Scale-dependent Growth of Structure in Viable f(R) Theories

and in Modified Source Gravity

Fermilab
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references: astro-ph/0709.0296
astro-ph/0611321, PRD'07
astro-ph/0607458, NJP'06

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Outline

- Cosmic Acceleration : Modified Gravity ?
- Can we distinguish it from the Cosmological Constant and (conventional) Dark Energy models?

↓

we need to study Large Scale Structure!

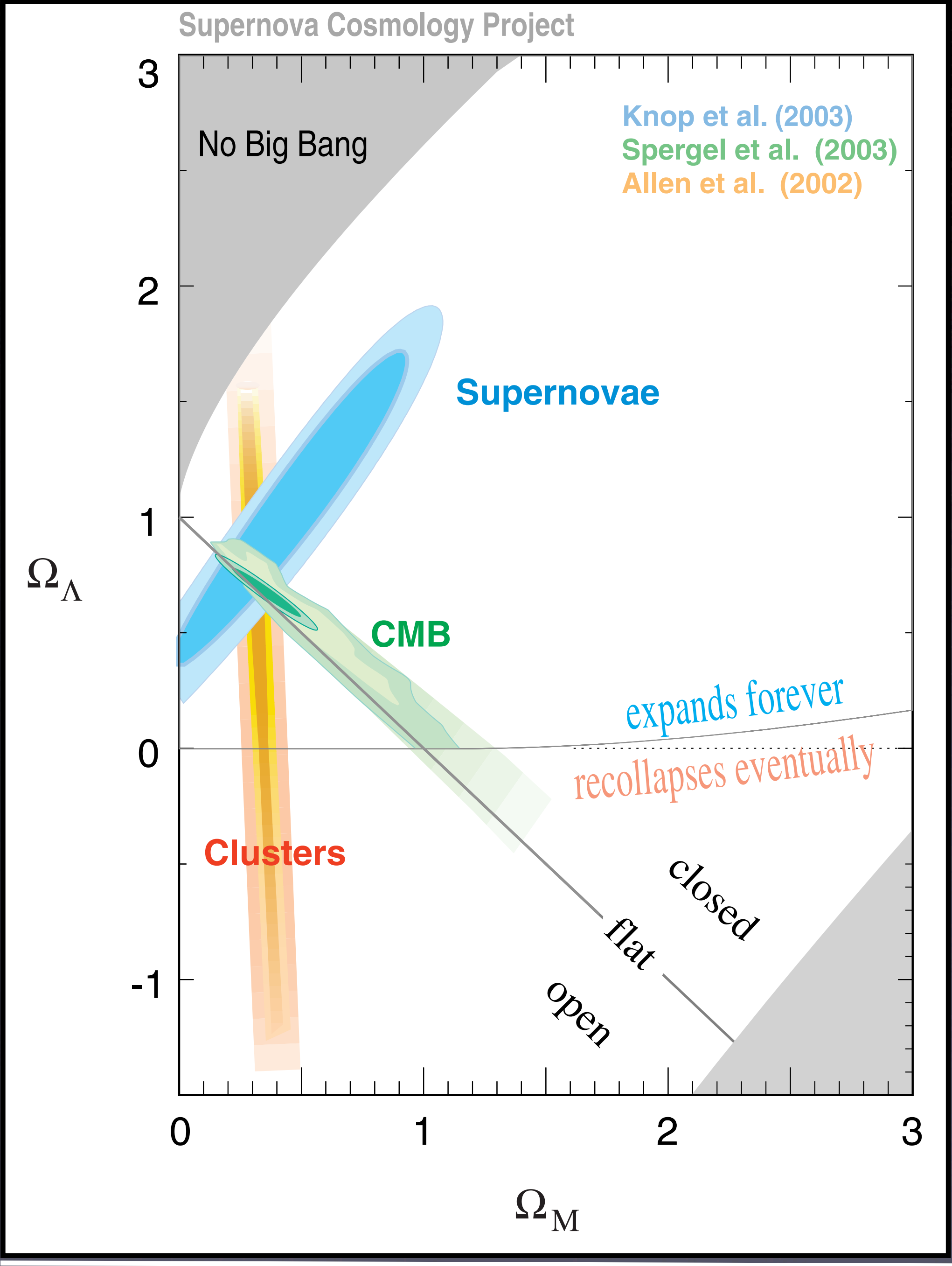
f(R) theories

- Background viability
  ↓
  $w_{eff} \approx -1$
- Growth of Structure
  ↓
  the dynamics is changed, leading to a characteristic scale-dependent pattern

Modified Source Gravity

- Conclusions!

Cosmic acceleration



SNeIa, CMB, LSS

$+$

standard GR applied to a homogeneous and isotropic Universe

$\downarrow$

$$\Omega_m^0 \approx 0.3$$

$$\Omega_X^0 \approx 0.7$$

$$\left(\Omega^0 \equiv \frac{\rho^0}{3H_0^2 M_P^2} \right)$$

Cosmic acceleration

A very good fit to all these data is a Universe in which 70% of the energy budget is in the COSMOLOGICAL CONSTANT, LCDM

Anyhow it is important to explore the whole space of explanations that fit these data and could have testable features...

Dark Energy

$$G_{\mu\nu} = \frac{1}{M_P^2} \tilde{T}_{\mu\nu}$$

X matter fields with dynamics such as to cause the late universe to accelerate (quintessence, k-essence, ...)

Modified Gravity

$$\tilde{G}_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$$

modification of GR on large scales, admitting self-accelerating solutions

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Modifying Gravity

- Modifications of the Friedmann equation, changing the dependence of the Hubble parameter on the matter density (Cardassian model,...)

(K.Freese & M.Lewis, Phys.Lett.B540,(2002))

- Extra-dimensional models giving modified 4-dim equations (DGP model,...)

(G.Dvali, M.Porrati & G.Gabadadze, Phys.Lett.B485(2000))

⟶ Degravitation

(G.Dvali, S.Hofmann & J.Khoury, hep-th/0703027)

- Covariant modifications of the 4-dim Einstein-Hilbert action (f(R),f(R,P,Q),...)

