Analysis of EIED Backoff Algorithm for the IEEE 802.11 DCF

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***Abstract*— Exponential Increase Exponential Decrease (EIED) backoff algorithm, a flexible backoff algorithm with a number of adjustable parameters, was proposed by Song et al. to enhance the performance of the IEEE 802.11 DCF, where the performance benefit of EIED was shown by simulation.**

In this paper, we extend on the previous work, and provide an analysis of EIED backoff algorithm and an optimization methodology of the parameters of EIED based on the analysis.

I. Introduction

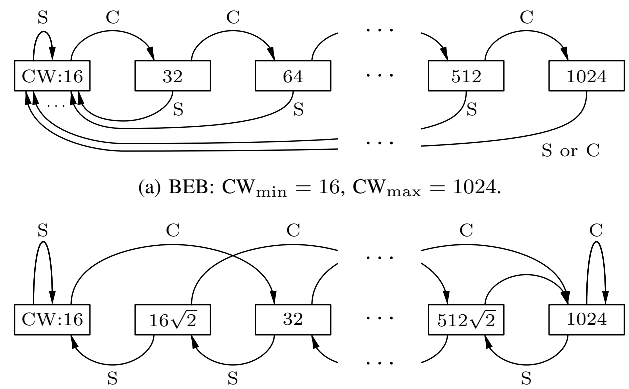
The Distributed Coordination Function (DCF) is the fundamental access mechanism in the IEEE 802.11 MAC. In DCF, Binary Exponential Backoff (BEB) algorithm is used as a contention resolution scheme. However, the performance of BEB suffers when the network is heavily loaded since the backoff procedure for every new packet starts with the *minimum* contention window.

In [1], Exponential Increase Exponential Decrease (EIED) backoff algorithm was proposed to enhance the performance of the IEEE 802.11 DCF. It was shown that while the throughput of EIED and BEB are the same and identical to the systemwide packet arrival rate when the packet arrival rate is small, as the packet arrival rate increases, BEB reaches saturation first and thus EIED has higher throughput than BEB does. EIED provides significant performance improvement without additional complexity.

EIED is a quite flexible backoff algorithm with a number of adjustable parameters. The optimal values of the parameters depend on the operating condition of the network such as the network load, packet length, etc. It was shown in [1] that the performance of EIED varies depending on the choices of the backoff factors using computer simulation. Even though the simulation results gave a good insight on how the backoff factors affect the performance of EIED, no analytical result was given to provide a means to make systematic decisions on the values of the backoff factors. In this paper, we extend on the previous work presented in [1], and provide an analysis of EIED backoff algorithm and an optimization methodology of the parameters of EIED based on the analysis.

II. EIED backoff algorithm

In EIED, the contention window size (CW) is exponentially increased by a backoff factor $r_I > 1$ whenever a packet is involved in a collision, and is exponentially decreased by a backoff factor $r_D > 1$ if a packet is transmitted successfully.



(a) BEB: $\mathrm{CW}_{\min} = 16$, $\mathrm{CW}_{\max} = 1024$.

(b) EIED with $r_I = 2$, $r_D = \sqrt{2}$: $\mathrm{CW}_{\min} = 16$, $\mathrm{CW}_{\max} = 1024$.

Fig. 1. Backoff mechanism of BEB and EIED. (S: success, C: collision)

EIED backoff algorithm can be expressed as follows:

$$\begin{aligned} \mathrm{CW} &= \min[r_I \cdot \mathrm{CW}, \mathrm{CW}_{\max}] \quad \text{on a collision}, \\ \mathrm{CW} &= \max[\mathrm{CW}/r_D, \mathrm{CW}_{\min}] \quad \text{on a success}, \end{aligned} \tag{1}$$

where $\mathrm{CW}_{\min}$ and $\mathrm{CW}_{\max}$ are the minimum and the maximum contention window sizes, respectively.

