Dynamo action and the nearly axisymmetric magnetic field of Saturn

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Abstract. We examine kinematic dynamo action in an electrically conducting spherical body of fluid with an overlying shell of differential rotation. It is thought that such flow types could explain the nearly axisymmetric magnetic field of Saturn. Although we find that an outer layer of differential rotation can 'axisymmetrize' the exterior magnetic field, surprisingly, sometimes it does not, giving magnetic fields that have no axisymmetric ingredients.

Introduction

Dynamo action in heavenly bodies is sustained by the flow of electrically conducting fluid. In a kinematic dynamo analysis, such as that discussed here, one investigates the types of fluid flows that sustain dynamo action by solving the magnetic induction equation,

$$\partial_t \mathbf{B} = R_m \nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla^2 \mathbf{B}, \qquad (1)$$

where $\mathbf{B}$ is the magnetic field and $\mathbf{u}$ is a prescribed dimensionless fluid velocity. The magnetic Reynolds number, R_m, is a measure of the relative influence on the magnetic field of advection and diffusion.

Fluid flow cannot generate axisymmetric magnetic fields [*Cowling*, 1934; *Hide and Palmer*, 1982; *Ivers and James*, 1984]. However, it is possible that dynamo fields can be nearly axisymmetric, an issue of relevance to Saturn. Spacecraft measurements have shown the Saturnian field to be remarkably axisymmetric [*Acuña and Ness*, 1980; *Smith et al.*, 1980]. Although it is not precisely axisymmetric [*Desch and Kaiser*, 1981], the Saturnian magnetic field is modelled well by zonal harmonics [*Connerney, Ness and Acuña*, 1982], a configuration that may have persisted for millions of years [*Northrop and Connerney*, 1987].

Dynamo action in Saturn probably arises from convection in a fluid interior made of metallic hydrogen and helium. *Stevenson* [1980] argues that, as a result of the planet's thermal evolution, overlying this convective region is a fluid layer which is depleted in helium and therefore stably stratified. In this layer *Stevenson* [1982] suggests that the nonaxisymmetric ingredients of the magnetic field, sustained by the interior dynamo, are sheared by differential rotation, thereby promoting ohmic dissipation of the surficial nonaxisymmetric magnetic field and giving an external magnetic field that is nearly axisymmetric. Alternatively, *Rheinhardt* [1997] has suggested that asymmetric field ingredients in the Saturnian field are of short lengthscale, and are therefore geometrically attenuated at spacecraft distances from the planet. Here we examine the viability of Stevenson's 'axisymmetrization' mechanism by calculating the kinematic dynamo action of convective flows with overlying differential rotation.

Method

We adopt spherical coordinates (r, θ, ϕ) and consider a system consisting of an electrically conducting incompressible fluid sphere of unit radius ($r = 1$) surrounded by an electrical insulator. Equation (??) is solved by the *Bullard and Gellman* [1954] method: discretizing the equation by expanding the velocity field and magnetic field in terms of toroidal and poloidal vector harmonics and radial grid points. For a prescribed velocity field, $\mathbf{u}$, the induction equation is then reduced to an algebraic eigenvalue problem. Standard numerical techniques are used to solve for the magnetic field, $\mathbf{B}$, and the critical magnetic Reynolds number, R_m^c, at which dynamo action occurs and above which the solutions grow exponentially. R_m^c is defined so that $\langle \mathbf{u} \rangle = 1$, where $\langle \cdots \rangle$ denotes a volumetric rms average.

