Use and abuse of fine-tuning: Dark matter at the LHC

Jonathan Roberts
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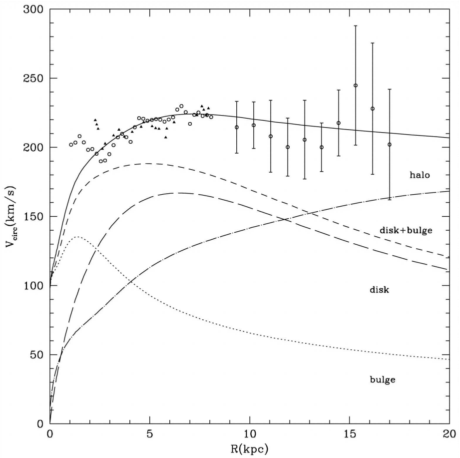
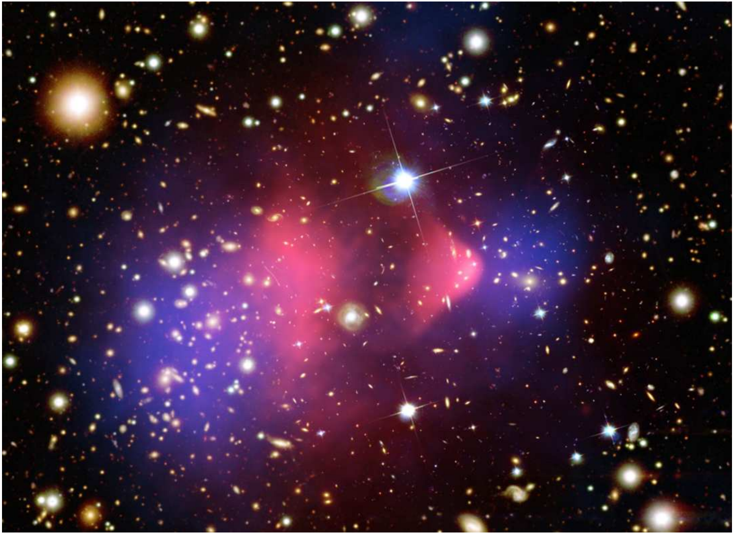
November 21, 2007

Rotation Curves of Galaxies



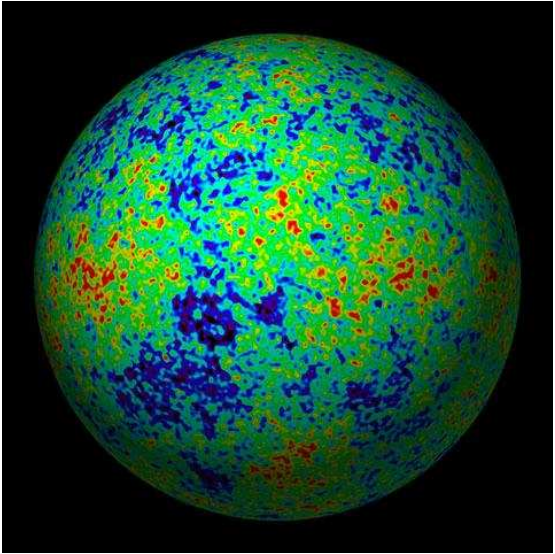
(From Klypin, Zhao and Somerville, 2002)

Lensing: Dark Matter in the Bullet Cluster



Lensing: Dark Matter in the Bullet Cluster

The Cosmic Microwave Background



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In contrast, the density of baryonic matter is measured as:

$$\Omega_b h^2 = 0.0224$$

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R-parity conserving supersymmetry requires a relic density of sparticles.

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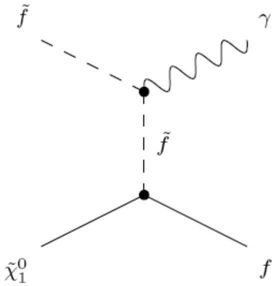
This sounds like fine-tuning.