In general, the relationship between r_I and r_D is given by

$$r_I^m = r_D^n, \tag{2}$$

where m and n are integers greater than or equal to 1. However, our previous simulation studies, part of which was published in [1], shows that there is no performance benefit when $r_D > r_I$, and thus we restrict our analysis and optimization effort only within a subset of the parameter space defined by

$$\mathrm{CW}_{\max} = r_I^M \cdot \mathrm{CW}_{\min}, \quad M \geq 0, \tag{3}$$

$$r_I = r_D^n, \quad n \geq 1, \tag{4}$$

where M is a non-negative integer. Fig. 1(a) shows the backoff mechanism of BEB, and Fig. 1(b) shows the backoff mechanism of EIED with $r_I = 2$, $r_D = \sqrt{2}$ when $\mathrm{CW}_{\min} = 16$, and $\mathrm{CW}_{\max} = 1024$. Note that for the given $\mathrm{CW}_{\min}$ and $\mathrm{CW}_{\max}$, these backoff parameters $r_I = 2$ and $r_D = \sqrt{2}$ correspond to $(M, n) = (6, 2)$, where integers M and n are defined in (3) and (4).

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In the subset of the parameter space of interest, there are $(Mn+1)$ backoff states. Let W_i be the contention window size of backoff state i, where W_i is given by

$$W_i = \mathrm{CW}_{\min} \cdot r_D^i, \quad i = 0, 1, 2, \cdots, Mn. \tag{5}$$

For integer W_i, a backoff counter value is chosen from $\{0, 1, \cdots, W_i - 1\}$ with equal probability $1/W_i$. That is, entering backoff state i, a node sets its backoff counter to a random number D_i of uniform distribution, where

$$\Pr\{D_i = k\} = \frac{1}{W_i}, \quad k = 0, 1, \cdots, W_i - 1. \tag{6}$$

In EIED, however, the contention window size is not always an integer due to the non-integer backoff factors r_I and r_D. For non-integer W_i, the probability distribution of D_i is given by

$$\begin{aligned} \Pr\{D_i = k\} &= \frac{X_i + 1 - Y_i}{X_i(X_i+1)}, \quad k = 0, 1, \cdots, X_i - 1, \\ \Pr\{D_i = X_i\} &= \frac{Y_i}{X_i + 1}, \end{aligned} \tag{7}$$

where X_i and Y_i are the integer and fractional parts of W_i defined by

$$X_i = \lfloor W_i \rfloor, \tag{8}$$
$$Y_i = W_i - X_i. \tag{9}$$

For integer W_i, this operation is equivalent to (6).

As an alternative to (7), for non-integer W_i, W_i can be rounded to the closest integer by approximating $Y_i = 0$. This approach will render the behavior of EIED very close to that of using the probability distribution in (7), and is a very practical choice for implementation. However, this approach makes the analysis extremely complicated. For example, (16) does not hold anymore with this approximation.

In the IEEE 802.11 DCF, the backoff counter is decreased at the end of each idle backoff time slot, and a node transmits its packet when the backoff counter reaches zero [2]. In this paper, however, we consider an additional enhancement of the IEEE 802.11 DCF in addition to EIED backoff algorithm. The IEEE 802.11 DCF can be further improved by changing the behavior such that the backoff counter is decreased also at the end of DIFS or EIFS. This minor enhancement of the IEEE 802.11 DCF has also been proposed and discussed by Vaidya in the mailing list of the MANET (Mobile Ad hoc Networks) working group in the Internet Engineering Task Force (IETF) [3]. This change makes the MAC protocol more efficient, even though the difference may not be very significant. As an additional benefit of the change, the analysis becomes much simpler. Note that Bianchi's analysis model of IEEE 802.11 DCF in [4], and other analytical works that follows Bianchi's model imply this modification without explicitly specifying it. In this paper, whenever EIED backoff algorithm is considered, this additional enhancement of the IEEE 802.11 DCF is always implied unless indicated otherwise.

