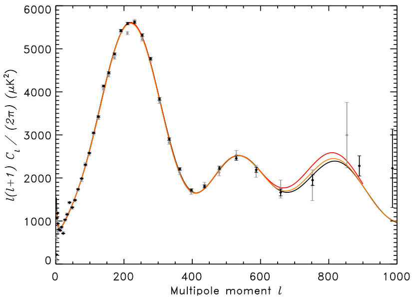
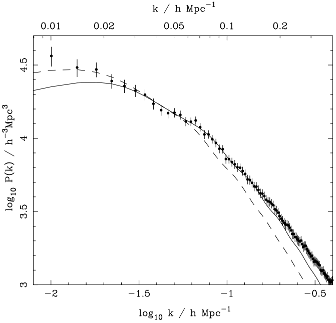
What do WMAP and SDSS really tell about inflation?

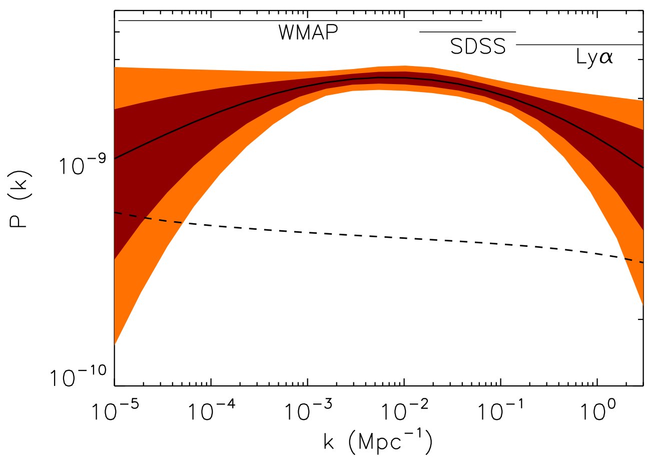
Wessel Valkenburg (LAPTH)
14 November, 2007

Phys.Rev.D75:123519, 2007, Julien Lesgourgues, WV
arXiv:0710.1630, Julien Lesgourgues, Alexei Starobinsky, WV





WMAP3, from Spergel et al, astro-ph/0603449
SDSS-LRG5, from Percival et al, astro-ph/0608636



Taken from Easther & Peiris, astro-ph/0609003.

$$\dot{\phi} = -\frac{m_P^2}{4\pi} H'(\phi)$$

$$\left[H'(\phi)\right]^2 - \frac{12\pi}{m_P^2} H^2(\phi) = -\frac{32\pi^2}{m_P^4} V(\phi)$$

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$$\partial_\eta^2 \mu_{\mathrm{S,T}} + \left[k^2 - \frac{\partial_\eta^2 z_{\mathrm{S,T}}}{z_{\mathrm{S,T}}}\right] \mu_{\mathrm{S,T}} = 0$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{8\pi^2} \left| \frac{\mu_{\mathrm{S}}}{z_{\mathrm{S}}} \right|^2$$

$$\mathcal{P}_h(k) = \frac{2k^3}{\pi^2} \left| \frac{\mu_{\mathrm{T}}}{z_{\mathrm{T}}} \right|^2$$

