Unveiling Nonlinear Gravitational Clustering

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Renormalized Cosmological Perturbation Theory

M. Crocce and R. Scoccimarro, Phys. Rev. D **73** 063519 & 063520

Nonlinear Evolution of Baryon Acoustic Oscillations

M. Crocce and R. Scoccimarro, arXiv:0704.2783

Higher Order Propagators in Gravitational Clustering

F. Bernardeau, M. Crocce and R. Scoccimarro, *in preparation*

Overview

* Why nonlinear clustering ?
  tools: simulations, halo model, perturbation theory (PT)

* Problems with standard PT

* Renormalized PT approach :

> Power Spectrum and two-point Propagator (Baryon Acoustic Oscillations)

> Bispectrum and three-point Propagator (work in *progress*)

Conclusions

Nonlinear Gravitational Clustering

scales much smaller than the Horizon (Hubble radius) $\longrightarrow$ Newtonian gravity

scales larger than strong clustering regime $\longrightarrow$ *single stream approximation*

no velocity dispersion or pressure
(prior to virialization and shell crossing)

$$\nabla^2\Phi(\mathbf{x},\tau) = \frac{3}{2}\Omega_m(\tau)\ \mathcal{H}^2(\tau)\ \delta(\mathbf{x},\tau)$$

$$\frac{\partial\delta(\mathbf{x},\tau)}{\partial\tau} + \nabla\cdot\{[1+\delta(\mathbf{x},\tau)]\,\mathbf{u}(\mathbf{x},\tau)\} = 0$$

$$\frac{\partial\mathbf{u}(\mathbf{x},\tau)}{\partial\tau} + \mathcal{H}(\tau)\ \mathbf{u}(\mathbf{x},\tau) + \mathbf{u}(\mathbf{x},\tau)\cdot\nabla\mathbf{u}(\mathbf{x},\tau) = -\nabla\Phi(\mathbf{x},\tau)$$

velocity field can be assumed irrotational $\quad \theta(\mathbf{x},\tau) \equiv \nabla\cdot\mathbf{u}(\mathbf{x},\tau)$

$$\frac{\partial\tilde{\delta}(\mathbf{k},\tau)}{\partial\tau} + \tilde{\theta}(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\tilde{\theta}(\mathbf{k}_1,\tau)\,\tilde{\delta}(\mathbf{k}_2,\tau),$$

$$\frac{\partial\tilde{\theta}(\mathbf{k},\tau)}{\partial\tau} + \mathcal{H}(\tau)\tilde{\theta}(\mathbf{k},\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\tilde{\delta}(\mathbf{k},\tau) = -\int d^3k_1 d^3k_2\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\beta(\mathbf{k}_1,\mathbf{k}_2)\,\tilde{\theta}(\mathbf{k}_1,\tau)\,\tilde{\theta}(\mathbf{k}_2,\tau)$$

Linear order $\delta_L(k,z) = D_+(z)\delta_0(k) \xrightarrow[\langle \delta(\mathbf{k})\delta(-\mathbf{k})\rangle]{} P_{\rm lin}(k,z) = [D_+(z)]^2 P_0(k)$

Standard perturbation theory expands the density contrast in terms of the linear solution,

$$P(k,z) = D_+^2(z)P_0(k) + \underbrace{P_{13}(k,z) + P_{22}(k,z)}_{\text{1 - loop terms}} + \ldots$$

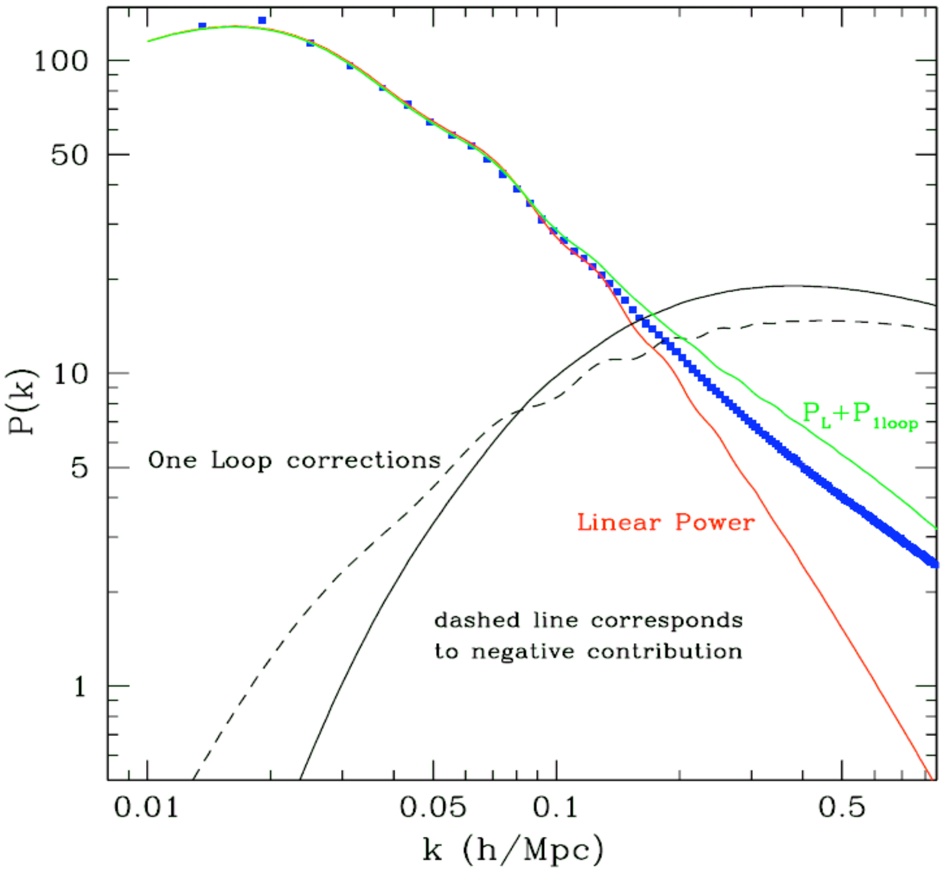
$$P_{\rm 1loop} \sim \mathcal{O}(P_{\rm lin}\,\Delta_{\rm lin})$$

$$P_{\rm 2loop} \sim \mathcal{O}(P_{\rm lin}\,\Delta_{\rm lin}^2)$$

$$\Delta_{\rm lin} \equiv 4\pi k^3 P_{\rm lin}$$

This expansion is valid at large scales where fluctuations are small, but it brakes down when approaching the nonlinear regime where $\Delta_{\rm lin} \gtrsim 1$.

One needs to sum up all orders to get meaningful answers !!



Renormalized Perturbation Theory

$$P(k,z) = D_+^2(z) P_0(k) + P_{13}(k,z) + P_{22}(k,z) + \ldots$$

$$P_{22}(k,z) \equiv 2\int [F_2^{(s)}(\mathbf{k}-\mathbf{q},\mathbf{q})]^2 P_{\rm lin}(|\mathbf{k}-\mathbf{q}|,z) P_{\rm lin}(q,z) \mathrm{d}^3\mathbf{q}$$

one loop irreducible

$$P_{13}(k,z) \equiv D_+^2(z)\, P_0(k)\, 6\int F_3^{(s)}(\mathbf{k},\mathbf{q},-\mathbf{q}) P_{\rm lin}(q,z) \mathrm{d}^3\mathbf{q}$$

one loop reducible

$$P(k,z) = D_+^2(z)\left[1 + 6\int F_3^{(s)} P_{\rm lin} \mathrm{d}^3\mathbf{q} + \ldots\right] P_0(k) + P_{\rm irreducibles}(k,z)$$

all orders can be systematically incorporated

$$\boxed{P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\rm Mode\,Coupling}(k,z)}$$

G is the nonlinear propagator
(or two-point propagator)

final density / vel divergence

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')} \right\rangle$$

Initial conditions

RPT behind the scene

$$\Psi_a(\mathbf{k},\eta) \equiv (\delta(\mathbf{k},\eta), -\theta(\mathbf{k},\eta)/\mathcal{H}), \qquad \eta \equiv \ln a(\tau).$$

The solution to the nonlinear equation of motion can be formally written as

$$\Psi_a(\mathbf{k},\eta) = g_{ab}(\eta)\ \phi_b(\mathbf{k}) + \int_0^\eta d\eta'\ g_{ab}(\eta-\eta')\ \gamma^{(s)}_{bcd}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\ \Psi_c(\mathbf{k}_1,\eta')\Psi_d(\mathbf{k}_2,\eta')$$

Initial Conditions

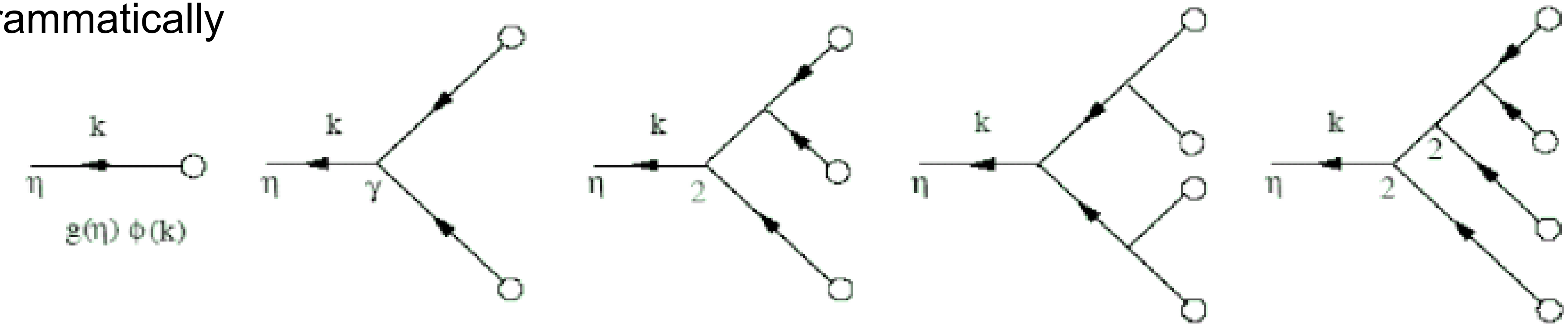
Linear propagator $$g_{ab}(\eta) = \frac{e^\eta}{5}\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \frac{e^{-3\eta/2}}{5}\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix},$$

growing mode
$\phi_a(\mathbf{k}) \propto (1,1)$

decaying mode
$\phi_a(\mathbf{k}) \propto (1,-3/2)$

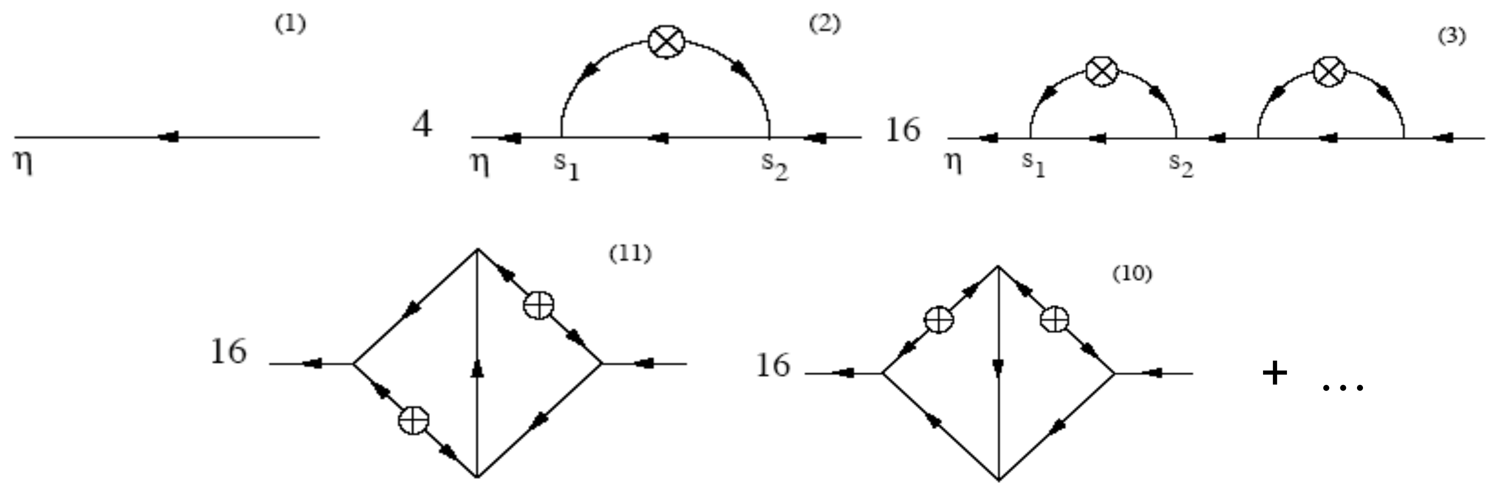
Diagrammatically

Ψ_a

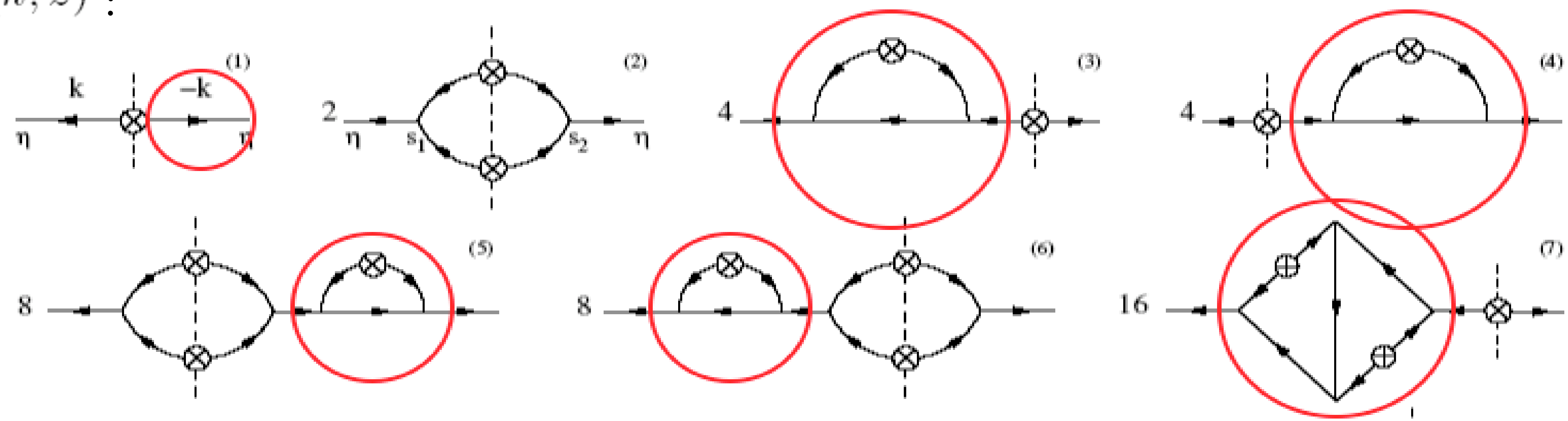


In this way we keep all sub-leading time dependencies (i.e. nonlinear corrections involve integrals of momentum and time). The series for the propagator and power spectrum are:

$G_\delta(k, z)$:



$P(k, z)$:



$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\mathrm{Mode\,Coupling}}(k,z)$$

$G_\delta(k,z)$ is well defined and has physical meaning

* The **propagator** is a measure of the *memory to the initial conditions*

At large scales it reduces to the usual growth factor in linear theory (i.e. memory is "preserved"): $G_\delta(k \longrightarrow 0) \longrightarrow D_+$

At smaller scales it receives nonlinear corrections due to mode coupling that drive it to zero (the final field "does not remember" the initial distribution): $G_\delta(k \longrightarrow \infty) \longrightarrow 0$

* The rest of the diagrams (still an infinite number) can be thought of as the effect of **Mode Coupling**. The propagator can be re-summed in them as well

They measure generation of structure at small scales

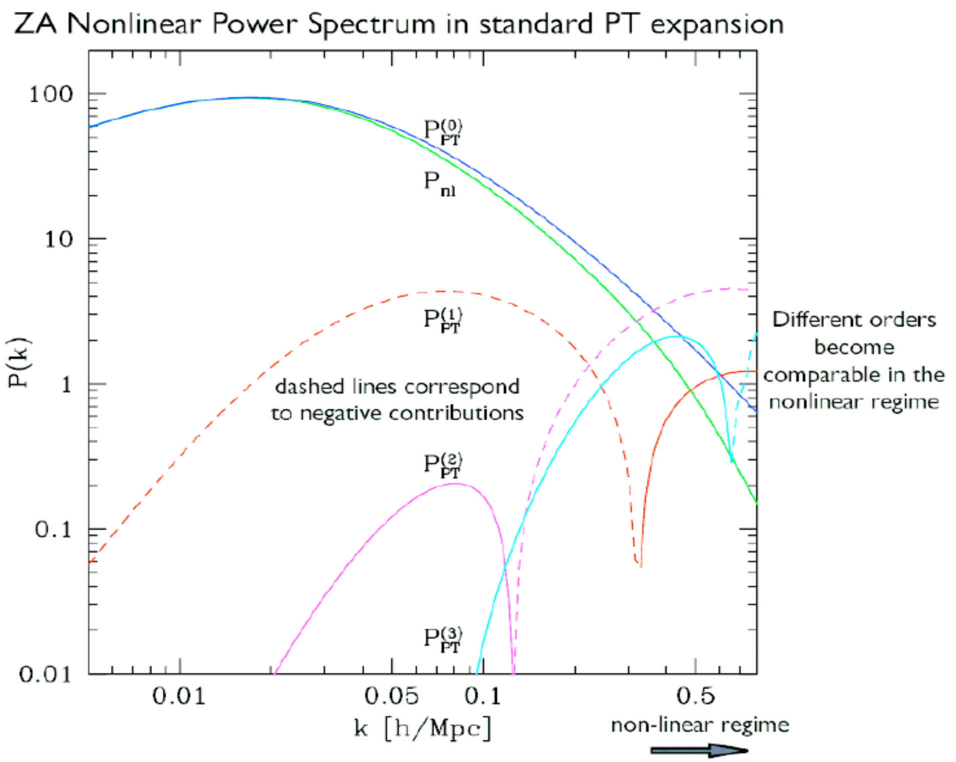
They dominate in a narrow range of scales drastically improving convergence. Now small scale large amplitude fluctuations are exponentially suppressed

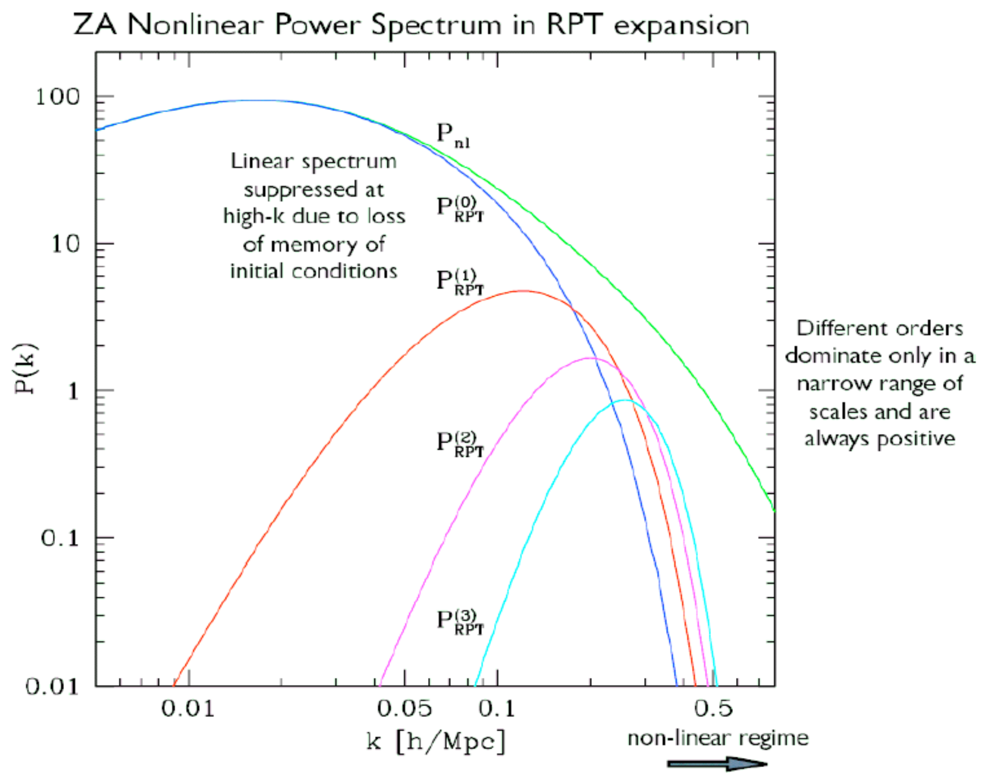
$$P_{\mathrm{MC}}(k,z) \sim \mathcal{O}([G^2(k',z')P_0(k')])$$

Example : Zel'dovich Approximation (particles moving in straight lines according to their primordial gravitational forces)

* In this approx. is possible to compute the nonlinear propagator and mode coupling power exactly

$$\boxed{G_\delta^{ZA} = a\ \exp(-k^2\sigma_v^2/2)} \qquad \left(\sigma_v^2 \equiv (1/3)\int d^3q P_L(q,a)/q^2\right)$$



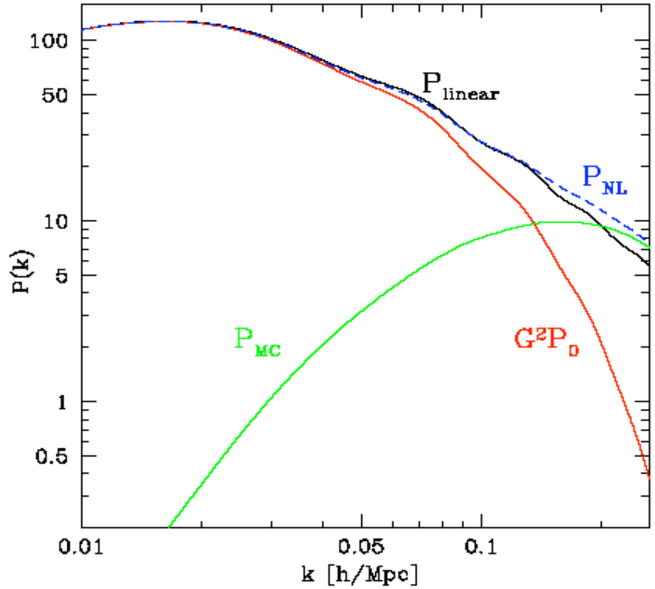


$$P(k,z) = [D_+(z)]^2\ P_0(k) + P^{(1)}(k,z) + \cdots$$

depends on integrals of $D_+^2(z)P_0(k)$

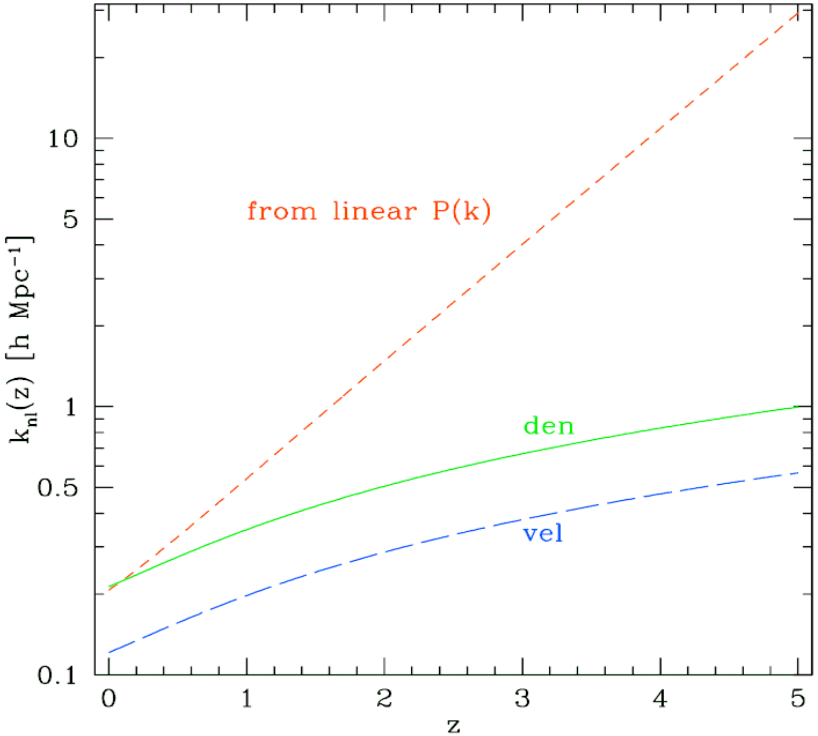
$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{1\,\mathrm{Loop}}(k,z) + \cdots$$

depends on integrals of $G^2(k,z)P_0(k)$



Perturbations are more non-linear than usually thought ..

Even at k ~ 0.05 h/Mpc the deviation from linear power is of order 5 %



$$4\pi k_{nl}^3 P_{\mathrm{L}}(k_{nl}) = 1$$

$$k_{nl} \sim \sigma_v^{-1} \qquad \sigma_v^2 = \frac{1}{3}\int \frac{P_{\mathrm{L}}(q)}{q^2} d^3q$$

1D velocity dispersion in linear theory

What can be said about the propagator in Real Dynamics ?

(as it became a key ingredient)

A. From the Cross-Correlation

* For Gaussian initial conditions it can be shown that
(and a similar expression holds for velocity)

$$G_\delta = \langle \delta(\mathbf{k}, z)\delta_0(-\mathbf{k}, z)\rangle / P_0(k)$$

Thus it can be measured from N-body !

In this sense the propagator measures the memory of perturbations to their initial values

B. From the Functional Derivative

* The definition involves

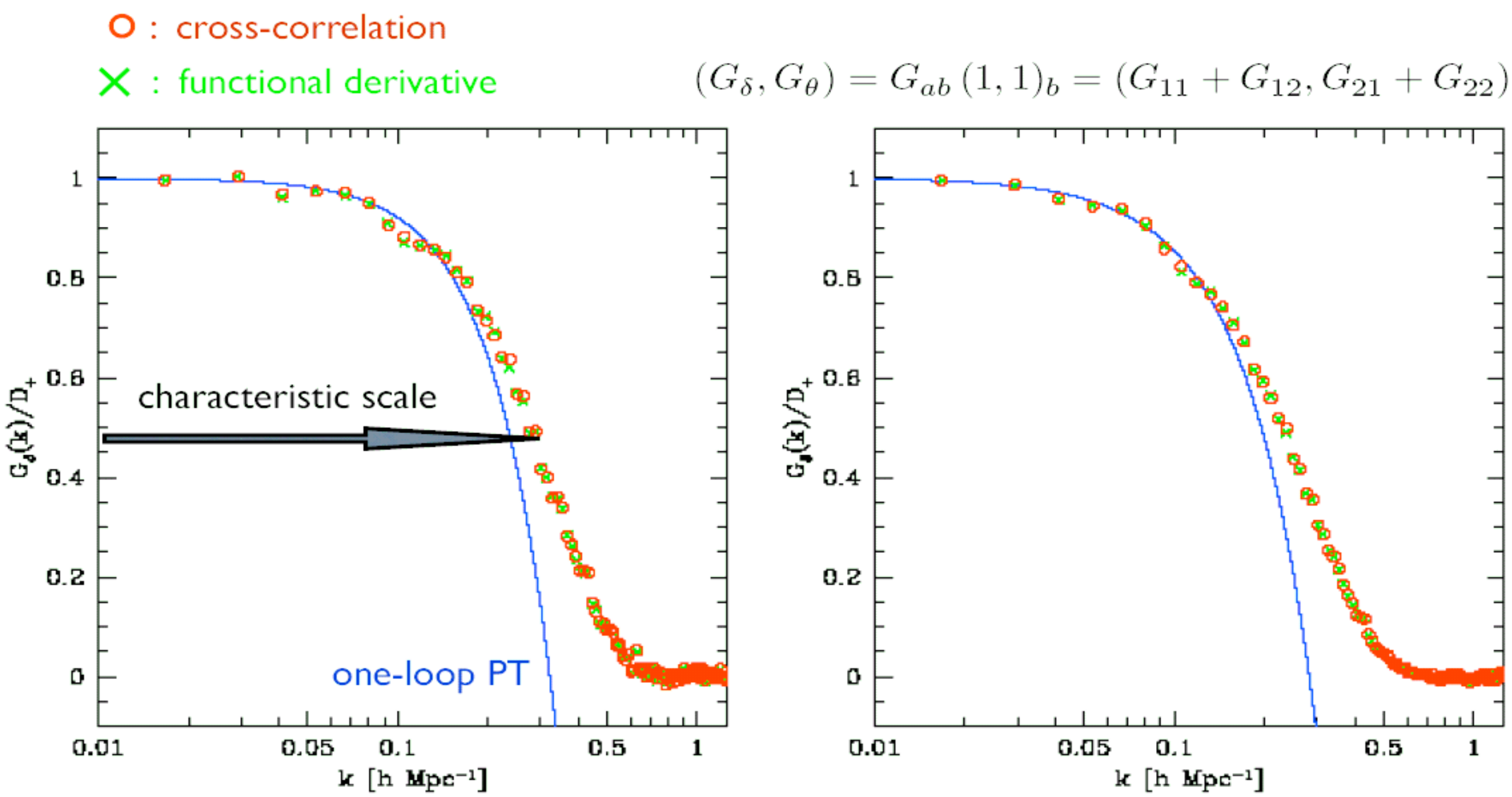
$$\frac{\delta \Psi_a(\mathbf{k})}{\delta \phi_b(\mathbf{k}')} = \lim_{\epsilon \to 0} \frac{\Psi_a[\phi_b(\mathbf{k}) + \epsilon\, \delta_{\rm D}(\mathbf{k} - \mathbf{k}')] - \Psi_a[\phi_b(\mathbf{k})]}{\epsilon}.$$