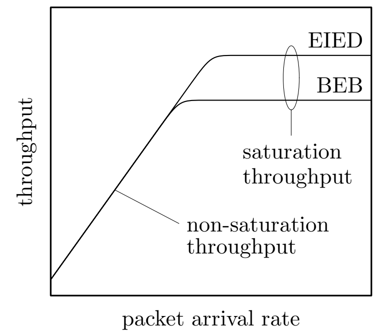

Fig. 2. Schematic comparison of the performance profile between BEB and EIED in terms of throughput vs. packet arrival rate.

III. Analysis of EIED

Analysis Model: As shown in [1], the performance difference between different backoff algorithms is insignificant under non-saturated condition. Fig. 2 illustrates typical performance profiles of BEB and EIED. (See Fig. 3 in [1] for simulation results showing this behavior.) As shown in the figure, the performance of the backoff algorithms only makes a practical difference when the network is saturated, and thus we analyze and optimize EIED under *saturation* condition in steady state. The saturation condition assumption is also made by Bianchi et al. in [5], where they used a 2-D Markov chain model to analyze the throughput of the IEEE 802.11 DCF. In our analysis, however, we use a 1-D Markov chain model, which is much simpler and easier to analyze without any loss of accuracy compared to the 2-D Markov chain model. This was achieved by removing deterministic behavior of the IEEE 802.11 DCF from the Markov chain model.

Let B_k be the backoff state that a node enters after k state transitions. Then, B_k is a 1-D Markov chain with $Mn+1$ states, where M and n are defined in (3) and (4). Assuming the system is in *steady state*, let P_i be a probability defined as

$$P_i = \Pr\{B_k = i\}, \quad i = 0, 1, 2, \cdots, Mn, \tag{10}$$

then P_i is the relative frequency that a node will enter state i in steady state, and satisfies

$$\sum_{i=0}^{Mn} P_i = 1. \tag{11}$$

Let p_c be the probability that a transmitted packet will experience a collision, then

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{Mn} \end{bmatrix}^T = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{Mn} \end{bmatrix}^T \begin{bmatrix} q_c & \cdots & \underset{\substack{\uparrow \\ (n+1)\text{-th} \\ \text{column}}}{p_c} & & \\ q_c & & & \ddots & \\ & \ddots & & & p_c \\ & & & \ddots & \vdots \\ & & & q_c & p_c \end{bmatrix}, \tag{12}$$

where $q_c = 1 - p_c$, and the $(Mn+1) \times (Mn+1)$ matrix on the right-hand side is the state transition matrix of the Markov chain. In the case shown in Fig. 1(b) where $M = 6$ and $n = 2$, (12) can be written as follows

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{12} \end{bmatrix}^T = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{12} \end{bmatrix}^T \begin{bmatrix} q_c & 0 & p_c & & & & \\ q_c & 0 & 0 & p_c & & & \\ & \ddots & & & \ddots & & \\ & & & & & & p_c \\ & & & & \ddots & & \vdots \\ & & & & & q_c & p_c \end{bmatrix}, \tag{13}$$

For example, (13) says that $P_2 = p_c P_0 + q_c P_3$, which means that the relative frequency that a node will will enter state 2 is the probability that a node in state 0 will experience a collision plus the probability that a node in state 3 will successfully transmit a packet.

When $n = 1$, P_i is obtained in a closed form from (12) as follows:

$$P_i = \frac{1}{\sum_{m=0}^{M} \left(\frac{p_c}{1-p_c}\right)^m} \left(\frac{p_c}{1-p_c}\right)^i, \quad i = 0, 1, 2, \cdots, M. \tag{14}$$

For general $n \geq 1$, a closed form solution for (12) does not exist, but P_i can be calculated numerically using the *power method* as follows:

$$\left[\ P_0 \quad P_1 \quad P_2 \quad \cdots \quad P_{Mn}\ \right] = \lim_{k \to \infty} e^T \mathcal{T}^k, \tag{15}$$

where $\mathcal{T}$ is the state transition matrix defined in (12), and the positive vector e is an initial condition such that $\mid e \mid_1 = 1$. Since $\mathcal{T}$ is a *regular* stochastic matrix, from the Perron-Frobenius Theorem, (15) converges exponentially to a unique solution, the left eigen-vector of $\mathcal{T}$ [6].

