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Calibration and Modification for the Pacific Northwest of the New Zealand Douglas-Fir Silvicultural Growth Model

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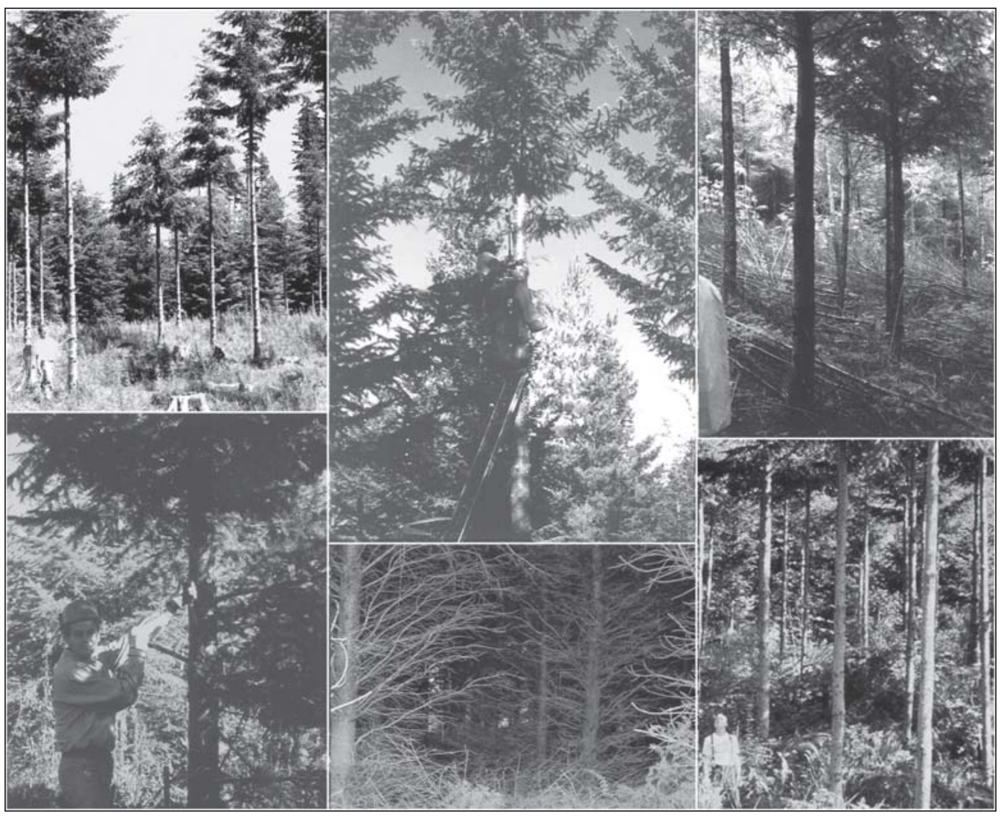

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Abstract

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This paper describes a growth model for young plantations of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) growing in the Pacific Northwest. The overall model has three major components. The first is a yield model for diameter and height distributions describing stands prior to pruning or precommercial thinning. The second component is an annual per-acre net increment model adapted from a recent model for Douglas-fir plantations in New Zealand; thinning and pruning are features of the model. The third component is growth equations for cohorts of individual trees; the results from this component are adjusted to match those from the second component. Fitting data are from Stand Management Cooperative experiments, with top heights generally below 75 ft. An intended use of the model is the evaluation of pruning regimes, in conjunction with the ORGANON model for growth at older ages, and TREEVAL model for clear-wood recovery and economic evaluation.

Keywords: Growth and yield, diameter distribution, pruning, thinning, *Pseudotsuga menziesii*.

Summary

A growth model is presented for young plantations of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) in the Pacific Northwest. The model is based on experimental data from the Stand Management Cooperative (SMC), headquartered at the University of Washington. A primary purpose of the model is to facilitate the evaluation of pruning regimes in conjunction with the TREEVAL program and another growth model capable of carrying the stands from a height of about 75 ft through harvest age.

Data include most of the SMC type I and type III installations. The former are spacing trials within young plantations. The latter installations have a wide range of planting densities. Both types of installations have had a variety of thinning and pruning treatments subsequently imposed; some plots are fertilized. Growth data subsequent to fertilization are excluded from the analysis. Top heights (HTOP) range from 12 ft to over 75 ft.

A “presilviculture” yield model predicts stand characteristics as a function of site index, age, and planting density; the yield model is applicable to stands prior to precommercial thinning with top heights in the range of 12 to 40 ft. The directly predicted characteristics include top height, basal area, surviving trees per acre, and other stand attributes sufficient to allow the calculation of parameters (recovery process) for a three-parameter Weibull distribution for diameter at breast height (d.b.h.).

Subsequent growth is predicted by an annual growth model using equation forms adapted from those in the New Zealand Douglas-Fir National Model. The New Zealand model was developed from an extensive array of pruning and thinning experiments, and was presumed to be a good prototype model for pruning effects. Even so, equation forms for stand-level growth were modified to better reflect the Pacific Northwest. Tree growth and mortality equations were fit and a reconciliation process formulated to scale their predictions so as to reconcile with stand-level growth.

Two identified deficiencies in the model relate to diameter distributions. The recovery process for the Weibull distribution attempts to recover three attributes: quadratic mean d.b.h., the coefficient of variation for basal area per tree, and the 10^{th} percentile for d.b.h. Predictions of those attributes are constrained to ensure that a Weibull recovery is possible. However, the resultant Weibull location parameter, the minimum d.b.h., is often unrealistically close to zero. This indicates a fundamental deficiency of the Weibull to accurately model the lower tail of the diameter distribution. The problem is mitigated by the use of a minimum-d.b.h. empirical equation, which is used in converting the Weibull distribution to a tree list. A second problem is that the magnitude of the decrease in coefficient of variation over time is underestimated. An indication of the problem is that change in the 10^{th} percentile of d.b.h. is underestimated by 0.017 inch per year.

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Introduction

The effects of early silvicultural treatment on Douglas-fir plantations in the Pacific Northwest, both in terms of growth and wood quality, are of great concern to the forest industry. The Stand Management Cooperative (SMC), headquartered at the University of Washington, has established coordinated series of experiments in Oregon, Washington, and British Columbia to study these effects. One series of experiments, the SMC type I installations, were established in juvenile stands with an array of spacing treatments followed by supplementary treatments of fertilization, pruning, and subsequent thinning. The SMC type III installations all include a standardized wide range of planting densities; the first of these installations was installed in 1992. The growth data, particularly those from the type III installations, are viewed as being extremely valuable because of the deliberate treatment contrasts imposed on a variety of initial densities. The SMC has other installations; two of these are used in the present analysis. The calibration of a growth model using these data, with an architecture similar to that found to be useful for Douglas-fir in New Zealand, is the objective of this project[1] and the subject of this report.

Portions of the SMC data have been used in many studies of growth and yield and wood quality. However, at the time this research was initiated, recent data from the type III studies had not been used in the calibration of any general-purpose growth model that is suitable for wood quality evaluations. One stand-level model that has used some of this data is Treelab;[2] some details are described by Pittman and Turnblom (2003). A general-purpose growth model that does use the older SMC data, but not the type III data, is ORGANON (Hann 2003). The version of ORGANON applicable to coastal plantations is SMC ORGANON, with key relationships by Hann and others (2003); a supplemental report[3] describes a dynamic link library (DLL) version of ORGANON 7.0. ORGANON 8.0 (Hann 2005) has since been released; the SMC equations in that model were fit to data sets that include measurements from type III installations.

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[1] Funded by contract 43-046W-3-1270. Develop Report/Calibration of Douglas-fir in the Pacific Northwest R6/PNW.

[2] Pittman, S.D.; Turnblom, E.C. 2001. TreeLab version 1.0. Users guide and model description. http://www.cfr.washington.edu/research.smc/treelab/website/treelab_home.htm (November 5, 2007).

[3] User's manual for the DLL version of ORGANON Version 7.0. Received from Mark Hanus in September, 2004.

Although the Pacific Northwest lacked a detailed general purpose growth model fit to data from intensively managed plantations throughout the entire age range, such a model has been developed in New Zealand. It is the New Zealand Douglas-Fir National Model; within this report we may refer to it as the "NZ model." This model has its origins in the STANDPAK model for *Pinus radiata* D. Don (Knowles and West 1986, West 1993). Current details of the Douglas-fir model were made available to us by the Forest Research Douglas-Fir Cooperative based in Rotorua,[4] an earlier version in an unpublished report.[5] Applicability to the Pacific Northwest is addressed in a report by Knowles and Hansen (2004) and in a presentation by these authors.[6]

The purpose of the present project is to calibrate the NZ model to young intensively managed plantations in the Pacific Northwest, making modifications and supplemental relationships where needed. The original intent of the newly fit young-stand model was to simulate the growth of young stands to an age where Version 7.0 of SMC ORGANON could reliably project the stands through rotation.

With Version 8.0 of SMC ORGANON now calibrated for younger ages, the need for the model described in this report is lessened. The present model does describe presilviculture stands based on minimal input, a feature not in ORGANON. Wood quality of the simulated stands can be assessed by the TREEVAL model (Fight and others 2001).

General Data Description

The data being used are exclusively from SMC installations that include pruning treatments on Douglas-fir plantations. The two major series of installations are the type I and type III series. The type I installations were established in juvenile plantations with uniform stocking. Most plots were reduced to one-fourth or one-half of initial density followed by thinning regimes; some plots were treated with fertilizer or pruning. Type III experiments were planted at fixed levels of initial

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[4] Principal references were two memos by L. Knowles and M. Kimberly "Douglas-fir growth model (NZ DF NAT) equations" and "Method—allocation of basal area increment to sand elements" (March, 2004, together with subsequent clarifications). On file with: James W. Flewelling, 9320 40th Avenue, Seattle, WA 98115-3715.

[5] Fight, R.; Knowles, L.; McInnes, I. 1995. Effect of pruning on early growth and stand dynamics in Douglas-fir plantations. Proceedings of the 20th IUFRO world congress. Tampere, Finland.

[6] Knowles, L.; Hansen, L. 2005. Application of the New Zealand Douglas-fir stand-level growth model to data from the Pacific Northwest. Presented at the Western mensurationist meeting, July 4-6, 2005, Hilo, HI. http://www.growthmodel.org/wmens/m2005/Knowles(a).ppt (August 16, 2006).

densities ranging from 100 trees per acre (TPA) to 1,210 TPA, to be followed by silvicultural regimes. Some of the installations have extra plots for very early thinning or pruning. In the three widest spacings, a matrix of pruning density (100 or 200 TPA pruned plus additional unpruned "followers") and level of pruning (50 percent of live crown removed or pruned to a 2.5-in top) are prescribed. In the three dense spacings, a matrix of thinnings is scheduled.

The establishment dates for the type I installations were 1986 through 1992, with breast-height ages at establishment from 4 to 8 years. The type III installations were established in plantations with planting dates from 1987 through 1990, with the first measurements 3 to 5 years after planting. Two other installations, 353 and 501, were established in 1995, and 1993, respectively, in plantations with planting dates some 18 years earlier. The final measurements used in this analysis were after the growing season in 2000, 2001, or 2002. All measurements were made during the dormant season.