Coannihilation



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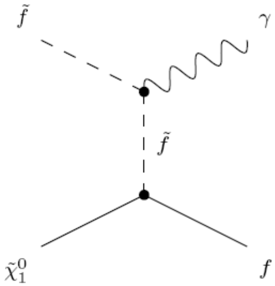

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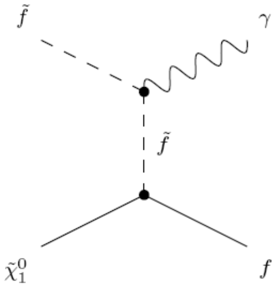

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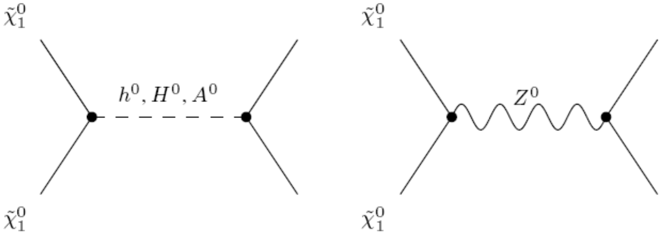

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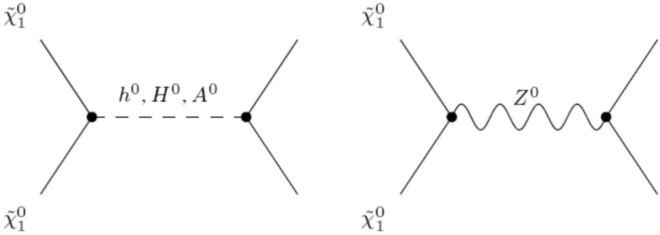

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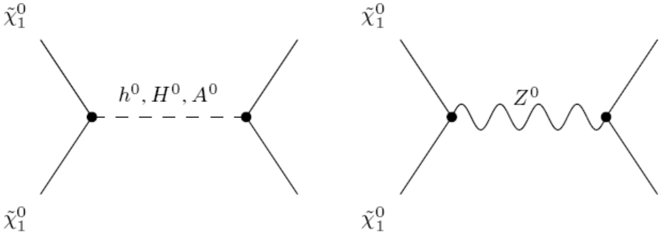

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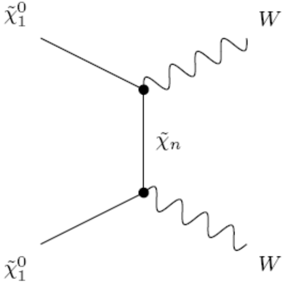

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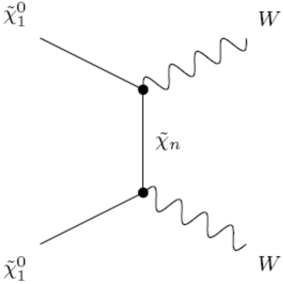

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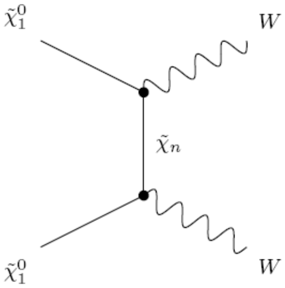

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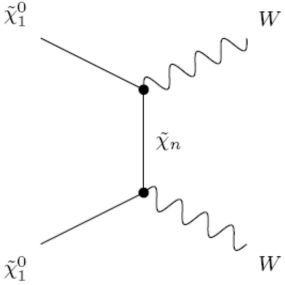

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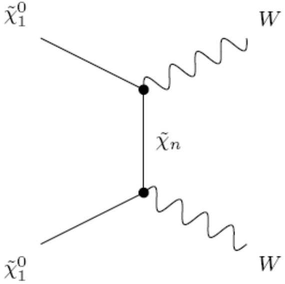

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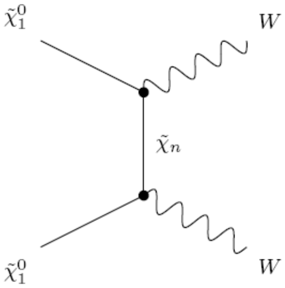

- Requires either:
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- or quantify the degree of fine-tuning involved.

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We use an analagous measure to the one used to measure the fine-tuning required for electroweak symmetry breaking:

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This **also** gives us a handle on the precision required of colliders to give a prediction of $\Omega_{CDM} h^2$ with precision comparable to WMAP.

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  - $m_{\tilde{\chi}_1^+}$, $(m_{\tilde{\chi}_3^0},\ m_{\tilde{\chi}_2^+})$
- Resonant annihilation
  - $\Delta m = m_{Z^0,h^0,H^0,A^0} - 2m_{\tilde{\chi}_1^0}$

t-channel sfermion exchange

For a typical Bino dark matter region, the sensitivity to low energy masses is:

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... though this sensitivity looks likely to remain small in dominantly bino regions.

Resonant annihilation

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To calculate the relic density to $\mathcal{O}(10\%)$, we need to determine the masses of $A^0, \tilde{\chi}_1^0$ to $\mathcal{O}(0.2\%)$.

