Transmission of Sound Waves in Gases at Low Pressures

Midway through World War II it became apparent that a better understanding of the motion and interaction of gaseous media with solid objects was needed. As aircraft flew at higher altitudes, approaching the speed of sound, strange shock wave effects interrupted normal flight. Propulsion mechanisms became challenged by the rapid increases in frictional drag. At the same time, the fledgling nuclear industry was trying to accelerate ion plasmas to higher speeds and energies, where confinement and wall heating were serious issues. The normal kinetic theory of gases was inadequate to describe these additional complex actions originating from the frictional and viscous forces of real compressive fluids. Two different parts of this problem existed. They are identified as nonlinearity, which occurs at high speeds, and *nonuniformity,* which is the behavior of the mechanics of rarefied gases.

Greenspan [1] described this behavior in the following way. "Nonuniform effects in gases are best studied at small acoustic amplitudes where relaxation effects can be observed. From kinetic-theoretical considerations in a gas of smooth rigid spheres, the speed of sound depends only on the mean speed of the molecules, provided that the gas is sufficiently dilute so that practically all of the molecular momentum is transferred and that the mean collision rate is very high. Laplace's formula states that the sound speed depends on the molecular mass and the temperature, and these determine the molecular speed. Any dispersion must depend on the ratio of collision rate to sound frequency. A suitable parameter for comparison is one proportional to this ratio (pressure/frequency). For example, a sound wave at an audio frequency of some kHz in a gas near atmospheric temperature and pressures will have a mean collision rate of order of 10^{10} s^{-1}. The medium is very fine grain and the dispersion is negligible. As the sound frequency is steadily increased, the frequency becomes comparable at first to the collision rate of the slower molecules. The collision rates becomes positive correlated with the molecular speed. The slower molecules can not transfer the acoustic momentum coherently, and this burden shifts to the faster molecules. Accordingly, the speed of sound steadily increases with frequency. The effect is negligible unless the frequency is very high."

In 1845 Stokes had first considered the effect of viscosity of gases on the propagation of sound. Since then, several approximations to the Boltzmann theory were developed to describe the action of frictional and viscous forces that affect gas dynamics. No singular closed solution exists that covers all cases of a rarefied or high speed gas. Some of the best known approximations are attributed to Stokes-Navier (1st order) and Burnett (2nd order). In the mid 1940s, experimental confirmation for these approximations was pursued using shock wave experiments. However, this technique cannot separate the combined effects of two distinct behaviors of gases, nonlinearity and nonuniformity. One way to make this separation is to use the small displacement condition of ultrasonic wave propagation to determine the nonuniformity behavior of the gas in the absence of any nonlinear effects.

The first endeavor that Martin Greenspan undertook after he began research in ultrasonics in 1946 was the investigation of the effects of gas viscosity on the acoustic propagation constant. At the suggestion of R. K. Cook, then Chief of the NBS Sound Section, "Moe" began the construction of an ultrasonic interferometer and necessary related gas control equipment. The first element of research was to measure the attenuation of sound as a function of pressure in rarefied helium at 1 MHz. This resulted in a paper describing the attenuation [2] over a pressure range of 0.5 kPa to 4 kPa and a comparison of the results to the Kirchhoff (Stokes-Navier) approximation.

This was the first of a succession of research projects; starting with attenuation measurements for sound waves in several rarefied monatomic gases and eventually continuing to diatomic gases. This research program covered a 14-year period and resulted in eight archival publications, two of which were review treatises of great significance [1-8].

Shortly after this first paper, Greenspan recognized the inadequacy of the theoretical explanation for the deviation of the gas behavior from the classic theory. He extended this work for the propagation of sound in diatomic gases. At the time, no theory for the behavior of more complex gas molecules existed. Greenspan extended the existing theory to cover both translational and thermal relaxational dispersions of sound energy

[5, 7]. He adapted a Stokes-Kirchhoff (S-K) function to represent a diatomic gas by converting the conventional S-K function into a dimensionless equation. This equation is described by two main variables: normalized viscosity and normalized thermal conductivity. The equation can be solved by eliminating the normalized thermal conductivity, but the solution is very formidable. However, an approximate solution can be obtained by factoring the standard S-K function into two parts; one associated with a sound wave and the other related to a heat wave. Under special values of γ, the ratio of specific heats, this relationship compares very favorably to the more exact solution. For the thermal relaxational part, Moe chose a relationship developed by Hertzfeld and Rice. He combined the translational and the thermal relaxation descriptions in a manner that produced two equations, a propagational one and an attenuative one.