Our aim is to investigate the kinematic dynamo action of flows similar to those considered by *Stevenson* [1982]. Specifically, we search for numerically convergent magnetic fields sustained by flows of the form

$$\mathbf{u} = \mathbf{u}_\bullet + \mathbf{u}_\circ, \qquad (2)$$

where the flow ingredients are shown in Figure 1. $\mathbf{u}_\bullet$ is an internal flow, being in sphere, $0 < r < r_\bullet$, and consisting of a mixture of toroidal and poloidal ingredients, those used originally by *Kumar and Roberts* [1975] and which have been demonstrated to sustain a magnetic field. $\mathbf{u}_\circ$ is in an overlying shell, $r_\bullet < r < 1$, and consists of toroidal motion; we consider two types: one of radial shearing (case I) and one of solid body rotation (case II). By adjusting the relative sizes of $\mathbf{u}_\bullet$ and $\mathbf{u}_\circ$ we can investigate the efficacy and efficiency of Stevenson's axisymmetrization mechanism.

Previous related studies have considered flows like $\mathbf{u}_\bullet$, but for the whole sphere ($r_\bullet = 1$), with various mixtures of toroidal and poloidal ingredients [*Gubbins et al.*, 2000; *Love and Gubbins*, 1996] and with flows

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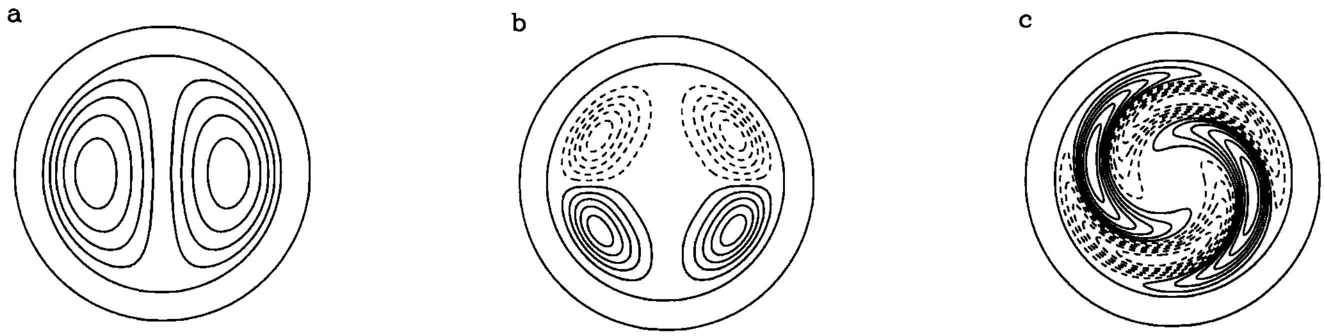

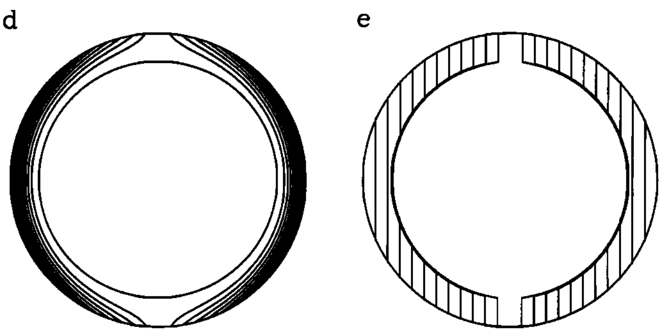

Figure 1. Fluid velocity components. The interior flow ($0 < r < r_\bullet$) consists of a mixture of toroidal and poloidal ingredients: $\mathbf{u}_\bullet = \mathbf{t}_1^0 + \epsilon_2^0 \mathbf{s}_2^0 + \epsilon_2^2 \left(\mathbf{s}_2^{2s} + \mathbf{s}_2^{2c}\right)$. (a) Contours of ϕ-component of the interior toroidal flow, $\mathbf{t}_1^0$, in meridional cross section. (b) Streamlines of the meridional circulation, $\mathbf{s}_2^0$, in meridional cross section. (c) Streamlines of convective motion, $\mathbf{s}_2^{2s} + \mathbf{s}_2^{2c}$, in equatorial cross section. The outer toroidal flow ($r_\bullet < r < 1$) is $\mathbf{u}_\circ = \epsilon_\circ \mathbf{t}_\circ$, where $\mathbf{t}_\circ = \nabla \times \nabla \times (t_\circ \cos\theta \hat{\mathbf{r}})$. We investigate two different outer flows similar to those considered by *Stevenson* [1982]: (d) ϕ-component of case I, where $t_\circ = (r - r_\bullet)^3$, which has radial shearing. (e) ϕ-component of case II, where $t_\circ = (r - r_\bullet)^2$, which is solid body rotation. For this discussion the parameters $(\epsilon_2^0, \epsilon_2^2)$ are fixed at $(1.028, 0.332)$; the radius $r_\bullet$ is fixed at 0.8. Adjusting $\epsilon_\circ$ changes the fraction of $\mathbf{u}_\circ$ within $\mathbf{u}$.