A state transition occurs after each packet transmission, and P_i represents only the probability (relative frequency) that a node *enters* state i. However, the expected time a node will *stay* in a state is different for each state due to the different contention window sizes. In state i, after choosing a random backoff counter value D_i, it takes $D_i + 1$ backoff time slots for a node to transit to another state after a successful or unsuccessful transmission. On average, a node will stay in state i for

$$\overline{d}_i = \mathrm{E}\{D_i + 1\} = \frac{W_i + 1}{2}, \quad i = 0, 1, \cdots, Mn, \tag{16}$$

backoff time slots. Let S_i be the probability that a node is in state i at an arbitrary time instance, then S_i defines the distribution of nodes over the backoff states, and is given by

$$S_i = \frac{P_i \overline{d}_i}{\overline{d}}, \tag{17}$$

where

$$\overline{d} = \sum_{i=0}^{Mn} P_i \overline{d}_i = \frac{1}{2} + \frac{W_0}{2} \sum_{i=0}^{Mn} P_i r^i \tag{18}$$

Define $s_{i,k}$, $i = 0, 1, \cdots, Mn$, $k = 0, 1, \cdots, X_i$, as the probability that a node is in backoff state i and its backoff counter value is k, then

$$S_i = \sum_{k=0}^{X_i} s_{i,k}. \tag{19}$$

Since the backoff counter is decreased by one at the elapse of every backoff time slot,

$$s_{i,k} = \sum_{d=k}^{X_i} S_i \Pr\{D_i = d \mid i\} \Pr\{t = k \mid i, D_i = d\} \tag{20}$$

Note that

$$\begin{aligned} \Pr\{t = 0 \mid i, D_i = 0\} &= \Pr\{t = 1 \mid i, D_i = 1\} \\ &\cdots \\ &= \Pr\{t = X_i \mid i, D_i = X_i\} \end{aligned} \tag{21}$$

and

$$\begin{aligned} \Pr\{t = 0 \mid i, D_i = d\} &= \Pr\{t = 1 \mid i, D_i = d\} \\ &\cdots \\ &= \Pr\{t = d \mid i, D_i = d\}. \end{aligned} \tag{22}$$

From (7), (20) can be written as

$$\begin{aligned} s_{i,k} &= S_i \frac{X_i + 1 - Y_i}{X_i(X_i + 1)} \sum_{d=k}^{X_i - 1} x + S_i \frac{Y_i}{X_i + 1} x \\ &= S_i \left(1 - \frac{X_i + 1 - Y_i}{X_i(X_i + 1)}\, k\right) x, \end{aligned} \tag{23}$$

where

$$x = \Pr\{t = k \mid i, D_i = d\}, \qquad \begin{aligned} k &= 0, 1, \cdots, d, \\ d &= 0, 1, \cdots, X_i. \end{aligned} \tag{24}$$

By substituting (23) into (19), we obtain

$$x = \frac{1}{\overline{d}_i}, \tag{25}$$

where $W_i = X_i + Y_i$ and (16) are used. From (17), (23), and (25), we have

$$s_{i,k} = \left(1 \; - \; \frac{X_i + 1 - Y_i}{X_i(X_i + 1)}\, k\right) \frac{S_i}{\overline{d}_i} \tag{26}$$

$$= \left(1 \; - \; \frac{X_i + 1 - Y_i}{X_i(X_i + 1)}\, k\right) \frac{P_i}{\overline{d}} \tag{27}$$

Note that $s_{i,0}$ is the probability that a node is in state i and the backoff counter is expired. Let p_t be the probability that a node will transmit a packet in an arbitrary time slot. Thus,

$$p_t = \sum_{i=0}^{Mn} s_{i,0} = \sum_{i=0}^{Mn} \frac{P_i}{\overline{d}} = \frac{1}{\overline{d}} \tag{28}$$

