Solar Zenith and Viewing Geometry Dependent Errors in Satellite Retrieved Cloud Optical Thickness; Marine stratocumulus case

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Abstract

The error in the domain averaged cloud optical thickness retrieved from satellite-based imagers is investigated using a cloud field generated by a cloud model and a 3D radiative transfer model. The cloud field used in the simulation is a relatively uniform (retrieved shape parameter of a gamma distribution averaged over all simulated viewing and solar zenith angles is 18) and nearly isotropic stratocumulus field. The retrieved cloud cover with a 1 km pixel resolution is 100%. The domain averaged optical thickness error is separated into two terms, the error caused by the extinction coefficient variability outside a pixel (external variability) and inside a pixel (internal variability). For the cloud field used in this study, the external variability term increases with solar zenith angle and the sign changes from negative to positive while the internal variability term is generally negative and becomes more negative as the solar zenith angle increases. At a small solar zenith angle, therefore, both terms are negative but the error partially cancels at a large solar zenith angle. When the solar zenith angle is less than 30°, both terms are small; the error in the viewing zenith angle and domain averaged cloud optical thickness derived from the relative azimuth angle smaller than 150° is less than 10%. However, if the optical thickness is derived from nadir view only for overhead sun, the domain averaged optical thickness is underestimated by more than 10%. When the solar zenith angle increases to 60°, the internal variability term exceeds 10% especially viewed from the forward direction but the domain and viewing zenith angle averaged optical thickness error can be less than 10% in the backward direction. When the solar zenith angle is 70°, both terms are greater than 10%. The shape parameter of a gamma distribution derived from retrieved optical

thicknesses increases with the viewing zenith angle but decreases with solar zenith angle. Based on this simulation and Moderate Resolution Imaging Spectroradiometer (MODIS) viewing geometry and solar zenith angle at the sampling time over the eastern Pacific, the error in the domain averaged retrieved optical thickness of uniform stratocumulus over eastern Pacific is less than 10% in March and September.

1. Introduction

Clouds reduce the global radiation budget (Ramanathan et al. 1989), increase its interannual variability (Kato 2008) and play a key role in climate feedback processes. Understanding their spatial and temporal variablities as a function of key meteorological variables is essential in modeling their response to and understanding their role in climate change. Observationally, global cloud properties can be estimated only by satellite-based instruments. Among cloud properties, cloud cover contributes most to the global top-of-atmosphere (TOA) irradiance variability (Loeb et al. 2007; Kato 2008). The cloud optical thickness is the most important of all cloud optical properties, and vital for any cloud-radiation parameterization. Its impact on radiative fluxes and therefore on climate is exceeded only by cloud cover. It is also the entry point of other retrieved cloud properties such as droplet size, liquid water and ice water contents (e.g. Minnis et al. 1998; Platnick et al. 2001) because the retrieval of these properties requires the optical thickness. Therefore, understanding the possible error in the satellite-derived cloud optical thickness is essential for assessing the error in climate data sets.

Besides imager calibration drifts and uncertainties in surface albedo and atmospheric correction, the error in satellite-derived cloud optical thickness depends on cloud type, illumination and viewing geometry: solar zenith angle, viewing zenith and relative azimuth angles. Comparing 2D and independent column approximation (ICA), it was found (e.g., Chambers et al., 1997 and Zuidema and Evans, 1998) that the retrieved optical thickness is smaller than the true optical thickness when the sun is overhead. In contrast, for oblique illumination, the retrieved optical thickness can be larger than the true one. Obviously, the retrieved optical thickness from horizontally inhomogeneous clouds decreases with increasing the imager pixel size (Zuidema and Evans 1998). Várnai and Marshak (2003) used both 3D and ICA to understand the mechanism causing the reflectance difference at nadir. They showed that, when the solar zenith angle is 60°, the nadir view 3D reflectance is typically larger than reflectance computed with ICA because the nadir view reflectance is enhanced by less scattering event in 3D than in ICA caused by side illumination of clouds in 3D.

Because the error in the retrieved cloud optical thickness depends on other cloud properties, it is difficult to understand the error by analyzing satellite-derived cloud optical thicknesses, although the error can be addressed through analyses of viewing angle dependence of retrieved optical thickness (e.g., Loeb et al. 1997; Loeb and Coakley 1998; Várnai and Marshak 2007). Surface-based and in-situ measurements can provide data for validation (e.g., Mace et al. 2005; Dong et al. 2008), but the field-of-view difference adds a complication in understanding the accuracy of retrievals. Another, somewhat less utilized, way to understand the error in the retrieved cloud optical thickness is by simulating the retrieval process with realistic cloud fields (e.g., Zuidema and Evans 1998; Zinner and Mayer 2006; Kato et al. 2006). The advantage of this approach is that the true cloud optical thickness is known and the exact error can be accurately estimated. The disadvantage is that it is unknown how well cloud fields used in the study represent real cloud fields. As a consequence, the result derived from simulations using a particular cloud field may not be directly applied to other cloud fields. However, if we are able to identify a viewing geometry that gives the smallest optical thickness retrieval error, we have a better chance of understanding the possible error when we analyze the viewing zenith and relative azimuth angle dependence of retrieved optical thicknesses. Therefore, the purpose of this paper is to understand solar zenith, viewing zenith, and relative azimuth angle dependent error in the optical thickness retrieved from relatively uniform low-level water clouds simulated with cloud resolving model (Stevens et al., 1999).

Instead of analyzing the error in the retrieved optical thickness from individual pixels, we will be focusing on the error in domain averaged retrieved optical thicknesses as a function of the imager viewing geometry and solar zenith angle in this study. Investigating domain-averaged errors instead of pixel-by-pixel errors makes the analysis less complicated because errors often partially cancel each other in an averaging process. In addition, understanding the error in the domain averaged optical thickness is more practical because averaged properties, such as regional, zonal, daily, or monthly means, are used to investigate climate problems.

