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Cross-references

Rock Magnetism, Hysteresis Measurements

FISHER STATISTICS

When describing the dispersion of paleomagnetic directions about some mean direction it is a standard practice within paleomagnetism to employ Fisher (1953) statistics. The theory of Fisher statistics assumes a mean direction, defined by a Cartesian unit vector $\hat{\mathbf{x}}_\mu$, and data, each defined by Cartesian unit vectors $\hat{\mathbf{x}}$. The off-axis angle θ between the mean direction and a datum is defined by

$$\cos\theta = \hat{\mathbf{x}} \cdot \hat{\mathbf{x}}_\mu. \quad \text{(Eq. 1)}$$

With this, then, the Fisher distribution gives the probability that a particular directional datum $\hat{\mathbf{x}}$ falls between θ and $\theta + \mathrm{d}\theta$ as

$$P_f(\theta|\kappa) = \int_\theta^{\theta+\mathrm{d}\theta} p_f(\theta'|\kappa)\mathrm{d}\theta', \quad \text{(Eq. 2)}$$

where the probability-density function is

$$p_f(\theta|\kappa) = \frac{\kappa}{2\sinh\kappa}\sin\theta\,\exp(\kappa\cos\theta), \quad \text{(Eq. 3)}$$

and where κ is a parameter that measures the directional dispersion of the data about the mean direction. If one considers the dispersion of directions over the unit sphere, then the probability of particular a datum falling onto a unit differential area

$$\mathrm{d}A = \sin\theta\mathrm{d}\theta\mathrm{d}\phi, \quad \text{(Eq. 4)}$$

where ϕ is the azimuthal angle symmetrically-distributed about $\hat{x}_\mu$, is just

$$P_f(A|\kappa) = \int_\phi^{\phi+\mathrm{d}\phi}\int_\theta^{\theta+\mathrm{d}\theta} p_f(A'|\kappa)\sin\theta'\mathrm{d}\theta'\mathrm{d}\phi', \quad \text{(Eq. 5)}$$

where the corresponding density function is

$$p_f(A|\kappa) = \frac{\kappa}{4\pi\sinh\kappa}\exp(\kappa\cos\theta). \quad \text{(Eq. 6)}$$

As with all probability-density functions, that a particular datum is realized is certain, we are, after all, describing data that exist, and therefore

$$\int_0^\pi p_f(\theta'|\kappa)\mathrm{d}\theta' = \int_0^{2\pi}\int_0^\pi p_f(A'|\kappa)\sin\theta'\mathrm{d}\theta'\mathrm{d}\phi' = 1. \quad \text{(Eq. 7)}$$

In Figure F9 we show examples of the Fisher density function.

Application

Let us consider now a set of paleomagnetic data, with the ith paleomagnetic direction $\hat{\mathbf{x}}_i$ defined by an inclination-declination pair (I_i, D_i), such that the Cartesian components are

$$x_i = \cos I_i \cos D_i, \quad y_i = \cos I_i \sin D_i, \quad z_i = \sin I_i. \quad \text{(Eq. 8)}$$

The mean unit direction $\bar{\mathbf{x}}$ given by N data is

$$\bar{x} = \frac{1}{R}\sum_{i=1}^N x_i, \quad \bar{y} = \frac{1}{R}\sum_{i=1}^N y_i, \quad \bar{z} = \frac{1}{R}\sum_{i=1}^N z_i, \quad \text{(Eq. 9)}$$

where

$$R^2 = \left(\sum_{i=1}^N x_i\right)^2 + \left(\sum_{i=1}^N y_i\right)^2 + \left(\sum_{i=1}^N z_i\right)^2. \quad \text{(Eq. 10)}$$

The mean vector $\bar{\mathbf{x}}$ is an estimate of the true mean $\hat{\mathbf{x}}_\mu$ corresponding to the underlying distribution p_f, and with this estimated mean direction we can measure the off-axis angle θ_i of each datum, where

$$\cos\theta_i = \bar{\mathbf{x}} \cdot \hat{\mathbf{x}}_i. \quad \text{(Eq. 11)}$$

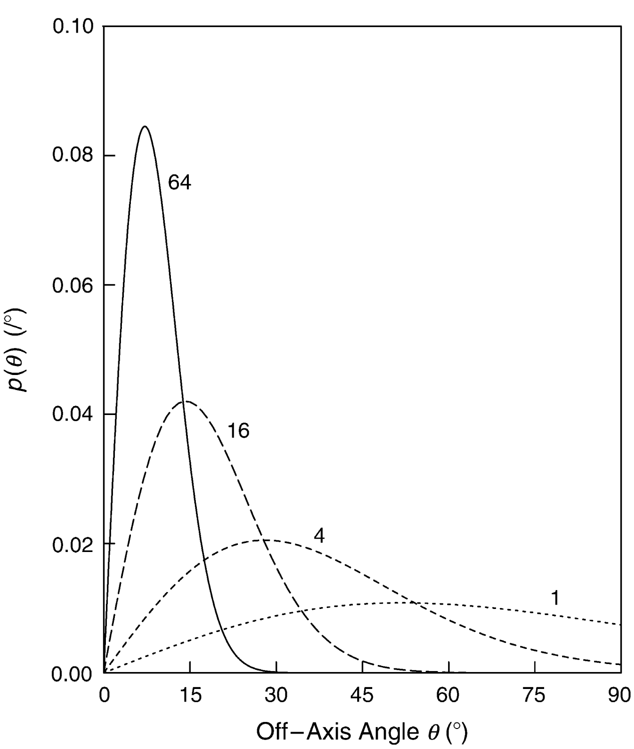

Figure F9 Examples of the Fisher probability-density function $p_f(\theta)$ for a variety of κ dispersion parameters: 1, 4, 16, 64. Note that as κ is increased the dispersion decreases.

