_{12\cdots n}$ are positive. Replacing the expressions in (4) with (5) and (6), and distributing $p(x_{i-1} \le x_i)$ throughout gives

$$\mathcal{E}(P_F^m) = n! \int_0^d f_x(x_1) \int_0^d p(x_1 \le x_2) f_x(x_2|x_1 \le x_2) \cdots$$

$$\int_0^d p(x_{n-1} \le x_n) f_x(x_n|x_{n-1} \le x_n) P'_{12\cdots n} dx_n \cdots dx_2 dx_1.$$

Since $\int_0^d p(x_{i-1} \le x_i) f(x_i|x_{i-1} \le x_i) dx_i = \frac{1}{d}\int_{x_{i-1}}^d dx_i$, the above expression simplifies to

$$\mathcal{E}(P_F^m) = \frac{n!}{d^n} \int_0^d \int_{x_1}^d \cdots \int_{x_{n-1}}^d P'_{12\cdots n} dx_n \cdots dx_2 dx_1. \quad (7)$$

The Appendix reduces (7) to the closed-form expression below, stated explicitly in terms of the parameters d and n for future reference:

$$\boxed{\mathcal{E}(P_F^m|d,n) = \frac{(n+1)!\, d^r}{(r+1)(r+2)\cdots(r+n)}} \quad (8)$$

III. Minimum Power between Random Source and Destination

This section extends the final result of the previous section to a more general scenario where the source and destination nodes have the same position density as the intermediate nodes, for a total of n nodes in the network. Now the minimum-power route includes only the m intermediate nodes between S and D for multi-hopping. In the sequel we compute the minimum power $\mathcal{E}(P_R^m)$ between random (R) source and destination.

Under the assumption of uniformly distributed nodes, the density function of the distance $y = |x_S - x_D|$ between any two source and destination nodes follows [6]:

$$f_y(y) = \begin{cases} \frac{2}{d^2}(d-y) & , 0 \le y \le d \\ 0 & , otherwise \end{cases} \quad (9)$$

Given the distance between S and D, we now seek the density function of the number m of intermediate nodes lying between the two. The probability that the position x of an intermediate node lies between the source and destination nodes is simply $p(x \in y) = \frac{y}{d}$, and since all the intermediate nodes are i.i.d., the probability that exactly m out of the remaining $n-2$ nodes lies between S and D follows the Bernoulli distribution for m:

$$f_m(\mu|y) = \sum_{m=0}^{n-2} \binom{n-2}{m} \frac{(d-y)^{n-m-2} y^m}{d^{n-2}} \delta(\mu - m) \quad (10)$$

Now $\mathcal{E}(P_R^m)$ can be found by averaging $\mathcal{E}(P_F^m|y,\mu)$ in (8) through the joint density function $f_{ym}(y,\mu)$ over all values of y and μ, as shown below:

$$\mathcal{E}(P_R^m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{ym}(y,\mu) \mathcal{E}(P_F^m|y,\mu) d\mu dy =$$

$$\int_0^d f_y(y) \int_0^{n-2} f_m(\mu|y) \mathcal{E}(P_F^m|y,\mu) d\mu dy = \quad (11)$$

$$\left[\frac{2}{d^n} \sum_{m=0}^{n-2} \frac{(m+1)!}{(r+1)(r+2)\cdots(r+m)} \binom{n-2}{m} \right] \int_0^d (d-y)^{n-m-1} y^{r+m} dy$$

Employing the identity in equation (A.6) for the integral above and simplifying gives

$$\boxed{\mathcal{E}(P_R^m) = \frac{1}{3} \frac{n!n\ d^r}{(r+1)(r+2)\cdots(r+n)}.} \tag{12}$$

This result extends readily to the cases for multiple source and/or destination routing: for example, if n nodes route to n destinations (n^2 total routes), the minimum power of the network is $n^2\mathcal{E}(P_R^m)$, and average power per node is $n\mathcal{E}(P_R^m)$.