The data were reviewed and cleaned with primary concern on correctly identifying treatments and obtaining good estimates of yield and growth rates. Records from each tree in the database were reviewed for completeness; consistency in codes for ingrowth, thinning, mortality, and pruning; and the presence of measurements for diameter at breast height (d.b.h.), height, and height to crown base. Missing values for height and height to crown base were imputed with a variety of interpolation and local-fitting techniques. The imputation procedures were applied to all measurements, including various "check" measurements made for the purpose of scheduling thinnings. Growth rates in d.b.h. and height received minimal checking with only the most serious errors being corrected; smaller inconsistencies, where it would be impossible to know which of a series of measurements was wrong, were generally not changed.

Site indices were estimated for plots based on top height at the latest measurement year for the installation. Only the plots that had not received a prior pruning were used for this purpose. The computation required use of the density history of the plot, current top height, and current total age by using the methods of Flewelling and others (2001). The site indices for each installation were averaged. Those installation average values are used as an independent variable in the fitting process.

A list of the installations, with details on the computation of site index for each installation, is in table 1. Table 2 gives details on how many growth periods there are for various combinations of thinning and pruning treatment; only the periods with initial top heights of 12 ft or greater are included in this table. That top height restriction was enforced in fitting most of the empirical growth equations.

Table 1—List of installations, showing details related to the plots used for site index computations

			Site index[b]			
Installations[a]	Name	Number of plots	Mean	Standard	Minimum	Maximum
			Feet			
353	Chandler pruning	2	82.5	1.2	82	83
501	Last Creek	4	62.0	3.8	56	65
703	Longbell Road	9	84.3	2.4	81	89
704	Ostrander Road	9	77.3	2.3	74	81
705	East Twin Creek	6	74.0	2.5	71	79
706	B & U Plantation	9	85.1	1.9	82	88
708	Copper Creek	9	88.6	2.4	84	92
711	Kitten Knob	7	96.7	1.8	94	99
713	Sauk Mountain	9	83.7	4.5	77	89
717	Grant Creek #1	7	97.2	1.7	94	99
718	Roaring River 100-REV	9	85.7	2.9	79	89
722	Silver Creek Mainline	9	71.4	2.5	68	76
724	Vedder Mountain	7	85.9	2.3	83	89
725	Sandy Shore	9	87.8	2.7	84	92
726	Toledo	9	90.0	3.5	83	94
729	Gnat Creek	9	93.5	1.7	92	96
732	100-Lens East	9	64.6	3.4	58	69
735	Rayonier Sort Yard	7	82.6	3.8	77	88
736	Twin Peaks	11	91.5	2.3	89	95
737	Allegany	7	86.3	2.5	82	89
905	LaVerne Park	8	89.7	7.8	73	99
910	King Creek	10	75.9	5.2	68	85
915	Big Tree	11	90.2	2.5	86	94
916	Bobo's Bench	11	75.4	4.3	69	82
919	Brittain Creek #1	12	87.8	4.0	82	94
926	R.F. Sale	9	86.4	2.2	82	89
932	Forks #3	10	85.7	2.0	82	88

[a] Installations 703-707 are type I, 903-932 are type III, 353 and 501 are from other series.

[b] Site index (ft) is based on a total age reference of 30 years.

Table 2—Treatment details for the 998 periods used modeling net basal area increment and mortality

					Thinning								
				Mean	None			Current		Previous			
		Total age		period	– – – – – – – – – – – Pruning – – – – – – – – – – –								
Installation	**Plots**	**Minimum**	**Maximum**	**length**	**N**	**C**	**P**	**N**	**C**	**N**	**C**	**P**	**All**
		– – – – – *Years* – – – – –			– – – – – – – – – *Number of periods* – – – – – – – – –								
353	4	22	26	2.0	2	3	3						8
501	13	21	27	2.0	3	32	4						39
703	12	10	26	3.8	38	6	3	3		1			51
704	12	17	31	3.5	25	4	2	3	1	5	1	1	42
705	9	14	27	3.3	24	2	1		2		2	2	33
706	12	13	27	0.4	26	2	1	3	2	6	2	2	44
708	12	10	22	3.1	32	4	4	2	1		1	2	46
711	10	9	23	3.1	25	2	2	2	2		2	4	39
713	12	11	23	3.4	29	4	4	1	1		1	2	42
717	10	9	21	3.1	24	2	2	2	2	1	2	4	39
718	12	11	23	3.1	32	2	2	3	2		2	4	47
722	12	15	27	2.6	40	4	4	2	1	1	1	2	55
724	10	10	22	3.1	23	4	4	3	1	1	1	2	39
725	12	11	23	2.9	33	4	4	3	1	1	1	2	49
726	12	9	21	3.3	31	2	2	1	2		2	4	44
729	12	10	22	3.3	33	2	4	2	1			2	44
732	12	14	26	3.7	30	2	1		2		2	2	39
735	10	12	22	2.9	19	4	2		1		1	1	28
736	16	11	21	2.9	28	6	9	1	1		1	1	47
737	10	9	19	3.0	17	2	2		2		2	4	29
905	11	9	17	2.3	23	3	3	1					30
910	13	10	18	1.9	25	6	3	2		1			37
915	11	8	16	2.3	19	3	3	3					28
916	12	10	17	2.0	20	3	3	2					28
919	13	8	15	2.0	18	3	3	3					27
926	12	10	14	2.0	14	3		1					18
932	12	8	16	2.4	18	3	3	2					26
All	12	8	31	2.9	651	117	78	45	25	17	24	41	998

Periods are classified as to thinning status at the start of the period: None (N) implies no current or previous thinning; previous (P) implies a previous thinning but no current thinning. Pruning status is indicated by aforementioned codes N, C (current) and P.

Model Overview

The New Zealand Douglas-Fir National Model is a per-unit-area growth model that disaggregates to tree cohorts. Disaggregation techniques are reviewed by Ritchie and Hann (1997); the techniques may be very simple as in the NZ model, or may be more complex and use independently developed individual-tree growth models as in the case of STIM (Bonnor and others 1995). The disaggregation techniques used here are intermediate in terms of their complexity; they make use of individual-tree growth and mortality equations, which are tightly tied to per-acre estimates. The tree cohorts are similar to those used in ORGANON; each cohort represents a certain number of trees per acre, all assumed to have identical characteristics. The number of trees per acre in a cohort can decline over time owing to mortality; similarly a cohort could be split by thinning or pruning some but not all of its members. The two distinct parts of the model are a presilviculture yield model, and an annual growth model. The term "presilviculture" is intended to refer to conditions after stand establishment but before the ages where precommercial thinning or pruning would be considered. Also discussed are methodologies by which this model can be linked to ORGANON and TREEVAL.

A complete planting-to-harvest simulation consists of several modeling components. First is the planting specification, with the primary variables being age of the seedlings, number planted per acre, and site index. A presilviculture yield model is invoked at the age where top height is about 15 to 20 ft, prior to any sivicultural treatments. The growth model specified here is then applied for any silviculture regime, growing the stand to a top height of about 75 ft. With the advent of ORGANON 8.0, earlier transition ages become possible. At the transition, the tree list, including d.b.h., tree height, crown information, and "shadow" crown information is passed to ORGANON for growth through harvest age. Shadow crown ratio is a conceptual variable within ORGANON equal to the predicted crown ratio of a tree if it had not been pruned. The linkage to TREEVAL is through a file of tree descriptors, including d.b.h., height profile histories at various ages, pruning lift history, and branch thickness estimators from the literature (Maguire and others 1999).

Terms and Symbols

Variable names are defined here. The names are used within this documentation and may also be used within a computer program implementing the model. Subscripts may sometimes be needed to indicate tree cohort, year, or pretreatment versus posttreatment.

Variable	Definition	Units
Δ	"Delta" operator, indicating a 1-year change	
ba_tree	Tree basal area	ft^2
BA	Basal area of a stand	ft^2/acre
BAL	Basal area of trees larger than subject tree	ft^2/acre
CL	Crown length for a tree	ft
CLL	Crown length sum of larger trees	ft/acre
CLSUM	Sum of crown lengths in a stand	ft/acre
CV	Coefficient of variation of basal area/tree	percent
D0	Zeroth percentile of d.b.h. distribution (superpopulation concept, not the lowest d.b.h. in a d.b.h. list for a particular plot)	in
D10	Tenth percentile of d.b.h. distribution	in
D90	Ninetieth percentile of d.b.h. distribution	in
DBH	Diameter at breast height (4.5 ft)	in
DQ	Quadratic mean d.b.h.	in
DTOP	Quadratic mean d.b.h. of largest 40 TPA	in
H	Tree height	ft
HBC	Height to base of the (live) crown for a tree	ft
HBCrs[7]	A mean height to base of crown for all the trees in a stand that is undergoing crown recession	ft
HBCrt[7]	Height to base of live crown for an individual tree undergoing crown recession	ft
HTOP	Top height (mean of 40 largest diameter trees)	ft
RD10	A relative diameter computed as D10/DQ	1
SAGE	Stand age, growing seasons since planting	years
SBAP	Stand basal area potential	(ft^2/acre per year)
SDI	Stand Density Index = $TPA \times (DQ/10)^{1.605}$	
SI	Site index, based on total age (since germination) of 30 (Flewelling and others 2001)	ft
TAGE	Total age from seed	years
TPA	Trees per unit area	No./acre
TPA0	Trees per unit area at planting	No./acre

Presilviculture Yield Model

The presilviculture yield model predicts sufficient statistics to derive a stand table including diameters and heights. The components having direct empirical predictions are survival, quadratic mean diameter (DQ), the coefficient of variation for basal area per tree (CV), the 10th percentile of the diameter distribution (D10),

[7] Variables used in modeling. May not always be measurable.

the zeroth percentile of the diameter distribution (D0) and a generalized height-diameter model. The intent is to apply this model at an age when top height is around 15 to 20 ft, prior to any silvicultural operations. At this age, it may be presumed that crown recession is negligible; alternatively the crown recession component of the growth model could be applied. Three selected diameter distribution statistics (DQ, CV, D10) are used in recovering a continuous Weibull distribution, which in turn is converted to a tree list; the prediction model for D0 may be used as a lower bound on acceptable values for the d.b.h. cohorts to be inferred from the Weibull distribution.

Growth Model

The growth model contains the following components: top height growth, stand-level crown recession, tree-level (individual tree) crown recession, net basal area growth, stand-level mortality (TPA), tree-level mortality, d.b.h. growth, and tree-level height growth. The stand-level growth components—top-height growth, net basal area, and mortality (TPA)—are primary; the tree-level components are secondary and are adjusted to make them agree, in aggregate, with the stand-level components. All the cited components are derived here with the exception of the top height growth model, where an existing model is already available, having been fit to a larger data set covering a wider span of ages, and including most of the unpruned data used here and data from other older plantations (Flewelling and others 2001). The growth equations are patterned after those in the New Zealand Douglas-Fir National Model; equation forms were modified if residual analyses indicated significant lack of fit for the original equation forms.

Other Features

In addition to predicting diameter and height distributions, the growth model should be able to provide input to ORGANON and TREEVAL. The input requirements for ORGANON include, by cohort, expansion factor, d.b.h., height, height to crown base, and shadow crown ratio. The ORGANON DLL documentation (see footnote 3) states, “it is strongly recommended that stands which had been pruned before the measurement of the input tree listing not be projected in ORGANON.” Two possible ways that may allow for this warning to go unheeded would be to grow the young pruned stands for a sufficiently long period after the pruning that the crown has again started to rise, or to supply shadow height predictions from the crown recession equations, which would be updated annually but which would disregard pruning operations. Additionally, treatment history and breast height age are required.