This is impractical but doable assuming ergodicity

Measuring G(k) in N-body simulations

Both methods give the same answer !

$$(G_\delta, G_\theta) = G_{ab}\,(1,1)_b = (G_{11} + G_{12}, G_{21} + G_{22})$$

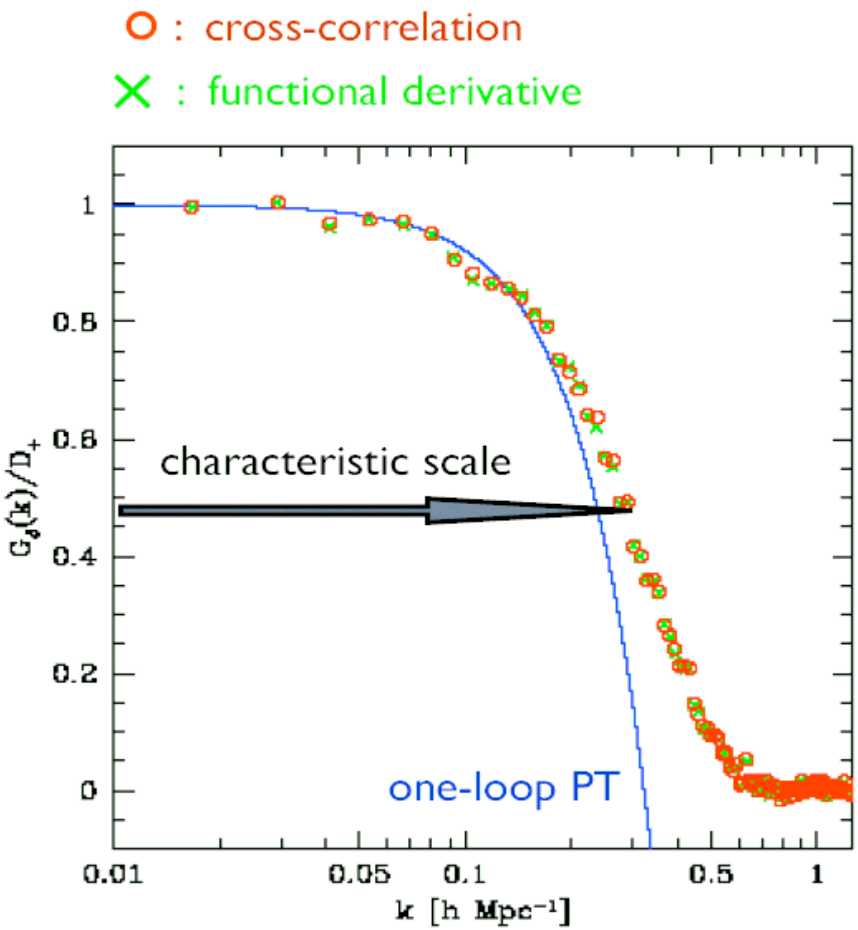


Density propagator between z =5 and z = 0

Velocity divergence propagator

Measuring G(k) in N-body simulations

Both methods give the same answer !



Density propagator between z =5 and z = 0

Nearly Gaussian decay

The departure from linear evolution at large scales is well described by the one-loop diagram (first nonlinear correction)

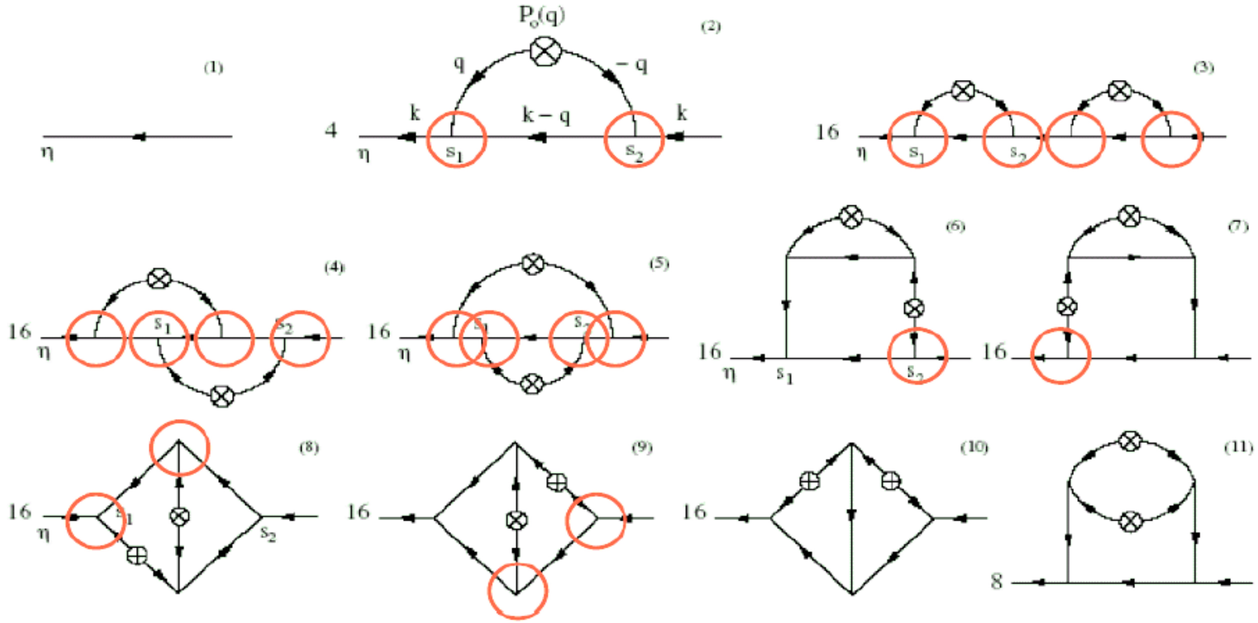
low-k limit :

$$G_\delta(k,z) \approx D_+(z)\left(1 - \frac{61}{210}k^2\sigma_v^2 D_+^2\right)$$

Calculating the propagator in RPT

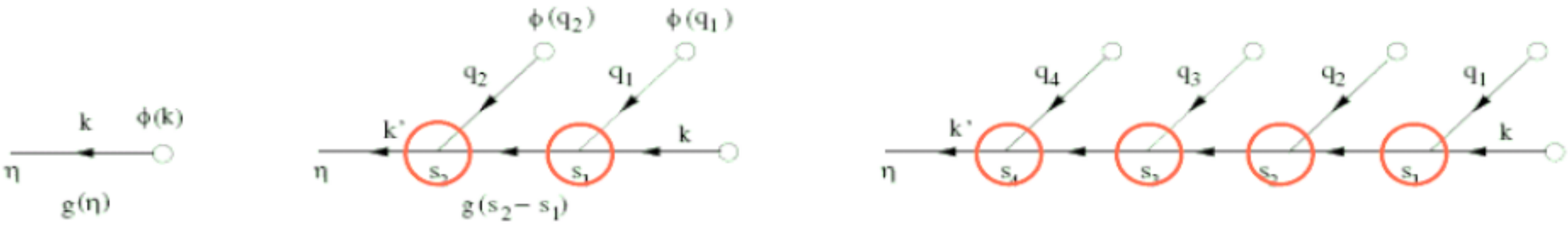
low-k : one loop RPT
large-k ??

$$\left\langle \frac{\delta \Psi_a(\mathbf{k}, z)}{\delta \phi_b(\mathbf{k}')} \right\rangle$$

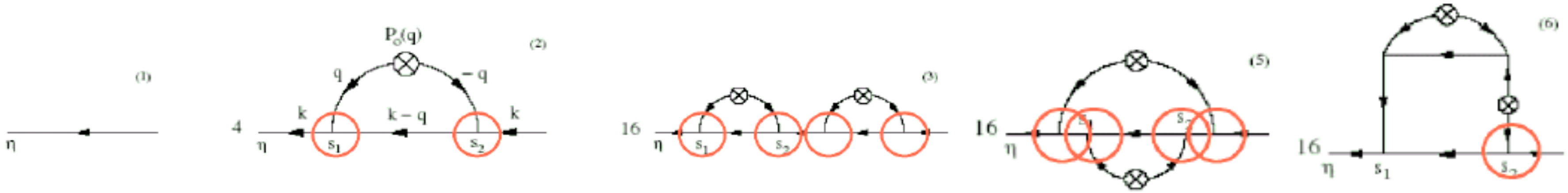


The dominant contribution have the simplest ramification possible in terms of the initial conditions (that's why they cross-correlate the most). They arise from

$$\Psi_a(\mathbf{k}, \eta) \sim$$

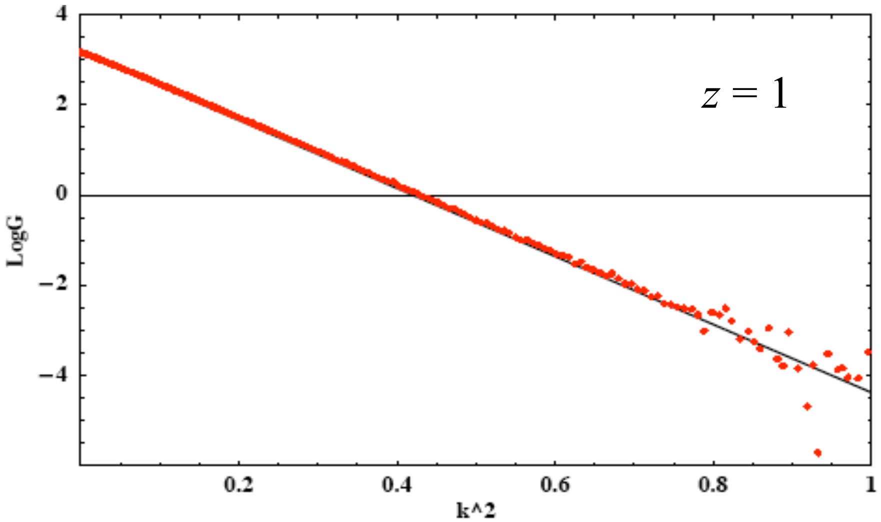


We were able to re-sum the dominant contribution to all orders
in the large-k limit ! (because the vertex simplifies considerably)



$$\boxed{G_\delta(k,z) \simeq D_+(z)\exp\left(-\frac{1}{2}k^2\sigma_v^2(D_+(z)-1)^2\right)}$$

$$\sigma_v^2 \equiv \frac{1}{3}\int d^3q\frac{P_0(q)}{q^2}$$

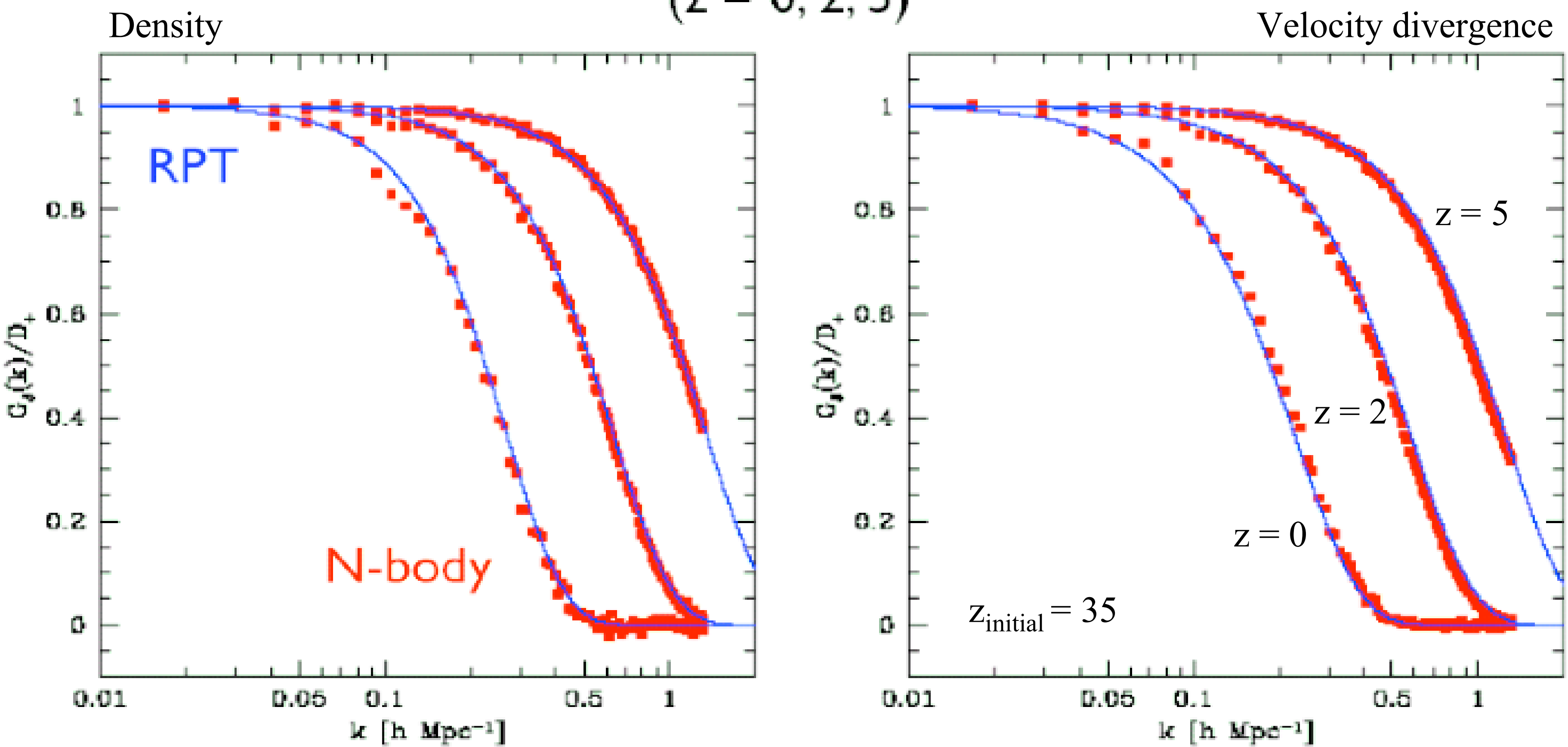


To recover the two-point propagator for all scales we match this asymptotic result with to the low-k (one-loop correction) expression by