After his first experimental effort, the ultrasonic interferometer was improved in several ways. The device was redesigned as a double crystal interferometer with constant gas volume that allowed the interferometer to operate in a nonresonant condition. This suppressed the transverse modes. New transducers were selected to run at the higher frequency of 11 MHz, improving the resolution greatly. Also, the measuring equipment was automated to measure continuously both the propagation constant (sound speed) and frictional component (attenuation) [4].

With the new equipment, Greenspan revisited the case of helium [3]. He compared the measured attenuation and speed of sound to the predictions of both the Stokes-Navier and the Burnett theories. He demonstrated how the measured data deviated from these theories at pressures below 0.5 kPa. Fig. 1 compares the real part α and the imaginary part β of the propagation constant for helium with computed functions of three approximate theories: Stokes-Navier, Burnett, and Super Burnett (Super Burnett theory is a separate approximation of higher order). The parameters shown on Figure 1 are: $\beta = \omega/c$, α = amplitude attenuation, $\omega = 2\pi f$, c is speed of sound at the frequency f, and p is the dynamic pressure. These variables are normalized to $\beta_0 = \omega/c_0$, the low frequency value. For comparison, Moe selected an independent variable that was proportional to the ratio pressure/frequency. This variable, r, is the traditional Reynold's number divided by γ and is used in place of gas pressure alone because it is a normalizing factor.

The next project was to measure sound speed and attenuation in four rarefied monatomic gases (neon, argon, krypton, xenon) [5]. When compared through the independent variable r all of the data for the individual monatomic gases collapse onto the same functional relationship. Again, Moe demonstrated the regions of agreement between the measurements and the various theories and identified the region where the measurements deviated significantly. The results show that simple monatomic gases can display sound dispersion associated with normal gas viscosity. Some representative results of these measurements on the five monatomic gases are shown in Fig. 2, which compares

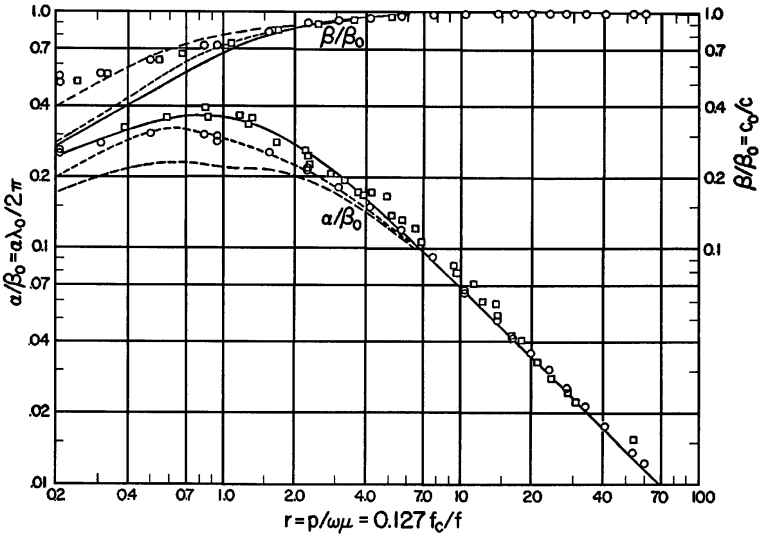

Fig. 1. Helium propagation constant as a function of r. Solid line: Stokes-Navier. Short dashed line: Burnett. Longer dashed line: Super Burnett. Circles: measurements at 11 MHz. Squares: previous measurements at 1 MHz.