(S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004)
S.Capozziello, S.Carloni & A.Troisi, astro-ph/0303041)

- Modified Source Gravity and Cuscuton

(S.Carroll, I.Sawicki, A.S. & M.Trodden, New J Phys. 8 323 (2006)
N.Afshordi, D.J.H.Chung & G.Geshnizjani, Phys.Rev.D75 083513 (2007))

Modifying Gravity

Typically, at the background level there is a degeneracy between Modified Gravity models and LCDM-Dark Energy

We need to move to the perturbations to break the degeneracy

In Modified Gravity the equations for the perturbations are changed, leading possibly to a richer dynamics

Galaxy count, Weak Lensing, ISW

f(R) Gravity

$$S = \frac{M_P^2}{2}\int dx^4\sqrt{-g}\,[R + f(R)] + \int d^4x\sqrt{-g}\mathcal{L}_m[\chi_i, g_{\mu\nu}]$$

early-universe

$$\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 g^{\mu\nu}\nabla_\mu R\nabla_\nu R + \ldots$$

→ Starobinsky's "inflation"

(A.A.Starobinsky, JETP Lett. 30, 682(1979))

late-universe

$$f(R) = -\frac{\mu^{2(n+1)}}{R^n}, \qquad n > 0$$

(S.Capozziello, S.Carloni & A.Troisi, astro-ph/0303041
S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004))

f(R) Gravity

$$S = \frac{M_P^2}{2}\int dx^4\sqrt{-g}\,[R + f(R)] + \int d^4x\sqrt{-g}\mathcal{L}_m[\chi_i, g_{\mu\nu}]$$

$$\begin{cases} (1+f_R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R+f) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R = \frac{T_{\mu\nu}}{M_P^2} \\ \nabla_\mu T^{\mu\nu} = 0 \end{cases}$$

$$f_R \equiv \frac{df}{dR}$$

The Einstein equations are fourth order.

The trace-equation becomes:

$$(1 - f_R)R + 2f - 3\Box f_R = \frac{T}{M_P^2}$$

dynamical !

The scalaron

$$\Box f_R = \frac{1}{3}(R + 2f - Rf_R) - \frac{\kappa^2}{3}(\rho - 3P) \equiv \frac{\partial V_{eff}}{\partial f_R}$$

$$m_{f_R}^2 \equiv \frac{\partial^2 V_{eff}}{\partial f_R^2} = \frac{1}{3}\left[\frac{1 + f_R}{f_{RR}} - R\right]$$

$|Rf_{RR}| \ll 1$

$f_R \rightarrow 0$

$$m_{f_R}^2 \approx \frac{1 + f_R}{3f_{RR}} \approx \frac{1}{3f_{RR}}$$

$$\lambda_C \equiv \frac{2\pi}{m_{f_R}} \approx 2\pi\sqrt{\frac{3f_{RR}}{1 + f_R}}$$

Designer f(R)

The fourth order nature of f(R) provides enough freedom to reproduce any cosmological background history by an appropriate choice of the f(R) function

We fix the expansion history

$$E \equiv \frac{H^2}{H_0^2} = \Omega_m a^{-3} + \Omega_r a^{-4} + \frac{\rho_{eff}}{\rho_{cr}} \qquad (w_{eff}(a))$$

and solve the Friedmann eq. as a second order differential equation for f(R)

$$\boxed{y'' - \left(1 + \frac{E'}{2E} + \frac{R''}{R'}\right) y' + \frac{E'}{2E} y = \frac{E'}{2E} E_{eff}}$$

$$y \equiv \frac{f(R)}{H_0^2}$$

there is a family of f(R) for each expansion history

Background Viability

1. $f_{RR} > 0$ to have a stable high-curvature regime, to have a non-tachyonic scalar field

2. $1 + f_R > 0$ to have a positive effective Newton constant

3. $f_R < 0$ negative, monotonically increasing function of R that asymptotes to zero from below

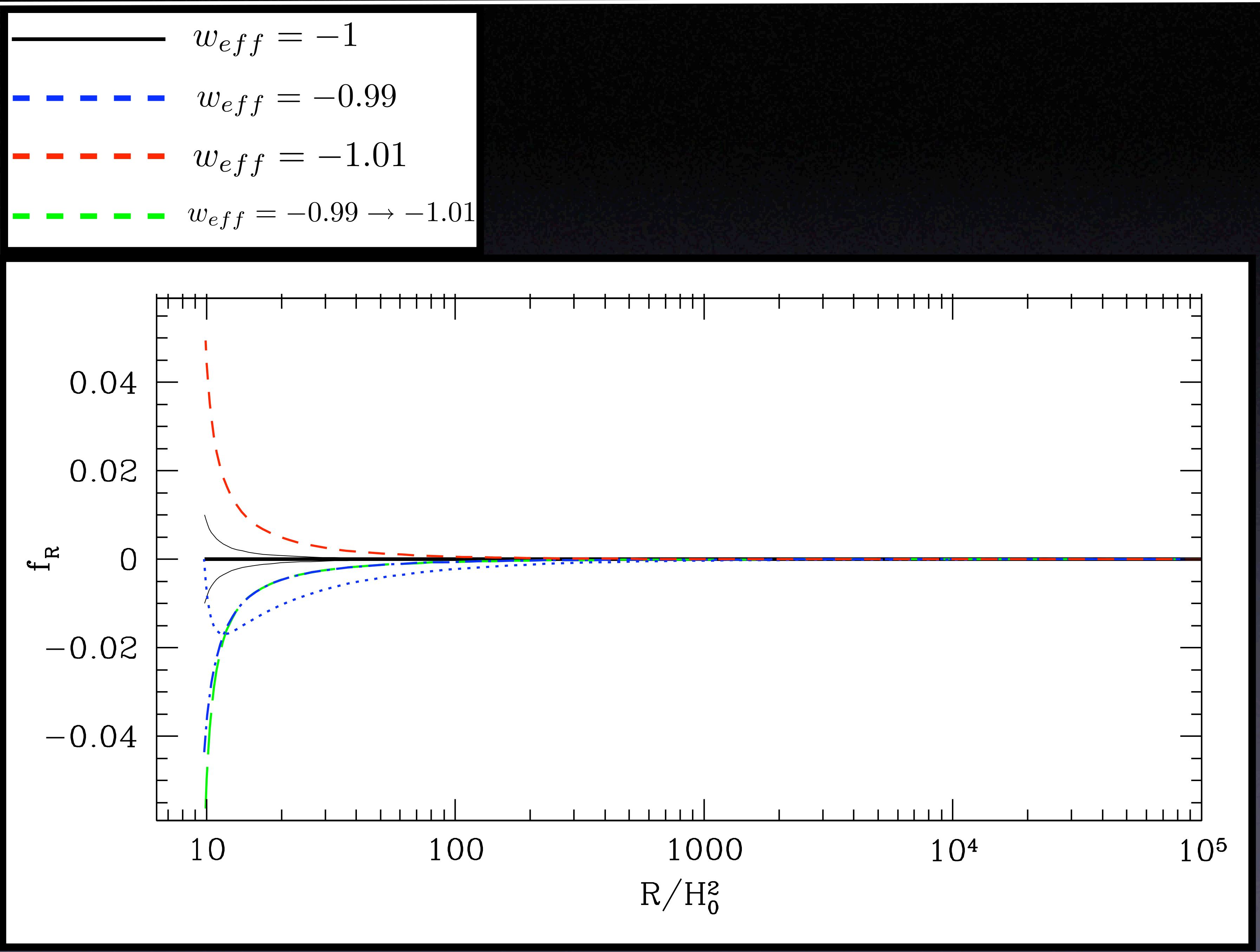
4. $|f_R^0| \leq 10^{-6}$ must be small at recent epochs to pass LGC