The TREEVAL model's primary input requirements are tree sizes at harvest, profile predictions based on those sizes, pruning history, branch diameter estimates, and the size of a juvenile core (defined as the bole at 20 years after germination). These will require tracking cohorts between the model developed here and older-age predictions from ORGANON, use of a taper equation, assumptions about tree-size history in the years prior to invoking the presilviculture yield model, and the use of branch size prediction equations, possibly from Maguire and others (1999). Tree dimensions for the years prior to the start of the model are also required.

Development of Equations

Presilviculture Survival

A model is required to predict mortality from time of planting to the age where the growth model is to take over. This will be an arbitrary model specified by two annual mortality rates: first year and subsequent years. Guidance on plausible aggregate mortality rates are developed from SMC data. The major effect of mortality will be in the prediction of surviving trees.

Data—

Data used to build the model came from type III plots. Initial measurements are summarized as:

89 observations (plots)		**7 installations**	
Variable	**Mean**	**Min**	**Max**
SAGE	7.65	3	14
TPA0	520	100	1,210
TPA	417.8	71	1,174

Sixty of the type III plots had no initial treatments. The first growth period on these plots are summarized as:

Variable	Mean	Minimum	Maximum	Standard deviation
SAGE(1)	6.55	3	14	2.66
TPA(1)	428.7	71	1174	277
TPA(2)	423.8	69	1160	275
Period length (years)	2.11	2	4	0.45

(1) refers to start of period.

(2) refers to end of period.

Model—

The survival from planting to some early presilviculture age is estimated as:

$TPA_i = TPA_0 \times (1 - a_0) \times (1 - a_1)^{(SAGE - 1)}$

where TPA_0 is density at planting, and TPA_i is the density at stand age SAGE. Using the type III plot data in the model results in parameters estimated as:

Parameter	Estimate	Standard error
a_0	0.1011	0.0159
a_1	0.00633	0.0012

The model predicts a first-year mortality of 10.11 percent, and subsequent annual mortality of 0.633 percent.

Fitting method—

The available data are insufficient to provide empirical evidence of the annual mortality rate in the first few years after planting. Most of the plots do not have reliable estimates of planting density. The most reliable data are from the type III installations. Of the 89 plots on these installations, 60 had no initial treatment. Their second measurement was 2 to 4 years after the first measurement; these initial growth periods (between first and second measurement) on the 60 untreated plots were used to estimate a_1, an annual mortality rate.

Coefficient a_1 was fit using the survival data for the first growth period as described above. Coefficient a_0 was fit to the data at the first measurement, treating a_1 as a known constant. The standard error for a_0 is a conditional standard error and understates the true uncertainty.

Discussion—

The model and parameter estimates presented provide plausible estimates of mortality, derived from a very limited set of plantations; these estimates may not be reflective of typical operation results. The model is generally in accord with the data in that the trend between survival fraction and stand age at the first measurement is roughly linear, with an intercept of about 0.9.

Presilviculture Quadratic Mean Diameter

Quadratic mean d.b.h. (DQ) is predicted as a function of HTOP, TPA, site index, and age.

Data—

Measurements of untreated plots with top heights between 12 and 40 ft are used as the regression data set. These are summarized for 6,649 observations (measurements) on 345 plots at 26 installations as:

Variable	Mean	Minimum	Maximum	Standard deviation
TPA	335	69	1,174	211
HTOP	24.2	12.20	39.9	7.4
SI	83.8	62	97	8.6
TAGE	13.18	8	22	2.93
DQ	3.89	0.96	7.36	1.51
CV (percent)	45.7	21.1	109.7	13.22
D10	2.62	0	6.20	1.27
D0	1.37	0	5.40	1.08

Model—

$DQ = a_1 \times [(HTOP)^{a_2 + a_3 \times HTOP}] \times \exp(Modifier)$

$Modifier = [a_4 \times TPA/100 + a_5 \times (TPA/100)^2 + a_6 \times TAGE + a_7 \times SI] \times (1 + a_8 \times TAGE)$

r-square = 0.949

MSE [log(DQ)] = 0.0092

In the nonlinear regression results throughout this paper, r-square is the proportion of variance about the mean that is explained by the model. It is calculated as 1 - (residual MSE) / (variance of the dependent variable about its mean). The parameter estimates are:

Parameter	Estimate	Standard error
a_1	0.014927	0.0030
a_2	1.69904	0.074
a_3	-0.0045249	0.00079
a_4	-0.030200	0.0117
a_5	0.0011987	0.00055
a_6	0.00034432	0.00137
a_7	0.0029480	0.000121
a_8	0.097437	0.0645

Discussion—

The form of the equation appears to extrapolate well to older stands. However, it is suggested that HTOP ≥12 should be an absolute requirement and that TPA should not go much higher than 1,200, which represent the range of the data used in modeling.

Presilviculture Diameter Distribution

These three components are discussed together as they are closely related and are fit to a common data set. Empirical prediction equations are developed for CV, D10, and D0. These are, respectively, the coefficient of variation of basal area per tree (percent), the tenth percentile of d.b.h., and an estimate of the zeroth percentile of d.b.h.

Data—
The data are limited to untreated plots with HTOP in the range of 12 to 40 ft; these are the same data as used for the presilviculture DQ model. Within each plot, CV is calculated as 100 times the standard deviation of basal area per tree divided by the mean; the standard deviation is the square root of the usual unbiased estimate of variance. The tenth percentile (D10) is estimated as a weighted average focused at the order position $0.1 \times (n + 1)$, where n is the number of live trees on the plot. The zeroth percentile (D0) is estimated as $DBH_1 - (DBH_2 - DBH_1)/2$ where DBH_1 and DBH_2 are the smallest and next-to-smallest trees on the plot. The D10 regression excluded two observations with D10 = 0.

Models—
The fitted model for CV is:

$$CV\ (\%) = a_0 + a_1 \times \exp[a_2 \times (TAGE/10)^{a_3} \times DQ]$$

r-square = 0.59

MSE = 72

Parameter	Estimate	Standard error
a_0	33.93	1.05
a_1	180.4	17.7
a_2	-1.0345	0.073
a_3	-0.9347	0.048

The model for D10 makes use of a pair of reference functions from which lower and upper limits can be calculated; limiting values are referred to as $D10_{low}$ and $D10_{high}$. The limits are in the context of recovering a set of three Weibull parameters that result in a diameter distribution having specified values of DQ, CV, and D10. If D10 were to be less than $D10_{low}$, the Weibull location parameter a would be negative. If D10 were to be greater than $D10_{high}$, the Weibull shape parameter c would be less than 1.0. Hence the imposition of the D10 limits precludes the possibility that the recovered Weibull parameters correspond to either of these two extreme conditions. The reference functions are documented in the appendix.

The fitted model for D10 is:

$$D10 = DQ \times \{[RD10_{high} - a_0 \times RD10_{low}] \times \exp(LTERM) / [1 + \exp(LTERM)] + a_0 \times RD10_{low}\}$$

where $LTERM = a_1 + a_2 \times HTOP + a_3 \times CV$
r-square = 0.937
MSE = 0.1017

The equation was fit by using the estimated, not actual, CV. The parameter estimates are:

Parameter	Estimate	Standard error
a_0	0.9815	0.0064
a_1	41.713	14.89
a_2	-0.1983	0.099
a_3	-1.0138	0.370

In application, predicted values of D10 will have to be compared with the appropriate lower limit, $RD10_{low} \times DQ$, and replaced by that limit whenever they violate it; or equivalently, set the Weibull location parameter (*a*) to zero. Eighty-seven percent of the observations have predictions that violate the limit; all the violations are by very small amounts. The cases without violations are those with the lowest CV values.

The D0 model is fit as:

$$D0 = \exp(LTERM) / [1 + \exp(LTERM)] \times (RD10_{low} \times DQ)$$

where $LTERM = a_0 + a_1 \times DQ$
r-square = 0.53
MSE =0.550

The equation was fit by using the estimated, not actual, CV. The parameter estimates are:

Parameter	Estimate	Standard error
a_0	-0.8790	0.163
a_1	0.20963	0.031

Discussion—

The CV model is the most important of three models controlling the diameter distribution. Its validity is not dependent upon the assumption of a Weibull distribution. Predictions from this model are plausible for an extended range of conditions; the model coefficients are similar to those obtained from fitting the model to all available data from untreated stands with no upper limit on HTOP. The D10 model is dependent upon the Weibull assumption, as demonstrated by the fact that most D10 predictions are limiting values derived from that distribution. The D0 function has a low r-square value, but is simple and well behaved.

Generalized Height-Diameter Model

The generalized height-diameter at breast height (d.b.h.) curve is the within-plot height-diameter relationship with a free parameter. The curve is used to describe the height-diameter relationship, subject to a specified top height and specified diameter distribution. It is used as part of the presilviculture yield model.

Data—
The data are limited to untreated plot measurements with HTOP of 12 ft or greater. Plots are required to have at least 30 measured heights. Generally all trees have been measured for height; a few individual tree heights will have been imputed either by interpolating between measurements or with local height-diameter curves. Data are summarized for 932 plot measurements on 328 plots at 27 installations as:

Variable	Mean	Minimum	Maximum	Standard deviation
DQ	5.27	0.96	13.36	2.61
HTOP	35.21	12.2	75.9	14.8
TAGE	15.6	8.0	29.0	4.7
TPA	343	68.5	1,175	200

Models—
The full height-diameter model is:

$$H = 4.5 + a_o \times \exp(a_1 \times \text{d.b.h.}^{a_2})$$

where a_0 is an indeterminate parameter, and a_1 and a_2 are empirical functions of stand variables. The parameter a_0 is easily recovered from any stand table for which HTOP is known. In this case, recovery is simply the process of finding the value of a_0 that will cause HTOP (as calculated using the diameter distribution and the above height model) to equal the value of HTOP determined from the site curves.

For each plot measurement, the general model described above was fit with nonlinear least squares allowing all three parameters to be fit; parameter a_2 was constrained to the range (-0.5, -2). The a_2 parameter estimate from each fit was subsequently regressed against several stand variables:

$a_2 = a_{20} + a_{21} \times TAGE + a_{22} \times TPA$
r-square = 0.149
MSE = 0.131

Parameter	Estimate	Standard error
a_{20}	-0.45271	0.048
a_{21}	-0.030135	0.0025
a_{22}	0.0001991	0.000059

In application, extrapolation problems may be avoided by limiting the TPA variable used in the equation to values less than or equal to the approximate maximum TPA in these data.

Given the foregoing equation for a_2, the individual plot measurements were refit to obtain estimates of a_0 and a_1. Values of a_1 were then related to the other stand-level variables:

$a_1 = -\exp(a_{10} + a_{11} \times TAGE + a_{12} \times DQ + a_{13} \times HTOP + a_{14} \times TPA)$
r-square = 0.588
MSE = 0.370

Parameter	Estimate	Standard error
a_{10}	0.48914	0.034
a_{11}	0.044212	0.0034
a_{12}	0.056215	0.0072
a_{13}	-0.009251	0.00173
a_{14}	-0.0003841	0.000053

Discussion—
Prior to applying the generalized height-diameter model, the diameter distribution and HTOP must be known. Then a_0, for which there is no empirical equation, is calculated so as to exactly recover HTOP. The calculation formula, to be applied to the portion of the stand table corresponding to the largest 40 TPA is:

$$a_0 = (HTOP - 4.5) \times \sum TPA_i / \{\sum[TPA_i \times \exp(a_1 \times DBH_i^{a_2})]\}$$

where the subscript *i* refers to the cohorts within a stand. The parameter estimates (a_0, a_1, and a_2) are then used in predicting the heights for association with all the diameters in the stand table.