We investigate the error as a function of viewing zenith, relative azimuth and solar zenith angles to answer three questions 1) whether or not the optimal geometry that gives a sufficiently small error in the satellite retrieved optical thickness for water clouds over ocean can be identified; 2) what is the viewing, relative azimuth, and solar zenith angles of the optimal condition for optical thickness retrievals?; 3) accounting for satellite and solar geometry, how accurate are the Moderate Resolution Imaging Spectroradiometer (MODIS, King et al. 1992) retrievals of cloud optical thickness for water clouds over ocean? Our emphasis is on identifying the optimal geometry that gives sufficiently small error in retrieved optical thicknesses instead of pointing out a large error in them.

In this study, we limit the analysis to the errors caused by a uniform overcast plane parallel cloud over an imager pixel and the independent column approximation (ICA). Therefore, our error analysis does not account for the uncertainty in the surface bidirectional reflectance function and atmospheric extinction above and below clouds.

In the following section, after a brief description of cloud field, we start with separating the error into two terms and focus on the difference in the radiance computed with ICA and full 3D in Section 4. Section 5 analyzes the error by viewing geometry and solar zenith angle and seeks for an optimal viewing geometry. Section 6 investigates whether or not the optimal viewing geometry actually occur in the data taken by MODIS on Terra over regions where low-level water clouds are often present.

2. Method

A cloud field of stratocumulus in a marine boundary layer with domain averaged optical thickness of 3.75 (Table 1) was generated by a cloud resolving model (Stevens et al. 1999) and described in Kato et al. (2006, ASTEX-Sc). The horizontal resolution of the modeled liquid water content field is 50 m and the domain size is 3.4 km by 3.4 km. The threshold of liquid water content is set to give the cloud fraction of 0.96 over the domain but the retrieved cloud fraction with 1 km pixels is 1. With this cloud field, we simulate a satellite-based cloud optical thickness retrieval process from narrowband visible radiances. Clouds are non-absorbing and the droplet effective radius is assumed to be 10μm everywhere. The albedo of the underlying surface is 0.05, which is a typical value for an ocean surface in a visible wavelength. We use the Spherical Harmonics Discrete Ordinate Method (SHDOM, Evans 1998) to compute radiances.

As indicated in Table 1 in Kato et al. (2006), the cloud field used in this study is not isotropic because the cloud field vertically tilts toward the direction of wind shear (Hinkelman et al. 2005). In other words, the radiance computed with a full 3D mode is a function of orientation of the cloud field relative to the sun position in addition to the viewing θ, relative azimuth ϕ, and solar zenith angles θ_0. Note that the relative azimuth angle is 0 when the imager views towards the sun position. The error in actual retrieved cloud optical thicknesses from imagers also depends on both the viewing geometry and cloud field orientation relative to the sun position. However, we expect that the dependence to the cloud field orientation affecting optical thickness retrievals becomes negligibly small when many retrieved optical thicknesses are averaged, as if the domain averaged cloud optical thickness is derived from isotropic cloud fields. The indication of apparent isotropic cloud fields in actual satellite data is that retrieved optical thicknesses sorted by viewing and relative azimuth angles is nearly symmetric about the principal plane if the temporal sampling among angles is uniform. Hence, we assume that there is no preferential cloud field orientation relative to the sun position in domain averaged data. The error in retrieved cloud optical thicknesses is, therefore, only a function of viewing zenith, relative azimuth, and solar zenith angles. To minimize the effect of anisotropic cloud fields, we rotate the original cloud field by 180°. We compute the reflectance at 7 relative azimuth angles with an increment of 30° from 0° through 180° for both the original and 180° rotated cloud field. We then average the reflectance pair of each relative azimuth angle.

The reflectance from an individual pixel observed by an imager does not obey the reciprocity principle because photons incident on the outside the pixel affect the radiance observed from the pixel (e.g. Davies, 1994; Aronson 1997, Di Girolamo et al 1998). However, under a periodic boundary condition, which does not have net photon transport through the boundary of the domain, the domain-averaged reflectance is supposed to obey the reciprocity principle (Di Girolamo 2002, Davis and Knyazikhin, 2005). Two reciprocal pairs of the reflectance in our simulation, even when the cloud field orientation relative to the sun is considered, are (30°,0°,60°) and (60°,0°,30°); (30°,180°,60°) and (60°,180°,30°), where angles in the parenthesis are (θ, ϕ, θ_0). In addition, if the cloud field is isotropic, the reflectance is symmetric around the principal plane so that more reciprocal pairs are possible. These reciprocal pairs that should obey the reciprocity principle, if the radiance is symmetric around the principal plane, are also listed in Table 2. The largest reflectance relative difference among these reciprocal pairs is 2.7%. Although we only average two radiation fields by rotating the cloud field by 180°, Table 2 indicates that the effect of anisotropic cloud field in averaged radiation fields is small. Because of this, the viewing zenith angle and solar zenith angle are interchangeable for the azimuthally averaged domain average reflectance from the simulation.

In this studies, we define the reflectance r as

$$r = \frac{\pi I}{\cos\theta_0 F_0}, \tag{1}$$

where θ_0 is the solar zenith angle, I is the radiance, and F_0 is the solar constant of the narrowband wavelength.

3. Optical thickness error

To better understand causes of the error in the domain averaged retrieved cloud optical thicknesses, we split it into two parts as follows:

$$\overline{\Delta\tau} = \overline{\tau_{3D}} - \overline{\tau_{true}} = \left(\overline{\tau_{3D}} - \frac{1}{N}\sum_{j=1}^{N}\tau_j\right) + \left(\frac{1}{N}\sum_{j=1}^{N}\tau_j - \overline{\tau_{true}}\right) = \overline{\Delta\tau_i} + \overline{\Delta\tau_e}, \quad (2)$$

where $\overline{\tau_{3D}}$ is the domain-averaged optical thickness retrieved from the 1 km resolution reflectance r_{3D}, $\overline{\tau_{true}}$ is the true domain-averaged optical thickness, τ_j is the subpixel optical thickness retrieved from a subpixel (50 m in our case) reflectance r_j; N is the total number of cloudy subpixels and finally, $\overline{\Delta\tau_i}$ and $\overline{\Delta\tau_e}$ stand for the error due to the internal and external variability, respectively. $\Delta\tau_i$ is also referred to the error due to unresolved variability and $\Delta\tau = \Delta\tau_i + \Delta\tau_e$ is referred to the error due to resolved variability (e.g., Marshak et al., 2006). The plane-parallel assumption of an uniform homogeneous cloud over the pixel is responsible for the first term while ICA is responsible for the second term (see Cahalan, 1994).