For the special case of $n = 1$,

$$S_i = \frac{(\frac{p_c}{1-p_c})^i (w_i + 1)}{\sum_{m=0}^{M} (\frac{p_c}{1-p_c})^m + W_0 \sum_{m=0}^{M} (\frac{r p_c}{1-p_c})^m} \tag{29}$$

$$\overline{d} = \frac{1}{2} + \frac{W_0}{2} \frac{\sum_{m=0}^{M} (\frac{r p_c}{1-p_c})^m}{\sum_{m=0}^{M} (\frac{p_c}{1-p_c})^m} \tag{30}$$

$$p_t = \frac{2}{1 + W_0 \sum_{m=0}^{M} (\frac{r p_c}{1-p_c})^m \;/\; \sum_{m=0}^{M} (\frac{p_c}{1-p_c})^m} \tag{31}$$

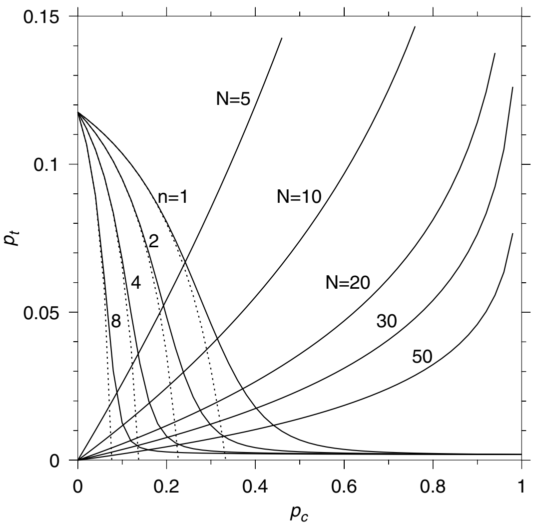

Fig. 3. Plots of (28) and (33). The monotonically decreasing functions represent (28) with $\mathrm{CW_{min}} = 16$, $\mathrm{CW_{max}} = 1024$ (dotted lines: $\mathrm{CW_{max}} = \infty$), $r_I = 2$, $n = 1, 2, 4, 8$. The monotonically increasing functions represent (33) with $N = 5, 10, 20, 30, 50$.

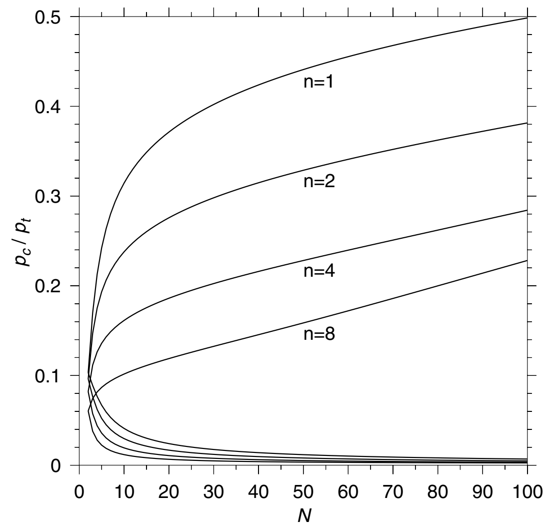

Fig. 4. Plots of p_c and p_t with respect to N. $\mathrm{CW_{min}} = 16$, $\mathrm{CW_{min}} = 1024$, $r_I = 2$, $n = 1, 2, 4, 8$.

In our analysis so far, we assumed that the probability of collision of a transmitted packet p_c was known, and p_t was calculated in terms of p_c. Now, assume p_t is known. A collision occurs when more than two nodes transmit at the same time, and the conditional probability that a transmitted packet will experience a collision can be expressed in terms of p_t as

$$p_c = 1 - (1 - p_t)^{N-1}, \tag{32}$$

where $(1 - p_t)^{N-1}$ is the probability that none of the other $N - 1$ nodes will transmit. Solving (32) for p_t gives