(Hu and Sawicki astro-ph/0705.1158)

$$w_{eff} \simeq -1$$

(Dolgov & Kawasaki, Phys.Lett.B 573 (2003), Navarro et al. gr-qc/0611127, Sawicki and Hu astro-ph/0702278
Starobinsky astro-ph/0706.2041, Chiba, Smith, Erickcek astro-ph/0611867
Amendola et al. astro-ph/0603703-0612180, Amendola & Tsujikawa astro-ph/0705.0396)

Designer f(R)



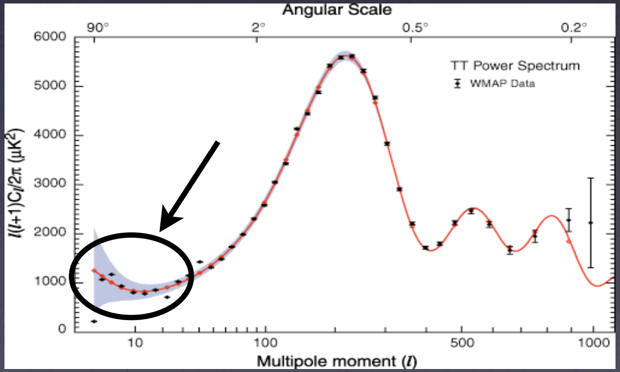
$w_{eff} = -1$
$w_{eff} = -0.99$
$w_{eff} = -1.01$
$w_{eff} = -0.99 \rightarrow -1.01$
0.04
0.02
0
−0.02
−0.04
f_R
10
100
1000
10^4
10^5
R/H_0^2

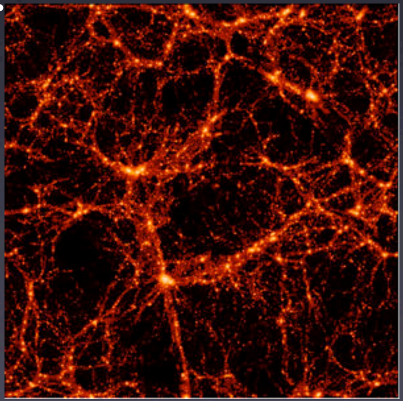
Can we distinguish them from LCDM?

Yes! While at the background level viable f(R) must closely mimic LCDM, the difference in their prediction for the growth of large scale structure can be significant

The scalaron will set a transition scale, inducing a characteristic scale-dependent pattern of growth

On scales below the Compton wavelength of the scalaron, the modifications contribute a slip between the Newtonian potentials and the growth is enhanced by the fifth-force.





ISW, P(k), ISW-galaxy & WL

Dynamics of Linear Perturbations

Scalar perturbations in Newtonian gauge

$$ds^2 = -a^2(\tau)\,(1+2\Psi)\,d\tau^2 + a^2(\tau)\,(1-2\Phi)\,d\vec{x}^2$$

$$\begin{cases} T^0_0 = -\rho(1+\delta) \\ T^0_j = (\rho+P)v_j \\ T^i_j = (P+\delta P)\delta^i_j + \pi^i_j \end{cases}$$

Einstein eqs.

anisotropy:

$$\Phi - \Psi = \frac{\delta(f_R)}{1+f_R} = \frac{f_{RR}}{1+f_R}\delta R$$

$$\Phi - \Psi = 0$$

Poisson:

$$\frac{k^2}{a^2}\Phi = -\frac{3}{2}E_i\Delta_i - \left[f_R\frac{k^2}{a^2}\Phi - \frac{1}{2}\frac{k^2}{a^2}f_{RR}\delta R + \frac{3}{2}H^2 f'_R\,(\Psi+\Phi') + \frac{3}{2}HH'f_{RR}\delta R\right]$$

$$\frac{k^2}{a^2}\Phi = -\frac{3}{2}E_i\Delta_i$$

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Dynamics of Linear Perturbations

New variables:

$$\begin{cases} \Phi_+ \equiv \dfrac{\Phi + \Psi}{2} \\ \chi \equiv f_{RR}\delta R = F(\Phi - \Psi) \end{cases}$$

→ ISW, Weak Lensing

→ effective anisotropic stress

$$F \equiv (1 + f_R)$$

$$\Phi'_+ = \frac{3}{2}\frac{aE_mV_m}{HkF} - \left(1 + \frac{1}{2}\frac{F'}{F}\right)\Phi_+ + \frac{3}{4}\frac{F'}{F^2}\chi$$

$$\chi' = -\frac{2E_m\Delta_m}{H^2}\frac{F}{F'} + \left(1 + \frac{F'}{F} - 2\frac{H'}{H}\frac{F}{F'}\right)\chi - 2F\Phi'_+ - 2F\left(1 + \frac{2}{3}\frac{k^2}{a^2H^2}\frac{F}{F'}\right)\Phi_+$$