* regarding the one-loop propagator as the power series expansion of a Gaussian

- must decay monotically as k increases for fixed time
- must decay monotically as time increases for fixed k

Comparison between RPT and N-Body Simulations
(z = 0, 2, 5)



- The RPT predictions match simulations, even into the nonlinear regime, for density and velocity fields, without introducing any free parameters.

Ready to model observable quantities and attack concrete problems ⟹ Application to Baryon Acoustic Oscillations (PRD 77 (2008) 023533)

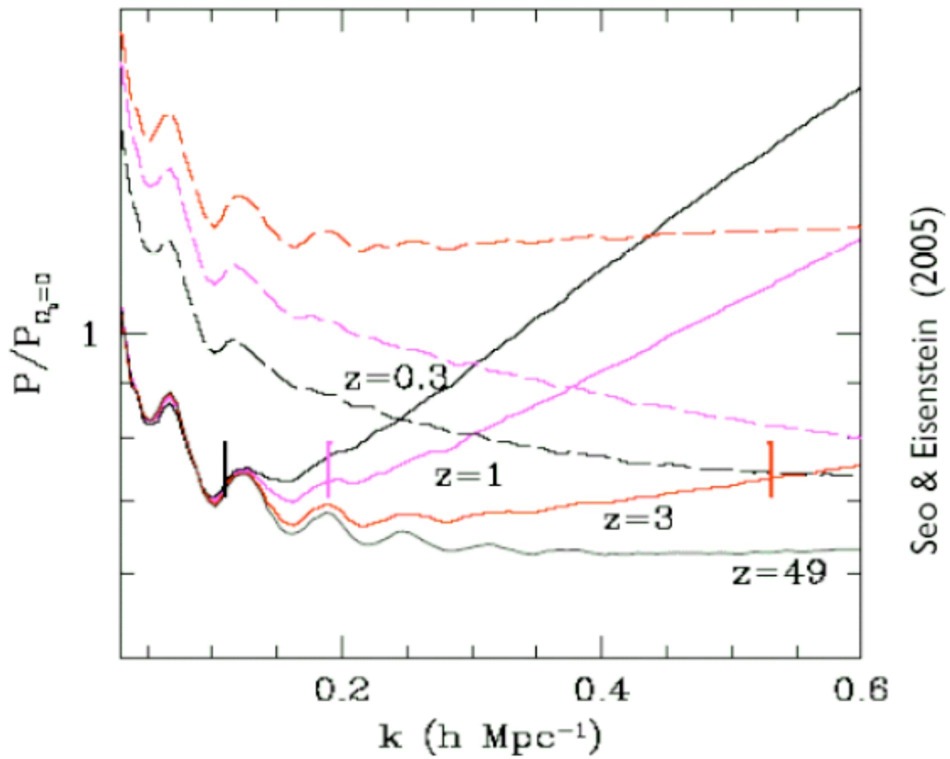
- One can use BAO imprinted in the dark matter power spectrum as a probe of the expansion history (to get to dark energy / modified gravity)

- This signature however, gets modified due to nonlinear evolution

Challenge:
1% error on wiggle position induces about 5% error in *w*

Hard to achieve for simulations :

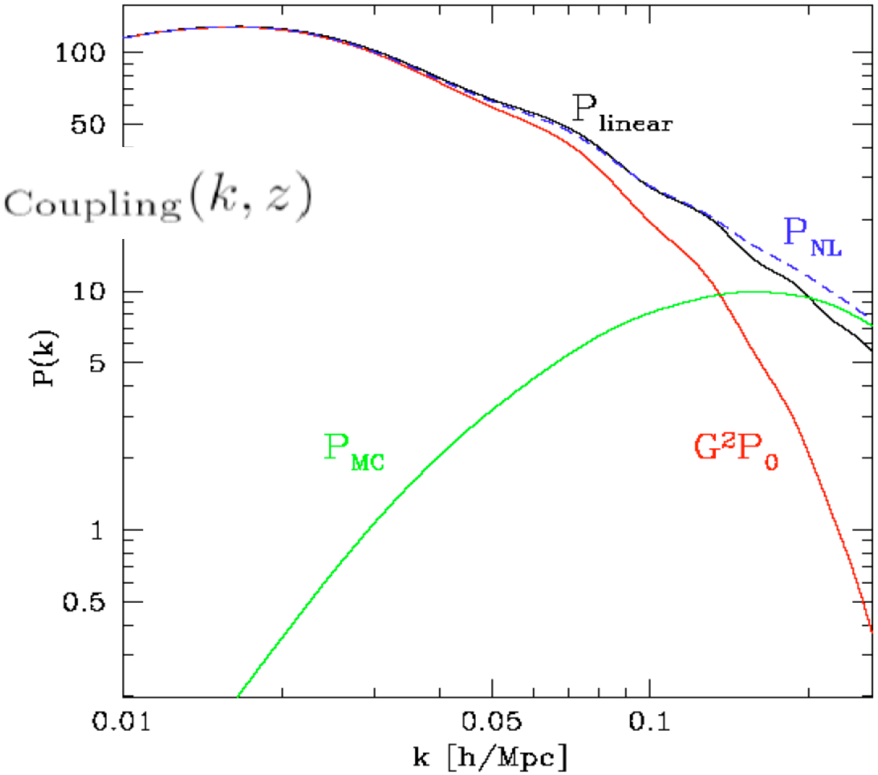
* Many big volume realizations are needed (BAO scale ~ 100 Mpc , large cosmic variance)
* Cosmology dependence



RPT is a perfect match for BAO because it can describe accurately the nonlinear scales where the acoustic signature extends to

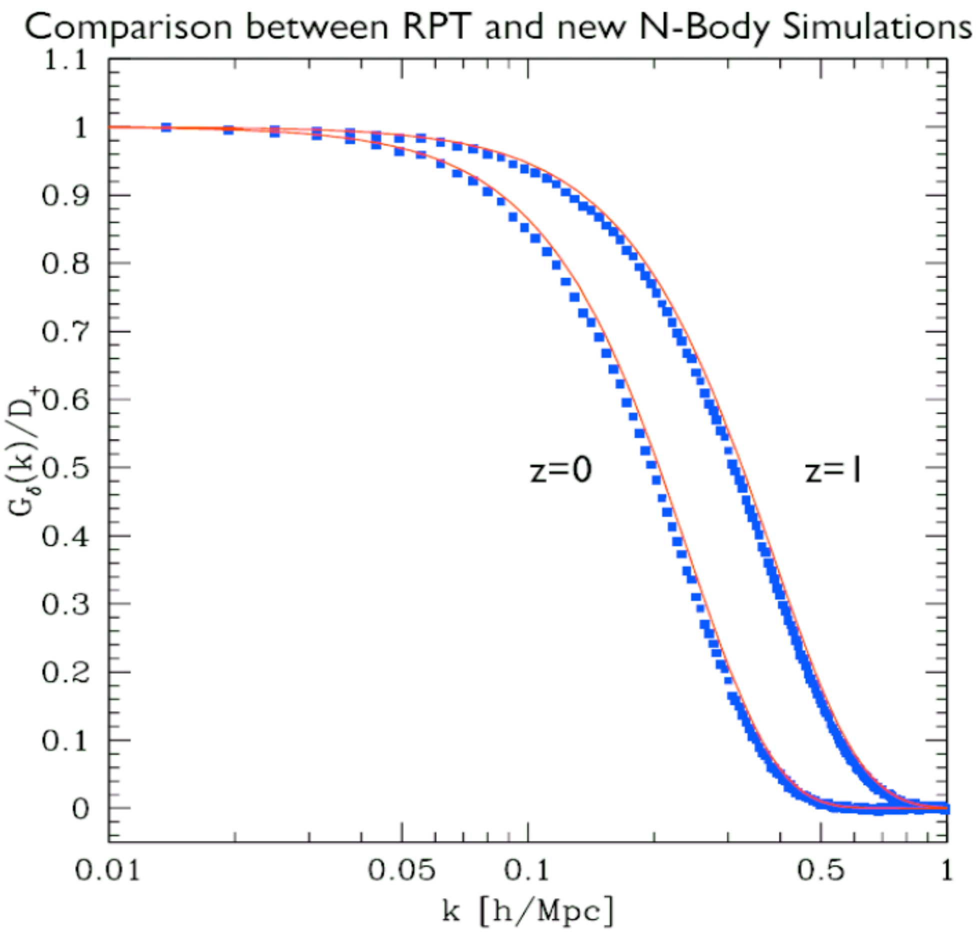
Provided with the prescription for the two-point propagator we computed $P_{\text{Mode Coupling}}$ "up to two loops" (only one irreducible contribution at each order)

$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\text{Mode Coupling}}(k,z)$$



We run a large set of simulations to test RPT accurately (50 realizations of 640^3 particles in 1280 Mpc/h aside box)

We found that RPT *slightly overestimates* the propagator

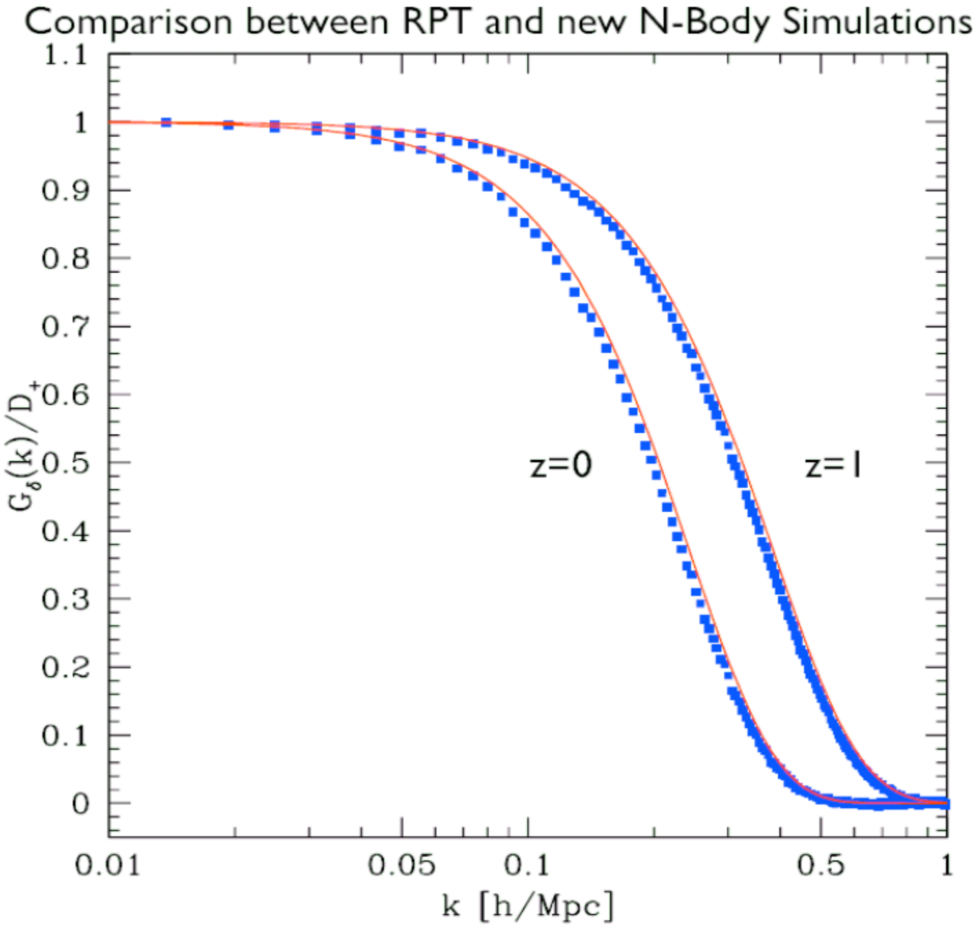


This could be due to several reasons

1) systematic in simulations or transients

2) cosmology dependence of decaying modes is important

3) sub-leading diagrams in large-k limit re-summation contribute slightly to the result