$$p_t = 1 - (1 - p_c)^{1/(N-1)}. \tag{33}$$

Note that p_t in (33) is a monotonically increasing function of p_c. This is because when there are more transmissions, there is more chance of a collision. On the other hand, in (28), p_t is a monotonically *decreasing* function of p_c, because higher p_c causes more backoff and thus larger average contention window size, which leads to smaller probability of transmission p_t. We do not prove the monotonicity of (28) in this paper and accept it as a conjecture. A proof of the monotonicity of a corresponding equation for the case of BEB can be found in [7]. Due to the monotonicity of (28) and (33), they have a unique intersection, and thus p_c and p_t can be obtained by finding the intersection of the two curves for given values of N, $\mathrm{CW_{min}}$, $\mathrm{CW_{max}}$, r_I, and r_D.

Figure 3 shows the plots p_t as a function of p_c. The monotonically decreasing curves in solid lines are plots of (28) for $n = 1, 2, 4, 8$, when $\mathrm{CW_{min}} = 16$, $\mathrm{CW_{max}} = 1024$, and $r_I = 2$. The figure shows that as n gets larger (more conservative reduction of the contention window size), the probability of transmission p_t has smaller value to the same p_c. For comparison purposes, corresponding curves with $\mathrm{CW_{max}} = \infty$ are also plotted in dotted lines.

Figure 4 shows the plots of p_t and p_c, obtained by finding the intersections of (28) and (33), with respect to the number of nodes N. As the load (N) increases, the probability of collision p_c increases while the probability transmission p_t decreases. Also note that more conservative backoff policy (larger n) gives smaller p_t, hence smaller p_c.

SATURATION THROUGHPUT: Let P_{succ} and P_{coll} be the probabilities that there is a successful transmission and a collision in an arbitrary time slot, respectively. Then, P_{succ} and P_{coll} can be calculated in terms of p_c and p_t as follows:

$$P_{\text{succ}} = \binom{N}{1} p_t (1 - p_t)^{N-1} = N p_t (1 - p_t)^{N-1} \tag{34}$$

$$P_{\text{busy}} = 1 - (1 - p_t)^N \tag{35}$$

$$P_{\text{coll}} = P_{\text{busy}} - P_{\text{succ}} \tag{36}$$

The saturation throughput (ST) can be calculated by taking into account the average time duration of data transmission:

$$\mathrm{ST} = \frac{P_{\text{succ}} E[P]}{P_{\text{succ}} T_s + P_{\text{coll}} T_c + (1 - P_{\text{busy}})\sigma}, \tag{37}$$

where $E[P]$ is the average packet payload, T_s is the average time duration the channel is busy when there is a successful transmission, T_c is the average time duration the channel is busy when there is a collision, and σ is the slot time.

IV. OPTIMIZATION OF EIED

We obtain the optimal backoff factors of EIED backoff algorithm that maximize the saturation throughput. Parameters defined in the IEEE 802.11 specifications that are not directly related to the backoff algorithm, such as $\mathrm{CW_{min}}$ and $\mathrm{CW_{max}}$, are not optimized. Specifically, given $\mathrm{CW_{min}}$ and $\mathrm{CW_{max}}$, we find optimal M and n that maximize the saturation throughput. Note that finding M and n is equivalent to finding r_I and r_D.

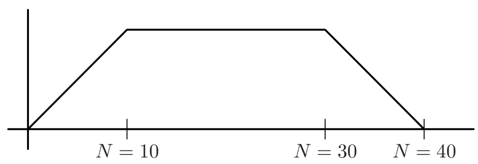

Fig. 5. Weighting function. N: number of nodes.