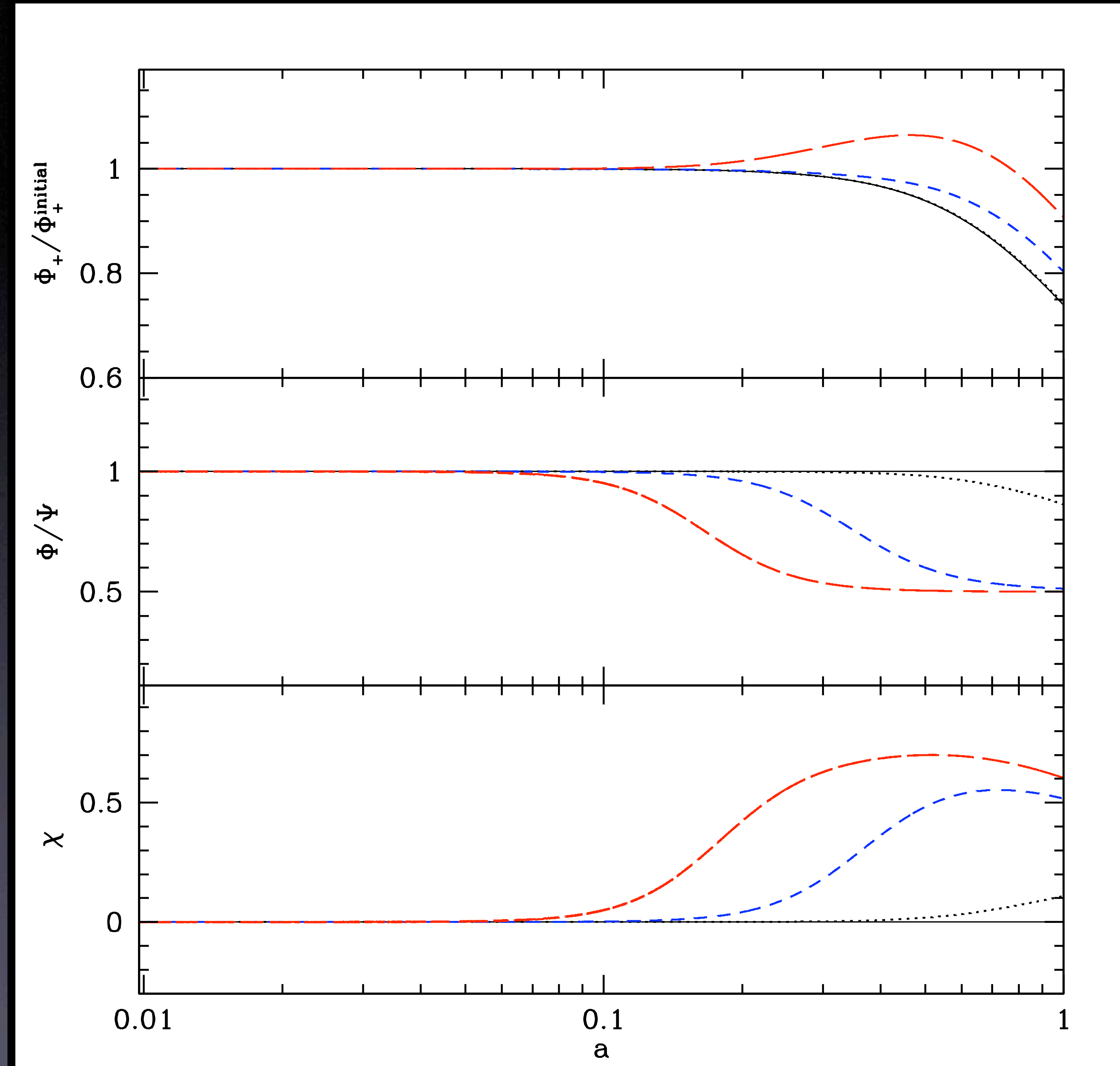
I.C.s:

at z=1000

$$\Phi_+ = -1 \qquad \Delta_m = -\frac{2k^2}{3a^2H^2}\Phi_+$$

$$\chi = 0 \qquad v_m = \frac{2k}{3aH}\Phi_+$$

$w_{eff} = -1$
$f_R^0 = -10^{-4}$



$k = 0.01h/Mpc$
$k = 0.1h/Mpc$
$k = 0.5h/Mpc$

Sub-Horizon

CDM equation:

$$\delta_m'' + \left(1 + \frac{H'}{H}\right)\delta_m' + \frac{k^2}{a^2H^2}\left(\Phi_+ - \frac{\chi}{2F}\right) = 0$$

Einstein equations:

$$\Phi_+ \simeq -\frac{3}{2}\frac{a^2 E_m}{k^2}\frac{\delta_m}{F}$$

$$\chi \simeq \frac{k^2}{k^2 + 3a^2H^2F'/F}\frac{a^2E_m\delta_m}{k^2}$$

$$\frac{k^2}{a^2}\frac{f_{RR}}{F}$$

There is a scale associated with the extra d.o.f. :

$$\lambda_C \equiv \frac{1}{m_{f_R}} \approx \sqrt{\frac{f_{RR}}{F}}$$

$$-\frac{3}{2}\ \frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{F}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{F}}E_m\delta_m$$

time and scale dependent rescaling of Newton constant $\equiv G_{eff}$

Sub-Horizon

The Compton wavelength of the scalaron separates two regimes

$\lambda \gg \lambda_C$

$$\chi \simeq 0, \quad G_{eff} \simeq \frac{G}{F}$$

$$\Psi \simeq \Phi$$

the scalaron is massive, the fifth-force is suppressed and the theory behaves similarly to standard GR

$\lambda \ll \lambda_C$

$$\chi \simeq -\frac{2}{3} F \Phi_+, \quad G_{eff} \simeq \frac{4}{3} \frac{G}{F}$$