It is well known that for horizontally inhomogeneous clouds,

$$\overline{r(\tau)} < r(\bar{\tau}), \quad (3)$$

because the reflected radiance $r(\tau)$, as a function of optical thickness τ, is a convex function. Therefore, if the resolution to compute the reflectance with ICA (hereinafter r_{ICA}) compared to the resolution of full 3D calculation (hereinafter r_{3D}) is coarse and the optical thickness is linearly averaged to compute the domain averaged reflectance, it is greater than $\overline{r_{ICA}(\tau)}$. To separate the error clearly, both r_{3Dj} and r_{ICAj} are computed for each pixel at the 50 m cloud model resolution. The optical thickness is then retrieved from 50 m resolution radiances. As follows from Eq. (2), $\overline{\Delta\tau_e}$ is the difference between domain averaged optical thickness derived from the 50 m resolution reflectance r_{3Dj} and the true domain averaged optical thickness. In addition, all r_{3Dj} are linearly averaged over 1 km $\times$ 1 km pixels and the optical thickness $\overline{\tau_{3D}}$ is derived from the 1 km resolution radiances. $\overline{\Delta\tau_i}$ can be defined as the difference between $\overline{\Delta\tau}$ and $\overline{\Delta\tau_e}$. Note that in real retrievals with a 1 km pixel resolution, both terms are not resolved unless high resolution imager retrievals are collocated.

As mentioned earlier, $\overline{\Delta\tau_e} = \overline{\tau_e} - \overline{\tau}_{true}$ is caused by the ICA and using Taylor expansion at each pixel can be approximately written as follows,

$$\overline{\tau_e} = \frac{1}{N}\sum_{j=1}^{N}\tau_j = \overline{\tau_{true}} - \frac{1}{N}\sum_{j=1}^{N}(r_{ICAj} - r_{3Dj})\frac{\partial\tau}{\partial r_j}. \quad (4)$$

Since $\frac{\partial\tau}{\partial r}$ increases as a function of r, the difference between r_{3D} and r_{ICA} at pixels with larger optical thicknesses contributes more than the difference at pixels with smaller optical thicknesses if the magnitude of $r_{3D} - r_{ICA}$ is the same. Understanding the difference between $\overline{r_{ICA}}$ and $\overline{r_{3D}}$ is the first step to understand $\overline{\Delta\tau_e}$ because the derivative of the optical thickness with respect to reflectance is known. In the following section, we investigate $\overline{\Delta\tau_e}$ by analyzing the difference between $\overline{r_{3D}}$ and $\overline{r_{ICA}}$.

4. ICA and 3D Reflectance Difference

Figure 1 shows the domain averaged reflectance difference $\overline{r_{ICA}} - \overline{r_{3D}} = \Delta r_{ICA}$ as a function of relative azimuth angle for different viewing zenith and solar zenith angles. Each point is the average of 5000 differences of 1 km reflectances except for nadir view with overhead sun (2500 differences). Prominent features in Figure 1 are:

a) the nadir view reflectance difference $\overline{\Delta r_{ICA}}$ decreases with increasing solar zenith angle (except near overhead sun).

b) In the forward direction ($\phi < 90°$), $\overline{\Delta r_{ICA}}$ at large ($\theta \geq 60°$) and small ($\theta \leq 30°$) viewing zenith angle show different solar zenith angle dependence: it decreases with solar zenith angle at small viewing zenith angles while it increases at large viewing zenith angles (except near overhead sun). In the backward direction ($\phi > 90°$) $\overline{\Delta r_{ICA}}$ decreases with solar zenith angle (except near overhead sun) for all viewing zenith angles.

c) At large viewing and solar zenith angles, $\overline{\Delta r_{ICA}}$ is positive in the forward direction and negative in the backward direction, monotonically decreasing with increasing relative azimuth angle.

d) The root mean square (RMS) $r_{ICA} - r_{3D}$ difference increases with solar zenith angle for all viewing zenith angles and with viewing zenith angle for all solar zenith angles.

e) The RMS $r_{ICA} - r_{3D}$ difference is smallest when clouds are viewed from relative azimuth angle near 90° for a large viewing zenith angle.

In Figure 2, the $\overline{\Delta r_{ICA}}$ from all azimuth angles are averaged and the difference is plotted as a function of viewing zenith angle (open circles). Applying the reciprocity principle, we also plot additional points (open squares). As mentioned earlier, azimuthally and domain averaged reflectance difference $\overline{\Delta r_{ICA}}$ obeys the reciprocity principle fairly well, which is a result of a nearly isotropic cloud field and a periodic boundary condition.

In this section, we examine the cause of the above features and investigate the reasons for the $\overline{r_{ICA}} - \overline{r_{3D}}$ difference. For overhead sun and a small viewing zenith angle, $\overline{r_{ICA}}$ is larger than $\overline{r_{3D}}$. When the solar zenith angle is small, photons leaks from the side of clouds in 3D computations while photons leave clouds only from top or bottom with ICA. This does not necessarily means that $\overline{r_{ICA}}$ at nadir is larger than $\overline{r_{3D}}$ because it says nothing about the direction in which photons leave. A larger $\overline{r_{ICA}}$ at nadir, however, implies that photons are reflected toward a smaller zenith angle with ICA than the 3D computation when the sun is overhead. Because cloud droplets scatter photons predominately in forward direction, photons reflected near nadir directions tend to have experienced more scattering events than those reflected at oblique angles. Therefore, for overhead sun, photons reflected toward an oblique angle tend to experience less scattering events than photons reflected toward nadir view for a given optical thickness (Figure 3). Slopes of the number of scattering events versus optical thickness in Figure 3 are approximately 1 as explained from the diffusion theory (Marshak et al. 1995). Davis and Marshak (1997) also showed that the average scattering angle