Stand-Level Crown Recession

This model predicts the mean height to the crown base in the absence of treatments. This model is used as an input to the tree-level crown recession model. Typically this model when applied in successive years will predict that each year's HBC will be the same or greater than that for the previous year. However, following a thinning or pruning, a height to crown base model derived from untreated stand data will typically estimate an HBC lower than that observed, and sometimes lower than that predicted for the previous year. Height to crown base seldom declines for real trees; this behavior is mimicked in the model by the imposition of a no-decline rule. That rule sets the current HBC to the greatest of the previous HBC, that predicted by the empirical equation developed here, or that computed from a stand table immediately following pruning. Hence, the HBC model predictions can provide a "shadow" height-to-crown base similar to that used within ORGANON.

Data—

The data are limited to untreated plots with HTOP of 12 ft or greater. HBCrs is the mean distance from the ground to the bottom of the crown. These are summarized for 1,011 observations (measurements) on 353 plots at 27 installations as:

Variable	Mean	Minimum	Maximum	Standard deviation
HTOP	35.3	12.0	75.9	14.8
SI	83.6	62.0	97.2	8.8
TPA	336	68.5	1,175	201
HBCrs	6.05	0.10	43.4	8.0

Model—

$$\mathrm{HBCrs} = \mathrm{MAX}(b_8 + b_4 \times \mathrm{HTOP}, \mathrm{HTOP} - b_3) / [1 + \exp(b_0 + b_1 \times \mathrm{HTOP} + b_2 \times \mathrm{TPA}/100 + b_5 \times \mathrm{HTOP} \times \mathrm{TPA}/100 + b_6 \times (\mathrm{TPA}/100)^2 + b_7/\mathrm{SI})]$$

r-square = 0.940

MSE = 3.86

The parameter estimates are:

Parameter	Estimate	Standard error
b_0	2.0515	0.37
b_1	-0.067197	0.0039
b_2	-1.05658	0.103
b_3	26.2649	1.38
b_4	0.38872	0.058
b_5	0.0049777	0.0011
b_6	0.063455	0.0062
b_7	270.12	20.9
b_8	-4.34	1.4

Discussion—

There still are some significant correlations between the residuals and independent variables TPA and HTOP, suggesting that further improvements in the model form should be possible. Visually the residuals appear to be homogenous about zero.

Individual Tree Crown Recession

The individual tree crown recession model predicts height to crown base for individual trees. The predictions at any age can be overridden by pruning to a greater height or by earlier crown recession, most likely prior to a thinning.

Data—

The data are limited to untreated plot measurements with HTOP of 12 ft or greater. These are the same plot measurements as were used for stand-level crown recession. All of the trees are used, including those with measured and those with imputed height to crown base. These are summarized for 134,799 tree measurements, 1,011 plot measurements on 353 plots at 27 installations as:

Variable	Mean	Minimum	Maximum	Standard deviation
DBH	5.13	0	19.0	2.61
H	32.6	0.8	87.1	14.9
HBCrt	7.73	0.10	59.7	8.94

Model—

As a base model, we could postulate that the height to crown base is the same for all trees in a stand. Under that assumption, and assuming HBCrs is known, the mean square error for the prediction of HBCrt is 2.51. An alternative model is now proposed:

$$\text{HBCrt} = c \times \text{Min}\{b_1 \times \text{H}, \text{HBCrs}\}$$

where c is set for each plot so as to force the plot mean value for HBCrt to equal HBCrs, and b_1 is an overall constant. The value of b_1 that minimizes the mean square error is $b_1 = 0.53$; here MSE = 2.37. The r-square value is 0.97.

Discussion—

For plots taken individually, the value of c ranged from 1.00 to 1.21 with a mean of 1.002. That the mean is so close to one implies that the b_1 term does not have a large impact on many trees. For the typical case where the b_1 term affects only a few trees in a stand, implying that c is close to 1, the b_1 term requires that HBC/H be at least 0.53, and that crown ratio not be less than 0.47. If in a particular stand a large fraction of the trees are affected by the b_1 term, c will increase, and the minimum predicted value for crown ratio will become less than 47 percent.

The proposed model provides a small improvement over the starting hypothesis, that all trees in a stand have the same height to crown base. Better models could probably be developed with more time for model exploration.

Net Basal Area Growth

Net basal area growth is the change in basal area from one year to the next.

Data—

The data are limited to growth periods on treated and untreated plots where the initial HTOP is 12 ft or greater. There are 998 such growth periods on 308 plots at 27 installations. These original growth periods have durations of 1 to 4 years. Linear interpolation is used to convert to 1-year periods; after dropping the 1-year periods with starting HTOP values of less than 12 ft, 2,919 1-year periods remain:

Variable	Mean	Minimum	Maximum	Standard deviation
BA	54.1	0.7	188	39
CLSUM	7,334	5,637	26,438	4,399
HTOP	38.2	12.2	77.0	13.3
SI	84.0	62.0	97.2	8.7
TAGE	16.5	8.0	29.5	4.4
ΔBA	8.34	.505	25.5	3.88
ΔTPA	-1.26	-39	5	2.89

Exploration of model forms used the periodic data, before the conversion to 1-year periods. A midpoint convention was used to adjust the independent variables as explained in the section on stand-level mortality.

After the model exploration phase, the model was fit to the linearly interpolated 1-year periods; the resultant parameter estimates were used in a preliminary version of the growth simulator. All the growth periods were then simulated, and the simulated annual results were used to calculate the relative increment of each of the years within each of the growth periods. This was done for basal area and for the other variables. Those relative annual increments were then used in an interpolation scheme to remake the annual growth database. That revised interpolated database was used in refitting the following model.

Model—

The following prediction model for annual increment in basal area is the product of several terms; all parameters are fit simultaneously with nonlinear regression.

$\Delta BA = s_2 \times \exp(s_3 \times HTOP) \times ATERM \times CROWN \times SITE \times COMP$

$ATERM = a_2 + (1 - a_2) \text{ x } \exp(a_3 \text{ x } TAGE)$

$CROWN = [1.0 - b_3 \exp(b_1 \times BA)]^{b_2} \times [1 - \exp(b_4 \times CLSUM/10000)]^{b_5}$

$SITE = (SI/80)\text{^}s_1$

$COMP = 1.0 - \exp(f + g \times BA/SITE) / [1.0 + \exp(f + g \times BA/SITE)]$

r-square = 0.85

MSE = 1.68

Parameter	Estimate	Standard error
a_2	0.1	
a_3	-0.029434	0.0051
b_1	-0.014743	0.0039
b_2	0.79680	0.15
b_3	0.91272	0.057
b_4	-5.03654	0.31
b_5	1.43513	0.113
f	-8.71384	1.71
g	0.046672	0.0097
s_1	0.23673	0.066
s_2	45.4609	4.1
s_3	-0.015602	0.0013

Discussion—

The parameter a_2 was set at 0.1; the regression would have been better by an insignificant amount if a_2 had been allowed to go to zero; keeping a_2 positive prevents the age term from controlling the asymptotic limit.

The CROWN term allows for increased growth as BA and CLSUM increase. Having CLSUM > 0 is a requirement for a positive growth prediction; however some positive growth can be predicted even if BA = 0. The interaction of BA and CLSUM is such that substantive growth prediction requires substantive BA and substantive crown. The SITE term predicts increased growth with increasing site index; the power coefficient on that relationship (s_1 = 0.24) seems to be quite low; however, this is not the only place that site index enters the relationship. A separate height term predicts that basal area growth declines with increasing height. The height term causes the predicted growth at 61 ft to be about half of that predicted at 15 ft. COMPTERM takes on value 1.0 at zero basal area and goes to zero as the ratio of BA to SITE approaches infinity; for SI 80, COMP has value 0.5 at a basal area of 187 ft^2/acre; higher sites will have lesser reductions in predicted growth rate for given values of basal area.

Stand-Level Mortality

This model predicts change in TPA in a year owing to mortality.

Data—
The data are the same as for the net basal growth model. However, midpoint conventions are used to adjust for multiyear periods. The midpoint convention is to linearly interpolate between starting and ending values for a period to arrive at estimates of the variables at 0.5 years prior to the mean of the starting and ending ages. The dependent variable is annual change in TPA owing to mortality. Mean mortality per year is 1.33 TPA. Another component of net change in TPA on the plots is ingrowth, with a mean value of 0.08 TPA per year; the ingrowth is ignored and is not modeled. The data are limited to growth periods on treated and untreated plots where the initial HTOP is 12 ft or greater. These are summarized for 998 growth periods on 308 plots at 27 installations as:

Variable	Mean	Minimum	Maximum	Standard deviation
BA	54.1	1.2	179	39
TPA	295	45	1,171	186
HTOP	38.1	13.9	77.0	13.2
SDI	118	4.5	373	77
$(\Delta TPA)_{net}$	-1.25	-39	5	2.86
ΔTPA	-1.33	-39	0	2.84
Period length	2.92	1	4	1.05

Model development—
The data exhibit a fairly low level of mortality; observed densities do not approach what would be the maximum stand density index (SDI) for Douglas-fir. An SDI value of 454 is a mean estimated asymptotic SDI (Hann and others 2003). Simply fitting an unconstrained mortality model to the data is unlikely to produce a model that will extrapolate well. Accordingly, a combined model is developed that includes an empirical model suitable for low densities, an SDI trajectory model for high densities, and an arbitrary smooth transitioning function. Although the model

ultimately used is a combined model, fit as such, we also present the two component models separately as an aid to understanding. The dependent variable in the regressions is mortality TPA per year, with a weight of the number of years in the period; this is true even though the notation may suggest that the dependent variable could include ingrowth. This notation is appropriate in the context of a model that does not provide for ingrowth. The two component models and the combined model are fit by minimizing the sums of error squared in TPA at the end of each period, with observation weights proportional to period length.

Model 1. An empirical model.

$\Delta TPA = -TPA \times Y$

$Y = a + (1.0 - a) \times X/(1.0 + X)$

$X = \exp(b_0 + b_1 \times \log(TPA + 1) + b_3 \times SI + b_4 \times HTOP)$

r-square = 0.40

MSE = 14.6

Fitted parameter values are not shown here because they are revised in the process of fitting the combined model.

Model 2. Direct model of mortality angle keyed to SDI.