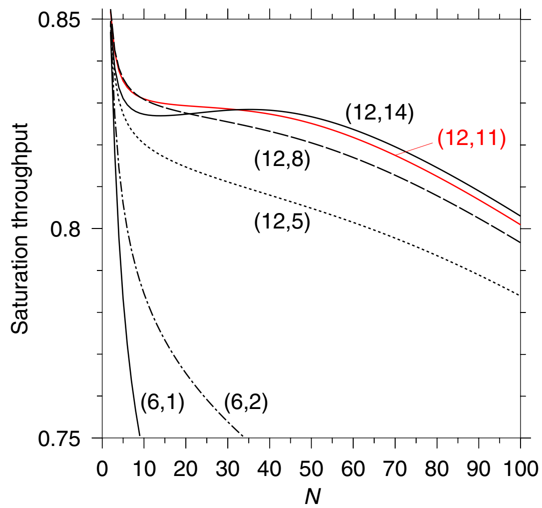

Fig. 6. Plots of the saturation throughput of EIED. Access mechanism: basic, $\mathrm{CW}_{\min} = 16$, $\mathrm{CW}_{\max} = 1024$, Optimum at $(M, n) = (12, 11)$.

As shown in (37), if the cost of collision is large (large T_c), the saturation throughput is very sensitive to the change of P_{coll}, and thus it is beneficial to have more conservative backoff policy (large r_I and/or small r_D) to reduce p_c. When the cost of collision is small (small T_c), we obtain better performance with a more aggressive backoff policy that yields small probability of idle channel.

In this paper, we present an example of optimization of the backoff parameters of EIED. For simplicity, we fix the packet payload to 1024 bytes. The optimization is conducted for $0 < N < 40$, where N is weighted with the weighting function shown in Fig. 5. The significance of the weighting function is that, the performance of EIED is equally important for any N between 10 and 30, inclusive, but it is less important for $N < 10$, or $30 < N < 40$, and $N \geq 40$ is of no interest. We calculate the saturation throughput in (37) for each N, and using the weighting function in Fig. 5, the optimum M and n are determined to maximize the weighted sum of the saturation throughput over N. For practical networks, which is often more complicated compared to our scenario, the methodology used in this paper can be easily applied using $E_N[\mathrm{ST}]$ instead of ST in (37), where $E_N[\cdot]$ represents an expectation over the distribution of N. In fact, in optimizing EIED for practical networks, the real challenge lies in collecting accurate statistics of the network such as the distributions of the packet length and network load N.

Fig. 6 shows the analytical saturation throughput vs. N when the basic access mechanism of DCF is used for various values of (M, n), including the optimum pair $(12, 11)$ which implies that $r_I = \sqrt{2}$, and $r_D = 2^{1/22}$. EIED with $(12, 11)$ offers the best overall performance in the range $0 < N < 40$, for which EIED is optimized. Note that the range of the saturation throughput in Fig. 6 is from 0.75 to 0.85. EIED with $(12, 11)$ shows quite consistent saturation throughput of about 0.83 for from $N = 10$ to $N = 40$. It is interesting to note that EIED with $(12, 14)$ displays peformace improvement as the number of nodes increases in the range $10 < N < 40$. This is because r_D is too small that it takes too many successful transmissions to return to the previous W_i after a collision, causing underutilization of the channel when the number of nodes is small. Note that it takes 14 successful transmissions, as oppose to 11 successful transmisison for the optimal EIED, to compensate a single collision.

The performance curve for BEB is not included in Fig. 6, because since the performance of BEB is so poor that including it will make it hard to compare the EIED performance curves for various parameter pairs. The saturation throughput of BEB is about 0.7 for $N = 10$, and 0.57 for $N = 40$. Note that even the two worst performers in Fig. 6, $(M, n) = (6, 1)$ and $(M, n) = (6, 2)$, easily outperform BEB.

V. CONCLUSION

EIED backoff algorithm was proposed in [1] to enhance the performance of the IEEE 802.11 DCF. EIED provides significant performance improvement without additional complexity. But the analysis of EIED was not provided to show its outperformance theoretically.

In this paper, we extend the work in [1] through an analysis of EIED backoff algorithm and present a methodology of obtaining the optimal parameters of EIED backoff algorithm in a given condition. The methodology used in this paper can be easily applied to real scenarios which is much more complicated in general. In fact, in optimizing EIED for practical networks, the real challenge lies in collecting accurate statistics of the network such as the distributions of the packet length and network load N.

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