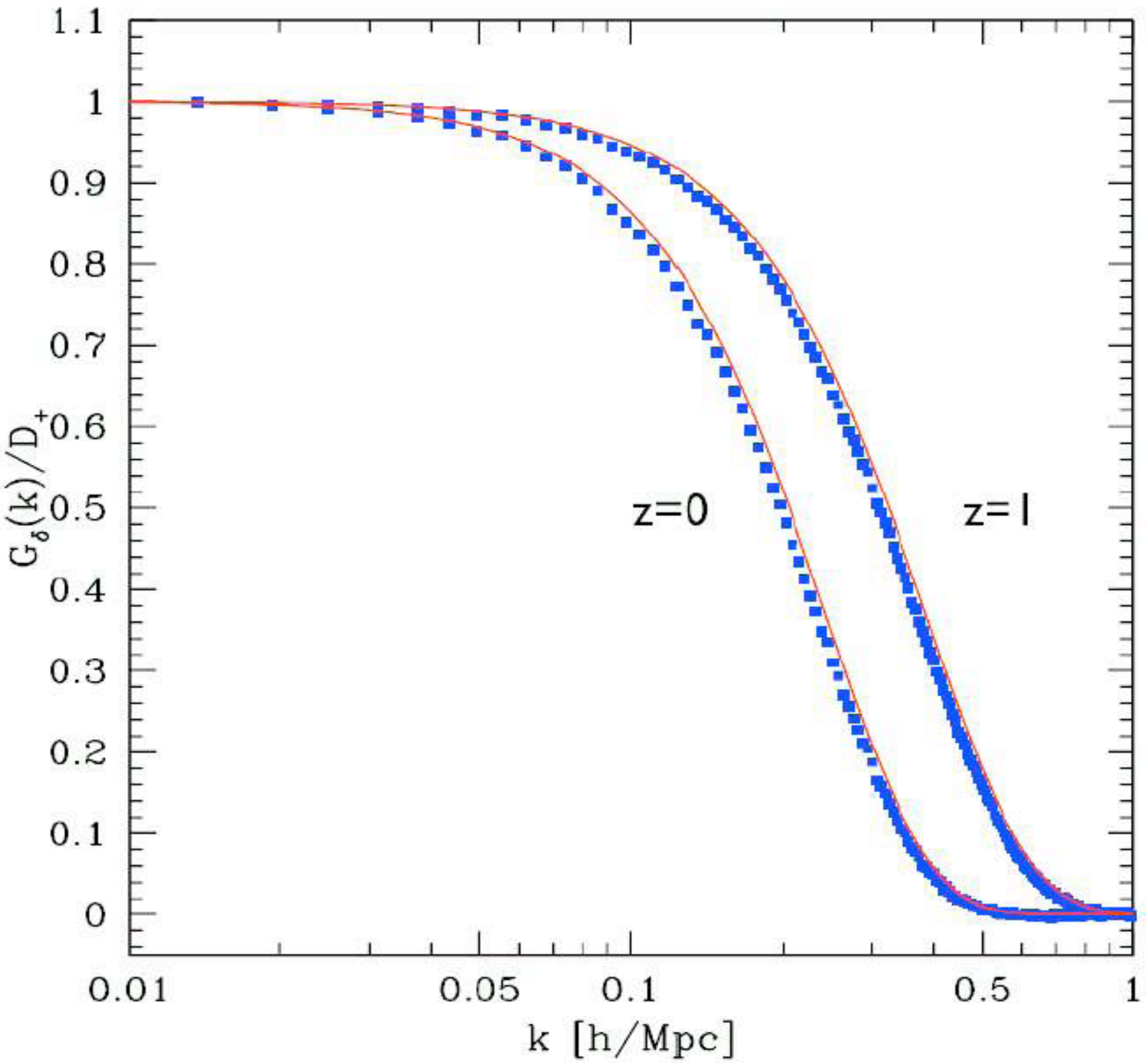
This could be due to several reasons

1) systematic in simulations and transients

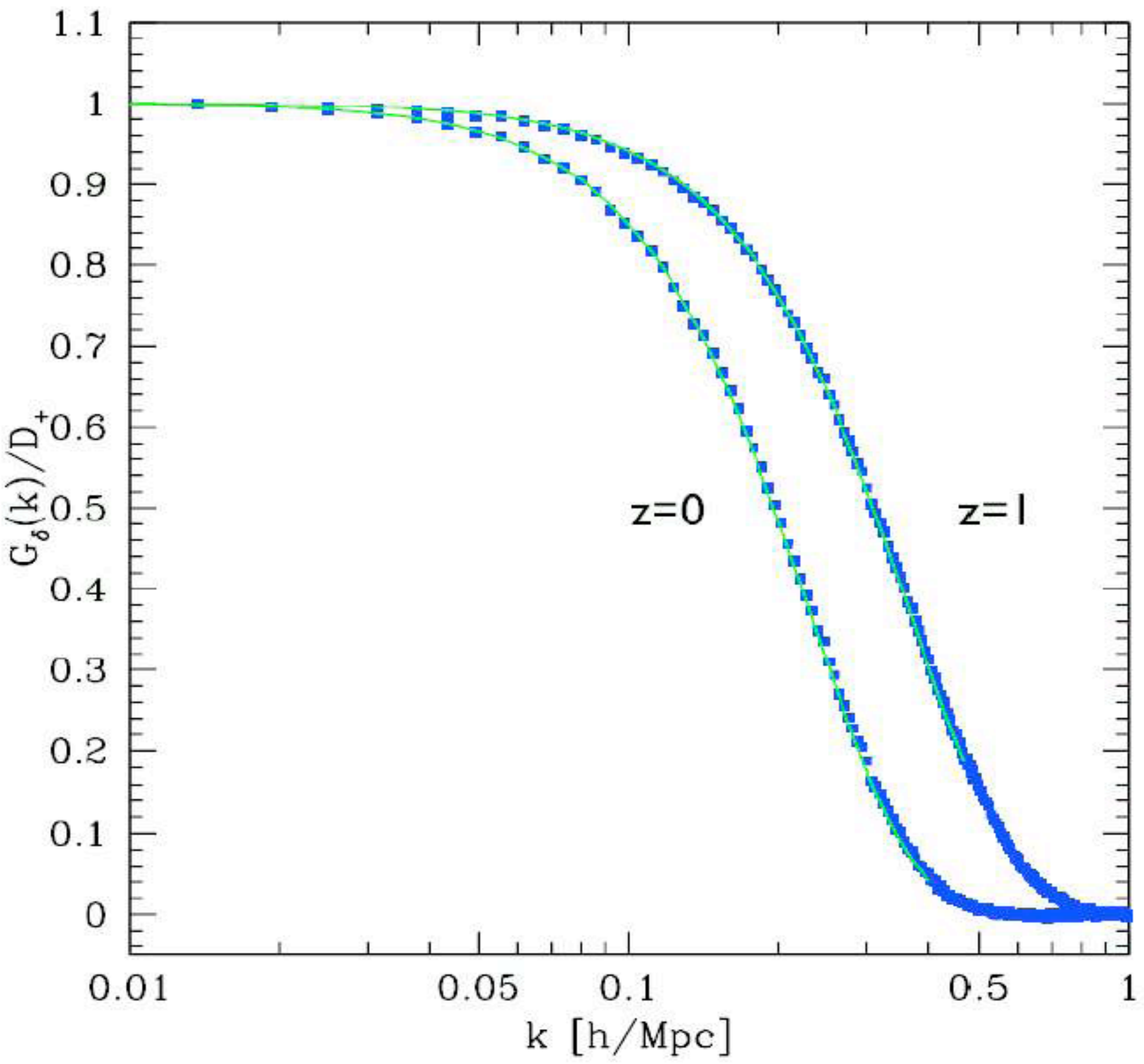
2) cosmology dependence of decaying modes is important

3) sub-leading diagrams in large-k limit re-summation contribute slightly to the result

At present #3 seems the most likely explanation. The correction coming from this sub-leading diagrams can be estimated.



1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$G_\delta(k)/D_+$
z=0
z=1
0.01
0.05
0.1
0.5
1
k [h/Mpc]



1.1
1
0.9
0.8
0.7
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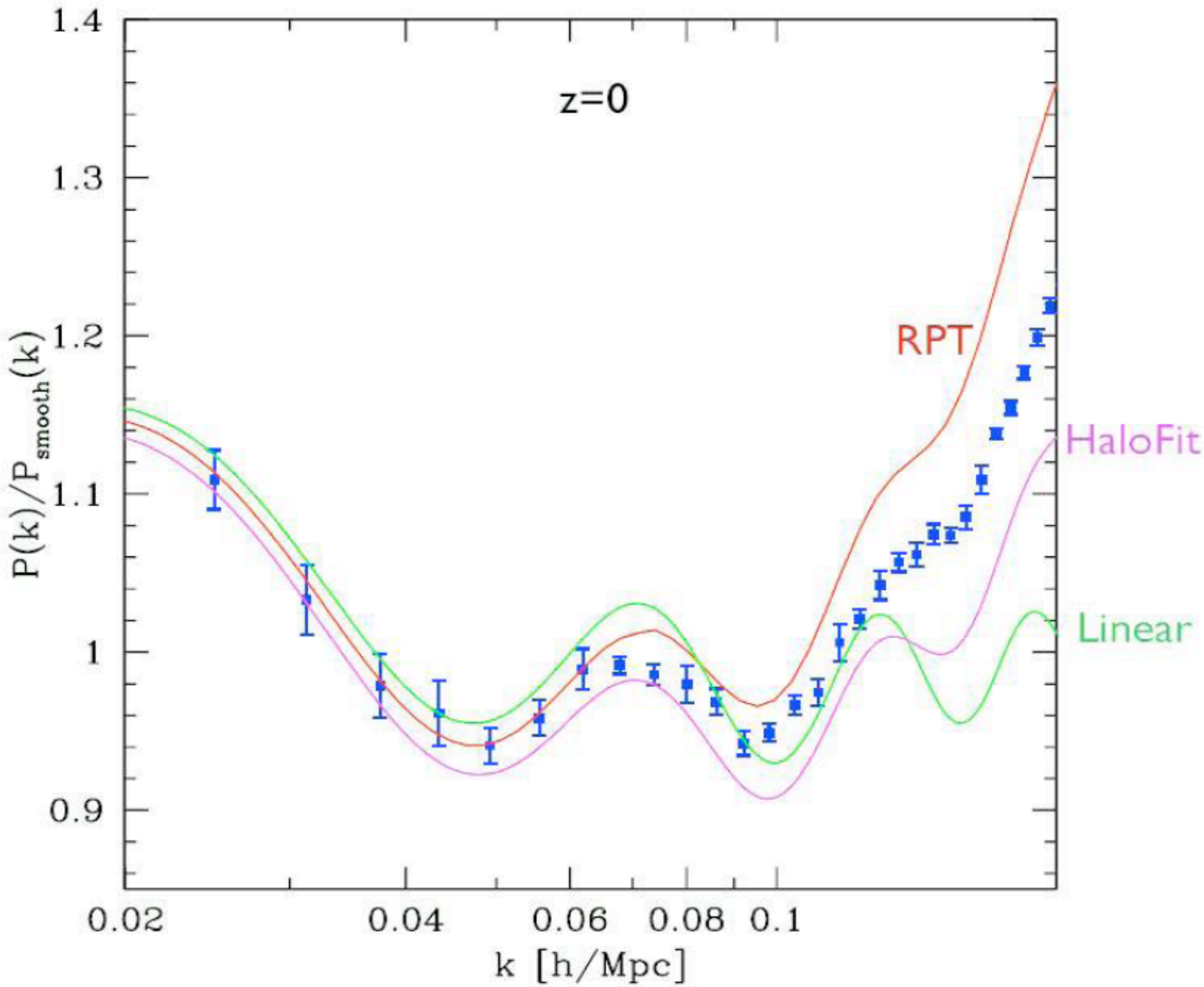
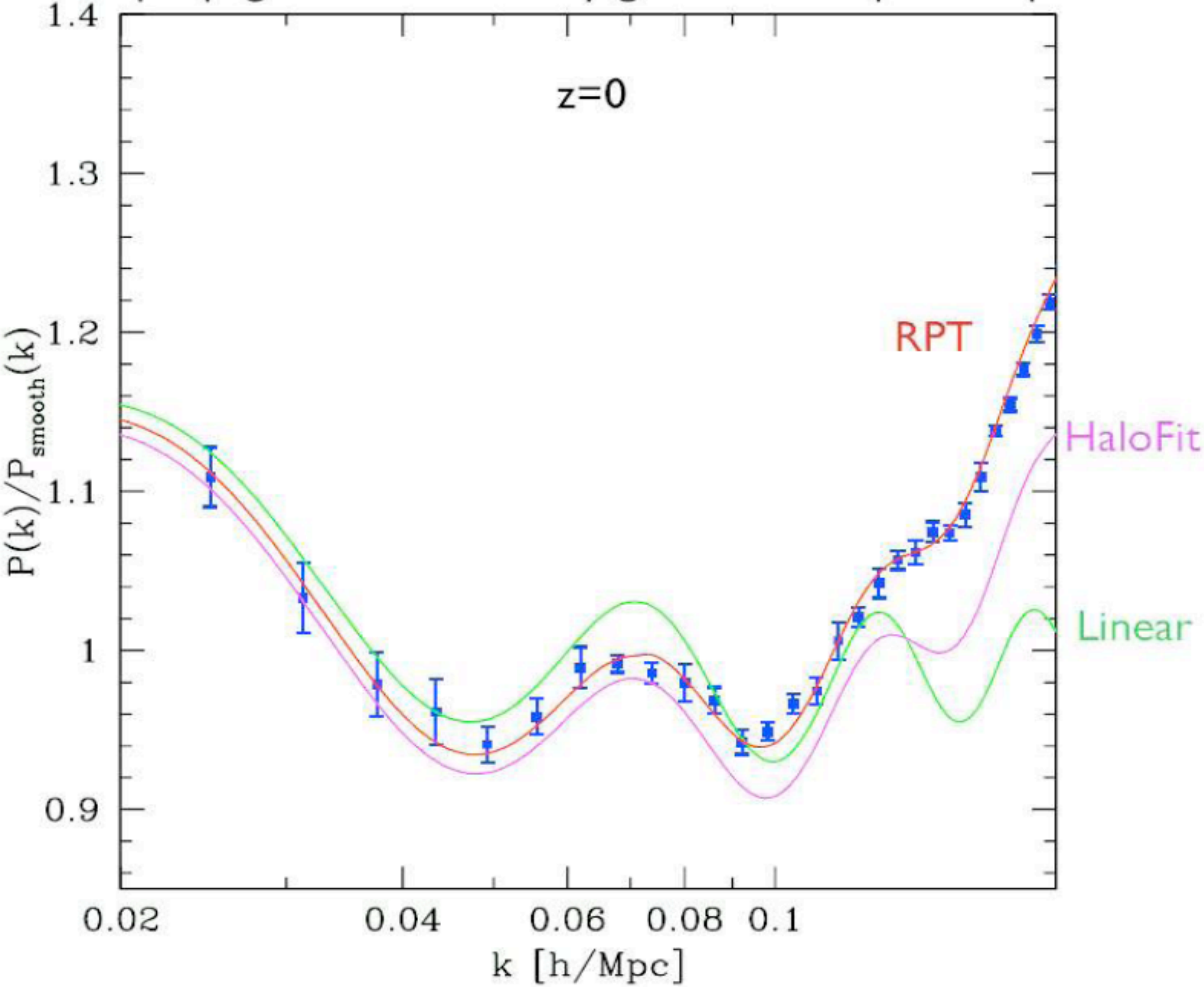
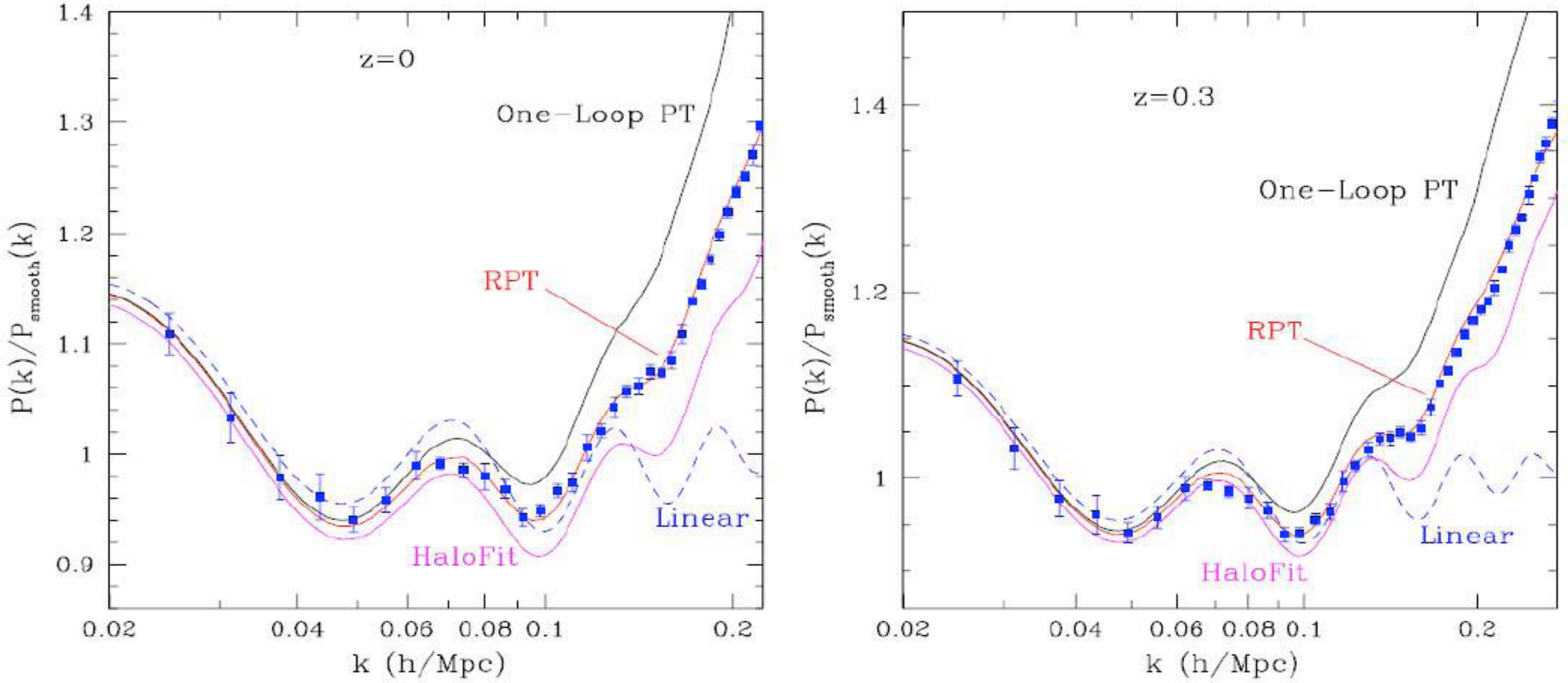
0.5
0.4
0.3
0.2
0.1
0
$G_\delta(k)/D_+$
z=0
z=1
0.01
0.05
0.1
0.5
1
k [h/Mpc]