When the reciprocity principal is applied, a smaller $\overline{r_{ICA}}$ than $\overline{r_{3D}}$ at oblique angles for overhead sun leads to a larger $\overline{r_{3D}}$ than $\overline{r_{ICA}}$ near nadir when the solar zenith angle is large. In addition, together with the above result of $\overline{\Delta r_{ICA}}$ for overhead sun, it leads that $\overline{\Delta r_{ICA}}$ at nadir decreases with solar zenith angle. When the solar zenith angle is large, relative azimuth angle dependence increases as the viewing zenith angle increases. In the forward direction where the imager detects transmitted photons and could see shadows, r_{ICA} is larger than r_{3D}. In the backward direction, the effect tends to be opposite because the imager views sunlit areas that are closer to perpendicular to direct solar radiation in 3D computations than with ICA. This effect is further pronounced when the actual cloud fraction is less than 1 (e.g. the true cloud fraction is 0.96 for the cloud field used in this study), especially near 180° relative azimuth angle because the cloud fraction projected in the direction of direct solar radiation increases with solar zenith angle in 3D computation while it is constant with ICA.

In summary, the ICA tends to increase the number of scattering events compared with 3D computations. As a consequence, $\overline{r_{ICA}}$ is larger than $\overline{r_{3D}}$ at nadir for overhead sun and the difference decreases with viewing zenith angle. Applying the reciprocity principal, this viewing zenith angle dependence of the difference for overhead sun is equivalent to decreasing $\overline{\Delta r_{3D}}$ at the nadir with increasing solar zenith angle. We can interpret the increase of the number of scattering events by the ICA as a larger apparent optical thickness in ICA computations. However, a simple correction to the optical thickness to match $\overline{r_{ICA}}$ with $\overline{r_{3D}}$ for all angles does not exist because the adjustment of $\Delta\tau$ depends on the viewing zenith angle for a given number of scattering events increase determine by the cloud field.

The sensitivity of viewing zenith dependence of number of scattering events to cloud top structure is weak, which is apparent in the study by Loeb et al. (1998) who analyzed the number of scattering events as a function of viewing zenith angle using various cloud top boundaries. Their result indicates that the difference in the number of scattering events between 3D and 1D computations is smaller than the difference caused by the viewing zenith angle. Therefore, the mean number of scattering events for a given direction is less sensitive to cloud top variability than to viewing zenith angle, except in the forward direction when the solar zenith angle is large.

Briefly, we consider whether or not above results are consistent with earlier result of the irradiance difference computed with ICA and 3D. Since the near nadir radiances contribute more to the reflected irradiance than oblique radiances, above results suggest that the difference of the ICA irradiance from 3D irradiance is positive when the solar zenith angle is small. This is consistent with the result by Davis and Marshak (2001). Our result also suggests that the irradiance difference decreases with solar zenith angle, which is consistent with Chambers et al. (1997) and Benner and Evans (2001).

The above results (d) and (e) are on the RMS difference of r_{ICA} and r_{3D}. The optical thickness along the path of the direct solar irradiance in 3D and ICA computations are the same for overhead sun. The difference in the optical thickness along the path of the direct irradiance increases with solar zenith angle, which also increases the RMS $r_{ICA} - r_{3D}$ difference. Increasing RMS $r_{ICA} - r_{3D}$ difference with viewing angle is also due to a similar reason; the optical thicknesses along the line of sight in 3D and ICA computations agree at nadir but the difference increases with viewing zenith angle. The larger RMS $r_{ICA} - r_{3D}$ difference in the forward direction especially at the viewing zenith angle of 60° for large solar zenith angles is caused by the fact that the imager detects more transmitted photons at this angle in the 3D calculation than any other simulated viewing angles while the imager detects reflected photon in the ICA calculation. In the forward direction, therefore, the difference of the sensitivity to the optical thickness for reflected and transmitted photons, in addition to the difference in the optical thickness along the path, increases the RMS difference.

5. The Error in the Retrieved Cloud Optical Thickness

Figure 4 shows the error in the retrieved optical thickness $\overline{\Delta\tau}(= \overline{\Delta\tau_e} + \overline{\Delta\tau_i})$ and error in the retrieved shape parameter ν as a function of relative azimuth angle. Note that a gamma distribution $P(\tau)$ of the optical thickness τ is expressed as

$$P(\tau) = \frac{1}{\Gamma(\nu)} \left(\frac{\nu}{\bar{\tau}}\right)^{\nu} \tau^{\nu-1} e^{-\nu\tau/\bar{\tau}}, \tag{5}$$

where $\Gamma(\nu)$ is the gamma function. We also plot two terms $\overline{\Delta\tau_e}$ and $\overline{\Delta\tau_i}$ separately in Figures 5 and 6. Figure 5 shows the error in the retrieved optical thickness as a function of solar zenith angle separated by viewing zenith angle and Figure 6 shows the error as a function of relative azimuth angle separated by the solar zenith angle. Based on (4), the dependence of $\overline{\Delta\tau_e}$ on solar zenith, viewing zenith, and azimuth angles should be consistent with that of the $\overline{r_{ICA}} - \overline{r_{3D}}$ difference with the opposite sign. Increasing $\overline{\Delta\tau_e}$ with solar zenith angle when the viewing zenith angle is 0° or 30° (Figure 5) agrees with the result of decreasing the $\overline{r_{ICA}} - \overline{r_{3D}}$ difference. However, $\overline{\Delta\tau_e}$ is large positive when the viewing zenith angle is 60° and solar zenith angle is 70° (Figure 5). When the solar zenith angle is large, the reflectance at edges of the cloud top can be very large (Evans and Marshak 2005). The reflectance, as a function of optical thickness, approaches an asymptote value and becomes nearly constant as the optical thickness increases. When the solar zenith angle is large, the reflectance asymptotes at smaller optical thickness than the reflectance for a smaller solar zenith angle does. As a consequence, it needs a large optical thickness to match a radiance observed in the forward direction at a large solar zenith angle with the radiance from 1D theory, if the forward reflectance peak is larger than that is given by 1D theory. In our simulation, some retrieved optical thicknesses are greater than 100 (Figure 7). These large retrieved optical thicknesses increase the domain averaged retrieved optical thickness.