CMB+LSS

$\downarrow$

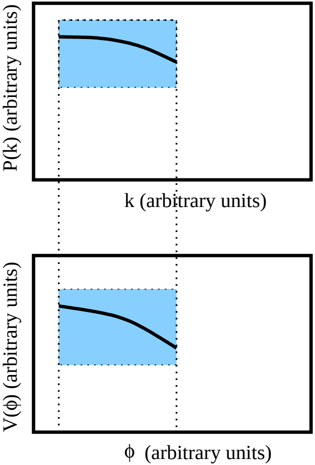
Primordial power spectrum

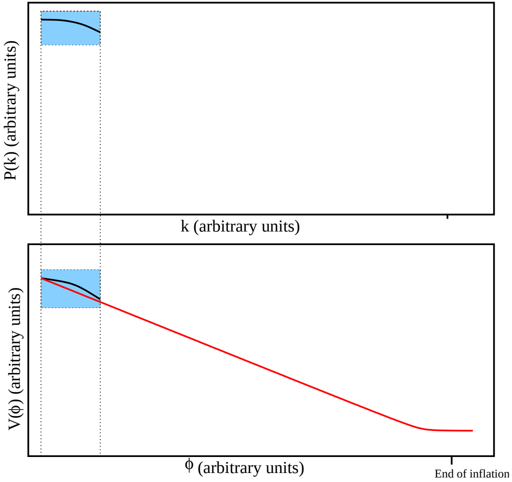
$\downarrow$

$H(\eta)$

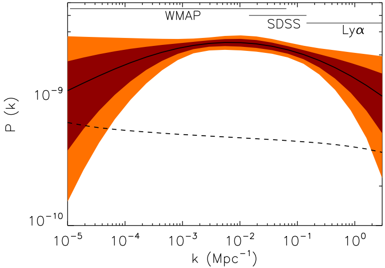
$\downarrow$

$V(\phi)$

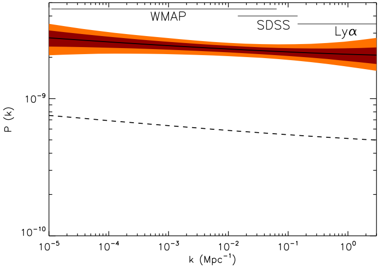

P(k) (arbitrary units)
k (arbitrary units)
V(ϕ) (arbitrary units)
ϕ (arbitrary units)


P(k) (arbitrary units)
k (arbitrary units)
V(ϕ) (arbitrary units)
ϕ (arbitrary units)
End of inflation

If you DID extrapolate:



Fitting $P(k) = k^{(n_S - 1 + \ldots)}$.



Fitting $P(k) = k^{(n_S - 1 + \ldots)}$, selecting SR-inflationary models with $N > 30$.
However: result heavily depends parametrisation.
(see e.g. Ballesteros et al. 2006 for large running and N$>$ 50)

Taken from Easther & Peiris, astro-ph/0609003.

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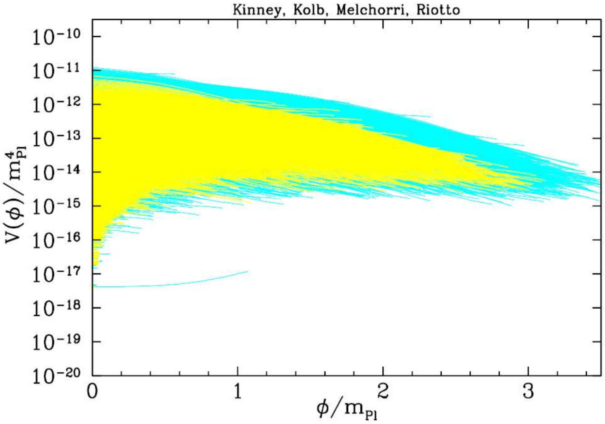
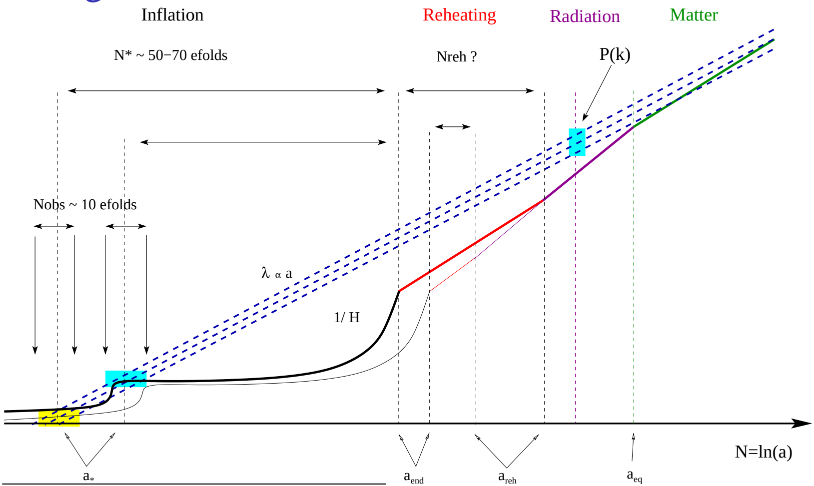


Taken from Kinney et al., astro-ph/0605338.