z=0
RPT
HaloFit
Linear
P(k)/P_smooth(k)
k [h/Mpc]
1.4
1.3
1.2
1.1
1
0.9
0.02
0.04
0.06
0.08
0.1

Corrected propagator automatically gives correct power spectrum

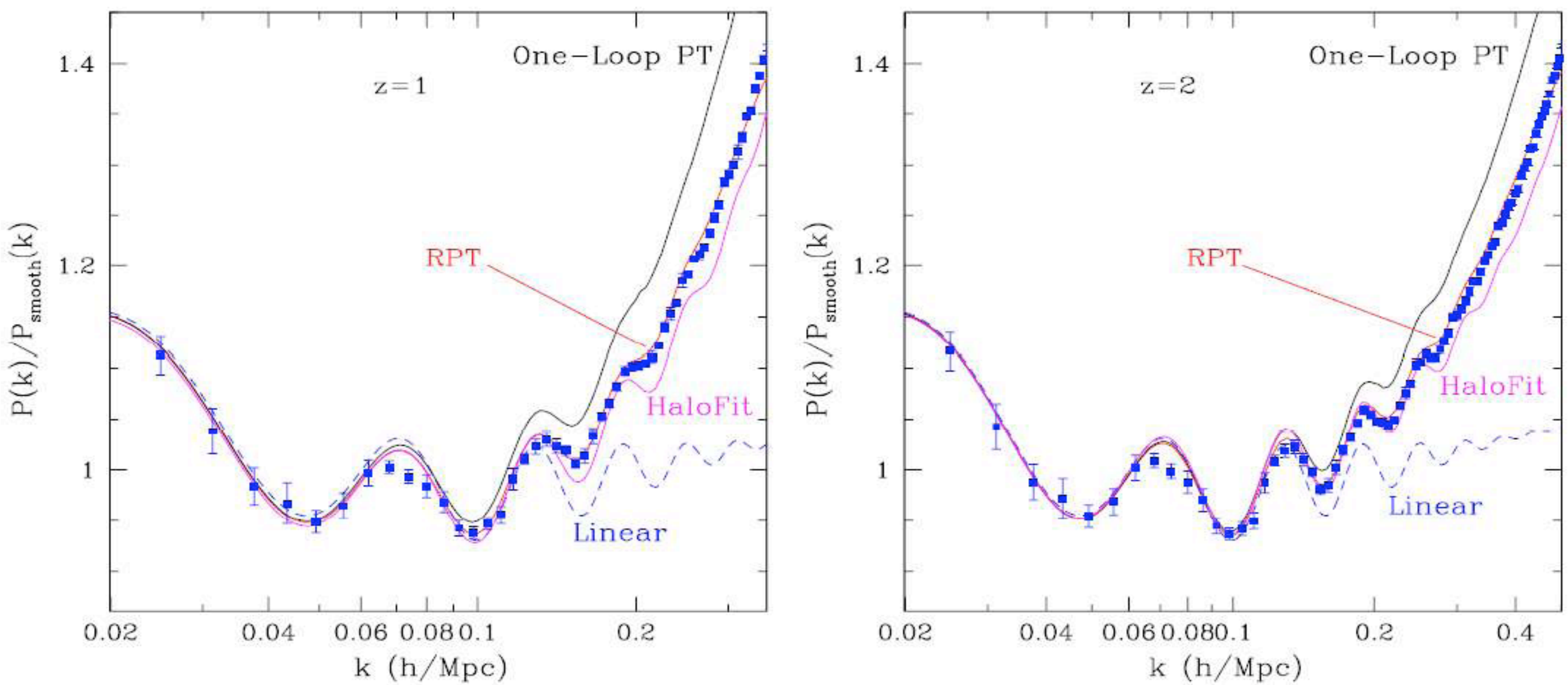


RPT vs. Simulations : Power Spectrum at low redshift



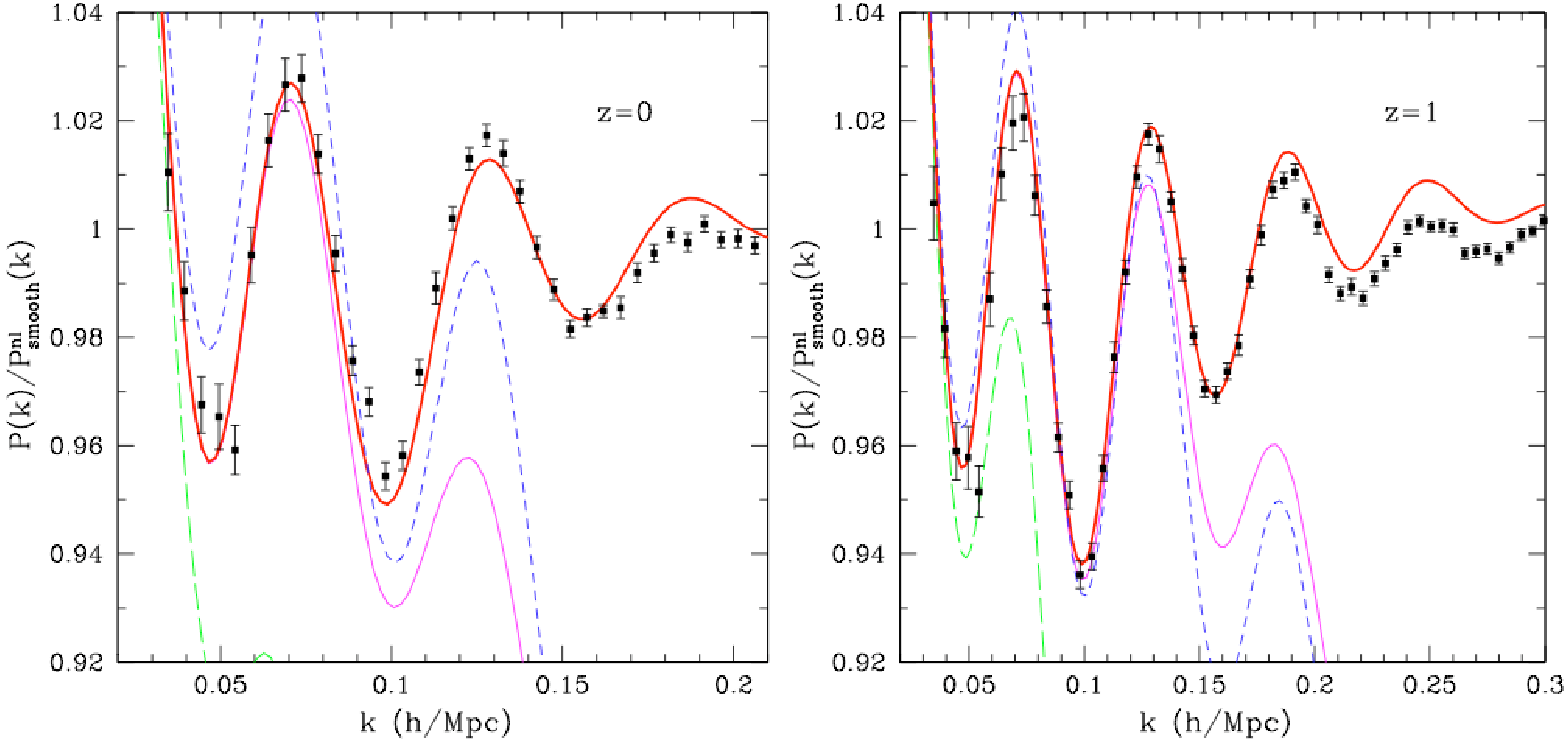
$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\rm Mode\,Coupling}(k,z)$$

RPT vs. Simulations : Power Spectrum at higher redshift



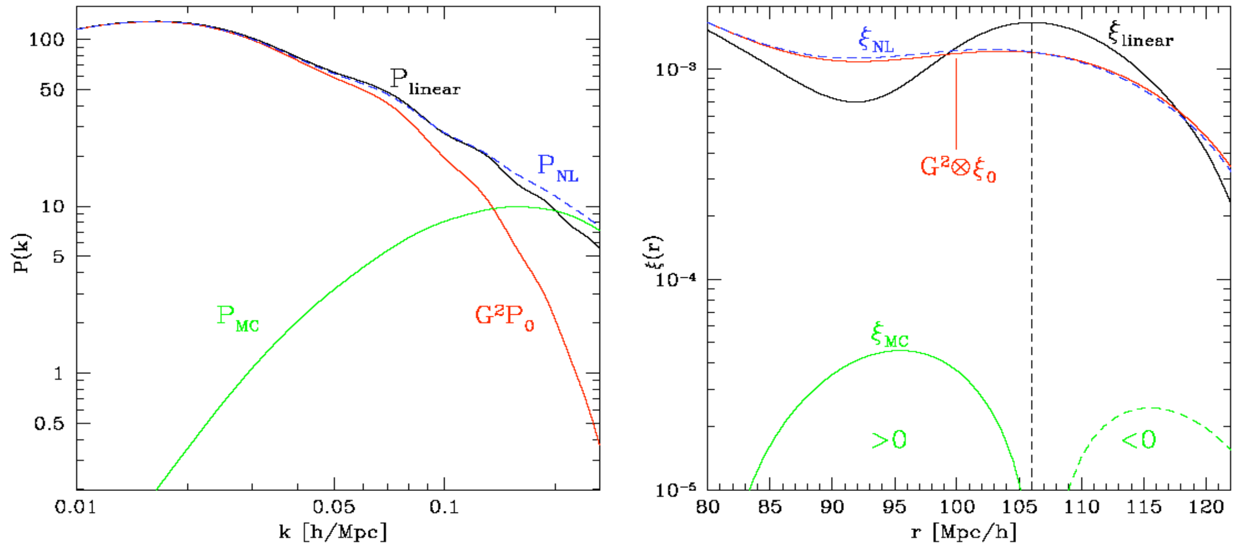
$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\mathrm{Mode\,Coupling}}(k,z)$$