IV. Maximum Power

In a mobile network, we assume that an update establishes the minimum-power route according to the node positions at the time of execution. As time progresses and in the absence of further updating, the nodes maintain that same route, which however may depart from minimum due to their mobility interfering with the order of the nodes at the last update. We define the *maximum power*, or the worst-case power, as the expected power when the route has not been updated for so long to completely destroy this order. At this time, we say the network has returned to *equilibrium*.

A. Fixed Source and Destination

This section adopts the same model as in Section II with $x_S = 0$ and $x_D = d$ fixed, and n intermediate nodes uniformly distributed between the two. Equation (13) represents the maximum power (M) for routing from S to D through n unordered intermediate nodes. Note that here we assume no position ordering, necessitating the absolute signs of $P_{ij\cdots k}$. Observing that the $n-1$ innermost terms feature the same probability density of y in (9) allows rewriting them compactly as a single term $(n-1)\mathcal{E}(y^r)$. Substituting the latter into (13) and expanding $\mathcal{E}$ leads to (14).

$$\mathcal{E}(P_F^M) = \mathcal{E}(P_{ij\cdots k}) = \mathcal{E}(|x_i - x_S|^r) + \underbrace{\mathcal{E}(|x_j - x_i|^r) + \cdots}_{innermost} + \mathcal{E}(|x_D - x_k|^r) \tag{13}$$

$$= \frac{1}{d}\int_0^d (x_i - 0)^r dx_i + \underbrace{(n-1)\frac{2}{d^2}\int_0^d (d-y)y^r\, dy}_{innermost} + \frac{1}{d}\int_0^d (d - x_k)^r dx_k \tag{14}$$

Evaluating the integrals above, invoking once again the identity in (A.6), reveals the following closed-form expression

$$\boxed{\mathcal{E}(P_F^M) = \frac{2d^r}{r+1} + \underbrace{\frac{2(n-1)d^r}{(r+1)(r+2)}}_{innermost}.} \tag{15}$$

B. Random Source and Destination

The derivation presented in the sequel for the maximum power between random source and destination closely follows the series of steps in Section III, however with some important differences. At an update, the minimum-power route includes the m nodes which lie within the distance y between S and D; at equilibrium, the route still contains exactly m intermediate nodes, but S and D may not be located at the same distance y. Hence we cannot just average $\mathcal{E}(P_F^M)$ through $f_{ym}(y,\mu)$ as in the previous section to obtain $\mathcal{E}(P_R^M)$. Rather we note that at equilibrium, we simply have total $n = m+2$ unordered nodes (including S and D) randomly distributed between 0 and d. The expected power of the source to destination route is $(n-1)\mathcal{E}(y^r)$ since there are $n-1$ hops between the nodes, and each one has the density of y. Note the correspondence of this expected power to the innermost component of (14). Replacing $n = m+2$ into this component of (15) yields a modified maximum power below, stated explicitly only in terms of m:

$$\mathcal{E}(P_{F'}^M|m) = \frac{2(m+1)d^r}{(r+1)(r+2)} \tag{16}$$

Now we can find the maximum power between random source and destination $\mathcal{E}(P_R^M)$ simply by averaging $\mathcal{E}(P_{F'}^M|\mu)$ through $f_m(\mu)$, as described in (17). Note the dependence of m on the distance y to determine the number of intermediate nodes between the source and destination at the update time. Substituting $f_m(\mu) = \int_0^d f_y(y) f_m(\mu|y)dy$ into (17) enables simplifying $\mathcal{E}(P_R^M)$ in the subsequent steps.

$$\mathcal{E}(P_R^M) = \int_0^{n-2} f_m(\mu)\mathcal{E}(P_{F'}^M|\mu)d\mu = \tag{17}$$

$$\int_0^d f_y(y) \int_0^{n-2} f_m(\mu|y)\mathcal{E}(P_{F'}^M|\mu)d\mu dy =$$

$$\left[\frac{4}{d^n}\frac{d^r}{(r+1)(r+2)}\sum_{m=0}^{n-2}\binom{n-2}{m}(m+1)\right]\int_0^d (d-y)^{n-m-1}y^m\, dy$$