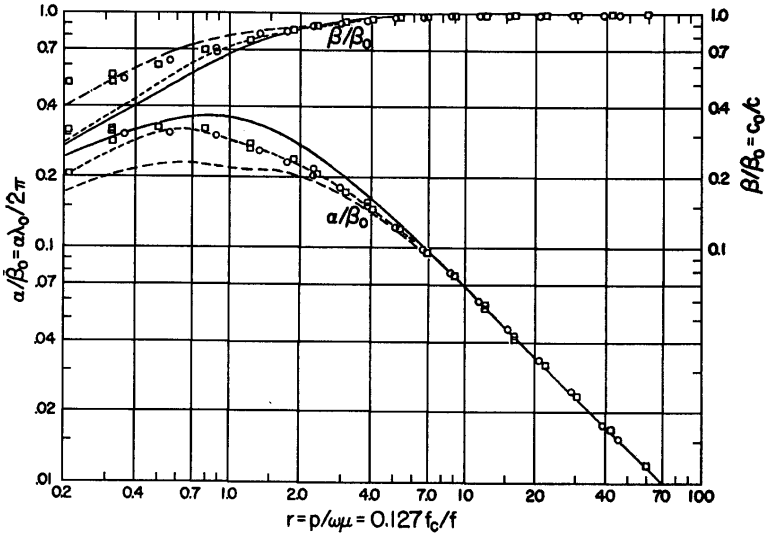

Fig. 2. Neon and argon at 11 MHz. Propagation constant as a function of r. Solid line: Stokes-Navier. Short dashed line: Burnett. Longer dashed line: Super Burnett. Circles: measurements in Ne. Squares: measurements in Ar.

α and β from measurements of neon and argon with three approximate theories: Stokes-Navier, Burnett, and Super Burnett. All five gases behave in substantially the same way. In the nondispersive range ($r >$ about 10) the measured values agree with the "classic" theory [3,6].

This work was extended to the measurement of viscosity effects for the diatomic gases nitrogen and oxygen and to dry air. Moe compared his theoretical predictions [5] to the attenuation and propagation constant of the diatomic gases as measured. The behavior at higher pressures follows the classic theory. As the mean free path approaches the ultrasonic wave length, Greenspan's constructed theory fitted at the intermediate pressures. For lower pressures significant deviations were found [7]

His work on propagation of sound in rarefied gases is a classic example of how to examine a complex system for relaxations resulting in a measured dispersion. He was able to show that the Stokes-Navier equation gave a surprisingly good quantitative account of attenuation and dispersion of sound in monatomic gases down to wavelengths approaching the mean free path. Moreover, he succeeded in making measurements to much lower pressures where the mean free path was significantly greater than the wavelength, and found substantial deviations from the theories. New theoretical many-body results are now judged by their agreement with these data. For diatomic and polyatomic gases, where molecular relaxation processes associated with vibrational and rotational modes occur in addition to the translation relaxation, he was able to demonstrate experimentally and theoretically how they combine to affect acoustic dispersion and attenuation.

In summary, Greenspan's work made a major contribution to the fields of acoustics and thermodynamics by allowing the effects of nonuniformity in a rarefied gas to be separated from those of nonlinear behavior. He provided the first experimental data on elastic and viscous behavior of a number of rarefied monatomic and diatomic gases. His papers on this subject received more than 250 citations, and his chapter in the prestigious *Physical Acoustics* series is still widely read.

Martin (Moe) Greenspan was born in New York City on May 8, 1912. He attended Cooper Union Institute of Technology and received a B. S. degree in 1934. He came to the National Bureau of Standards in 1935 and was employed until his retirement in 1975. As a guest scientist, he continued to contribute to NIST actively until his death in 1987. He was a very productive, prolific, and innovative researcher, not only in the area described here, but in several other fields. From 1935 until 1945 he worked in engineering mechanics, performing theoretical calculations on complex structures such as rigid knee bends, box girders, and perforated plates. Such calculations were coordinated with experimental tests producing a higher assurance of confidence in the safety of these structures. Moe's work on stress distribution in perforated plates, for which he received a Department of Commerce Silver Medal, was an important contribution to modern elastic fracture mechanics. During this period he authored at least 15 archival articles on theoretical engineering.