RPT vs Simulations , dividing by a "nonlinear" smooth reference spectrum

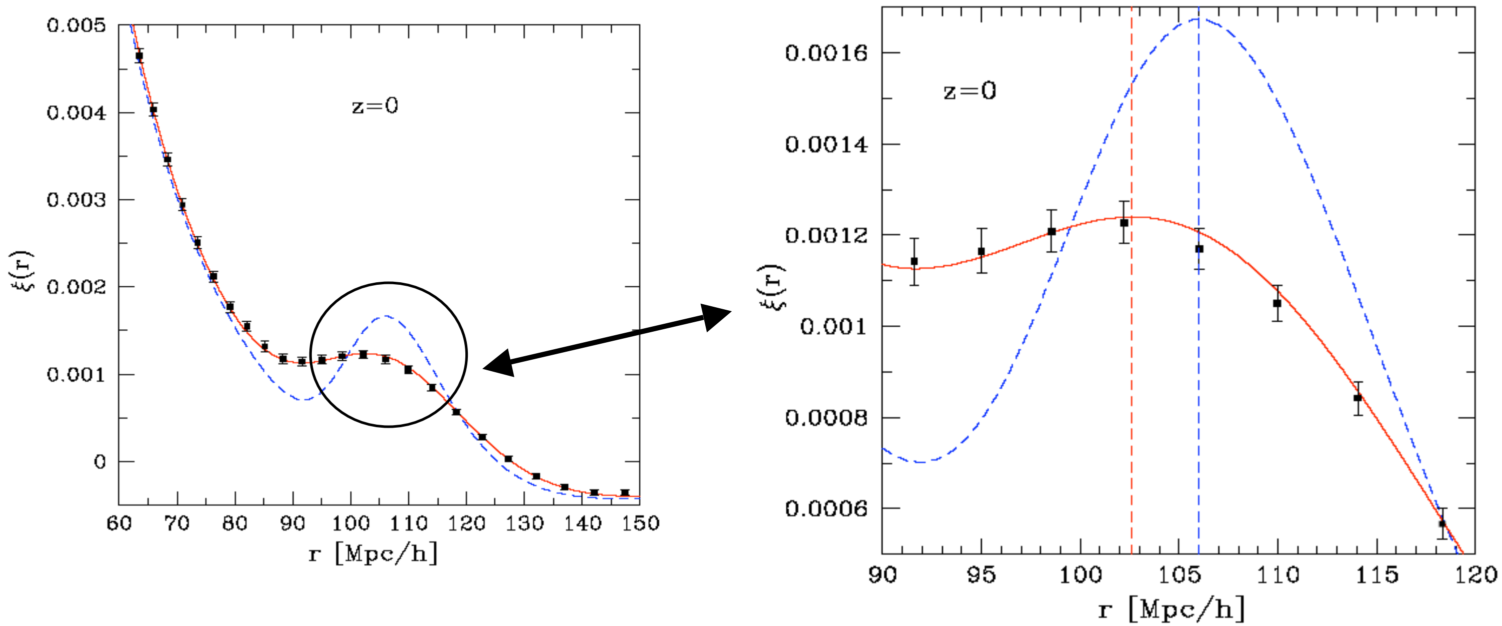


Correlation Function

$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\mathrm{Mode\,Coupling}}(k,z) \longleftrightarrow \xi(r) = [\xi_{\mathrm{linear}} \otimes G_\delta^2](r) + \xi_{\mathrm{Mode\,Coupling}}(r)$$



comparison to simulations

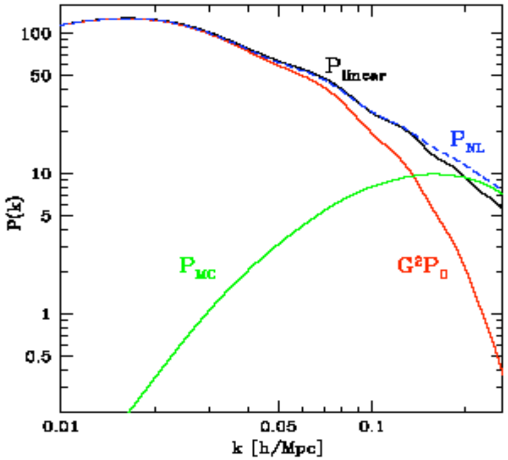


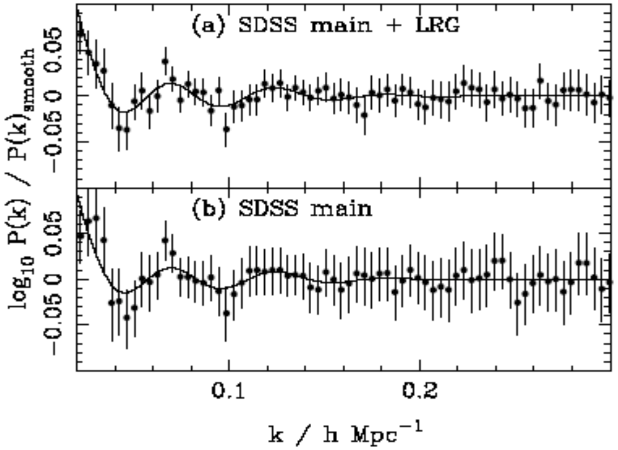
Motivates fitting formulas (e.g Gaussian suppression of the baryon wiggles)

Study systematic effects with accuracy

Revise Halo Model?

Gaussian smoothing of the wiggles



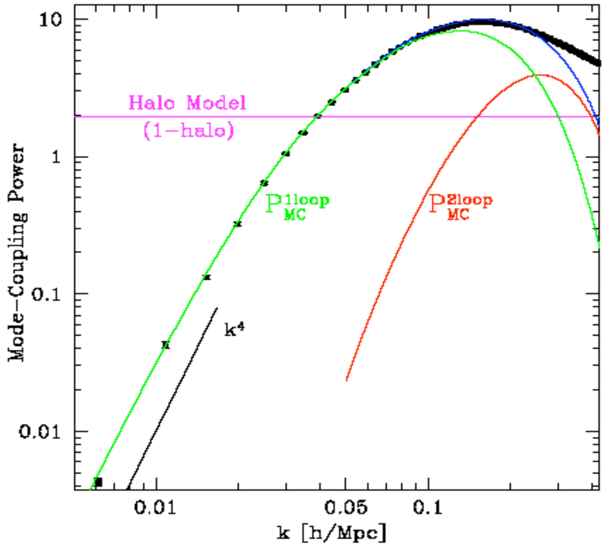


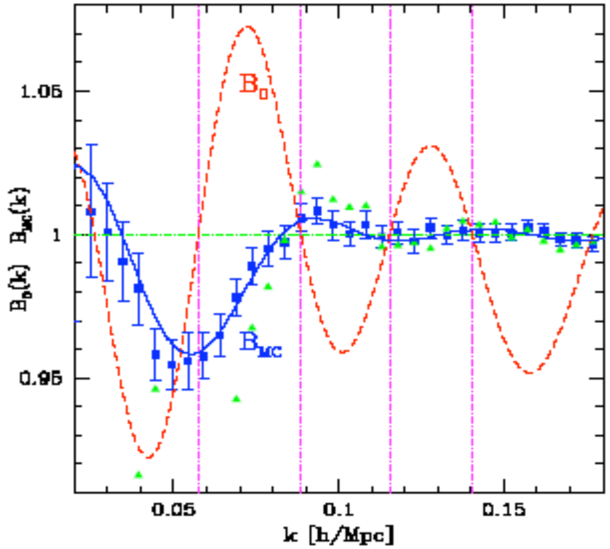
$$B_{\rm obs}(k) = g(k) B_{\rm lin}(k) + [1 - g(k)],$$

Where g(k) is a Gaussian window

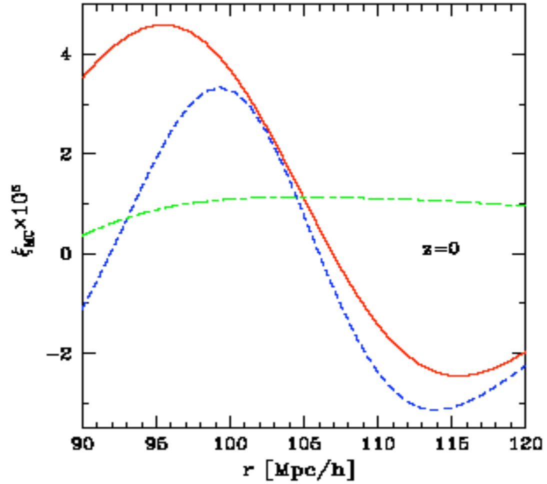
Percival et. al. 2007

Systematic in the wiggles location



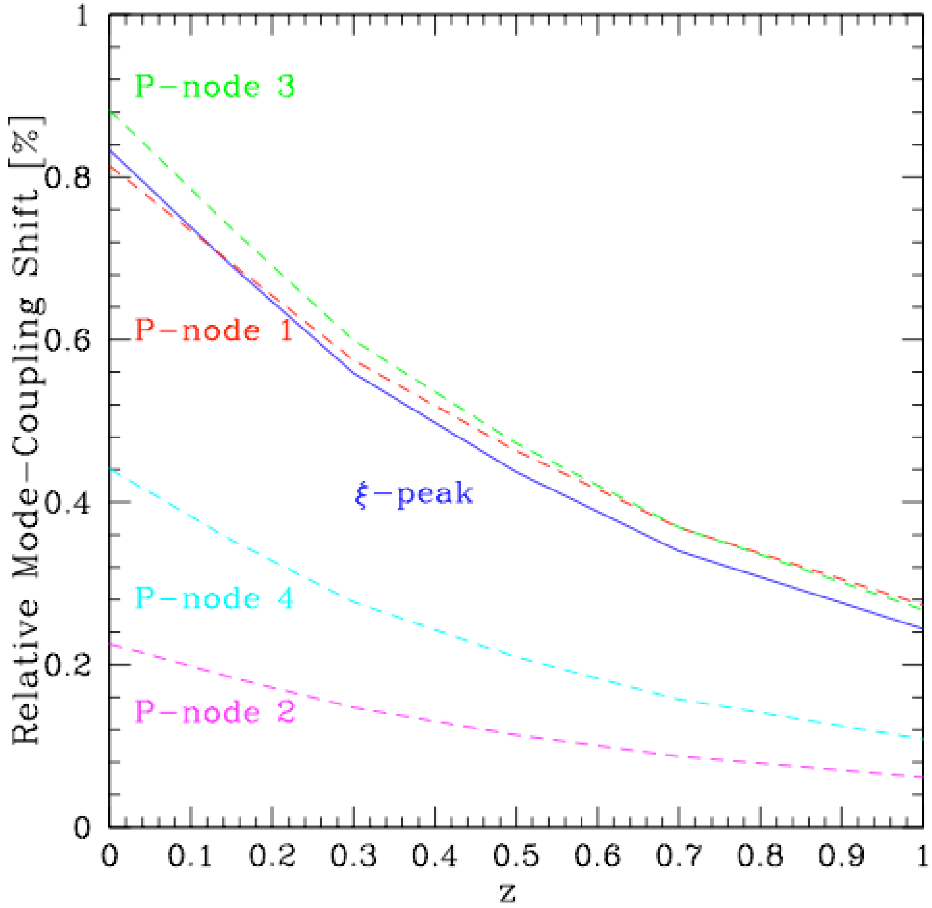


Back to real space



Motivates fitting formula

$$\xi_{\rm obs}(r) = A\,[{\rm e}^{-r^2/\sigma^2} \otimes \xi_{\rm lin}](r) + B\,\xi'_{\rm lin}(r)$$



P−node 3
P−node 1
ξ−peak
P−node 4
P−node 2
Relative Mode−Coupling Shift [%]
z
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

RPT and three-point statistics : Bispectrum

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv D_+^4(z)\, P_0(k_1)\, P_0(k_2)\, 2\, F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + \underbrace{B_{222} + B_{321}^{I} + B_{321}^{II} + B_{411}}_{\text{one-loop corrections}} + \text{permutations}$$

$$B_{321}^{II} \equiv 6 D_+^4(z) P_0(k_1) P_0(k_2) F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) \int P_{\rm lin}(q,z) F_3^{(s)}(\mathbf{k}_1,\mathbf{q},-\mathbf{q})\, \mathrm{d}^3\mathbf{q}$$ → renormalises D_+

$$B_{411} \equiv 12 D_+^4(z) P_0(k_1,z) P_0(k_2,z) \int P_{\rm lin}(q,z) F_4^{(s)}(\mathbf{q},-\mathbf{q},-\mathbf{k}_2,-\mathbf{k}_3)\, \mathrm{d}^3\mathbf{q}$$ → renormalises $F_2^{(s)}$

B_{222} , B_{321}^{I} → one loop irreducible

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv G_\delta^2(k_1,z) P_0(k_1)\, G_\delta^2(k_2,z) P_0(k_2) \left(2\, F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + 12 \int P_{\rm lin}\, F_4^{(s)} \mathrm{d}^3\mathbf{q} + \ldots\right) + B_{\rm irreducibles}$$

$$\Downarrow$$

$$\boxed{B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv G_\delta^2(k_1,z) P_0(k_1)\, G_\delta^2(k_2,z) P_0(k_2)\, \Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) + B_{\rm irreducibles} + \text{perm}}$$

three-point propagator $\quad \Gamma_{abc}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,z)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \equiv \left\langle \dfrac{\delta^2 \Psi_a(\mathbf{k},z)}{\delta\phi_b(\mathbf{k}_1)\delta\phi_c(\mathbf{k}_2)} \right\rangle$

