Who Hits Harder: The Nordic Skier or Aerial Jumper?

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Abstract

Computer modeling is used to estimate physical quantities that are difficult to measure, in this case, the landing shock experienced by ski jumpers. The landing shock is difficult to measure because attaching instruments to the skiers would affect their balance and possibly cause them to fall or reduce their performance. Using basic principals of physics, the flight trajectories of two forms of competitive ski jumping, Freestyle Aerialist and Nordic Jumpers, were modeled and the landing shocks compared. The model uses physical quantities such as the takeoff inclination, takeoff height, the shape of the hill, and the vertical drop to the landing. The landing shock is calculated from the velocity of the skier and angle which he or she hits the hill. For comparison to other sports, the landing shock is converted to a vertical drop to a flat surface.

The model shows that Freestyle Aerialist have landing shocks equivalent to a vertical drop of 20 feet to a flat surface. This is over twice the landing shock of the Nordic Jumpers even though the Nordic Jumpers travel at twice the velocity and fall twice as far.

Introduction

In the winter Olympics, two types of skiers, Nordic Jumpers and Freestyle Aerialists, are known for their dazzling displays of aerial maneuvers that most spectators find incredible and possibly dangerous. The Aerial Jumpers in the Freestyle Skiing competition perform very high leaps with twists and somersaults. The 1998 Olympic winner, Eric Bergous (Figure 1), executed three complete rotations with four twists. This required extensive time in the air, which he would later reconcile with when gravity pulled him back to earth. At the Nordic Ski Jumps, Masahiko Harada and Takanobu Okabe both flew 137 meters (449 ft) in distance.

Competition drives the skiers to fly higher and farther. The athletes risk falling upon impact to win the competition. Fortunately, the Federation of International Skiing[1] (FIS) has imposed rules on the designs of ski jumps to insure safety and minimize injuries. The FIS specifications for Freestyle Aerial jumps is shown in Figure 2.

Computer Model of Skier's Flight

Nordic ski jumping has been modeled by Muller et al. (1995 and 1996). They found that most jumpers have a very small impact on the landing hill equivalent to a vertical drop of < 1 meter, even though they fly distances over 100 meters (328 ft.) with vertical drops over 30 m (98 ft.). The Nordic skiers experience light landings because they are flying close to the hill which is shaped for their flight trajectory (Figure 1).

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[1] Federation Internationale de Ski, Blochenstrasse 2, CH-3653 Oberhofen/Thunersee, Switzerland and web site http://www.zip.com.au/~birdman/fis.html.

The Freestyle Aerial Jumpers use a much higher trajectory to perform rotations and twists. Judges award points for the heights of the jumps along with the difficulty of the rotations and twists. Distance is not a factor in Aerial Jumping as it is in Nordic Jumping.
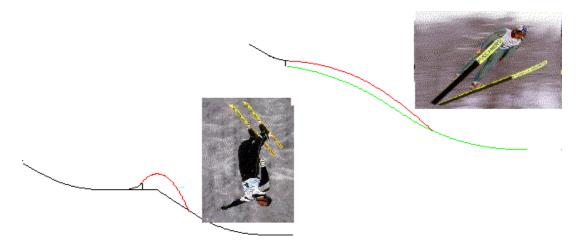

Figure 1: Freestyle Aerial jumper Eric Bergous (left) and Nordic jumper Masahiko Harada (right).

To answer the question, "How hard do the Freestyle Aerialists land?" a smaller version of the Muller model was used. The impact of a skier on the landing hill is not simply the vertical distance he/she drops because they land on a steep slope (~37°). The skier also does not stop on the hill. The impact the skier feels is the force it takes to change the skier's direction from their flight to the slope of the hill. This is illustrated in Figure 3. The skier's direction of motion is shown by the vector A-B. The hill has a slope from A-C. The skier's velocity (V_{A-B}) can be expressed as two orthogonal (right angle) components. The component along the hill is V_{A-C} whereas the other component is V_{C-B}. Upon impact, the skier's velocity will change from V_{A-B} to V_{A-C}. The change in direction is angle (a) and the velocity component V_{C-B} will be absorbed in the landing impact. The impact the skier feels is the dissipation of V_{C-B}.
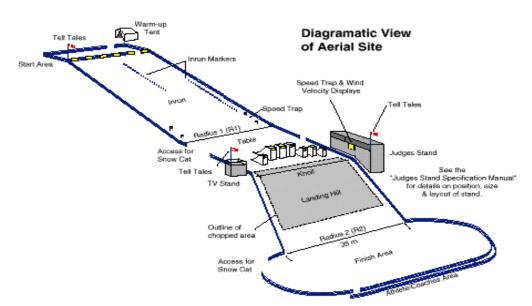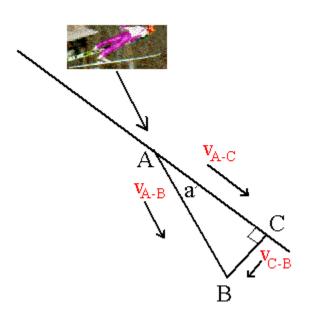