Define the angle *M* (radians) on a log-log graph of (TPA, DQ) such that an angle of zero represents no mortality. Hence

$M = \tan^{-1}[\log(TPA_1/TPA_2)/\log(DQ_2/DQ_1)]$

where subscripts 1 and 2 refer to the start and end of a 1-year period. *M* may be considered to be a function of SDI, which is itself defined as:

$SDI = TPA \times (DQ/10)^{1.605}$

The angle *M* that corresponds to a constant SDI equals $\tan^{-1}(1.605) = 1.013598$ radians. The asymptotic SDI of 454 is imposed by the trajectory angle model:

$M = b_0 + (1.013598 - b_0) \times (SDI/454)^{b_1}$

r-square = 0.153

MSE = 20.65

Parameter	Estimate	Standard error
b_0	0.0182	0.0023
b_1	3.601	0.119

Representative trajectories predicted by this model are shown in figure 1. To apply the model, first compute *M* and the grown basal area (BA_2). The grown quadratic mean d.b.h. (DQ_2) is computed as:

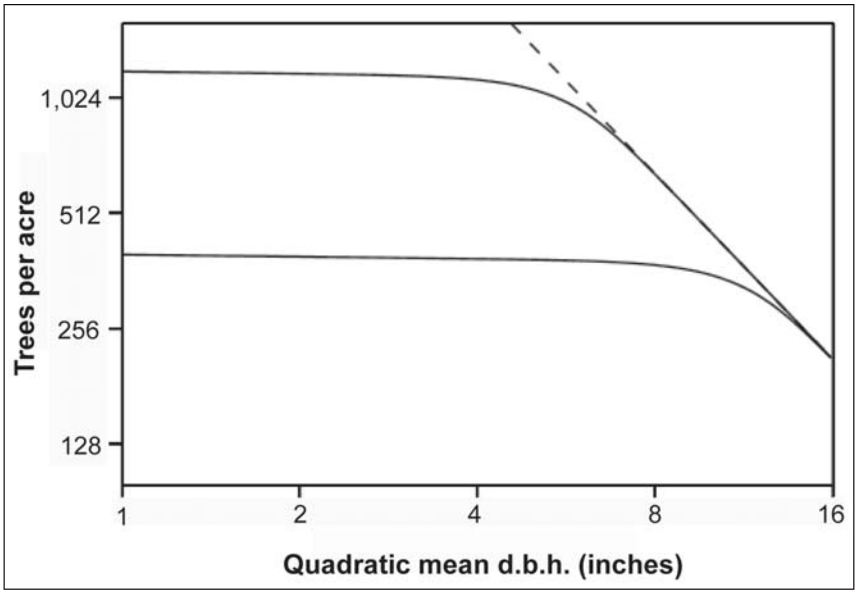

Figure 1—Two mortality trajectories derived from the direct model of mortality angle, with both trajectories approaching the dashed line having a stand density index of 454.

$$\log(DQ_2) = [\log(BA_2) - \log(k) - \tan(M) \times \log(DQ_1) - \log(TPA_1)] / [2 - \tan(M)]$$

where k is the units constant 0.005454154, and the subscripts 1 and 2 refer to the start and end of a 1-year period. TPA_2 can be determined from BA_2 and DQ_2. The parameters were estimated by minimizing the mean square error in TPA_2; observations were weighted by period length.

Model 3. Weighted combination model.

This model is a weighted combination of a Model 1 (with parameters re-estimated here), and the angle model, Model 2 (with the parameter estimates shown above). The overall model is:

$\Delta TPA = w \times (\Delta TPA)_{\text{Model 2}} + (1 - w) \times (\Delta TPA)_{\text{Model 1}}$
where
$w = \text{MAX}\{[\text{MIN}(SDI/454,\ 1)]^{w_1},\ \text{MIN}[(HTOP - 40)/60,\ 1]\}$
r-square = 0.421
MSE = 14.1
and the parameter estimates for the refit Model 1 are:

Parameter	Estimate	Standard error
a	0.003189	0.00028
b_0	-29.39*	2.42
b_1	4.5837	0.41
b_3	-0.15900	0.020
b_4	0.15223	0.0093
w_1	1.57	0.40

* The b_0 parameter was originally estimated as -29.49. However, with that value the combined model had a mean residual of 0.03. Changing b_0 to the value shown in the table forces the mean residual to zero.

The transition weighting is entirely arbitrary except for w_1, the power on [MIN(SDI/454, 1)]. The effect of the transition weighting is that Model 2 is assigned a weight of at least $(SDI/454)^{1.57}$, and at least (HTOP - 40)/60, subject to an upper bound of 1. Hence Model 2 dominates for any stand with SDI approaching 454 or HTOP approaching 60 ft.

The resulting mean value of w, the weight for Model 2, is 0.158; w exceeds 0.2 in only 319 of the observations. Hence, Model 1 is definitely dominating the performance of the combination model within the range of the data.

The residuals for the combined model are examined in table 3. Trends in predicted and observed mortality are similar to one another across the range of each of the shown independent variables. For example, the lowest quartile of the data with respect to site index (RANK SI = 0), has observed mortality of 1.92 TPA/year, and the highest quartile of the data (RANK SI = 3) has mortality of only 0.91 TPA/year; the trend in predicted mortality across the SI range is similar, with the lowest and highest quartiles of the data having predicted mortality of 2.06 and 0.90 TPA/year, respectively. Residual tables of this kind offer some assurance that predicted trends with respect to key independent variables are approximately correct.

Discussion—

None of the growth plots are fully into a self-thinning mode. Model 1 correctly captures the mortality patterns within the data, but there is no reason to believe that extrapolations from that model would be valid. Even within the range of the data, the possibility exists that the observed mortality trends will be substantively different from trends on operational nonresearch plantations. One reason to suspect that the data are not representative is the field protocol, which called for hardwood competition to be removed periodically.

Table 3—Residual analysis for stand-level mortality model

	Mean mortality				t value	Mean					
	N	Observed	Predicted	Residual	Residual	SI	BA	TPA	TAGE	HTOP	SDI
	– – – – – – – – – *Trees per acre per year*				– – – – – – – –	*Feet*	Ft^2/ac		*Years*	*Ft*	
All	998	1.34	1.34	-0.00	-0.0	84	55.7	300	16.5	38.2	121
Rank by SI											
0	273	1.92	2.06	-0.15	-0.9	71	53.8	314	19.7	39.9	119
1	246	1.37	1.29	0.08	0.5	84	66.6	321	16.9	41.0	143
2	238	1.13	1.08	0.05	0.5	88	49.4	303	14.5	34.4	110
3	241	0.91	0.90	0.01	0.1	93	51.4	261	14.6	36.8	111
Rank by BA											
0	249	0.74	0.66	0.08	0.9	85	13.4	204	12.6	24.3	36
1	250	1.07	0.95	0.12	1.0	83	32.1	292	15.1	32.2	78
2	250	1.18	1.09	0.10	0.7	84	60.1	303	17.5	41.4	131
3	249	2.28	2.56	-0.29	-1.5	84	109.0	387	20.2	52.2	225
Rank by TPA											
0	251	0.64	0.34	0.30	3.7	85	30.0	110	16.7	36.9	61
1	248	0.55	0.64	-0.09	-1.4	84	50.5	202	16.8	38.6	104
2	250	1.18	1.14	0.04	0.3	84	68.7	313	17.0	41.1	146
3	249	2.92	3.14	-0.22	-1.0	83	71.0	554	15.7	36.1	168
Rank by TAGE											
0	264	1.10	1.08	0.02	0.1	88	22.3	330	11.5	23.6	61
1	247	1.16	1.14	0.02	0.1	85	45.8	316	14.8	33.5	107
2	263	1.35	1.24	0.11	0.7	85	71.0	279	18.2	44.7	149
3	224	1.86	2.04	-0.18	-1.1	77	90.0	270	22.8	53.8	180
Rank by HTOP											
0	249	0.91	1.03	-0.11	-1.1	85	18.6	313	11.8	22.5	52
1	250	1.39	1.21	0.18	1.2	84	39.3	324	14.7	32.0	96
2	250	1.15	1.15	0.00	0.0	83	62.2	281	17.9	42.0	135
3	249	1.92	1.99	-0.07	-0.4	84	101.2	285	21.5	55.7	200
Rank by SDI											
0	249	0.66	0.59	0.07	0.8	84	13.5	184	13.1	25.3	35
1	250	1.07	0.84	0.23	1.8	83	32.5	262	15.3	33.0	77
2	250	1.17	1.10	0.07	0.6	84	59.7	322	17.1	40.5	130
3	249	2.35	2.69	-0.34	-1.8	84	108.3	410	19.9	51.3	225

Note: Mortality is defined in terms of trees per acre per year. The t value is student's t for the residual. The reported independent variables are period midpoint values for site index (SI), basal area (BA), trees per acre (TPA), top height (HTOP) and stand density index (SDI). Results are presented overall, and by quartiles for each of the independent variables. RANK 0 is the lowest quartile of the data, and RANK 3 is the highest quartile of the data with respect to the selected variable

The angle-based mortality model is not a precise model; but it will predict plausible self-thinning patterns similar to those predicted by ORGANON at high densities. The model works by predicting a mortality angle that allows SDI to increase if SDI is < 454, and requires SDI to decrease if SDI is > 454. The transition between the fully empirical model and the angle model is smooth and plausible. Recognizing the absence of high-density data at older ages, and a need to transition to ORGANON projections, we see the combined model as offering a reasonable compromise.

Diameter Growth

Diameter growth is to be computed from the regression equation described here. This equation makes use of the stand-level prediction ΔDQ.

Data—
Data from all growth periods were used, provided that the midpoint HTOP was at least 12 ft; these are the same growth periods used in the net basal area growth analysis. The data include treated and untreated observations, but no fertilization. Only trees that survive the growth period are used. Variables are summarized for 1,134,392 observations (tree growth) in 998 growth periods on 308 plots at 27 installations:

Variable	Mean	Minimum	Maximum	Standard deviation
DBH	5.45	0	16.25	2.38
H	34.1	1.44	81.9	13.6
HBC	9.2	0.1	47.6	8.9
ΔDBH	0.45	-1.1*	3.0	0.23
Survival (period)	0.9857	0	1	0.119
Period length	3.15	1	4	1.07

* This negative DBH growth is presumably a measurement error.

A stand-level independent variable (ΔDQ) is the predicted change in DQ, given the midpoint conditions and the empirical models to predict annual change in BA and TPA.

Several individual variables are also defined, referencing the subject tree with respect to other trees in the plot.

X1 = CL/[Mean CL]
X2 = DBH/DQ
X3 = CL/H
X4 = BAL/BA where BAL = basal area of larger trees
X5 = CLL/CLSUM where CLL = sum of crown length of larger trees

BAL and CLL include larger trees plus one-half the attribute for the current tree. The computations are cumulative from a sorted (descending) list of diameters; no special consideration is given for ties in d.b.h. Trees that died during the period do contribute to these variables, even though dying trees are not included in the regression data.

Model—

$$\Delta DBH = \Delta DQ \times \exp[b_0 + b_1 X2 + b_2(X2)^2 + b_3 X3 + b_4 X4 + b_5 X5 + b_6 X4 \times X5 + b_7 (X3)^2]$$

r-square = 0.70
MSE = 0.047

Parameter	Estimate	Standard error
b_0	-1.75663	0.0321
b_1	1.42374	0.0347
b_2	-0.39149	0.0148
b_3	1.23969	0.0610
b_4	-0.42492	0.0290
b_5	0.52479	0.0462
b_6	-0.19377	0.0209
b_7	-0.43028	0.0393

Discussion—

Residuals for the model are in table 4. The observations are broken into classes on the basis of recent pruning, and on the basis of rank-ordered sets of the independent variables. Generally, the pattern of predicted mean d.b.h. increments closely follows the pattern of the observed increments. The mean errors for the 5,031 observations immediately following a pruning are a trivial value, indicating there is no large bias in predicting the growth of pruned trees.

The NZ model has growth proportional to crown length. This would be approximated by a model having the single variable X1 on the right-hand side. X1 was originally in the model; however, with the addition of the other terms, the X1 term became nonsignificant and was dropped. Hence there is a reasonable assurance that this model functions better than one allocating growth proportional to crown length.