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... but the sensitivities are coupled. If we could measure Δm (which we can) then this situation is greatly improved.

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&+ \left(A_{ij}^{u}\,Y_{ij}^{u}\right)\tilde{u}_{iR}^{*}\tilde{Q}_{jL}H_u - \left(A_{ij}^{d}\,Y_{ij}^{d}\right)\tilde{d}_{iR}^{*}\tilde{Q}_{jL}H_d - \left(A_{ij}^{e}\,Y_{ij}^{e}\right)\tilde{e}_{iR}^{*}\tilde{L}_{jL}H_d \\
&+ \tilde{Q}_{iL}^{\dagger}\left(m_{\tilde{Q}}^{2}\right)_{ij}\tilde{Q}_{jL} + \tilde{L}_{iL}^{\dagger}\left(m_{\tilde{L}}^{2}\right)_{ij}\tilde{L}_{jL} \\
&+ \tilde{u}_{iR}^{*}\left(m_{\tilde{u}}^{2}\right)_{ij}\tilde{u}_{jR} + \tilde{d}_{iR}^{*}\left(m_{\tilde{d}}^{2}\right)_{ij}\tilde{d}_{jR} \\
&+ \tilde{e}_{iR}^{*}\left(m_{\tilde{e}}^{2}\right)_{ij}\tilde{e}_{jR} + m_{H_u}^{2}H_u^{*}H_u + m_{H_d}^{2}H_d^{*}H_d - \left(B\mu H_u H_d + h.c.\right)
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- Therefore we **must** consider the impact of the RGEs.
- We also expect there to be relations between the soft masses, rather than the > 100 free parameters of the MSSM.

Let's see how this affects the tuning.

CMSSM

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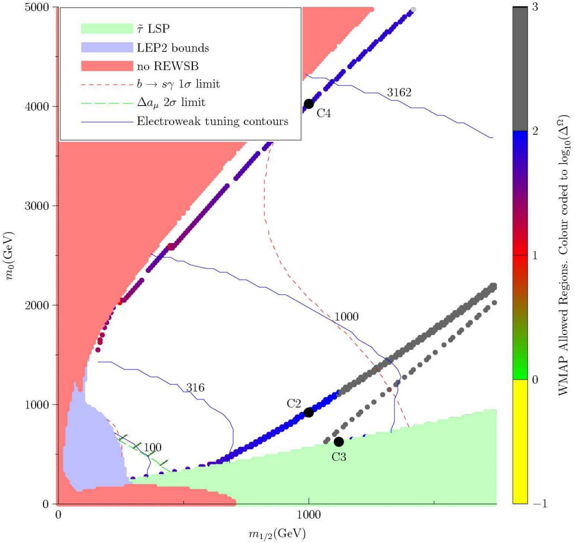
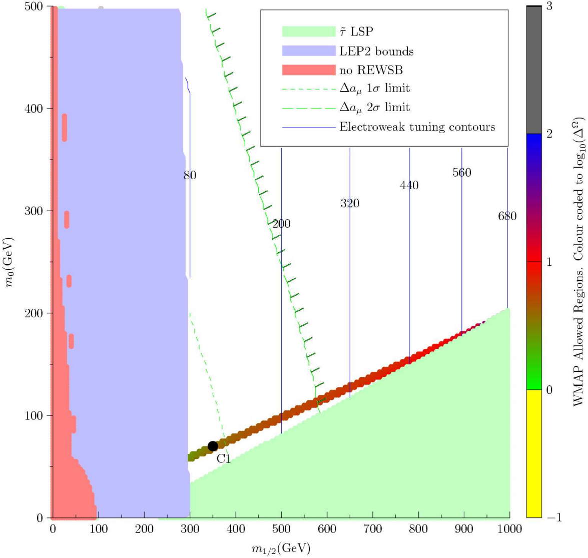
The masses are set at m_{GUT} and run (using `SoftSusy`) to m_{EW}.