Employing once more the identity in equation (A.6) for the integral above and simplifying gives

$$\boxed{\mathcal{E}(P_R^M) = \frac{2}{3}\frac{n\ d^r}{(r+1)(r+2)}.} \tag{18}$$

C. Comparison of Minimum to Maximum Power

Figure 1 compares $\mathcal{E}(P_R^m)$ to $\mathcal{E}(P_R^M)$ versus the number of nodes in the network with $d = 10$, for $r = 2.5$ and $r = 3.5$ Increasing the number of nodes in the network divides the total transmission distance into many smaller distances, enabling a reduction in the minimum power for $r > 1$[1]. On the contrary, the maximum power

[1]Note that for $r = 1$, the minimum power remains nearly constant with respect to n.

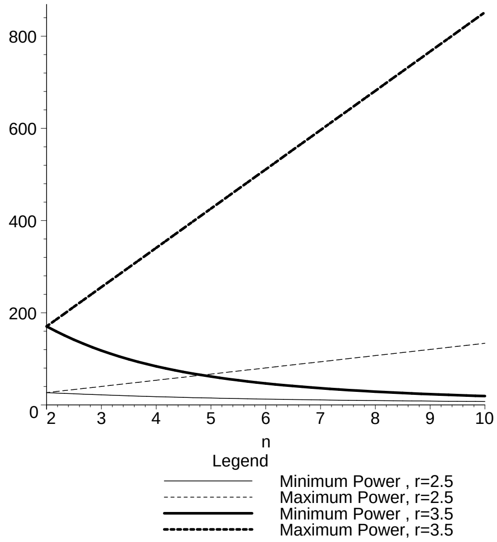

Figure 1: Expected Minimum and Maximum Power versus n.

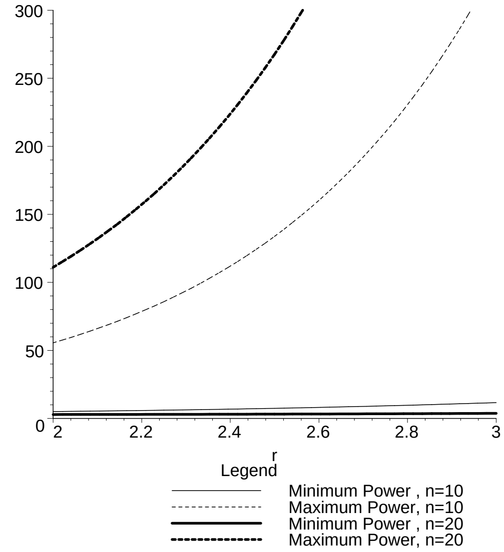

Figure 2: Expected Minimum and Maximum Power versus r.

increases with n since the average distance of each hop remains independent of the number of nodes in the network, but the number of hops grows.

Figure 2 shows the same curves as in the previous figure, however varying as a function of r, for $n = 10$ and $n = 20$. While all four curves increase with greater r, the effect on the maximum power is much more severe, again because the average distance of each hop is much larger for the maximum power.

V. Transient Power

An update establishes the minimum-power route at time $t = 0$, and in the absence of further updates this route grows to the maximum-power route, as explained in the previous section. We define the *transient power* as the expected power as a function of time, which increases from its minimum to its maximum value. This section develops an expression for it and studies its behavior by introducing a mobility model.

The subsequent derivation for the transient power between fixed source and destination closely follows the one for the minimum power in Section II, enabling us to omit most details. Equation (19) expresses the transient power (T) with x_i now a variable of t and conditioned upon the ordering of the nodes at the update time, where x_i^0 denotes the position of node i at $t = 0$.

$$\mathcal{E}(P_F^T) = n!\ p(x_1^0 \le x_2^0 \le \cdots \le x_n^0)\mathcal{E}(P_{12\cdots n} | x_1^0 \le x_2^0 \le \cdots \le x_n^0) = \quad (19)$$

$$n! \int_0^d \int_0^d \cdots \int_0^d f_x(x_1) \left[p(x_1^0 \le x_2^0) f_x(x_2 | x_1^0 \le x_2^0)\right] \cdots \left[p(x_{n-1}^0 \le x_n^0) f_x(x_n | x_{n-1}^0 \le x_n^0)\right] P_{12\cdots n}\, dx_n \cdots dx_2 dx_1 \quad (20)$$