In August 1946, at the invitation of its Chief, Richard K. Cook, Moe joined the NBS Sound Section and began his work on physical acoustics, as described above. In the early 1950s he was approached by the Navy and asked to help develop a means of measuring speed of sound in the sea. Moe and Carl Tschiegg collaborated in the development of an accurate, reliable and rugged instrument now called an acoustic velocimeter. This device measured the sound speed in a continuous fashion in the deepest part of the sea. Since 1960, all Navy submarines have been equipped with a variation of this original device, which has also found extensive use in the chemical industry. This work also won the Department of Commerce Silver Medal. In the 1960s, Greenspan and Tschiegg investigated ultrasonic cavitation in water. They directed their attention to the effects of dissolved gases and neutron irradiation on the threshold of cavitation, finding that both dissolved gas and recoil atomic nuclei from neutron irradiation weaken the water and lower the cavitation threshold.

In 1966 Moe became chief of the Sound Section and in the 1970s turned his attention to problems in acoustic emission. Acoustic emission (AE) is transient in character. The usual calibration procedure is inadequate for AE transducer calibration. Moe was instrumental in proposing the use of special transfer blocks which could be excited by a step force function and which would follow a simple Lamb's solution. This was the basis of the acoustic emission calibration facility that was developed by Frank Breckenridge and Carl Tschiegg in collaboration with Moe. After his retirement, he examined the theoretical problem of an acoustic piston radiator. In a 1979 paper he succeeded in extending and improving the theory. Over his professional career (1935-1986) Moe authored more than 60 technical papers that were published in scholarly journals.

Despite the fact that Moe's formal education ended with his B. S. degree, he built a professional reputation of knowledge of acoustics matched by few others. His lecturing earned him an enviable international reputation. In addition to his professional duties for NBS, he lectured and taught courses in mathematics, engineering and physics at the University of Maryland, Catholic University, George Washington University, NBS Graduate School, and the Department of Agriculture Graduate School. During the 1958-1959 academic year he was a visiting lecturer at the UCLA Physics Department. At the end of this duty he was offered a full professorship by UCLA, but he declined, preferring to return to his research duties at NBS.

Greenspan was a Fellow of the Acoustical Society of America and was elected President for 1966-1967. He chaired the Technical Council of the Society for 1967-1968 and was the Society's representative to the Council of the AAAS. He was an Associate editor (1961-1966) of the *Journal of the Acoustical Society of America and the Editor* (1962-1973) of the *Journal of Research of the National Bureau of Standards,* Section C. In addition to the two Silver Medals awarded by NBS, he received the Acoustical Society Silver Metal in Physical Acoustics (1977), their Gold Metal (1983), and the Harry Diamond Award of the IEEE (1980).

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Bibliography

[1] M. Greenspan, Transmission of Sound Waves in Gases at Very Low Pressures, in *Physical Acoustics,* Vol. 2A, Academic Press, New York (1965) pp. 1-45.

[2] M. Greenspan, Attenuation of sound in rarefied helium, *Phys. Rev.* **75**, 197-198 (1949).

[3] M. Greenspan, Propagation of sound in rarefied helium, *J. Acoust. Soc. Am.* **22**, 568-571 (1950).

[4] M. Greenspan and M. C. Thompson, Jr., An eleven megacycle interferometer for low pressure gases, *J. Acoust. Soc. Am.* **25**, 92-96 (1953).

[5] M. Greenspan, Combined translational and relaxational dispersion of sound in gases, *J. Acoust. Soc. Am.* **26**, 70-73 (1954).

[6] M. Greenspan, Propagation of sound in five monatomic gases, *J. Acoust. Soc. Am.* **28**, 644-648 (1956).

[7] M. Greenspan, Rotational relaxation in nitrogen, oxygen, and air, *J. Acoust. Soc. Am.* **31**, 155-160 (1959).

[8] M. Greenspan, Translational Dispersion in Gases, in *Dispersion and Absorption of Sound by Molecular Processes,* Estratto da Rendiconti della Scuola Internazionale di Fisica Enrico Fermi, XXVII Corso, Academic Press, New York (1963) pp. 73-100.

[9] Isadore Rudnick, Citation to Martin Greenspan, *J. Acoust. Soc. Am.* **73**, S33 (1983).