The CMSSM with $A_0 = 0$, $\tan\beta = 50$; S.F.King, J.P.R.: hep-ph/0609147,



The CMSSM with $A_0 = 0$, $\tan\beta = 10$



CMSSM $\tilde{\chi}_1^0 - \tilde{\tau}$ Coannihilation

Remember that at the EW scale we had the sensitivity:

Parameter a	Δ_a^Ω
$m_{\tilde{\tau}}$	41
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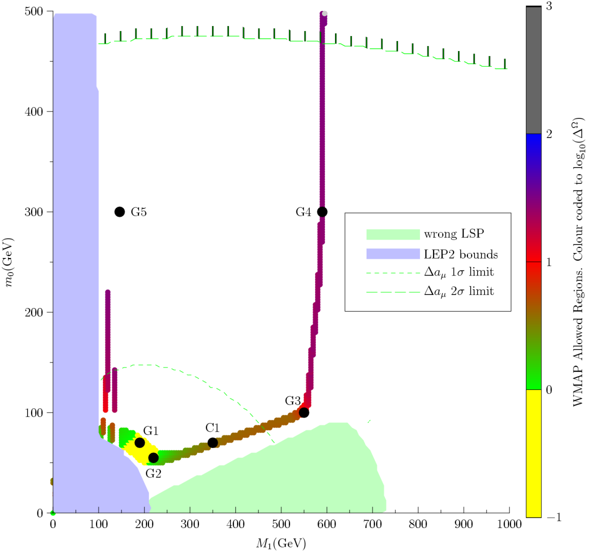
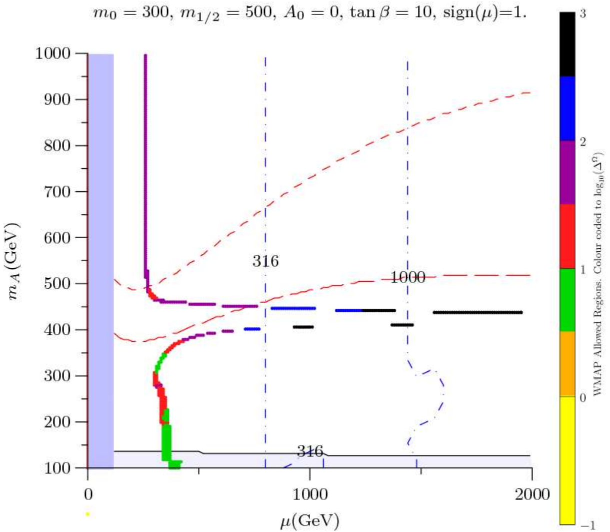
This can be understood by considering the $\tilde{\tau}$ RGE:

$$\frac{d(m_{\tilde{\tau}_R}^2)}{dt} = \frac{1}{8\pi^2}\left(-4g_1^2 M_1^2 + 2h_\tau^2\left(m_{\tilde{L}_{3L}}^2 + m_{\tilde{\tau}_R}^2 + m_{H_1}^2 + A_\tau^2\right) + 4S\right)$$

Relaxing the CMSSM: non-universal gauginos



Non-Universal Higgs Masses; J. Ellis, S. F. King, 0711.2741[hep-ph]



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m_{H_1}	5.1
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We find typical tuning scales for different dark matter channels.

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Mixed bino/higgsino ($m_{\tilde{\chi}_1^0} < m_{A,H}$)	$30 - 60$
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slepton coannihilation (low M_1, m_0, $\tan\beta$)	$3 - 15$
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Should we throw away regions that are fine-tuned at this stage?

Structures of SUSY breaking

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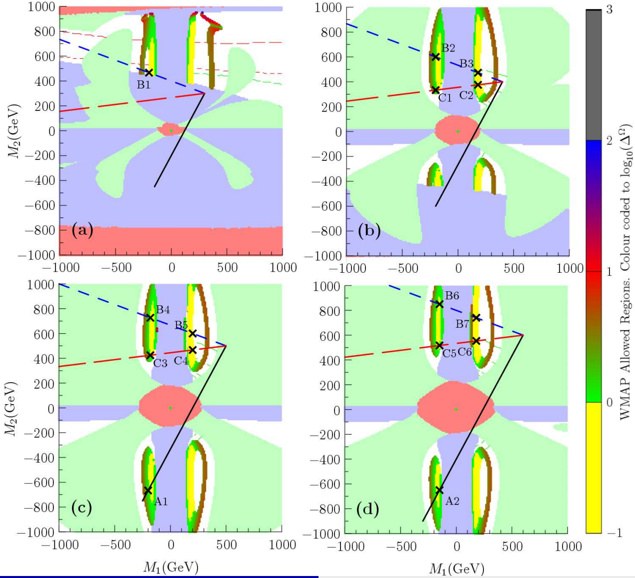
If we minimise the coefficients, we minimise the dark matter tuning.

An $SU(5)$ GUT model; S.F.King, JPR, D.P.Roy: arXiv:0705.4219



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