Reheating:



taken from Ringeval, astro-ph/0703486

Different purposes:

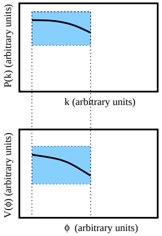
SR - extrapolate $\ddot{\phi} \approx 0$	No extrapolation $\ddot{\phi} = -3H\dot{\phi} - V'$
Elegant / simple	Conservative about unobservable epoch
Very predictive / constraining	Relies on data only
	Independent of reheating.

Directly fit the inflaton potential, numerically, using COSMOMC[I] and our own freely available module[II].

CBM + LSS

$\updownarrow$

$H(\phi) \rightarrow V(\phi)$



[I] Lewis & Bridle, 2002
[II] see astro-ph/0703625

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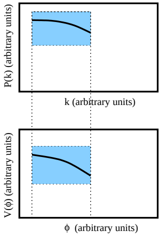

Result applies to any theory of inflation which, during the observable window, has effectively one scalar degree of freedom.

[I] Lewis & Bridle, 2002
[II] see astro-ph/0703625

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- Fit to data using an MCMC.

Numerically fitting $V(\phi - \phi_*)$:

$$\dot{\phi} = -\frac{m_P^2}{4\pi} H'(\phi)$$

$$\left[H'(\phi)\right]^2 - \frac{12\pi}{m_P^2} H^2(\phi) = -\frac{32\pi^2}{m_P^4} V(\phi)$$

- No unique $\dot{\phi}$ for $V(\phi)$ → one option is to always start in attractor solution.

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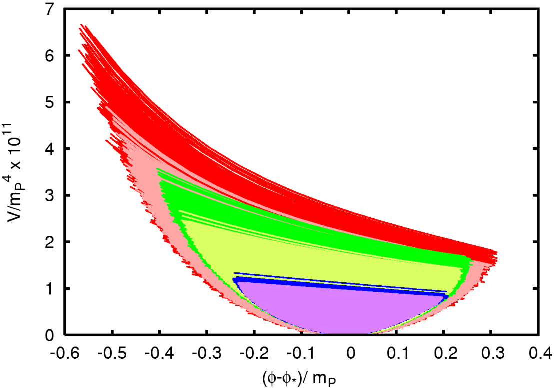
- Everything uniquely defined.
- Potentials equivalent to $V({}^{\ell}\lambda_H(\phi))$, but numerical calculation is the most accurate.

Directly fit the inflaton potential, numerically

Slow Roll	$V(\phi)$	$H(\phi)$
$\ln[10^{10}\mathcal{P}_{\mathcal{R}}^{k_*}]$	$\ln\left[\frac{128\pi 10^{10} V_*^3}{3V_*'^2 m_P^6}\right]$	$\ln\left[\frac{4\times 10^{10} H_*^4}{H_*'^2 m_P^4}\right]$
r	$\left(\frac{V_*'}{V_*}\right)^2 m_P^2$	$\left(\frac{H_*'}{H_*}\right)^2 m_P^2$
n_{S}	$\frac{V_*''}{V_*} m_P^2$	$\frac{H_*''}{H_*} m_P^2$
α_{S}	$\frac{V_*'''}{V_*}\frac{V_*'}{V_*} m_P^4$	$\frac{H_*'''}{H_*}\frac{H_*'}{H_*} m_P^4$
β_{S}	$\frac{V_*''''}{V_*}\left(\frac{V_*'}{V_*}\right)^2 m_P^6$	$\frac{H_*''''}{H_*}\left(\frac{H_*'}{H_*}\right)^2 m_P^6$