Figure 2: Federation of International Skiing specifications for Freestyle Aerial hills. Full details are available at http://www.zip.com.au/~birdman/FTP.html under "1997/98 Freestyle Course Standards".

Using the triangle in Figure 3, the skier's velocity component perpendicular to the face of the hill, V_{C-B}, can be calculated from the skier's velocity, V_{A-B}, and the change in direction (a).

$$V_{C-B} = V_{A-B}.SINE(a) \tag{1}$$

Figure 3: The skier lands at A traveling toward B, but will abruptly change direction toward C.

The flight trajectory model calculates the skier's velocity and position for a series of small time steps (Figure 4). The model starts with a speed and direction (velocity) at the point of takeoff and then calculates how far the skier moves in each time increment. For this model, time increments of 0.05 seconds were used which is about 0.5 m in displacement. At the end of each increment, a new velocity has to be calculated because forces have acted on the skier. The process proceeds to a second step where a new position is calculated using the average velocity of the time increment followed by a re-calculation of the velocity. This process is successively repeated many times (60-100) until the skier arrives on the landing hill.
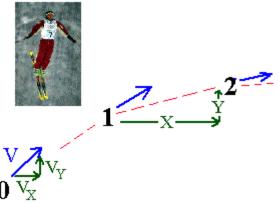

Figure 4: The large numbers are time steps of 0.05 seconds. The non-orthogonal blue arrows show the velocities at each point in time. Orthogonal green arrows indicate velocity and distance component vectors.

To simplify the calculations, the skier's displacement vector is decomposed into horizontal and vertical components (Figure 4). The horizontal component X increases to the right and the vertical component Y increases upward. The skier's position at the end of each time step is:

$$X_2 = X_1 + 0.05 \times V_x \tag{2}$$
$$Y_2 = Y_1 + 0.05 \times (V_{y1}+V_{y2}) / 2 \tag{3}$$

Subscripts 1 and 2 refer to the start and end of one time increment.

The velocities are changed by the accelerations of the forces on the skier. Since the speed of the Aerial Jumper is relatively slow, aerodynamic lift and drag are not considered. However, Muller had to consider aerodynamic forces for the Nordic jumper because of the greater velocities (23-27 m/s or 51-57 mph). For the Aerial jumper, we assume the horizontal component (V_X) is constant in time and gravity is the only acceleration on the vertical component (V_Y). Therefore, the formulas used to calculate velocity components for the Aerial Jumper are:

$$V_{x2} = V_{x1} \tag{4}$$
$$V_{y2} = V_{y1} - G \times 0.05 \tag{5}$$

G is 9.8 m/s^2 (32.3 ft/s^2).

The model starts with velocities at the takeoff of V_{x0} and V_{y0}, and then calculates V_{x1} and V_{y1} using equations (4) and (5). The position of point one, X_1 and Y_1, are calculated from equations (2) and (3). Then V_{x2} and V_{y2} are calculated followed by X_2 and Y_2, etc.

For the Aerial jumper we know the location of the takeoff point and the inclination angle, but not the velocity. Aerial jumpers can choose their velocity by selecting any starting position on the take off ramp. They vary their velocity for the air time needed for their maneuvers. The jumper's velocity can be estimated by assuming that the jumper intends to land in the lower part of the landing slope. This will give the maximum air time and also allow for the jumper to recover their balance on the steep slope before skiing through the curve at the bottom of the hill and stopping in the out run area.

A backward iterative method is used to find the takeoff velocity from the landing point. It begins with a guess of the takeoff velocity and running the model to find a landing point. If the skier lands too short with this velocity, the velocity is increased and the model re-run. Conversely, if the skier lands too long (or close to the bottom curve), the starting velocity is decreased and the model is rerun. The takeoff velocity can be found with a few iterations (<10).