Rewriting

$$f_x(x_1) = \int_0^d f_x(x_1|x_1^0) f(x_1^0) dx_1^0 = \frac{1}{d}\int_0^d f_x(x_1|x_1^0) dx_1^0 \quad (21)$$

and

$$p(x_{i-1}^0 \le x_i^0) f_x(x_i | x_{i-1}^0 \le x_i^0) =$$

$$p(x_{i-1}^0 \le x_i^0) \left[\int_0^d f_x(x_i|x_i^0) f_x(x_i^0 | x_{i-1}^0 \le x_i^0) dx_i^0\right] = \quad (22)$$

$$\int_0^d f_x(x_i|x_i^0)\left[p(x_{i-1}^0 \le x_i^0) f_x(x_i^0|x_{i-1}^0 \le x_i^0)\right] dx_i^0 = \frac{1}{d}\int_{x_{i-1}^0}^d f_x(x_i|x_i^0) dx_i^0,$$

and substituting (21) and (22) into (20) gives the final expression

$$\boxed{\mathcal{E}(P_F^T) = \frac{n!}{d^n} \int_0^d \int_0^d \cdots \int_0^d \left[\int_0^d f_x(x_1|x_1^0) \int_{x_1^0}^d f_x(x_2|x_2^0) \cdots \int_{x_{n-1}^0}^d f_(x_n|x_n^0) P_{12\cdots n} dx_n^0 \cdots dx_2^0 dx_1^0\right] dx_n \cdots dx_2 dx_1.} \quad (23)$$

The derivation for the expression $\mathcal{E}(P_R^T)$, the transient power between random source and destination, follows directly as presented in Section III and Subsection IV-B and is omitted here due to lack of space.

A. Diffusion Mobility Model

Equilibrium assumes time independence of the node positions, hence equal probability of finding a node anywhere between 0 and d. An update perturbs equilibrium and so the time independence of the node positions: the ordering condition constrains $x_{i-1}^0 \leq x_i$ at this time; however the network eventually returns to equilibrium in the absence of further updating through a transient state governed by the mobility model of the nodes.

The general nature of the expression in equation (23) lends to a broad range of mobility models expressed in terms of the node density $f_x(x_i|x_i^0)$. As an example, we present the diffusion mobility model below with mobility parameter c (unit area/time) [7].

$$\frac{\partial^2 f_x(x_i,t)}{\partial x_i^2} c^2 = \frac{\partial f_x(x_i,t)}{\partial t} \tag{24}$$

The density of a node now varies as a function of time, seeking to redistribute itself uniformly from its state at $t = 0$ forward. In this model, the density changes at a faster (slower) rate at positions where the profile imbalance is greatest (least): increasing where the density is smallest and decreasing where it is greatest. A fitting scenario depicts a troop in a military operation assigned to survey an area. When deployed at a certain position, the probability of finding the troop at that same position falls in time, while the probability of finding him/her at other points in the area rises. At a long enough time after, dependent on the value of the mobility parameter, it will be equally probable to find the troop anywhere in the survey area.

Applying the boundary conditions $\frac{\partial f_x(x_i=0,t)}{\partial x_i} = \frac{\partial f_x(x_i=d,t)}{\partial x_i} = 0$ to (24) ensures that the density of the node lies between 0 and d for all time, and yields the following solution:

$$f_x(x_i|x_i^0,t) = \sum_{h=0}^{\infty} A_h \cos\left(\frac{h\pi x_i}{d}\right) e^{-\lambda_h^2 t}, \quad \lambda_h = \frac{ch\pi}{d}, \tag{25}$$

$$A_0 = \frac{1}{d}\int_0^d f_x(x_i,t=0)dx_i, \quad A_h = \frac{2}{d}\int_0^d f_x(x_i,t=0)\cos\left(\frac{n\pi x_i}{d}\right)dx_i$$