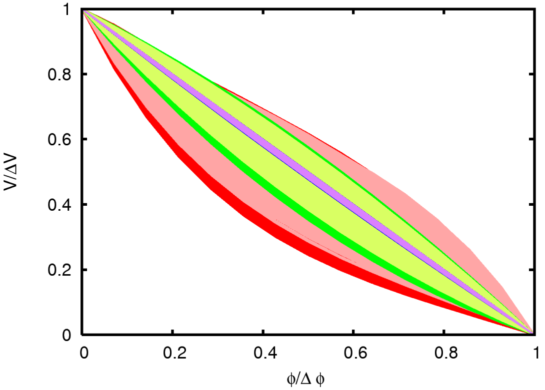
$$+\ \Omega_b h^2,\ \Omega_{cdm} h^2,\ \theta\ \&\ \tau$$

$H(\phi) \rightarrow P(k)$



The inflaton potential at 68% and 95% confidence level

$H(\phi) \rightarrow P(k)$



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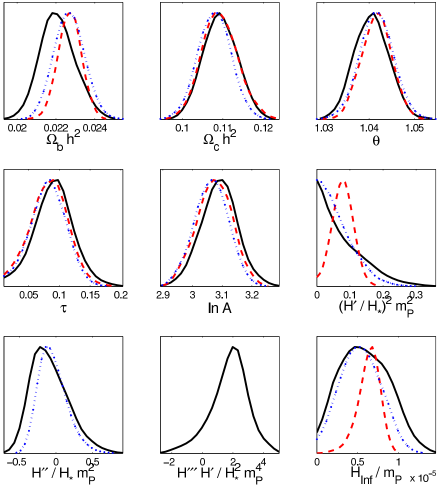
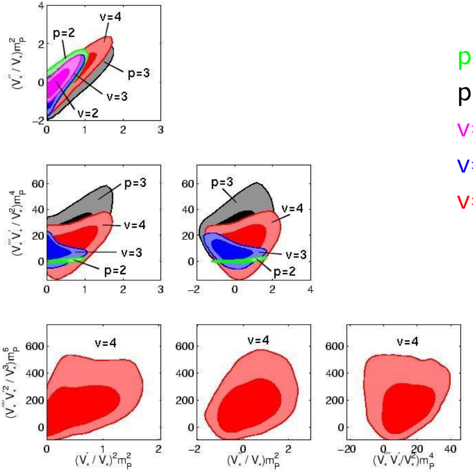

0.02
0.022
0.024
Ω_b h^2
0.1
0.11
0.12
Ω_c h^2
1.03
1.04
1.05
θ
0.05
0.1
0.15
0.2
τ
2.9
3
3.1
3.2
ln A
0
0.1
0.2
0.3
(H′ / H_*)^2 m_P^2
−0.5
0
0.5
H″ / H_* m_P^2
−2
0
2
4
H‴ H′ / H_*^2 m_P^4
0
0.5
1
H_Inf / m_P
x 10^−5