The internal variability term $\Delta\tau_i$ is the error due to cloud inhomogeneity within a pixel. It follows from (3) that the retrieved optical thickness is negatively biased because the size of a pixel is finite (1 km in this simulation) and optical thickness to match the 1 km resolution radiance is smaller than a linear mean of optical thicknesses in the pixel. An extreme case of this is a pixel with partially filled with clouds. The optical thickness retrieved from a partially filled pixel is also smaller than the true optical thickness (e.g. Coakley 2005). Therefore, $\Delta\tau_i$ is generally negative. However, when the cloud optical thickness is small, the derivative of reflectance with respect to the optical thickness decreases with decreasing the optical thickness and the reflectance function $r(\tau)$ is concave rather than convex. Figure 8 shows the derivative computed by DISORT (Stamnes et al. 1988) with a plane parallel non-absorbing clouds for overhead sun. The exact optical thickness at which the reflectance function becomes concave depends on solar zenith and viewing zenith angles. This means that $\overline{\Delta\tau_i}$ can be positive if clouds are optically thin. In figure 6, the vertical error bar on $\overline{\Delta\tau_i}$ indicates that this happens in our simulation when the solar zenith angle is 0° and 30°, although $\overline{\Delta\tau_i}$ averaged all viewing angles is negative. $\overline{\Delta\tau_i}$ becomes more negative as the solar zenith angle increases because clouds look more inhomogeneous. This is apparent in Figure 4 showing that the retrieved shape parameter of a gamma distribution decreases as the solar zenith angle increases.

In summary, the magnitude of $\overline{\Delta\tau_e}$ and $\overline{\Delta\tau_i}$ increases with solar zenith angle. They, however, tend to have opposite signs so that the error can partially cancel (Figures 5 and 6). Both terms are originated from horizontal inhomogeneity and the magnitude decreases as horizontal inhomogeneity decreases. However, $\Delta\tau_e$ and $\Delta\tau_i$ are caused by different assumptions in the retrieval process cause. The assumption of a uniform overcast cloud inside a pixel results in $\Delta\tau_i$ while neglecting horizontal flux causes $\Delta\tau_e$. The magnitude of both terms depends on the degree of inhomogeneity, the shape of reflectance function, and the size of pixel, but $\Delta\tau_e$ and $\Delta\tau_i$ have different dependence to them. $\Delta\tau_i$ is generally negative but can be also positive for a very small optical thickness. $\Delta\tau_i$ can be negligibly small if the pixel size decreases but the magnitude of $\Delta\tau_e$ increases with decreasing pixel size (e.g. Davis et al. 1997). $\overline{\Delta\tau_e}$ changes the sign from negative to positive as the solar zenith angle increases. A large positive $\overline{\Delta\tau_e}$ for $\theta = 60°$ and $\theta_0 = 70°$ is due to some large retrieved values at cloud top edges and the fact that reflectance function approaches an asymptote value at a smaller optical thickness when the solar zenith angle is large. Increasing $\overline{\Delta\tau_e}$ with solar zenith angle for $\theta = 0°$ and 30° is due to decreasing $r_{ICA} - r_{3D}$ with solar zenith angle, which is caused by the nature of the ICA that tends to increase the number of scattering events.

6. Optimum viewing geometry and solar zenith angle

Based on the result discussed above, we seek optimal viewing geometries and solar zenith angles that give a small $\overline{\Delta\tau}$. To determine whether or not $\overline{\Delta\tau}$ is sufficiently small, we use a 10% criterion of the optical thickness error required for climate data (Ohring et al. 2005). At a smaller solar zenith angle, both $\overline{\Delta\tau_e}$ and $\overline{\Delta\tau_i}$ terms derived from the cloud field used in our simulation are small (Figures 5 and 6). As a consequence, when the solar zenith angle is small ($\theta_0 \leq 30°$), the error is negative but less than 10% (except for $\phi = 180°$) for the cloud field we analyzed (Figure 6). However, if the optical thickness is derived from nadir view only for overhead sun, the domain averaged optical thickness is underestimated by more than 10% (Figure 5). The azimuthally averaged $\overline{\Delta\tau}$ is less than

10% in the range of the viewing zenith angle from 0° to 60° when the solar zenith angle is around 30° (Figure 5). When the solar zenith angle increases to 60°, viewing zenith averaged $\overline{\Delta\tau}$ exceeds 10% especially if viewed from the forward direction while it can be less than 10% in the backward direction (Figure 6). The azimuthally averaged $\overline{\Delta\tau}$ is less than 10% when the viewing zenith angle is less than 30° and solar zenith angle is 60°. When the solar zenith angle further increases to 70°, both terms are greater than 10% but with the opposite sign (Figures 5 and 6). As a consequence, the optical thickness error retrieved from the cloud field used in our study is less than 10%. In the forward direction, for example, $\overline{\Delta\tau}$ is smaller when θ_0 is 70° than $\overline{\Delta\tau}$ when θ_0 is 60°. Because the small error is achieved by two large terms with the opposite sign when solar zenith angle is 70°, $\overline{\Delta\tau}$ at a large solar zenith angle possibly highly depends on cloud field.

We check whether or not this optimal viewing geometry and solar zenith angle combination can actually occur in the data taken from MODIS on Terra where low-level stratocumulus clouds are often present. Figure 9 shows the frequency of occurrence of viewing zenith and relative azimuth angles of MODIS and solar zenith angle over a $1° \times 1°$ region centered at 32.5N and 134.5W. The solar zenith angle centered at about 30° occur in March and September. Since the viewing zenith angle is nearly uniformly distributed from 0° to 60°, we refer to Figure 6 for the domain averaged optical thickness error. Figure 6 indicates that the errors are likely to be less than 10% in March and September when solar zenith angle is near 30° and relative azimuth angles near 60° and 140° are sampled. The relative azimuth angle close to 0° occurs in June but the solar zenith angle is small so that the error in the forward direction is likely to be less than 10%. A possible larger error occur in December when the solar zenith angle is about 60° and relative azimuth angle is about 60°.