Table 4—Residual analysis for diameter growth equation

	N	Mean diameter growth per year			Mean					
		Observed	**Predicted**	**Residual**	**X1**	**X2**	**X3**	**X4**	**X5**	**DQ**
		– – – – – – – – *Inches* – – – – – – – –								*Inches*
All	111,817	0.47	0.47	-0.001	1.00	0.98	0.79	0.61	0.53	5.52
Prune now										
0	106,786	0.48	0.48	-0.002	1.00	0.98	0.79	0.61	0.53	5.50
1	5,031	0.37	0.37	0.003	0.98	1.00	0.53	0.58	0.51	6.24
Rank by X1										
0	13,977	0.32	0.34	-0.019	0.70	0.64	0.75	0.94	0.91	5.00
1	13,977	0.41	0.42	-0.010	0.87	0.82	0.77	0.86	0.79	5.45
2	13,977	0.44	0.45	-0.005	0.95	0.90	0.78	0.78	0.70	5.72
3	13,977	0.47	0.47	-0.002	1.00	0.96	0.78	0.68	0.59	5.92
4	13,978	0.50	0.49	0.003	1.04	1.02	0.78	0.57	0.47	5.96
5	13,977	0.52	0.51	0.006	1.08	1.08	0.79	0.46	0.36	5.75
6	13,977	0.54	0.53	0.008	1.13	1.15	0.80	0.36	0.27	5.47
7	13,977	0.58	0.57	0.006	1.25	1.26	0.84	0.26	0.19	4.88
Rank by X2										
0	13,977	0.30	0.32	-0.022	0.73	0.60	0.77	0.97	0.94	4.98
1	13,977	0.39	0.40	-0.010	0.90	0.79	0.77	0.91	0.85	5.47
2	13,978	0.44	0.44	-0.000	0.96	0.89	0.78	0.83	0.74	5.66
3	13,976	0.47	0.47	0.001	1.00	0.96	0.78	0.72	0.62	5.79
4	13,978	0.50	0.50	0.003	1.04	1.02	0.78	0.59	0.48	5.81
5	13,977	0.53	0.52	0.002	1.07	1.09	0.79	0.45	0.35	5.74
6	13,977	0.56	0.55	0.005	1.12	1.16	0.80	0.30	0.21	5.58
7	13,977	0.59	0.58	0.008	1.20	1.33	0.81	0.14	0.09	5.14
Rank by X3										
0	13,977	0.23	0.24	-0.012	0.94	0.94	0.48	0.67	0.59	7.38
1	13,977	0.32	0.31	0.005	1.00	0.98	0.61	0.60	0.52	7.24
2	13,977	0.37	0.38	-0.008	0.99	0.97	0.70	0.63	0.55	6.65
3	13,977	0.43	0.43	-0.002	1.00	0.97	0.78	0.62	0.53	5.89
4	13,978	0.52	0.53	-0.006	0.99	0.97	0.85	0.63	0.55	5.13
5	13,977	0.57	0.58	-0.009	1.01	0.98	0.89	0.61	0.53	4.47
6	13,977	0.61	0.60	0.005	1.03	0.99	0.93	0.59	0.51	4.17
7	13,977	0.68	0.66	0.015	1.06	1.02	0.96	0.57	0.48	3.71

Table 4—Residual analysis for diameter growth equation (continued)

		Mean diameter growth per year			Mean					
	N	Observed	Predicted	Residual	X1	X2	X3	X4	X5	DQ
		– – – – – – – – *Inches* – – – – – – – –								*Inches*
Rank by X4										
0	13,977	0.61	0.60	0.014	1.18	1.31	0.80	0.11	0.07	5.66
1	13,977	0.55	0.55	0.002	1.12	1.17	0.80	0.30	0.22	5.62
2	13,977	0.52	0.52	0.001	1.08	1.09	0.79	0.46	0.35	5.59
3	13,977	0.49	0.49	0.001	1.05	1.03	0.79	0.60	0.48	5.57
4	13,978	0.46	0.46	0.001	1.01	0.96	0.79	0.72	0.61	5.53
5	13,977	0.43	0.43	-0.002	0.96	0.88	0.78	0.83	0.73	5.50
6	13,977	0.39	0.40	-0.006	0.89	0.79	0.78	0.91	0.85	5.44
7	13,977	0.31	0.34	-0.022	0.74	0.60	0.77	0.98	0.95	5.26
Rank by X5										
0	13,977	0.61	0.60	0.013	1.18	1.31	0.80	0.11	0.07	5.60
1	13,977	0.55	0.55	0.002	1.12	1.17	0.80	0.30	0.22	5.58
2	13,977	0.52	0.52	0.001	1.08	1.09	0.79	0.46	0.35	5.56
3	13,977	0.49	0.49	0.001	1.04	1.02	0.79	0.60	0.48	5.55
4	13,978	0.46	0.46	0.001	1.00	0.95	0.79	0.72	0.61	5.53
5	13,977	0.43	0.43	-0.001	0.96	0.88	0.78	0.83	0.73	5.50
6	13,977	0.39	0.40	-0.005	0.89	0.79	0.78	0.91	0.85	5.47
7	13,977	0.32	0.34	-0.024	0.74	0.61	0.76	0.98	0.95	5.38
Rank by DQ										
0	14,115	0.56	0.54	0.015	1.00	0.96	0.92	0.65	0.56	2.53
1	14,026	0.59	0.59	0.003	1.00	0.98	0.89	0.61	0.54	3.54
2	13,911	0.59	0.59	0.002	1.00	0.97	0.88	0.62	0.54	4.14
3	13,859	0.43	0.44	-0.009	1.00	0.98	0.80	0.62	0.54	5.05
4	14,123	0.41	0.43	-0.024	1.01	0.98	0.75	0.61	0.53	5.85
5	13,842	0.38	0.39	-0.015	1.01	0.98	0.71	0.61	0.53	6.64
6	14,118	0.40	0.40	0.004	1.00	0.99	0.66	0.60	0.53	7.51
7	13,823	0.41	0.39	0.016	1.00	0.99	0.66	0.59	0.52	9.12

Dependent variables, calculated at the period midpoints, are:

DQ = quadratic mean of the diameters at breast height (DBH) for trees in the plot.

X1 = CL/[Mean CL] where CL is the crown length for a tree, and mean CL is the mean crown length of trees on the plot.

X2 = DBH/DQ

X3 = CL/H where H is a tree height.

X4 = BAL/BA where BAL = basal area of larger trees (on a per acre basis), and BA is basal area per acre for the plot.

X5 = CLL/CLSUM where CLL = sum of crown length of larger trees and CLSUM is the sum of all crown lengths on the plot.

The use of ΔDQ as the leading term in the model is a short-cut that somewhat disguises the intended usage of the model. This model is used only to allocate growth among the trees in a stand, with the net increase in basal area having been predetermined. Hence a reconciliation step is also required. That step would multiply the predicted d.b.h. growth of each tree by a constant so as to obtain the desired increase in DQ; alternatively, the multiplication could be on the increments in basal area per tree. The reconciliation method is not reflected in the model parameter estimates. In that the ultimate multipliers will be close to 1, the optimal coefficients would be almost the same as those presented here.

Individual Tree Mortality

A model to predict the annual probability that a tree dies is developed here. In application, it predicts the fraction of the trees in a cohort that will die in a given year. The total mortality of all the cohorts is subsequently constrained to match the predicted stand-level mortality.

Data—

The data are the same as described in the "Diameter Growth" section—including the trees that die during the period.

Model—

The 1-year model for the probability of mortality is:

$$L = b_0 + b_1 \times \log(MORTF/(1 - MORTF)) + b_2 \times \exp[b_3 \times (\Delta DBH/\Delta DQ)]$$

$$Pr(\text{Mortality per year}) = \exp(L)/[1 + \exp(L)]$$

where ΔDBH is the predicted d.b.h. growth, ΔDQ is the predicted change in DQ, and MORTF is the stand-level prediction of the fraction of the trees to die in the year. In the fitting process, the probability of mortality in a growth period of *n* years is computed as:

$$Pr(\text{Mortality per } n \text{ years}) = 1 - [1 - Pr(\text{Mortality per year})]^n$$

Fitting is by maximum likelihood.

Parameter	Estimate	Standard error
b_0	-4.5567	0.220
b_1	0.5876	0.014
b_2	13.3877	0.1972
b_3	-2.2781	0.116

Discussion—

As with the diameter growth model, reconciliation with the stand-level prediction of TPA mortality is required. That requires that for each 1-year growth period, the logit values driving the mortality equation be set as:

$L = k + b_2 \times \exp[b_3 \times (\Delta DBH)/(\Delta DQ)]$

where k is whichever value is required to recover the stand-level predicted mortality in TPA, and estimates for b_2 and b_3 are from the above table.

As was conjectured for the diameter growth model, improved estimates of b_2 and b_3 could probably be made by incorporating the reconciliation process into the fitting process. Another deficiency of the fitting process is the use of midpoint conventions. This convention probably introduces some bias but avoids the complication of iteratively applying growth and mortality equations during the fitting process.

Residuals are summarized in table 5. Some variables that had been tested as possible independent variables are included in the residual analysis. These are:

X1 = CL/[Mean CL]
X2 = DBH/DQ
X3 = CL/H
X4 = BAL/BA where BAL = basal area of larger trees
X5 = CLL/CLSUM where CLL = sum of crown length of larger trees

In the residual table, the data are broken out according to the rank classification of these variables. These residuals are generally satisfactory in that the trends of the actual and predicted mortality are similar. Still there are some notable mean errors within categories.

One problem is that the recently pruned trees have an average predicted annual mortality rate of 0.0126 (about 1.25 percent), whereas the observed rate is 0.0077. Attempts to bring the X variables into the logit equation did not resolve the patterns in the residuals. The primary concern is that predicted mortality rates may be too high for the pruned trees. Possibly that could be resolved by explicitly bringing pruning or the shadow crown length into the prediction equation. Because of the reconciliation logic, the pruning bias in the equation will have little consequence for stands where all trees are pruned; predictions of relative survival between pruned and unpruned trees in partially pruned stands are of greater concern. For

Table 5—Residual analysis for tree mortality model, overall and by ranks of independent variables