$V(\phi) \rightarrow P(k)$



p=2 - A_S, n_S

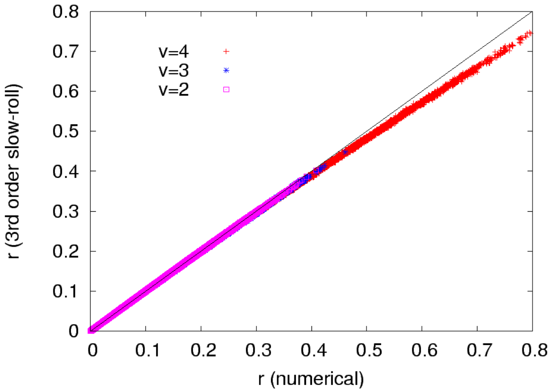
p=3 - A_S, n_S, α_S

v=2 - V'_*, V''_*

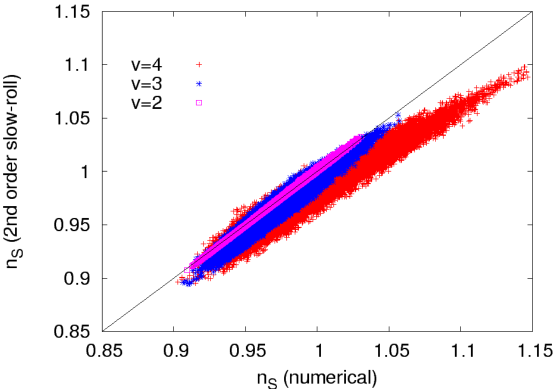
v=3 - V'_*, V''_*, V'''_*

v=4 - V'_*, V''_*, V'''_*, V''''_*

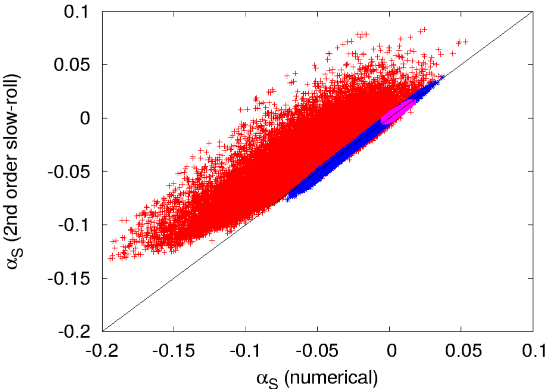
$V(\phi) \rightarrow P(k)$



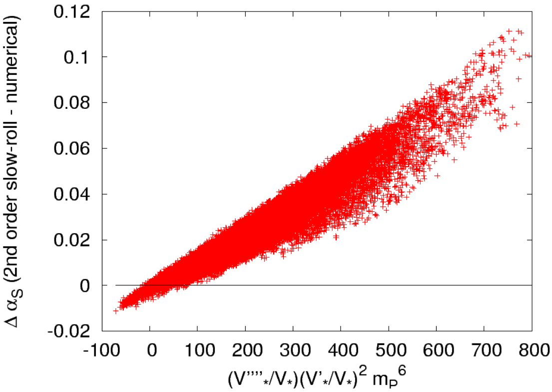
$V(\phi) \rightarrow P(k)$



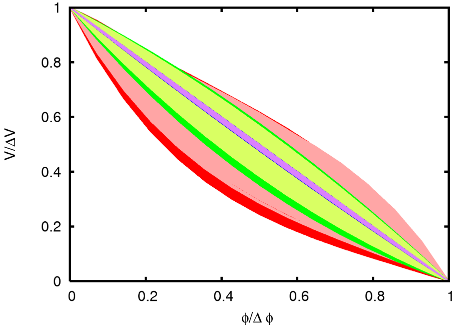
$V(\phi) \to P(k)$


0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
αS (2nd order slow-roll)
αS (numerical)

$V(\phi) \rightarrow P(k)$



Conclusion



- Previously obtained info on $V(\phi)$ depends on strong assumptions
- Hint to go to one order higher in SR
- Conservative analysis of data constrains $H(\phi)$ up to H''' and thereby $V(\phi)$.

Parameters: Slow Roll

$$\ddot{\phi} \ll 1 \rightarrow \dot{\phi} = -\frac{V'(\phi)}{3H}$$

$$^{\ell}\lambda_H \equiv \left(\frac{m_{\rm Pl}^2}{4\pi}\right)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{d^{(\ell+1)}H}{d\phi^{(\ell+1)}}; \ \ell \geq 1$$

$$^{\ell}\lambda_H = 0 \qquad \text{for } \ell > n$$

$$A_s = A_s(^{\ell}\lambda_H), \quad n_s = n_s(^{\ell}\lambda_H), \quad \text{etc}$$

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$$P_k = P_k(A_s, n_s, \alpha_s, \ldots)$$

$$ {}^{\ell}\lambda_H = {}^{\ell}\lambda_H(A_s, n_s, \alpha_s, \ldots)$$

$$V(\phi) = V_0({}^{\ell}\lambda_H) + V'({}^{\ell}\lambda_H)\phi + \ldots$$

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or define[III] $H(\phi - \phi_*)$,

$$P_k = P_k\left({}^{\ell}\lambda_H(\phi - \phi_*)\right)$$

$$V(\phi) = V\left({}^{\ell}\lambda_H(\phi - \phi_*)\right)$$

[III] Easther & Peiris, astro-ph/0603587.