As mentioned earlier, $\overline{\Delta\tau}$ discussed here is for relatively uniform water clouds. One could filter out highly inhomogeneous cloud fields using the shape parameter of a gamma distribution. The true shape parameter for the cloud field used in this study computed with a 1 km resolution is 15. The retrieved shape parameter averaged over all simulated viewing geometries and solar zenith angle is 18 and retrieved cloud cover with 1 km pixel is 100%. Therefore, the result obtained in this study is applicable for a domain average computed from relatively uniform overcast clouds of which retrieved shape parameter is greater than about 15. Although we only studied one isotropic cloud field and whether or not the result can be extrapolated to uniform marine stratocumulus clouds is an open question, we simulated more than 270000 1 km optical thickness retrievals. The above result indicates that the error in the domain averaged retrieved optical thickness of uniform stratocumulus over eastern Pacific is less than 10% in March and September when the solar zenith angle is around 30°.

To investigate cloud properties similar to those used in this simulation actually happens under similar viewing geometry and solar zenith angle, we sort low level clouds (cloud top height greater than 680 hPa) derived from MODIS by the Clouds and the Earth's Radiant Energy System (CERES) cloud algorithm (Minnis et al. 1998; Minnis et al. 2008) over 134°W to 135°W and 30°N to 35°N as a function of retrieved optical thickness and shape parameter (Figure 10). The shape parameter is derived from linear and logarithmic mean of optical thicknesses derived over a CERES footprint with 1 km MODIS pixels

(Kato et al. 2005), which is approximately 20 km at nadir. Note that the actual MODIS pixel size increases with viewing zenith angle while it is constant in our simulation. When the solar zenith angle is 30° and the relative azimuth angle is 60° or 120° as sampled by MODIS in March and September, the retrieved domain averaged optical thickness from the cloud field used in this study is 3.4 and retrieved shape parameter from linear and logarithmic mean of retrieved optical thicknesses is 27 if they are averaged over all viewing zenith angles. Figure 10 indicates that the optical thickness and shape parameter used in this study actually occur, although the mode of the distribution is shifted toward optically thicker and less uniform clouds. Therefore, if we limit the domain averaged retrieved cloud optical thickness and shape parameter to a similar range of those from the cloud field and if they show a similar, viewing, relative azimuth, and solar zenith angle dependence to those studied in this study, the retrieval error is likely to be less than 10%. A potential critical issue is that averaged cloud fields need to be nearly isotropic. Therefore, a significant amount of retrieved optical thicknesses needs to be averaged.

Because the error depends on season and region (solar zenith angle), separating seasonal variation of cloud optical thickness is critical to understand the error in the retrieved optical thickness. While earlier studies simulate the retrieval process with a broader range of cloud properties (e.g. Kato et al. 2006; Zinner and Mayer 2006) We also need to extend simulations to optically thicker and less uniform clouds in future to broaden the cloud type for estimating the error in domain averaged retrieved cloud optical thicknesses. In addition, we can compare the modeled TOA irradiance with these relatively uniform clouds and that derived from CERES radiances by angular distribution models to check a consistency, although calibration of MODIS and CERES instruments affect the result of this kind of comparisons.

7. Conclusions

We investigated the error in the retrieved cloud optical thickness as a function of viewing zenith, relative azimuth, and solar zenith angles for a relatively uniform cloud field. The retrieved cloud fraction with 1 km pixels is 1 and the retrieved shape parameter averaged over all simulated solar zenith angle is 18 while the true values are 3.75 and 15, respectively. The error in the retrieved optical thickness is separated into two terms, the error due to external variability $\overline{\Delta\tau_e}$ and the error due to internal variability $\overline{\Delta\tau_i}$. $\overline{\Delta\tau_e}$ is caused by the independent column approximation and $\overline{\Delta\tau_i}$ caused by the assumption of a uniform overcast clouds within a pixel. We determine the optimal viewing geometry and solar zenith angle that gives less than 10% error of the domain averaged retrieved optical thickness from the cloud field used in the simulation. When the solar zenith angle is small (less than $\approx 30°$), the azimuthally averaged $\overline{\Delta\tau}$ is most likely less than 10%. In addition, the $\overline{\Delta\tau}$ averaged over viewing zenith angle is less than 10% when optical thicknesses are derived from the relative azimuth angle smaller than $\approx 150°$. However, if the optical thickness is derived from nadir view only for overhead sun, the domain averaged optical thickness is underestimated by more than 10%. When the solar zenith angle increases to 60°, the viewing zenith angle averaged $\overline{\Delta\tau}$ is also likely to be less than 10% in the backward direction. The azimuthally averaged $\overline{\Delta\tau}$ is also likely to be less than 10% if viewing zenith angle is small (less than $\approx 30°$) but exceeds 10% for large viewing zenith angles (greater than $\approx 60°$). The viewing zenith averaged $\overline{\Delta\tau}$ also exceeds 10% in the forward direction.

When the solar zenith angle is further increased to 70°, both terms $\overline{\Delta\tau_e}$ and $\overline{\Delta\tau_e}$ are greater than 10% with the opposite sign so that $\overline{\Delta\tau}$ is smaller, although the magnitude of $\overline{\Delta\tau}$ possibly highly depends on cloud field. We checked MODIS viewing geometry from *Terra* satellite and showed that the optimal viewing geometry over eastern pacific where low level stratocumulus clouds are often present actually happens. We expect the domain averaged error in MODIS retrieved cloud optical thickness from cloud fields similar to the cloud field used in this study to be less than 10%, if retrieved optical thicknesses show a similar viewing angle and solar zenith angle dependence when a significant amount of optical thicknesses are averaged.