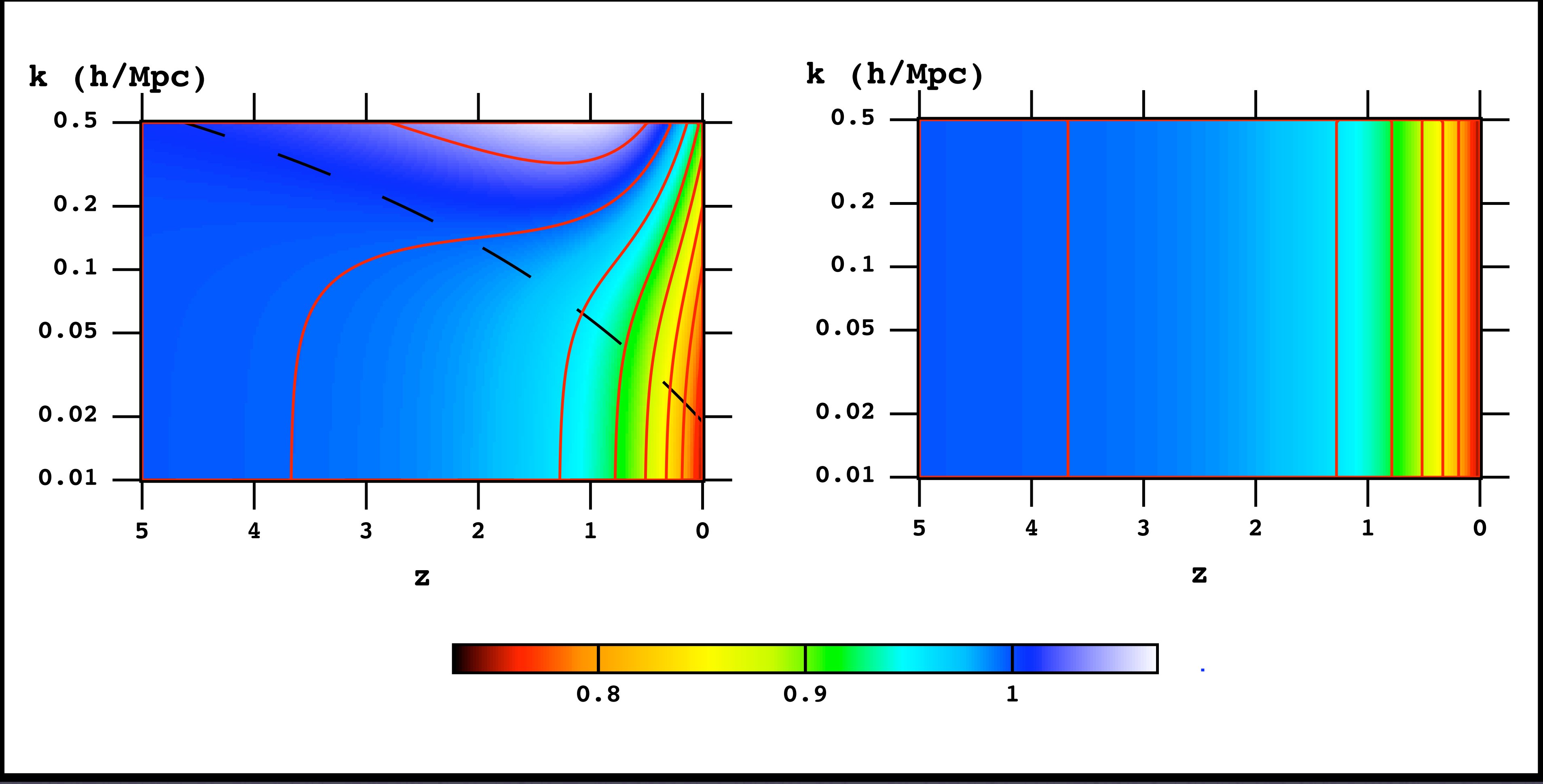
$$\Psi \simeq 2\Phi$$

the scalaron is light, the fifth-force enhances the growth

$$w_{eff} = -1$$
$$f_R^0 = -10^{-4}$$

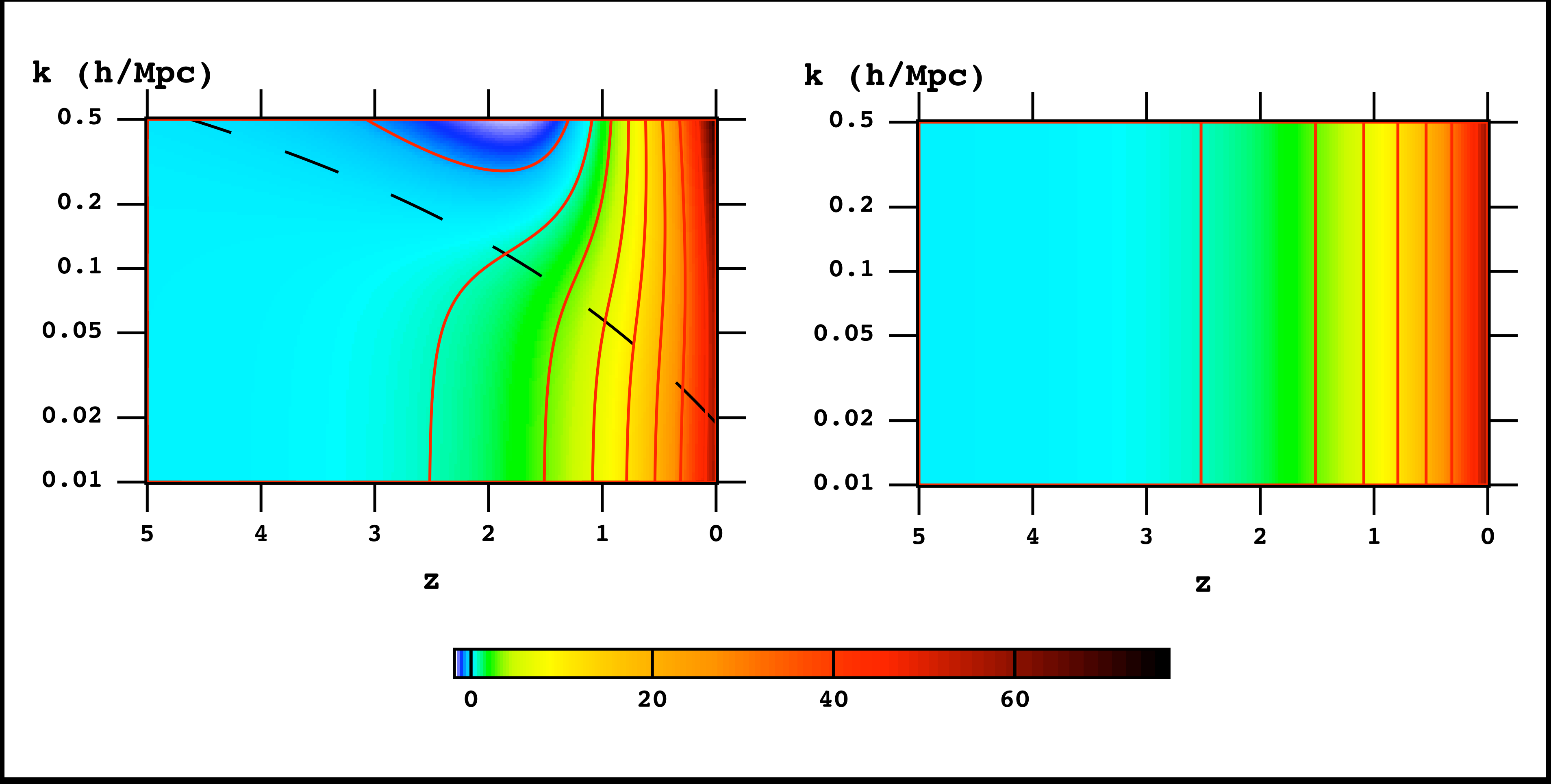
$$\frac{\Delta_m(k,a)/a}{\Delta_m(k,a_i)/a_i}$$