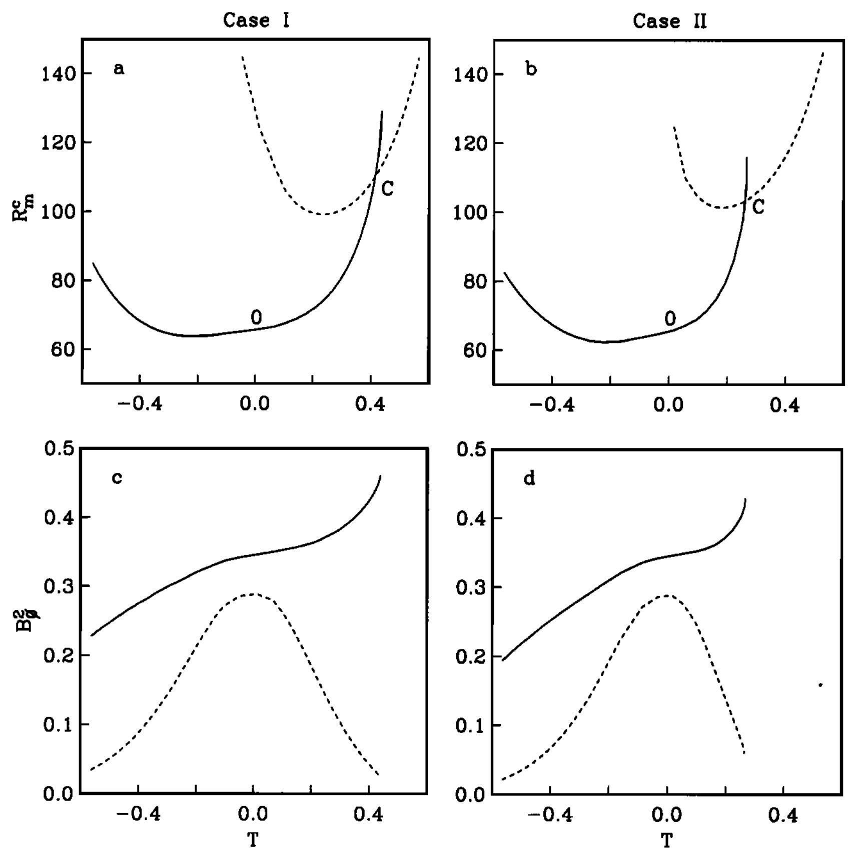

Figure 2. (a,b) Critical magnetic Reynolds number, R_m^c, as a function of overlying toroidal motion for cases I and II. $T = \text{sgn}(\epsilon_\circ)\langle \mathbf{u}_\circ \rangle / \langle \mathbf{u} \rangle$, so that T is positive (negative) if $\mathbf{u}_\circ$ is in the same (opposite) direction as the interior toroidal flow $\mathbf{t}_1^0$. Axial-dipole (equatorial-dipole) type solutions are shown by a solid (dashed) line. (c,d) Fraction of nonaxisymmetric field energy as a function of T for cases I and II. $B_\emptyset^2 = \langle \mathbf{B}_\emptyset \rangle^2 / \langle \mathbf{B} \rangle^2$, where $\mathbf{B}_\emptyset$ is the nonaxisymmetric part of the magnetic field. (c) and (d) apply only to axial-dipole solutions since $B_\emptyset^2 = 1$ for the equatorial-dipole solutions. Solid (dashed) line is for volumetric $0 < r < 1$ (surficial $r = 1$) average. Results for case II are similar to case I.