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Table 1: LES model generated Cloud Properties

Mean optical thickness	3.75
Shape parameter (ν) with 1 km res.	14.9
Cloud fraction (50 m res.)	0.960
Domain size (km)	3.4×3.4

Table 2: 3D Reflectance at Reciprocity Angles

Angle (θ, ϕ, θ_0)	3D Reflectance	Relative Difference (%)
(30,0,60)	0.2519	
(60,0,30)	0.2506	0 5
(30,180,60)	0.2671	
(60,180,30)	0.2665	0.2
(30,any,0)	0.1838	
(0,any,30)	0.1845	0.4
(60,any,0)	0.1764	
(0,any,60)	0.1799	2.0
(30,30,60)	0.2386	
(60,30,30)	0.2355	1.3
(30,60,60)	0.2151	
(60,60,30)	0.2107	2.1
(30,90,60)	0.2031	
(60,90,30)	0.1981	2.5
(30,120,60)	0.2145	

(60,120,30)	0.2088	2.7
(30,150,60)	0.2784	
(60,150,30)	0.2749	1.3

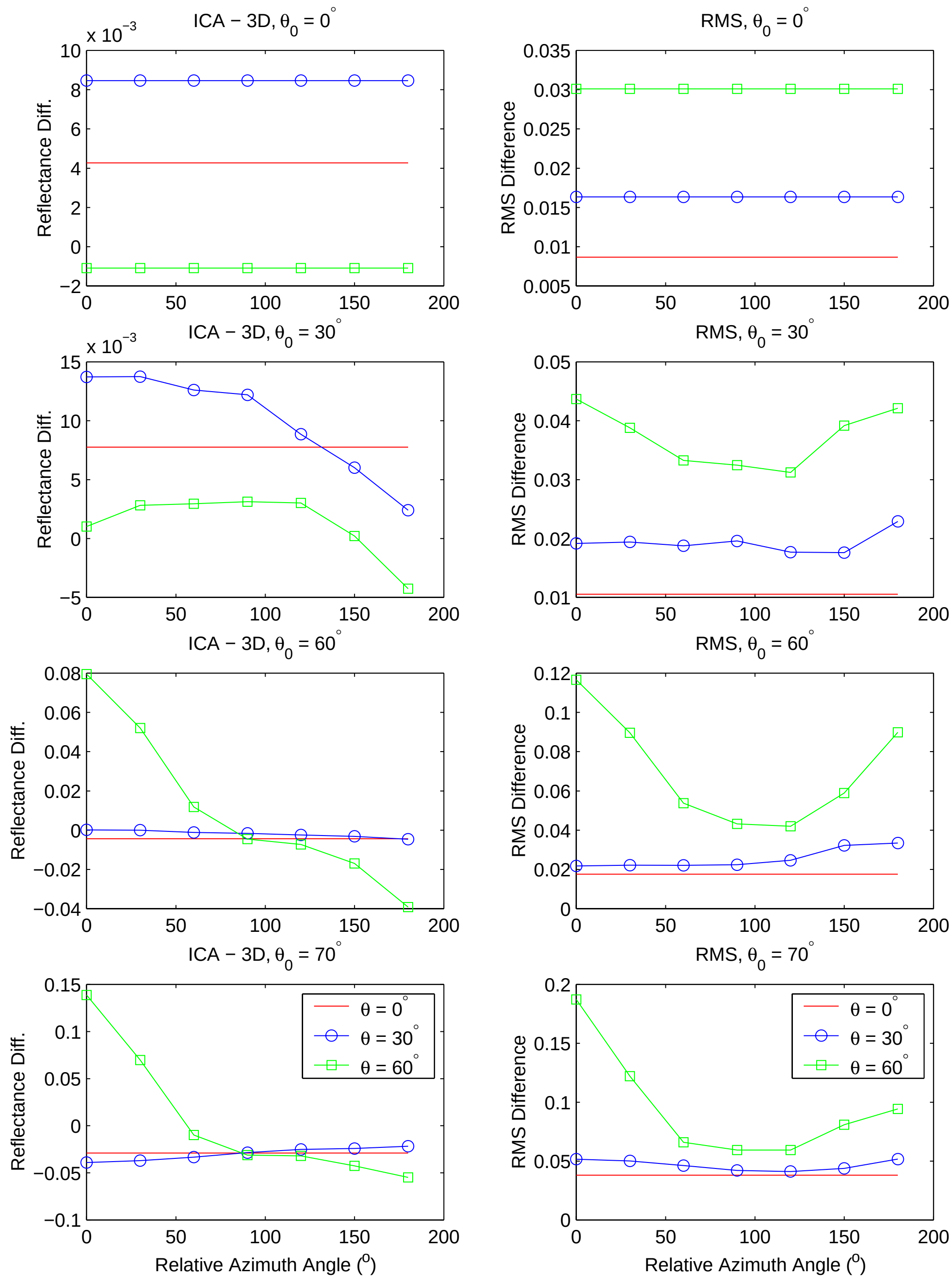

Figure 1 Left column) Difference of the domain averaged reflectance computed with the independent column approximation (ICA) from 3D reflectance as a function of relative azimuth angle. The relative azimuth angle is 0 when the imager looks into the sun. Right column) the root mean square difference of the ICA and 3D reflectance. ICA and 3D reflectances are computed with a 50 m resolution.