		Individual tree survival per year				Mean					
	N	Observed	Predicted	Residual	*t*	X1	X2	X3	X4	X5	DQ
											Inches
All	113,439	0.9841	0.9839	0.0002	0.6	1.00	0.97	0.78	0.62	0.54	5.53
Prune now											
0	108,373	0.9838	0.9838	0.0000	0.1	1.00	0.97	0.79	0.62	0.54	5.50
1	5,066	0.9923	0.9864	0.0059	5.0	0.98	1.00	0.52	0.58	0.51	6.23
Rank by X1											
0	14,179	0.9370	0.9416	-0.0046	-2.4	0.69	0.62	0.74	0.95	0.91	5.05
1	14,180	0.9832	0.9803	0.0029	2.7	0.87	0.81	0.77	0.86	0.80	5.43
2	14,180	0.9875	0.9864	0.0011	1.2	0.94	0.89	0.78	0.79	0.70	5.72
3	14,180	0.9890	0.9896	-0.0005	-0.6	1.00	0.96	0.78	0.69	0.59	5.91
4	14,180	0.9917	0.9915	0.0003	0.4	1.04	1.02	0.78	0.57	0.47	5.96
5	14,181	0.9933	0.9926	0.0007	1.1	1.08	1.08	0.79	0.46	0.37	5.76
6	14,179	0.9952	0.9936	0.0017	2.9	1.13	1.15	0.80	0.36	0.28	5.48
7	14,180	0.9951	0.9949	0.0002	0.3	1.25	1.26	0.84	0.26	0.19	4.88
Rank by X2											
0	14,179	0.9352	0.9394	-0.0042	-2.2	0.72	0.58	0.76	0.98	0.95	5.02
1	14,180	0.9834	0.9800	0.0034	3.2	0.89	0.79	0.77	0.92	0.85	5.47
2	14,180	0.9856	0.9868	-0.0012	-1.2	0.96	0.88	0.78	0.83	0.74	5.66
3	14,186	0.9903	0.9901	0.0002	0.2	1.00	0.95	0.78	0.72	0.62	5.78
4	14,174	0.9916	0.9919	-0.0003	-0.3	1.04	1.02	0.79	0.59	0.49	5.80
5	14,180	0.9943	0.9932	0.0011	1.7	1.07	1.08	0.79	0.45	0.35	5.74
6	14,180	0.9961	0.9944	0.0017	3.2	1.12	1.16	0.80	0.30	0.22	5.59
7	14,180	0.9965	0.9954	0.0011	2.3	1.20	1.33	0.81	0.14	0.09	5.14
Rank by X3											
0	14,179	0.9607	0.9483	0.0123	8.9	0.93	0.92	0.47	0.68	0.61	7.36
1	14,180	0.9857	0.9794	0.0063	6.6	1.00	0.98	0.61	0.61	0.53	7.24
2	14,180	0.9880	0.9833	0.0047	5.3	0.99	0.96	0.70	0.63	0.55	6.66
3	14,180	0.9845	0.9860	-0.0015	-1.5	1.00	0.97	0.78	0.62	0.54	5.89
4	14,183	0.9815	0.9885	-0.0070	-6.3	0.99	0.96	0.85	0.63	0.55	5.14
5	14,177	0.9862	0.9909	-0.0047	-4.8	1.01	0.98	0.89	0.61	0.53	4.48
6	14,180	0.9878	0.9925	-0.0048	-5.2	1.03	0.99	0.93	0.60	0.52	4.18
7	14,180	0.9945	0.9936	0.0009	1.4	1.06	1.02	0.96	0.57	0.48	3.71

Table 5—Residual analysis for tree mortality model, overall and by ranks of independent variables (continued)

		Individual tree survival per year				Mean					
	N	Observed	Predicted	Residual	*t*	X1	X2	X3	X4	X5	DQ
											Inches
Rank by X4											
0	14,179	0.9970	0.9955	0.0015	3.2	1.18	1.31	0.80	0.11	0.08	5.66
1	14,180	0.9961	0.9944	0.0016	3.1	1.12	1.17	0.80	0.30	0.22	5.62
2	14,180	0.9936	0.9933	0.0003	0.5	1.08	1.09	0.79	0.46	0.36	5.60
3	14,180	0.9910	0.9918	-0.0008	-1.0	1.04	1.02	0.79	0.60	0.49	5.56
4	14,180	0.9904	0.9896	0.0008	1.0	1.01	0.95	0.79	0.73	0.62	5.53
5	14,180	0.9857	0.9860	-0.0003	-0.3	0.96	0.88	0.78	0.83	0.74	5.49
6	14,180	0.9824	0.9788	0.0036	3.3	0.89	0.78	0.78	0.92	0.85	5.44
7	14,180	0.9363	0.9413	-0.0050	-2.6	0.73	0.59	0.76	0.98	0.96	5.30
Rank by X5											
0	14,179	0.9971	0.9955	0.0016	3.5	1.18	1.31	0.80	0.12	0.07	5.60
1	14,180	0.9955	0.9944	0.0011	1.9	1.12	1.17	0.80	0.30	0.22	5.58
2	14,180	0.9939	0.9932	0.0007	1.1	1.08	1.09	0.79	0.46	0.36	5.56
3	14,180	0.9919	0.9917	0.0002	0.3	1.04	1.02	0.79	0.60	0.49	5.55
4	14,180	0.9895	0.9894	0.0000	0.0	1.00	0.95	0.79	0.72	0.62	5.53
5	14,180	0.9861	0.9858	0.0002	0.2	0.96	0.88	0.78	0.83	0.74	5.50
6	14,180	0.9824	0.9788	0.0036	3.3	0.89	0.78	0.78	0.92	0.85	5.48
7	14,180	0.9365	0.9421	-0.0056	-3.0	0.73	0.60	0.75	0.98	0.96	5.42
Rank by DQ											
1	14,231	0.9918	0.9886	0.0032	4.2	1.00	0.96	0.92	0.65	0.56	2.53
2	14,169	0.9890	0.9901	-0.0011	-1.3	1.00	0.98	0.89	0.62	0.54	3.54
3	14,149	0.9847	0.9891	-0.0045	-4.4	1.00	0.97	0.88	0.62	0.54	4.14
4	14,207	0.9889	0.9853	0.0035	4.1	1.00	0.97	0.80	0.62	0.54	5.06
5	14,209	0.9742	0.9800	-0.0057	-4.6	1.00	0.97	0.75	0.62	0.54	5.85
6	14,138	0.9758	0.9734	0.0024	2.1	1.00	0.98	0.70	0.61	0.54	6.64
7	14,201	0.9782	0.9772	0.0010	0.9	1.00	0.98	0.66	0.60	0.53	7.50

Dependent variables, calculated at the period midpoints, are:

DQ = quadratic mean of the diameters at breast height (DBH) for trees in the plot.

X1 = CL/[Mean CL] where CL is the crown length for a tree, and mean CL is the mean crown length of trees on the plot.

X2 = DBH/DQ

X3 = CL/H where H is a tree height.

X4 = BAL/BA where BAL = basal area of larger trees (on a per acre basis), and BA is basal area per acre for the plot.

X5 = CLL/CLSUM where CLL = sum of crown length of larger trees and CLSUM is the sum of all crown lengths on the plot.

the simulation of stands that contained pruned and unpruned trees, results should be viewed with some skepticism if the predicted mortality of the pruned trees is high enough to affect management decisions; for many stand conditions this will not be the case.

Tree Height Growth

A height growth prediction equation is developed here. Subsequent to the predictions, the tree growth must be reconciled with predicted top height increment.

Data—

Variables for these 111,175 observations (tree growth) for 998 growth periods on 308 plots at 27 installations:

Variable	Mean	Minimum	Maximum	Standard deviation
DBH	5.41	0.02	16.25	2.39
H	34.1	2.35	81.9	13.6
HBC	9.2	0.1	47.6	8.9
ΔHTOP	3.38	0.15	7.39	0.64
ΔH*	3.11	-11.4	16.2	0.95
Period length	3.14	1	4	1.08

* Extreme values are probably due to uncorrected measurement errors.

The data are the same as were used for diameter growth, less a few excluded cases. Two periods with negative growth in top height were omitted, as were 10 trees with midpoint diameter of zero. Midpoint conventions are used for all variables. The dependent variable is height growth per year on an individual tree. All eligible trees were used in the fitting process without regard to whether the heights were measured or imputed.

Model—

The prediction model for trees with H > 8 ft:

$$\Delta H = (\Delta HTOP) \times a_0 \times \{1 - \exp[a_1 \times (DBH/DTOP)^{a_2 + a_3 \times DQ}]\}$$

and for short trees (H ≤ 8 ft):

$$\Delta H = (H/HTOP) \times (\Delta HTOP)$$

with overall fit statistics of:

r-square = 0.33

MSE = 1.57

Fitting is by nonlinear least squares with observation weights set to years in the period.

Parameter	Estimate	Standard error
a_0	1.0006	0.0019
a_1	-4.7434	0.088
a_2	0.4058	0.019
a_3	0.2126	0.0037

Discussion—

This model is simple and stable. However, it does not take pruning into account. Furthermore, the residual analysis indicates some problems. The residuals have a correlation of -0.17 with the model predictions. This might suggest that minor changes in model form could lead to an improved model. In this case, the residual correlation is most likely due to the expected correlation between ΔH and ΔHTOP. Hence the negative correlation serves as a reminder that the statistical assumptions in separately fitting each equation in a system of related equations are not being met, and that the resultant error statistics for the parameter estimates are suspect. The residual analyses do not suggest that the truly independent variables could somehow be used to make substantially better predictions.

The special equation for very short trees (H ≤ 8 ft) takes effect for only a handful of trees in the data, and will seldom, if ever, come into effect in simulations. It is prudent to have some sort of alternative model for trees with d.b.h. near zero, as the base model would predict negligible height growth.

A suitable reconciliation process is to revise a_0 to be whatever value is required to achieve the top height increment specified by the site curves. This is a non-iterative process; it mitigates the problem of the residual correlation with ΔHTOP.

Integration of Equations

Presilviculture Stand Description

Empirical equations have been shown for DQ, TPA, CV, D10, D0, and a height-diameter model. To develop a stand description, a Weibull distribution recovery process (estimation of parameters) and a stand table generator are required.

The Weibull distribution recovery process is viewed here as an exact mathematical solution to an intractable problem. Mathematically, the problem is formulated as a root-finding problem with three equations (CV, DQ, D10), and three unknowns (the three parameters of the Weibull distribution). If the lower bound on

D10 (see app.) is not imposed, the recovered Weibull parameter, a, will often be less than zero. This is unacceptable; two approximately equivalent solutions are to replace the D10 objective with a constraint that the Weibull location parameter must equal zero, or to set D10 equal to its lower bound. In the data, roughly half the plots have (CV, DQ, D10) combinations that violate the lower bound for D10 of a Weibull distribution.

After a Weibull distribution is parameterized, it must be converted to a tree list. The possible mechanisms for deciding how to divide a continuous distribution into a finite number of class sets are numerous, the easiest being to use predefined fixed-width diameter classes. The empirical equation for D0 may be useful in making rules for aggregating the smallest diameter classes. Each class can have its basal area and trees per acre calculated in accord with the Weibull distribution; in general the mid-point of the class should not be used as the nominal class diameter because that will preclude the exact tree-list recovery of basal area.

Crown Base Updates

The crown recession model operates at the tree level. Each year the crown base is set to the higher of the previous year's crown base or the current prediction of crown base from the tree-level crown recession equation.

Tree and Stand Reconciliation

The proposed tree and stand reconciliation processes are fairly simple, although d.b.h. growth requires an iterative process. The reconciliation methods have been separately described and are to be applied in this order: d.b.h. growth, individual tree mortality, and tree height growth.

Identification of Trees for Thinning and Pruning

Identification of trees for pruning and thinning is more of a programming issue than a modeling issue. Complete flexibility may be offered to the user, as is the case with ORGANON, or no flexibility as is the case with the NZ model, which instead provides an empirical solution. An intermediate solution would have the user supply the number of trees per acre to thin or prune and the d/D ratio: the ratio of the DQ of the selected trees divided by the DQ of all the trees. User inputs such as these are sufficient to parameterize a selection model that can differentially favor large or small trees.

Validation

A growth simulator has been constructed from the empirical growth models described here. The simulator was used to predict growth on all of the 998 growth periods where the starting top height was at least 12 ft. Growth per year is calculated for several attributes and is compared with the observed growth rates. Results are in table 6; period length is used as a weight in calculating means and variances.

Discussion

Model components that rely heavily upon the formulation of the NZ model include net basal area growth, the empirical portion of the tree mortality model, and the stand-level crown recession model. The most important of these is the basal area growth model. That model component follows the general formulation of the NZ model with two exceptions. First, the NZ model's independent productivity variable, SBAP, is replaced with site index. The reason for that was to simplify the use of the model, as well as the fitting process. The second major difference is the insertion of extra terms allowing for a somewhat greater interaction between basal area effects and crown length effects. Still, the crown length effect in the current model is similar to that in the NZ model and will necessarily predict pruning effects on growth similar to what would be predicted by the New Zealand model. Likewise the general form of the NZ crown recession model has been used, although the choice of independent variables has expanded.