Lastly, applying the initial condition $f_x(x_i,t=0) = \delta(x_i - x_i^0)$ and dropping the notation for t gives the final solution

$$f_x(x_i|x_i^0) = \frac{1}{d} + \frac{2}{d}\sum_{h=1}^{\infty}\cos\left(\frac{h\pi x_i^0}{d}\right)\cos\left(\frac{h\pi x_i}{d}\right)e^{-\lambda_h^2 t}. \tag{26}$$

Figure 3 displays the transient density function $f_x(x_i|x_{i-1}^0 \leq x_i^0)$ (see eq. 22) for $x_{i-1}^0 = 7$ at five points in time, with parameters $d = 10$ and $c = 1$. The ordering condition requires x_i to lie to the right of x_{i-1}^0 at $t = 0$. As time progresses, the chance of finding the node in this constrained area diminishes, augmenting the chance of finding it in the complement area. The network returns to equilibrium by $t = 60$.

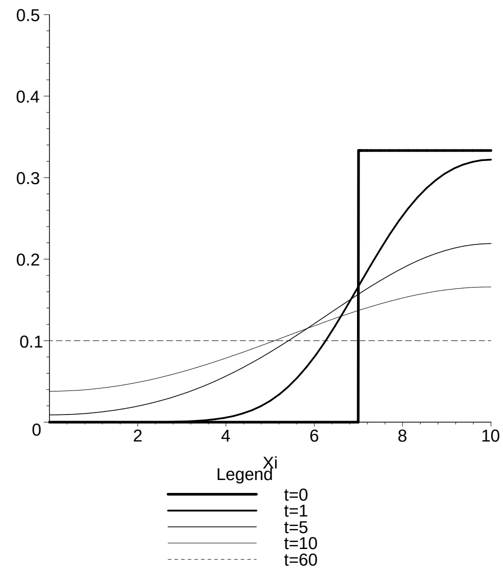

Figure 3: Transient Density Function $f_x(x_i|x_{i-1}^0 \leq x_i^0)$.

Figure 4 displays $\mathcal{E}(P_F^T)$ versus time bounded by its minimum and maximum values, with parameters $d = 10$, $c = 1$, $r = 2$, and $n = 10$. Updating the network at $t = 0$ sets the power to the minimum value of 16.67, as predicted through (8). The dark solid curve illustrates the transient power if no update occurs within the time range shown; it achieves the maximum power 216.67 as predicted through (15) by $t = 60$. The light solid curve illustrates the transient power with updates every $t = 10$; it realizes a significant reduction in power consumption over the other. The area between the two curves represents the energy savings in battery life of the network within the time range shown, which is almost 41% for this example; the transient energy can be found exactly through analytical integration over the transient power.

VI. Conclusions and Further Work

This paper develops closed-form upper and lower bounds on the expected power required to route through multi-hops in a one-dimensional Mobile Ad-Hoc Network. We illustrate and interpret plots on how these quantities vary with the number of nodes in the network and the transmission power exponent, most notably that the lower bound decreases and upper bound increases with increasing number of network nodes. The diffusion mobility model enables the analysis of the expected power as a function of the update period, enabling the computation of the expected battery life as a function of the update period as well.

Current work underway employs the basic equations presented here to expand the study to two dimensions.

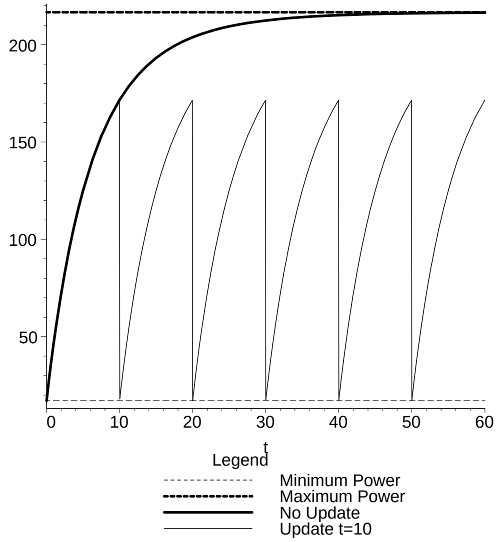

Figure 4: Transient Power versus t.