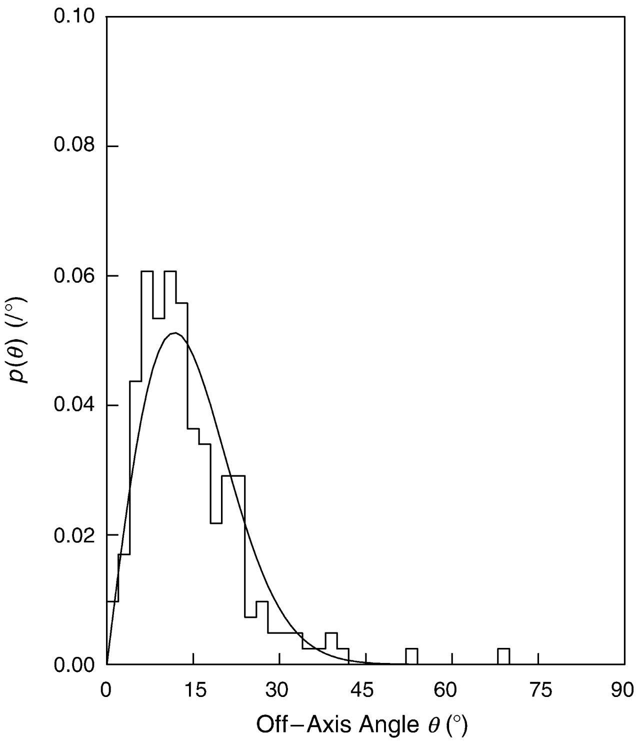

Figure F10 Comparison of a Fisher distribution fit to a histogram of Réunion data recording secular variation during the Brunhes.

The precision parameter κ is given approximately by (McFadden, 1980):

$$\kappa = \frac{N-1}{N-R}, \quad \text{(Eq. 12)}$$

As $R \to N$ the precision parameter κ increases, and the distribution of directions becomes more tightly clustered about the mean direction.

As an example of a fit of the Fisher distribution to real data, in Figure F10 we show a histogram of off-axis angles corresponding to a compilation of Brunhes-age paleomagnetic data collected at or near the island of Réunion (Love and Constable, 2003). Note that the Fisher distribution fitted using the procedure outlined here captures most of the actual distribution of the data, but that the fit is also not perfect. Indeed, although the Fisher distribution is often used in paleomagnetism, only rarely does paleaomagnetic data actually show a strict Fisher distribution. Both the secular variation of the geomagnetic field and the process by which rocks obtain their paleomagnetic signatures are extremely complicated, and it is, therefore, not too surprising that there is some misfit to a Fisher distribution. Although the Fisher distribution does arise from first principles in the context of the Langevin theory of paramagnetism, more generally, there often is very little reason to expect a set of paleomagnetic data to exhibit perfect Fisher statistics. The real utility of the Fisher distribution comes as a benchmark for comparison, the deviation from its relatively simple mathematical form that is of interest. Further review material on Fisher statistics can be found in the books by Butler (1992) and Tauxe (1998).

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Cross-references

Magnetization, Natural Remanent (NRM)
Paleomagnetic Secular Variation
Statistical Methods for Paleovector Analysis

FLEMING, JOHN ADAM (1877–1956)

Few individuals influenced geophysics in the 20th century more profoundly than John A. Fleming (Figure F11): geophysicist, engineer, scientific organizer, and administrator. He devoted his life to promoting the study of geomagnetism and building its professional organizations, and played a leading role in organizing magnetic and electric surveys of the Earth during the first half of the 20th century. Yet, as *Sydney Chapman* (*q.v.*) wrote of him, "He was so self-effacing that only those who knew and worked with him can properly assess… what he did for geophysics in USA and in the world at large" (Chapman, 1957).

Fleming was born in Cincinnati, Ohio on January 28, 1877. Educated as a civil engineer at the University of Cincinnati (1895–1899), he worked briefly in construction after receiving his B.S. degree, then joined the U.S. Coast and Geodetic Survey's Division of Terrestrial Magnetism. He advanced steadily at the Survey from 1899 to 1903 and was involved with the planning and construction of magnetic observatories in Alaska, Hawaii, and Maryland. In 1904, he was appointed "Chief Magnetician" at the newly established *Department of Terrestrial Magnetism* (DTM, *q.v.*) of the Carnegie Institution of Washington. He would be associated with the Institution for the next 50 years.

Figure F11 John Adam Fleming (*photograph:* Carnegie Institution, Department of Terrestrial Magnetism).