$$\left(\frac{\Phi_+(k,a)}{\Phi_+(k,a_i)}\right)$$



$$w_{eff} = -1$$
$$f_R^0 = -10^{-4}$$

$$-\frac{d\Phi_+}{dz} \cdot \Delta_m$$



Effective Dark fluid

$$\tilde{G}_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu} \longrightarrow G_{\mu\nu} = \frac{1}{M_P^2}\left(T_{\mu\nu} + T_{\mu\nu}^{eff}\right)$$

$$\Pi_{eff} \propto \chi$$

on sub-horizon scales

$$Q \equiv \frac{\lambda_C}{\lambda}$$

$$c_{eff}^2 \simeq \frac{2}{3}\frac{Q}{(1-F)(1+2Q)+QF}$$

$Q \ll 1$: $\simeq 0^+$

$Q \gg 1$: $\simeq -\frac{2}{3}$

$$c_{an}^2 \equiv \Pi/\delta$$

$$c_s^2 - c_{an}^2 \geq 0$$

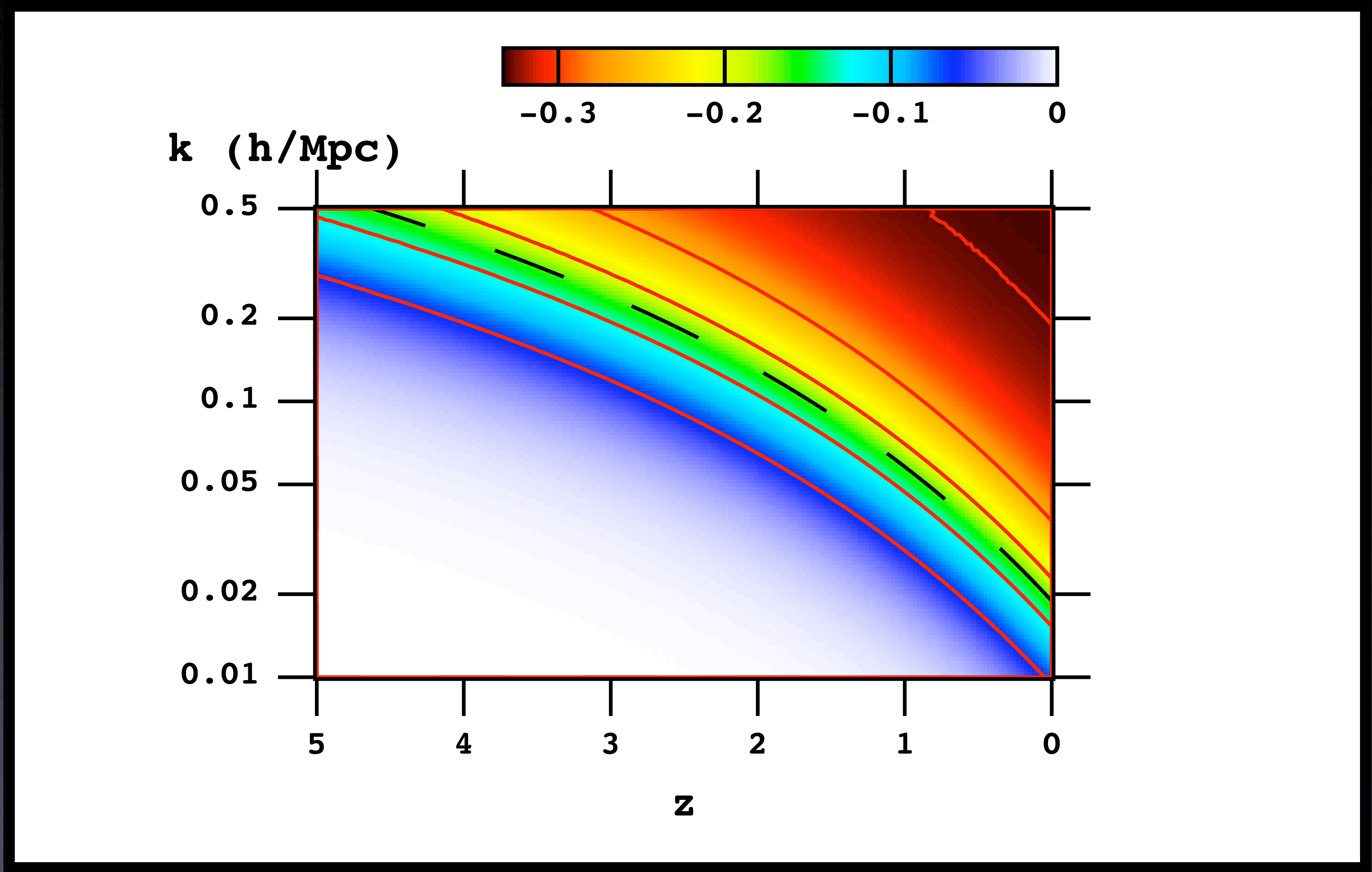
$$\frac{E_{eff}\Delta_{eff}}{E_m\Delta_m} \simeq -\frac{Q(3F-2)+(F-1)}{1+3Q}$$