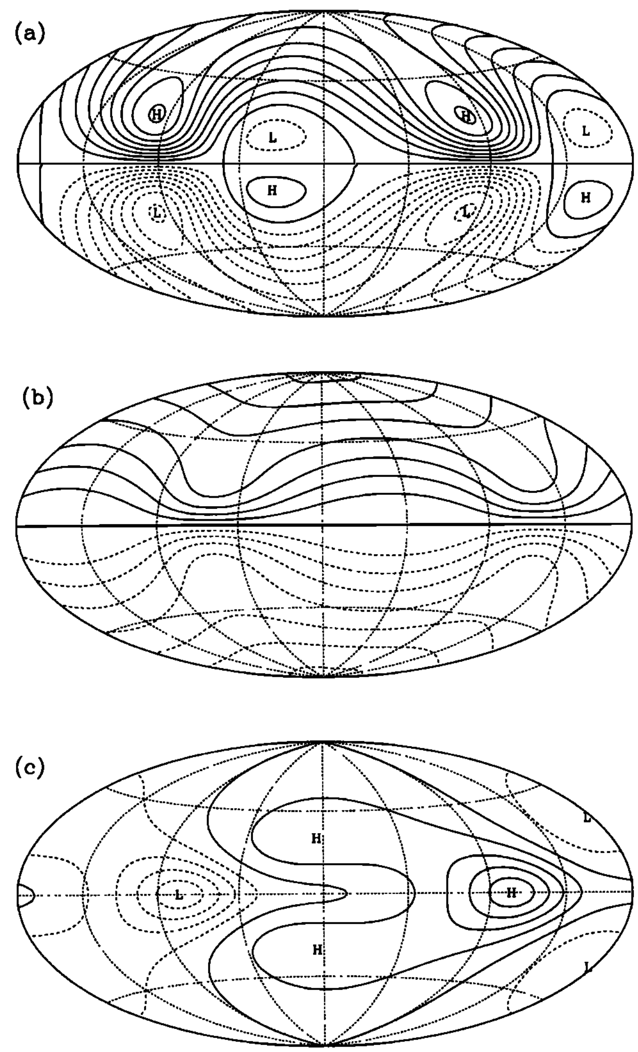

Figure 3. The radial component of the surficial magnetic field, B_r, for case I. (a) Axial-dipole type solution at point 0 in Figure 2(a), with no overlying toroidal motion, $T = 0$. (b) Axial-dipole type solution at point C in Figure 2(a). (c) Equatorial-dipole type solution at point C in Figure 2(a). Note that the surficial field is more axisymmetric in (b) than in (a), but that the field in (c) is an equatorial-dipole type and therefore has no axisymmetric ingredients. The magnetic fields for case II (not shown) are similar. All contour levels are the same. Projections are Mollweide.

like $\mathbf{u}_\bullet$, but with stagnant overlying conducting shells ($r_\bullet < r < 1, \mathbf{u}_\circ = 0$) of varying thickness [*Hutcheson and Gubbins*, 1994; *Sarson and Gubbins*, 1996]. There has also been a kinematic study of flows where the toroidal and poloidal ingredients were separated in space [*Serebrianaya*, 1988], but for which axisymmetrization was not explored. From such kinematic studies we learn that the configuration of the magnetic field, and even its symmetry, is a sensitive function of the details of the flow.