Bi-spectrum

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv G_\delta^2(k_1,z)P_0(k_1)\,G_\delta^2(k_2,z)P_0(k_2)\,\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) + B_{\rm irreducibles}$$

Three-point propagator : $\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = 2F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + \text{nonlinear corrections}$

* The low-k behavior can be obtained computing the one-loop diagrams

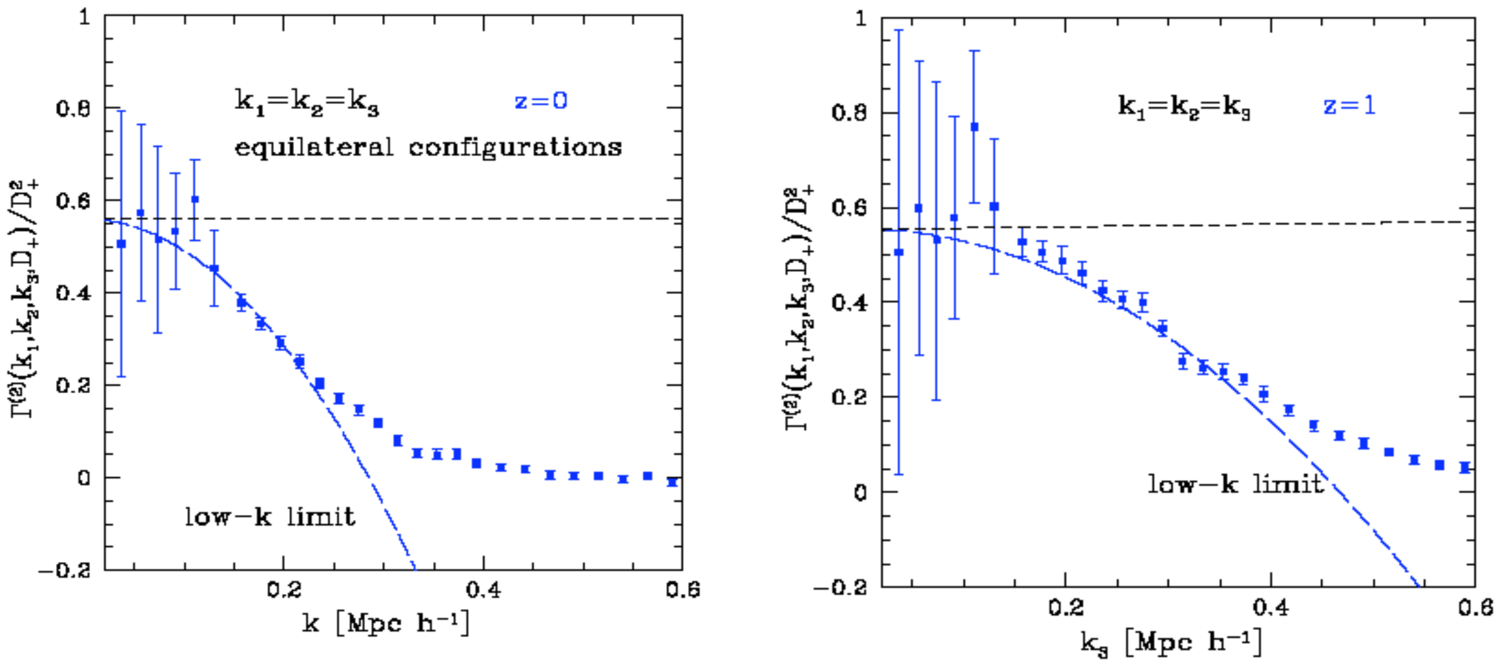
$$\frac{\Gamma^{(2)}(k,z)}{D_+^2(z)} \approx \frac{4}{7}\left(1-\frac{1219}{7840}k^2\sigma_v^2 D_+^2\right)$$

low-k limit for equilateral triangles

$$\sigma_v^2 \equiv \frac{1}{3}\int d^3q\frac{P_0(q)}{q^2}$$

* It can also be measured !

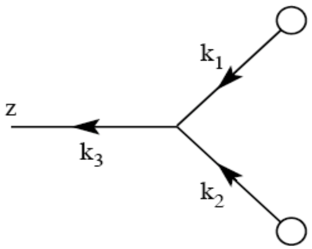
$$\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{12}) = \frac{\langle\delta(\mathbf{k},z)\delta_0(-\mathbf{k}_1,z)\delta_0(-\mathbf{k}_2,z)\rangle}{P_0(k_1)P_0(k_2)}$$



What about the asymptotic at large-k ? $\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = 2F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)$

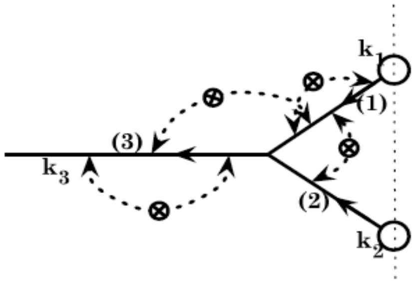
$+$ nonlinear corrections

$F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)$



$$\Rightarrow \quad \frac{2\left(\mathbf{k}_1.\mathbf{k}_2\right)^2}{7k_1^2k_2^2} + \frac{\mathbf{k}_1.\mathbf{k}_2}{2k_1^2} + \frac{\mathbf{k}_1.\mathbf{k}_2}{2k_2^2} + \frac{5}{7}$$

nonlinear corrections

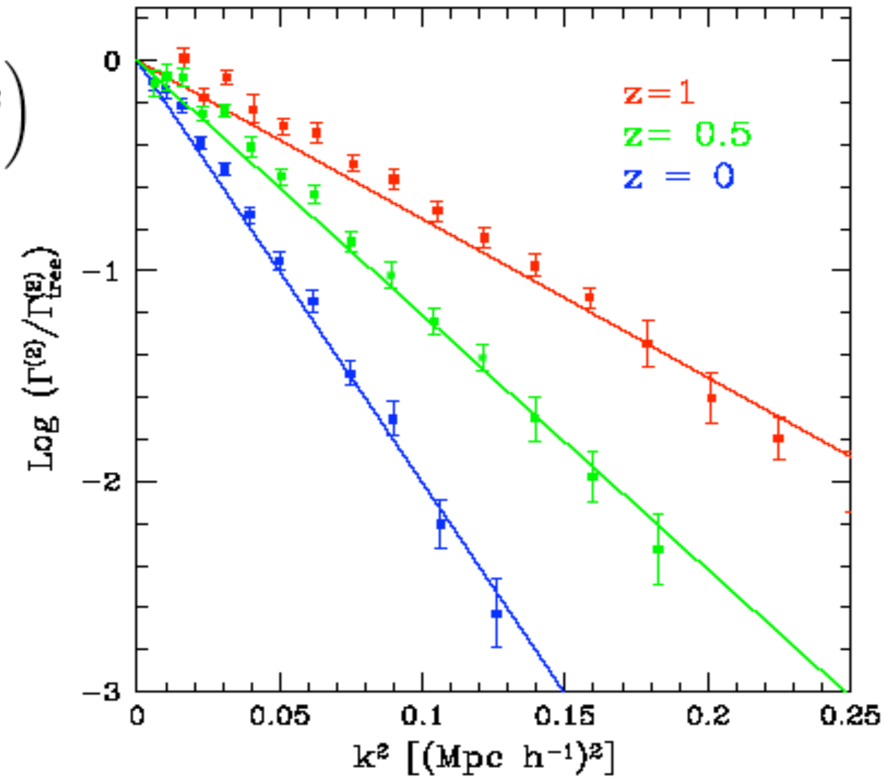


Dominant loop contribution for the $\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2)$propagator

We were able to sum this set of contributions over the infinite number of such loops and over the corresponding interaction times

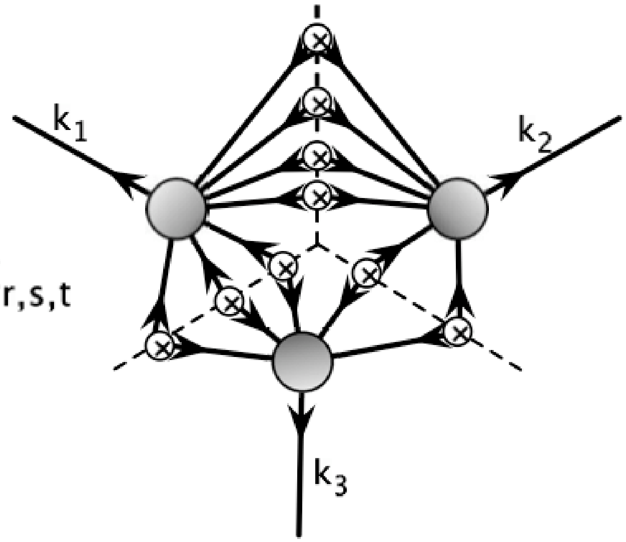
$$\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2,z) = \underbrace{2\,F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2,z)}_{\text{tree order}} \exp\left(-\frac{k_3^2\sigma_v^2}{2}(D_+(z)-1)^2\right)$$

$$\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2,z) = \underbrace{2\,F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2,z)}_{\text{tree order}} \exp\left(-\frac{k_3^2\sigma_v^2}{2}(D_+(z)-1)^2\right)$$



In fact, it is possible to generalize to propagators of arbitrary order, and express Power Spectrum and Bi-spectrum in term of them,

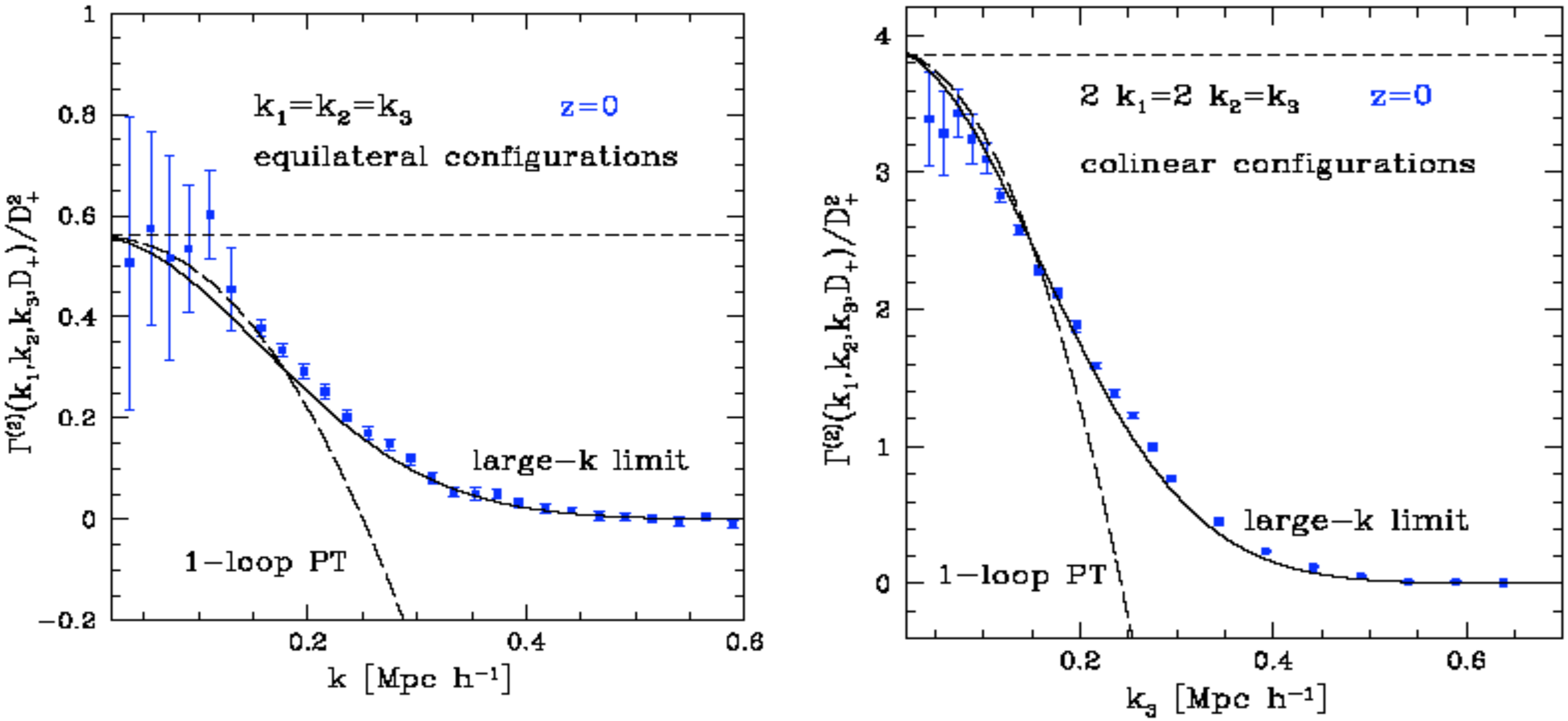
$B(k_1,k_2,k_3) = \Sigma_{r,s,t}$



Leading contribution in high-k limit

$$B^{(0)}(\eta,k_1,k_2,k_3) = B_{\rm tree}(k_1,k_2,k_3) \times \exp\left[-(k_1^2+k_2^2+k_3^2)(\eta-1)^2\sigma_v^2/2\right].$$

(see also Pan, Coles and Szapudi 2007)



Since we again know the low-k behavior and the large-k asymptotic we are trying to find an ansatz to match these two limits, in the same fashion as with the two-point propagator

In turn this will enable us to compute the bispectrum in its transition to the nonlinear regime

Conclusions

* New formalism to study nonlinear clustering of dark matter particles well defined , provide physical insight , ..

* Several other groups have started to look at similar approaches (Valageas 2004 , McDonald 2007, Valageas 2007, Matarrese & Pietroni 2007, Inumi & Soda 2007, Taruya & Hiramatsu 2007, Matsubara 2007) giving strength for (R) Pert. Theories

* Play an important role in modelling structure accurately in forthcoming surveys?

* Perfect match for BAO $\longrightarrow$ predicts P(k) and ξ(r) at percent level at all scales of interest !

* Important to study systematic effects, step to physically motivated fitting formulae

* We could also re-sum the Bispectrum perturbative series, emergence of two-point propagator (suppression of primordial features / non-gaussianities)

$\longrightarrow$ we have a clear path to follow