$Q \ll 1$: $\simeq 0^+$

$Q \gg 1$: $\simeq -\frac{1}{3}$

$w_{eff} = -1$
$f_R^0 = -10^{-4}$

$$\frac{E_{eff}\Delta_{eff}}{E_m\Delta_m}(k,z)$$



Modified Source Gravity

i.e. modifying gravity without introducing any new d.o.f.

$$S = \int dx^4 \sqrt{-g} \left[\frac{M_P^2}{2} e^{2\psi} R + 3e^{2\psi} \left(\nabla\psi\right)^2 - U(\psi) \right] + S_m[g_{\mu\nu}, \chi_i]$$

(S.Carroll, I.Sawicki, A.S. & M.Trodden
astro-ph/0607458, NJP'06)

EOMs

$$\Box\psi + (\nabla\psi)^2 + \frac{1}{6M_P^2} e^{-2\psi} \frac{dU}{d\psi} - \frac{1}{6} R = 0$$

trace of Einstein eq.

$$\frac{dU}{d\psi} - 4U(\psi) = -T$$

↓

$$\psi = \psi(T) = \psi(\rho_m)$$

$$G_{\mu\nu} = \tilde{T}_{\mu\nu}(\rho_m)$$

Structure formation

anisotropy eq.

$$\Phi - \Psi = 2\delta\psi = -\frac{1}{3}\frac{d\psi}{dlna}\delta_m$$

CDM eq.

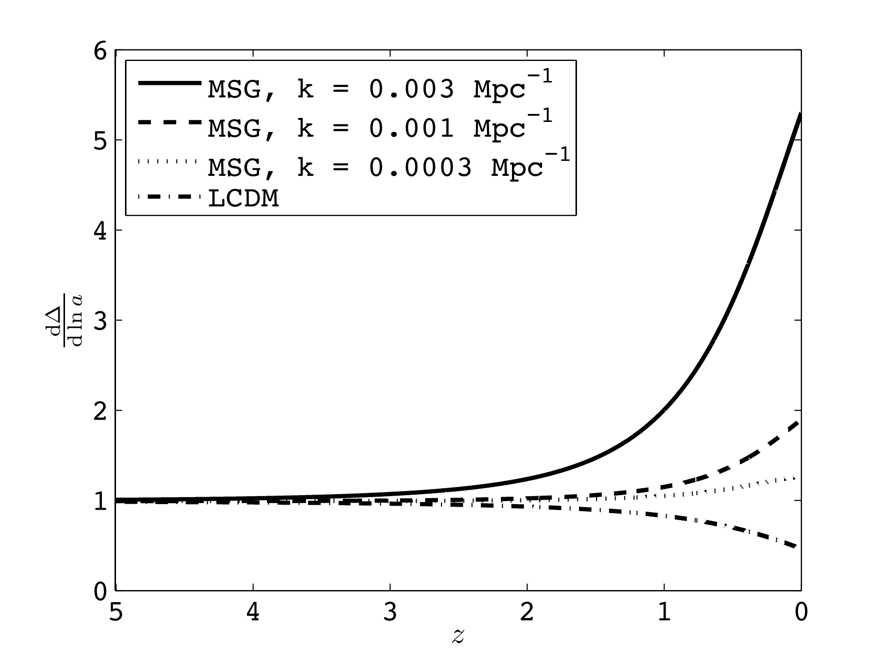
negative sound speed

$$\delta'' + \mathcal{H}\delta' - \left[\frac{e^{-2\psi}}{2M_P^2}\left(1 + \frac{d\psi}{dlna}\right)\rho - \frac{k^2}{3a^2}\frac{d\psi}{dlna}\right]\delta = 0$$

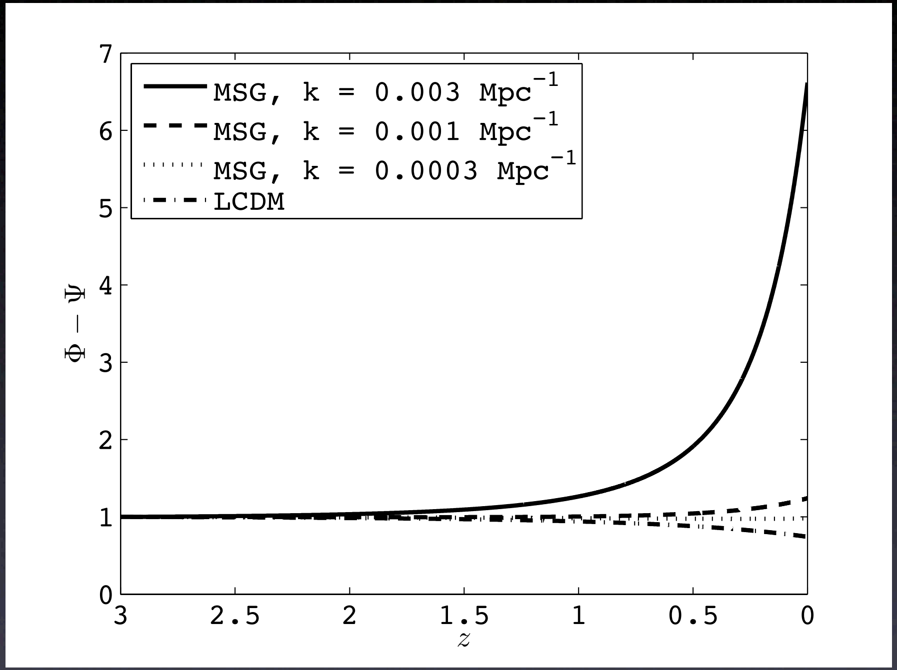
- scale-dependence
- runaway growth for small scales