Results and Discussion

A suite of steady dynamos was found for varying amounts of overlying toroidal motion $\mathbf{u}_\circ$, for cases I and II. Interestingly, the sustained magnetic fields are of two symmetries, either with surficial fields being primarily axial-dipolar, or with surficial fields being primarily equatorial-dipolar [*Dudley and James*, 1989; *Gubbins*, 1973; *Holme*, 1997]. In Figure 2(a,b) the critical magnetic Reynolds number R_m^c is shown as a function of T, which is a measure of the size of $\mathbf{u}_\circ$. Supercritical magnetic Reynolds numbers give exponentially growing solutions; the realizable ('preferred') field configuration is that with the smallest R_m^c. For the case of no overlying conducting shell ($r_\bullet = 1$), the critical magnetic Reynolds number for the axial-dipole (equatorial-dipole) type solution is 140.55 (259.52); since this is higher than for the case with the stagnent overlying conducting shell ($r_\bullet = 0.8$) considered here, 65.81 (124.88) for $T = 0$, the overlying shell makes dynamo action more efficient. In Figure 2(a,b) we see that over most of the range of T the preferred solution is an axial-dipolar type, but for case I (case II) with $T > 0.41$ ($T > 0.26$) there is a crossover of critical magnetic Reynolds numbers at point C, where the preferred solution changes to an equatorial-dipolar type.

In the Figure 2(c,d) we show the nonaxisymmetric magnetic energy, $B_\emptyset^2$, as a function of the amount of overlying toroidal motion, T. Over the volume of the sphere the total field for the axial-dipole type solutions becomes more (less) axisymmetric for $T < 0$ ($T > 0$). In the same figure we see that the surficial field of the

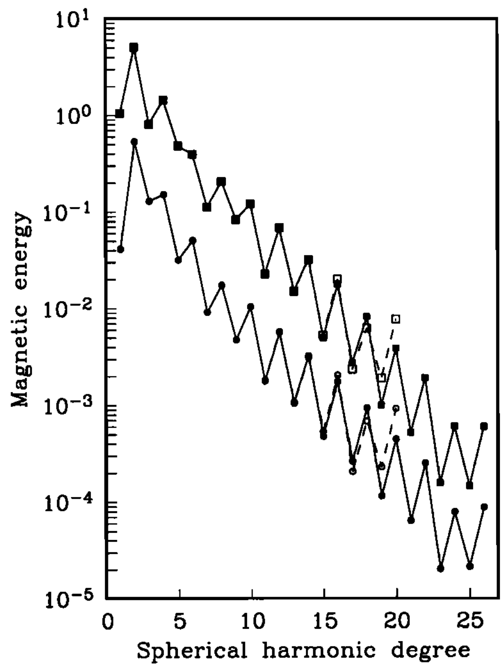

Figure 4. Magnetic energy (arbitrary units) of the dynamo fields at the point C for case I. Squares (circles) are for axial-dipole (equatorial-dipole) type solutions. Open (solid) symbols are for spherical harmonic expansions truncated at degree 20 (26). The energy decreases with increasing harmonic degree, and since the models are adequately converged, the inclusion of higher degree terms, those above degree 20, has little effect. The spectra for case II (not shown) are similar.

axial-dipole type solutions becomes more axisymmetric for $|T| > 0$. Stevenson's axisymmetrization mechanism works rather well for axial-dipole type magnetic fields; compare Figure 3(a) with 3(b). It fails dramatically, however, at point C, where the overlying differential rotation forces a change in symmetry from a nearly axisymmetric axial-dipole solution to an equatorial-dipole solution, Figure 3(c), which has no axisymmetric ingredients ($B_\emptyset^2 = 1$)! Convergence is verified in Figure 4.

Conclusions

Our analysis has been an idealization; we do not suggest that the details of our model flows resemble those of Saturn. Our goal was simply to investigate the ability of overlying differential rotation to 'axisymmetrize' a dynamo magnetic field. In fact, it is somewhat artificial to suppose that the dynamo resides in a particular part of the flow; dynamo action occurs over the volume of the flow. By changing the field at the top by differential rotation the field in the interior is inevitably changed as well, sometimes even forcing a change in the symmetry of the magnetic field which is contrary to the expected axisymmetrization. We conclude that sometimes overlying differential rotation acts to 'axisymmetrize' a dynamo field, and sometimes it doesn't.