In the future we also plan to develop analogous results for hierarchical networks involving clusters.

Appendix

In order to simplify the expression in (7), we consider the i^{th} term of its integrand individually, as described through (A.1). Successively integrating over the innermost variables x_n to x_{i+1}, variables which do not appear in the integrand of (A.1), reduces the expression to (A.2). Now the region of integration over x_i appears in Figure A.1, with x_i as the dependent variable of x_{i-1}. By switching the order of integration of x_i and x_{i-1}, effectively transforming x_{i-1} to the dependent variable of x_i, and applying the appropriate limit changes as in (A.3), we can integrate first over x_{i-1} which evaluates to (A.4). Successive switching of order between x_i and x_j and integration over x_j for $j = i-2, i-3, \ldots, 1$ reduces to (A.5), leaving x_i as the sole variable.

$$\frac{n!}{d^n}\int_0^d\int_{x_1}^d\cdots\int_{x_{n-2}}^d\int_{x_{n-1}}^d (x_i-x_{i-1})^r\, dx_n dx_{n-1}\cdots dx_2 dx_1 = \tag{A.1}$$

$$\frac{n!}{d^n}\int_0^d\int_{x_1}^d\cdots\int_{x_{i-2}}^d\int_{x_{i-1}}^d \frac{(d-x_i)^{n-i}}{(n-i)!}(x_i-x_{i-1})^r\, dx_i dx_{i-1}\cdots dx_2 dx_1 = \tag{A.2}$$

$$\frac{n!}{d^n}\int_0^d\int_{x_1}^d\cdots\int_{x_{i-2}}^d\int_{x_{i-2}}^{x_i} \frac{(d-x_i)^{n-i}}{(n-i)!}(x_i-x_{i-1})^r\, dx_{i-1} dx_i\cdots dx_2 dx_1 = \tag{A.3}$$

$$\frac{n!}{d^n}\int_0^d\int_{x_1}^d\cdots\int_{x_{i-3}}^d\int_{x_{i-2}}^d \frac{(d-x_i)^{n-i}}{(n-i)!}\frac{(x_i-x_{i-2})^{r+1}}{(r+1)}\, dx_i dx_{i-2}\cdots dx_2 dx_1 = \tag{A.4}$$

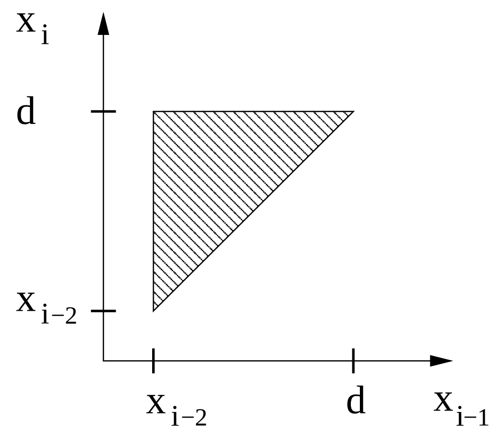

Figure A.1: Region of Integration.

$$\left[\frac{n!}{d^n}\frac{1}{(n-i)!}\frac{1}{(r+1)(r+2)\cdots(r+i-1)}\right]\int_0^d (d-x_i)^{n-i} x_i^{r+i-1} dx_i \tag{A.5}$$

Substituting for the integral above the identity

$$\int_0^a (a-u)^b u^c du = a^{b+c+1}\frac{\Gamma(b+1)\Gamma(c+1)}{\Gamma(b+c+2)} \tag{A.6}$$

and simplifying leads to

$$\frac{n!\; d^r}{(r+1)(r+2)\cdots(r+n)}. \tag{A.7}$$

The independence of the above expression from i indicates that the $(n+1)$ terms have equal value, and so multiplying (A.7) by $(n+1)$ renders the closed-form solution for (7)

$$\mathcal{E}(P_F^m) = \frac{(n+1)!\; d^r}{(r+1)(r+2)\cdots(r+n)}. \tag{A.8}$$

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