Structure formation

$$\frac{d\Delta_m}{dlna}(k,z)$$



$$\Phi_+(k,z)$$



- scale-dependent runaway growth
- rapid structure formation drives the growth of gravitational potentials
- the ISW effect is enhanced at the lowest multipoles
- negative LSS-ISW correlation

Characteristic signatures

- scale-dependence
- slip between potentials
- clustering: $G \rightarrow G_{eff}$

overall the dynamics is richer, and different observables are described by different functions, not by a single growth factor!

combining different measures, e.g. weak lensing and redshift space distortion, we can build discriminating probes of gravity

(Zhang et al. astro-ph/0704.1932)

Effective shear and ratio between the potentials

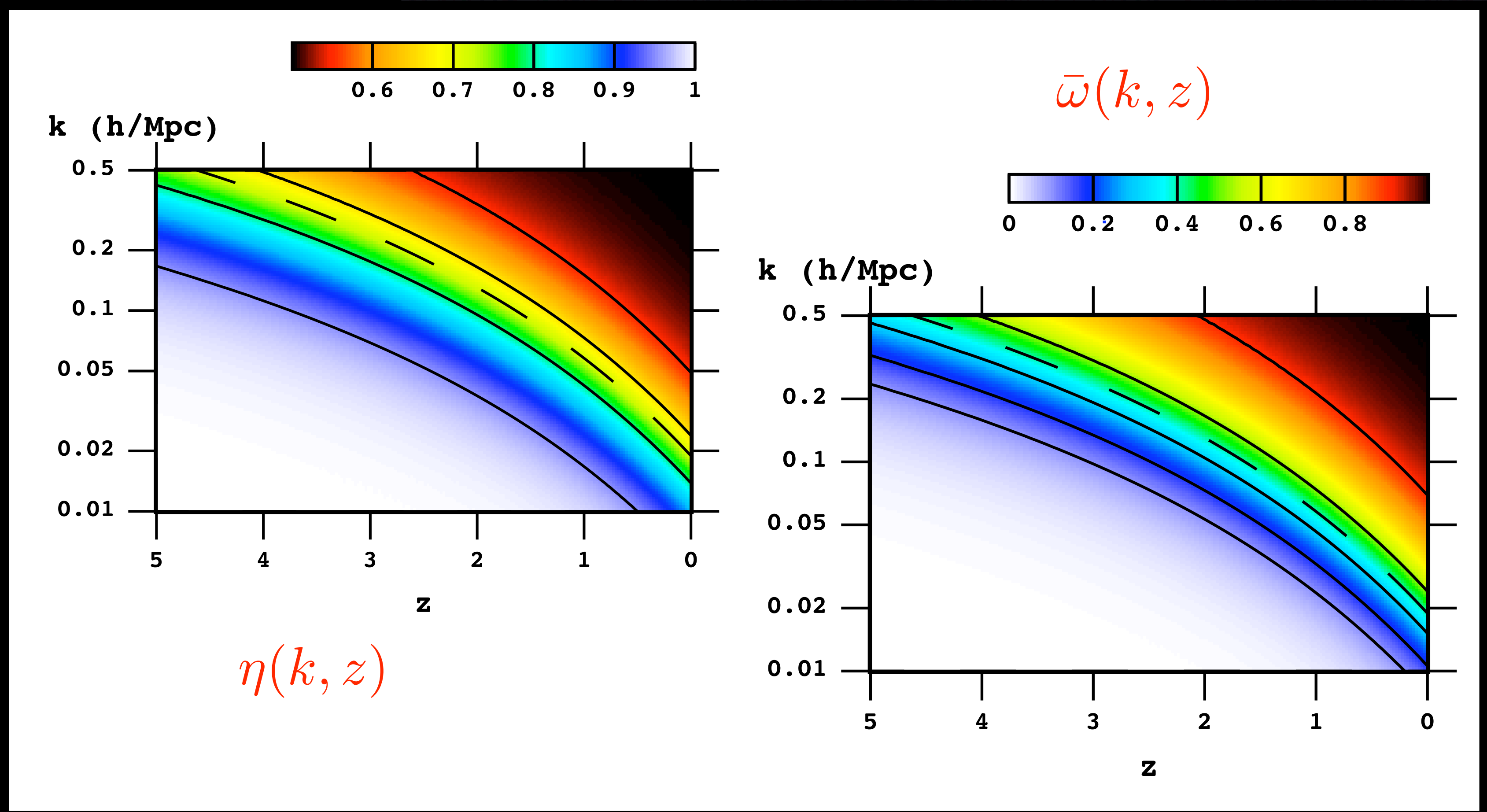
$$\eta(k,z) \equiv \frac{\Phi}{\Psi}$$

(Zhang et al. astro-ph/0704.1932)

$$\bar{\omega}(k,z) \equiv \frac{\Psi - \Phi}{\Phi} = -\frac{\chi}{F\Phi}$$

(Caldwell et al. PRD76 023507(2007))

f(R) theories



Summary I

f(R) theories

- viable expansion history + Local Constraints of Gravity

↓

viable f(R) closely mimic LCDM with $w_{eff} \approx -1$

- the degeneracy is broken at the level of perturbations

- we observe a characteristic scale-dependent pattern, with the scalaron Compton wavelength separating two regimes

Summary II

- on scales $\lambda \gg \lambda_C$ the fifth force is exponentially suppressed and things behave similarly to standard GR

- on scales $\lambda \ll \lambda_C$ there is an enhancement of the growth due to the fifth force

↓

- slip between Newtonian potentials $\Psi \to 2\Phi$

- $G_{eff} \simeq \frac{4}{3}\frac{G}{F}$

- the rate of growth depends on the balance between the fifth force and the acceleration of the background

- the effect of modifications is maximized on modes below Compton wavelength during matter era

Summary III

Modified Source Gravity

- scale-dependent runaway growth
- negative LSS-ISW correlation

Characteristic features:

- $\Phi \neq \Psi$
- Φ_+ grows ⇒
  - ISW effect during matter era (f(R))
  - negatve ISW-galaxy
- Highly accurate 3D maps of Φ_+ from Weak Lensing Surveys

THANK YOU !