The largest takeoff kicker in Figure 2 has an inclination of 55° and is positioned 8.1 meters back from the start of the landing slope, which has a constant pitch of 37° and is 25 meters long. From successive iterations we found that, at a takeoff velocity of 12.5 m/s (28 mph), would send the skier 26 meters to within 10 meters of the bottom of the slope. For this flight the skier had an air time of 3.0 seconds and rose 5.5 meters (18 ft) above the take-off. He/she dropped 18.7 meters (64 ft) from the apex height to the landing. The skier was traveling 20.5 m/s (45 mph) at an declination of 69° when hitting the hill. From Figure 3, the impact velocity, V_{C-B}, is 11 m/s.

Muller converted the vector V_{C-B} to an equivalent drop from a fictitious height to a flat surface so that comparisons could be made to other sports. The conversion uses the two basic equations for acceleration and distance in a gravitational field, $V = G \times T$ and $H = 0.5 \times G \times T^2$ resulting in:

$$H = 0.5 \times V_{C-B}^2 / G \tag{6}$$

Using equation (6) the Aerial jumper's impact velocity, V_{C-B}, is equivalent to a vertical drop of 6.1 meters (20 ft). A comparison between the landing impacts of Aerial and Nordic jumpers is given in Table 1. Nordic jumper flights are calculated from a replica of the Muller model, which includes lift and drag terms in equations 4 and 5 (see Muller et al., 1996 for details). Two landings are shown for each type of skier, a short jump and a long competitive jump. The Aerial jumps differ only in takeoff velocity whereas the Nordic Jumps differ in the leg spring of the skiers and the aerodynamic forces in flight. The first jump represents a junior skier weighting 80# with a vertical leg spring of 9 inches, and aerodynamic lift of 40# and drag of 48#. The second is an Olympic competitor weighting 130# with a vertical spring of 18 inches and aerodynamic lift of 78# and drag of 65#.

Table 1. Jump Parameters

Take off Angle Deg	Velocity m/s (mi/hr)	Jump Distance meters (feet)	Total vertical fall distance meters (feet)	Landing V_{A-B} m/s	V_{C-B} m/s	Equivalent drop to flat surface meters (feet)	Flight Time seconds
Aerial hill							
55°	9.7 (21.3)	15 (49)	10 (33)	15.1	7.9	3.2 (10)	2.2
55°	12.5 (27.5)	26 (85)	19 (62)	20.5	11.0	6.1 (20)	3.0
Nordic hill							
-10°	23.1 (50.8)	62 (203)	28 (92)	17.3	3.3	0.6 (2)	3.5
-10°	23.1 (50.8)	109 (358)	54 (177)	21.7	6.9	2.4 (8)	5.9

Discussion

The Aerial jumper model shows landing impacts of 3.2-6.1 m (10-20 ft) in vertical drop whereas Nordic jumpers experience much smaller impacts of 0.6-2.4 m. A 20-foot fall could cause serious injury. It's about the height of the roof on a two-story house. However, Freestyle Aerial jumping has an exceptional record in that they have never had a debilitating injury in 22 years[2].

The reason for Aerial jumping's safety record is in the preparation of the landing hills and the training and guidance by the coaches. Aerial landing hills have to be soft with two or more feet of soft snow that is never packed by skiers or grooming machines. Before competitions, hill crews loosen the snow with shovels and during competitions, they add extra snow between jumps. The competitors use different launching kickers based on the skill of the skiers. Also, Freestyle Aerialists start their training using water pools in the summer where air is bubbled into the pools to soften the landings.

Future Applications of the Model

The model described here can be applied to other sports for the same purpose. Two possible applications are water ski jumping and motorcycle stunts. To calculate the landing impacts in these sports, you need to find the inclination angle of the take-off ramp (~5°-10°), the velocity, the distance traveled, and the inclination of the landing ramp. Aerodynamic drag also should be included in Equations 4 and 5. With a model, you could try different ramp inclinations and velocities to estimate the best jump.

The computer code for this example is written in Fortran and available from the author. The author is a scientist by profession (meteorology) with a son who competes in Nordic Jumping. He also is a volunteer member of the hill crews at the Intervale Olympic Ski Jumps in Lake Placid, NY and the Blackhawk Ski Club in Madison, WI, and a member of the National Ski Patrol.

References
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_____, 1996: Dynamics of human flight on skis: Improvements on safety and fairness in ski jumping, *J. Biomechanics*, **29**, 1061-1068.

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[2] Peter Judge, Chairman, FIS Freestyle Subcommittee for Rules and Technical, e-mail PRJudge@compuserve.com