The diameter distribution model provides an easy startup mechanism for simulations. It is based on the Weibull distribution for diameters; this is the most commonly used distribution for diameters in planted stands. The need to impose tight constraints on the location parameter is indicative of some lack of fit of the Weibull model. A Johnson's S_B distribution, as discussed by Rennolls and Wang (2005) would possibly have been a better choice for the distribution function.

The validation code is derived from the equations and coefficients in this report. The facts that the r-squares are generally reasonable and that the mean errors in growth are generally small offer some support to the hypothesis that the equations function together in a reasonable manner. These results also offer a minimal assurance that the equations and coefficients in the report were recorded properly or at least without huge mistakes. The growth variables with the lowest r-squares are ΔHTOP and plot means for ΔH; the poor performance here is attributable to two causes. First, the site index values used in the validation are installation averages; hence within-installation variances in top height growth owing to site

Table 6—Validation results of growth simulation

Variable	Mean			Variance of	t value	r-square
	Observed	Predicted	Residual	residual		
ΔBA	8.338	8.328	0.010	5.70	0.2	0.822
ΔTPA	-1.345	-1.374	.029	12.80	0.4	0.470
ΔHTOP	3.278	3.349	-.071	0.99	-3.8	0.179
ΔH (mean)	3.123	3.151	-.029	0.76	-1.8	0.160
ΔHBC (mean)	1.148	1.175	-.027	1.95	-1.1	0.539
ΔCL (mean)	1.975	1.976	-.001	2.26	-0.1	0.503
ΔCV	-1.326	-0.998	-.327	6.41	-7.0	0.633
ΔCV of d.b.h.	-0.788	-0.598	-.190	2.22	-6.9	0.680
ΔD10	0.415	0.398	.017	0.0394	4.6	0.681
ΔD90	0.629	0.629	.000	0.0407	0.1	0.671

ΔBA = change per year in basal area (ft^2/acre).

ΔTPA = change per year in trees per acre.

ΔHTOP = change per year in top height (ft).

ΔH (mean) = change per year in mean height (ft).

ΔHBC (mean) = change per year in mean height to base of crown (ft).

ΔCL (mean) = change per year in mean crown length (ft).

ΔCV = change per year in the coefficient of variation (as a percentage) for basal area per tree.

ΔCV of d.b.h. = change per in the coefficient of variation (as a percentage) for tree diameter.

ΔD10 = change per year in the tenth percentile of the d.b.h. distribution (in).

ΔD90 = change per year in the ninetieth percentile of the d.b.h. distribution (in).

productivity differences are unexplained. Second, the ages in the data tend to center on the ages of maximum height growth, and to exclude the early ages of accelerating growth and later ages of declining height growth. Under these conditions, the proportion of the variance that can be explained by a site curve is necessarily lower than would be the case over a full range of ages. The most notable problem with mean increments is that the CVs are predicted to decline too slowly; the problem appears to be in the lower diameter classes, although the validation shows that the bias for ΔD10 is not great. An examination of the source of bias in ΔCV indicates that it is due to large errors in a small number of plots, and is confined mainly to plots with low DQ; this particular problem is likely to become less noticeable in long projections. The results for all other variables are reasonably good.

The presilviculture yield model meets a simulation need that is presently unmet by any other model. Although the investigation of the Weibull recovery process leads to some doubts as to whether the Weibull is the best distribution model for young Douglas-fir plantations, the recovered distributions are plausible. The growth portion of the model makes growth predictions that can be compared with three other models: ORGANON Version 8, TreeLab, and the PNW calibration of the

New Zealand Douglas-Fir National Model. Although comparisons of the three models are not made here, forest managers may find it instructive to compare the various model predictions to learn whether critical management decisions would be altered depending on the model selection. As the architecture of the various models is substantially different from one another, areas of agreement in predictions can be taken as a likely indication of adequate data support in those areas. Areas of disagreement can be viewed as opportunities for a more critical look at predictions and residuals. The growth model should not be used for predictions beyond a top height of 75 ft unless the purpose of doing so is simply to explore and compare model forms.

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Metric Equivalents

When you know:	Multiply by:	To find:
Inches(in)	2.54	Centimeters (cm)
Feet (ft)	0.3048	Meters (m)
Square feet (ft^2)	0.0929	Square meters (m^2)
Square feet per acre (ft^2/ac)	0.229	Square meters per hectare (m^2/ha)

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Appendix: Bounds for Diametric Parameters in Weibull Distributions

Overview

Diameter distributions for plantations can be modeled with the three-parameter Weibull distribution. For both yield and growth models, parameter recovery is generally preferable to direct predictions of distribution parameters. Code is available for recovery based on quadratic mean diameter at breast height (d.b.h.) (DQ), coefficient of variation in basal area per tree expressed as a percentage (CV), and the tenth percentage of the d.b.h. distribution (D10). The prediction of the CV and D10 statistics can be guided by suitable joint limits. This appendix defines those limits; statistics within the delimited space are such that an exact recovery of parameters is possible. This solves what is possibly the most significant problem with the parameter recovery technique for Weibull diameter distributions. The lower limit for D10 is used in fitting empirical equations for D10 and D0 (zero$^{\text{th}}$ percentile of d.b.h.).

Details

The cumulative three-parameter Weibull distribution is:
$F(x) = 1 - \exp\{-[(x - a)/b]^c\}$ for $x \geq a$

Recovery code is available that will find the Weibull parameters (a, b, and c) that exactly recover the statistic set DQ, CV, D10. The code will also indicate whether a proposed statistic set cannot be recovered with the Weibull distribution. The code was used to explore the space of RD10 (D10/DQ) and CV, finding solutions and identifying the recoverable domain. The limits of that domain were recovered (fig. 2). The use of RD10 changed the problem from three dimensions to two.

The bound of the domain has two curves: an upper curve and a lower curve. The upper curve is where c = 1.0, which is an extremely left-skewed distribution, having the mode of the probability density function (pdf) close to the location parameter (a). The lower curve represents the extreme limit of the recoverable distribution. Here we require that a ≥ 0; this is, in fact, the limiting condition identified by most of the domain for the lower curve; higher *a* values occur at the lowest values of CV.

For each CV, a range of RD10 values were tested. The recovered values were then examined. The lowest acceptable RD10 value was used as a fit point for the

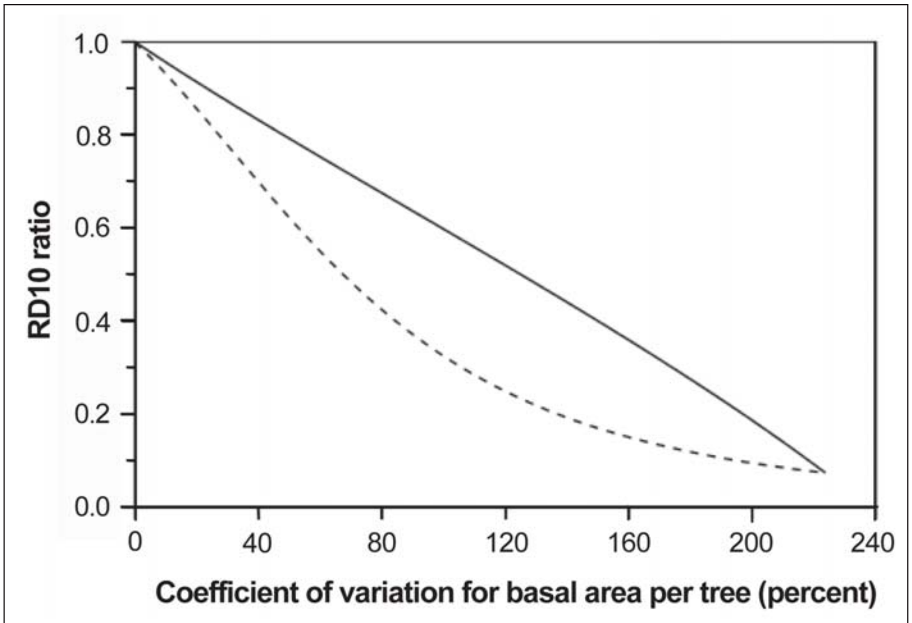

Figure 2—Upper and lower bounds for RD10 (D10/DQ) versus CV where RD10 = D10/DQ, D10 is the 10th percentile of the diameter distribution, DQ is the quadratic mean diameter at breast height, and CV is the coefficient of variation for basal area per tree.

lower curve. The interpolated RD10 value (where c = 1.0) was used for the upper curve. After identifying these points, equations for the curves had to be fit. We could not find any simple curve forms that would fit the entire CV domain. Therefore, we used segmented functions with preselected tie points, and a common equation form for all the segments.

Each segment is defined between a left and right value of CV; at the end of each segment, nodal values for RD10 are used as constraints. Within each segment, the equation form is:

$$RD10 = RD10_{right} + [-1 + a_0\, x + (1 - a_0)\, x^{a_1}] \times (RD10_{right} - RD10_{left})$$

where

$$x = (CV - CV_{left})/(CV_{right} - CV_{left}).$$

The nodal values and coefficients (a_0, a_1) are given for the upper curve and lower curves in tables 7 and 8 respectively.

Table 7—Nodal points and coefficients for upper boundary line ($RD10_{right}$) in figure 2

Segment	CV_{left}	CV_{right}	$RD10_{left}$	$RD10_{right}$	a_0	a_1
1	0	4.514	1.00	0.9800	1.00000	1.00000
2	4.514	23.323	0.98	0.9000	1.03368	1.82550
3	23.323	48.086	0.90	0.8000	1.02716	1.74763
4	48.086	73.632	0.80	0.7000	1.01894	1.42557
5	73.632	125.056	0.70	0.5000	0.99582	4.00000
6	125.056	174.197	0.50	0.3000	0.96969	2.40488
7	174.197	197.086	0.30	0.2000	0.96976	2.02603
8	197.086	218.440	0.20	0.1000	0.96210	2.03725
9	218.440	223.609	0.10	0.0745	0.98496	1.62135

CV = Coefficient of variation for basal area/tree.

RD10 = D10/DQ, where D10 = 10^{th} percentile of d.b.h. and DQ = quadratic mean diameter.

Table 8—Nodal points and coefficients for the lower boundary line ($RD10_{left}$) in figure 2

Segment	CV_{left}	CV_{right}	$RD10_{left}$	$RD10_{right}$	a_0	a_1
1	0	3.020	1.00	0.9800	1.40000	2.00000
2	3.020	14.210	0.98	0.9000	0.94207	1.82073
3	14.210	27.167	0.90	0.8000	0.94681	1.40000
4	27.167	39.917	0.80	0.7000	1.00965	4.00000
5	39.917	67.480	0.70	0.5000	1.08954	2.24868
6	67.480	106.000	0.50	0.3000	1.24833	1.93342
7	106.000	137.125	0.30	0.2000	1.24235	1.89434
8	137.125	195.667	0.20	0.1000	1.57522	1.61327
9	195.667	223.609	0.10	0.0745	1.20884	1.81821

CV = Coefficient of variation for basal area/tree.

RD10 = D10/DQ, where D10 = 10^{th} percentile of d.b.h. and DQ = quadratic mean diameter.

Within the specified bounds, recoveries should always be possible. However, the present software may have some problems in the extreme conditions. The CV domain where computational problems are least likely is $4 \leq CV \leq 220$. CVs beyond that range should not be occurring in plantations; at the low end, this would imply almost no variation in tree size; at the high end, it would require $RD10 < 0.08$.

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