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References

Acuña, M. H., and N. F. Ness, The magnetic field of Saturn, *Science*, *444*, 444–446, 1980.

Bullard, E. C., and H. Gellman, Homogeneous dynamos and terrestrial magnetism, *Philos. Trans. R. Soc. Lond. Ser. A*, *247*, 213–278, 1954.

Connerney, J. E. P., N. F. Ness, and M. H. Acuña, Zonal harmonic model of Saturn's magnetic field, *Nature*, *298*, 44–46, 1982.

Cowling, T. G., The magnetic field of sunspots, *Month. Not. R. Astr. Soc.*, *94*, 39–48, 1934.

Desch, M. D., and M. L. Kaiser, Voyager measurement of the rotation period of Saturn's magnetic field, *Geophys. Res. Lett.*, *8*, 253–256, 1981.

Dudley, M. L., and R. W. James, Time-dependent dynamos with stationary flows, *Proc. R. Soc. Lond. Ser. A*, *425*, 407–429, 1989.

Gubbins, D., Numerical solutions of the kinematic dynamo problem, *Philos. Trans. R. Soc. Lond. Ser. A*, *274*, 493–521, 1973.

Gubbins, D., N. Barber, S. Gibbons, and J. J. Love, Kinematic dynamo action in a sphere: II Symmetry selection, *Proc. R. Soc. Lond. Ser. A*, *456*, 1669–1683, 2000.

Hide, R., and T. N. Palmer, Generalization of Cowling's theorem, *Geophys. Astrophys. Fluid Dyn.*, *19*, 301–309, 1982.

Holme, R., Three-dimensional kinematic dynamos with equatorial symmetry: Application to the magnetic fields of Uranus and Neptune, *Phys. Earth Planet. Inter.*, *102*, 105–122, 1997.

Hutcheson, K. A., and D. Gubbins, Kinematic magnetic field morphology at the core-mantle boundary, *Geophys. J. Int.*, *116*, 304–320, 1994.

Ivers, D. J., and R. W. James, Axisymmetric antidynamo theorems in compressible non-uniform conducting fluids, *Philos. Trans. R. Soc. Lond. Ser. A*, *312*, 179–218, 1984.

Kumar, S., and P. H. Roberts, A three-dimensional kinematic dynamo, *Proc. R. Soc. Lond. Ser. A*, *344*, 235–258, 1975.

Love, J. J., and D. Gubbins, Dynamos driven by poloidal flow exist, *Geophys. Res. Lett.*, *23*, 857–860, 1996.

Northrop, T. G., and J. E. P. Connerney, A micrometeorite erosion model and the age of Saturn's rings, *Icarus*, *70*, 124–137, 1987.

Rheinhardt, M., Untersuchungen kinematischer und dynamisch konsistenter Dynamomodelle in sphärischer Geometrie, Ph.D. Thesis, Univ. Potsdam, Potsdam, 1997.

Sarson, G. R., and D. Gubbins, Three-dimensional kinematic dynamos dominated by strong differential rotation, *J. Fluid Mech.*, *306*, 223–265, 1996.

Serebrianaya, P. M., Kinematic stationary geodynamo models with separated toroidal and poloidal motions, *Geophys. Astrophys. Fluid Dyn.*, *44*, 141–164, 1988.

Smith, E. J., L. Davis Jr., D. E. Jones, P. J. Colman, D. S. Colburn, P. Dyal, and C. P. Sonett, Saturn's magnetic field and magnetosphere, *Science*, *207*, 407–410, 1980.

Stevenson, D. J., Saturn's luminosity and magnetism, *Science*, *208*, 746–748, 1980.

Stevenson, D. J., Reducing the non-axisymmetry of a planetary dynamo and an application to Saturn, *Geophys. Astrophys. Fluid Dyn.*, *21*, 113–127, 1982.

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