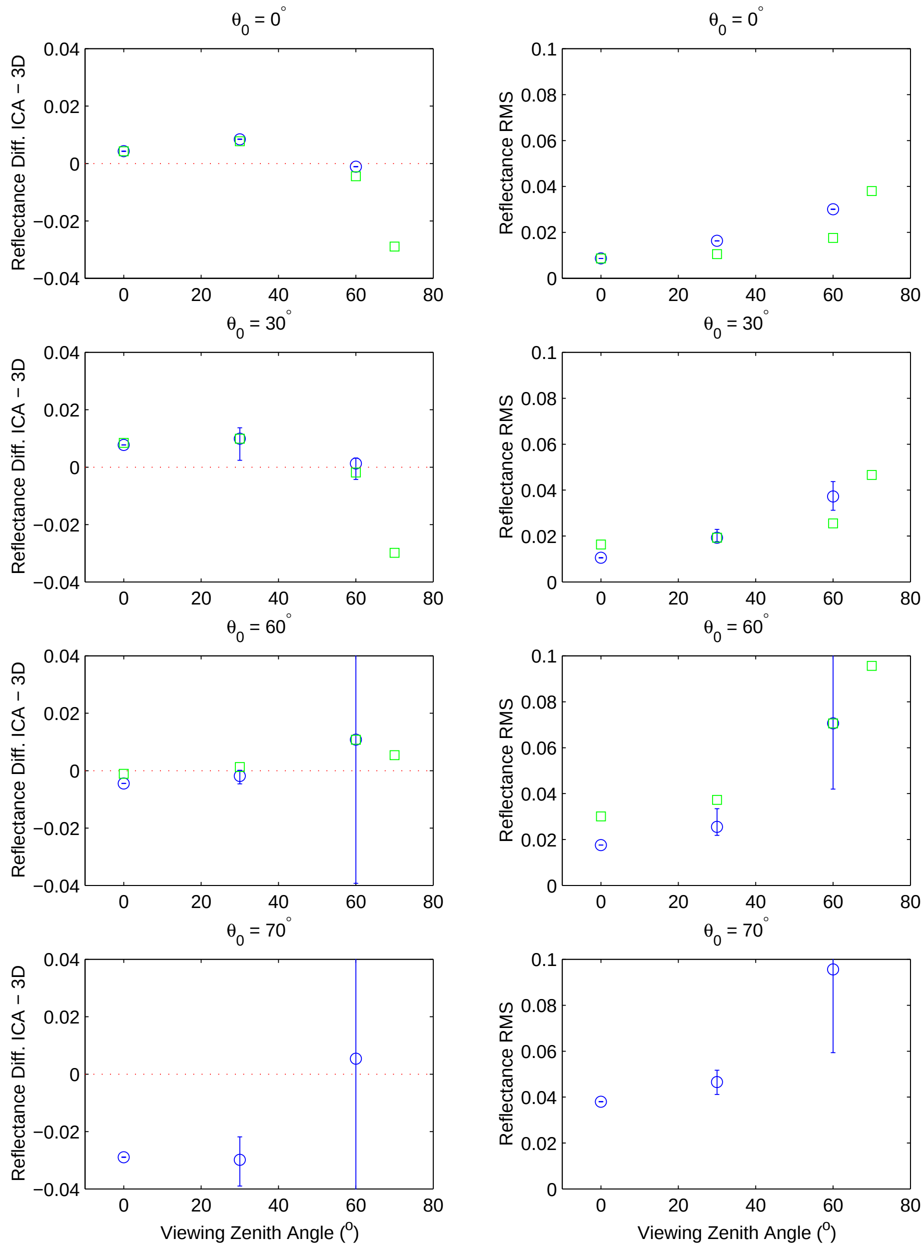

Figure 2 Left column) Difference of the azimuthally and domain averaged reflectance computed with the independent column approximation (ICA) from 3D reflectance as a function of viewing zenith angle (open circles). Reflectances at 7 different relative azimuth angles shown in Figure 1 are averaged for each point. The error bar indicates the maximum and minimum reflectances among different relative azimuth angles. Right column) Same as the left column but for root mean square difference of ICA and 3D radiances. Values indicated by open squares are obtained with the reciprocity principle (i.e. with interchanging solar and viewing zenith angles).

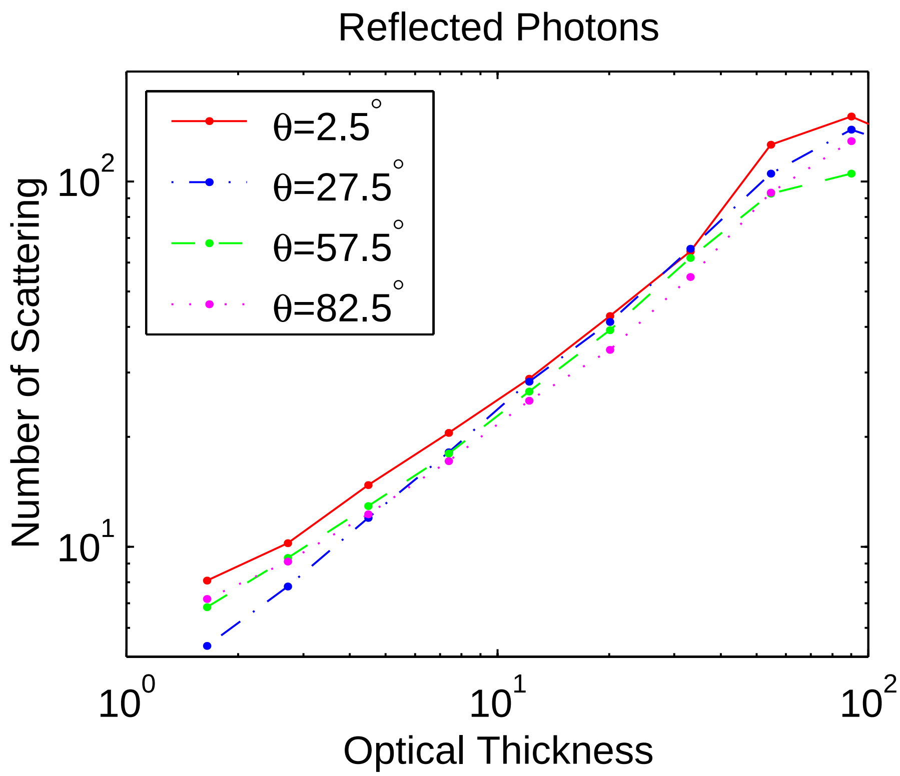

Figure 3 Number of scattering events as a function of optical thickness for reflected photons. The cloud is plane parallel and non-absorbing. The asymmetry parameter of cloud droplet is 0.86 and the Henyey-Greenstein phase function is used.

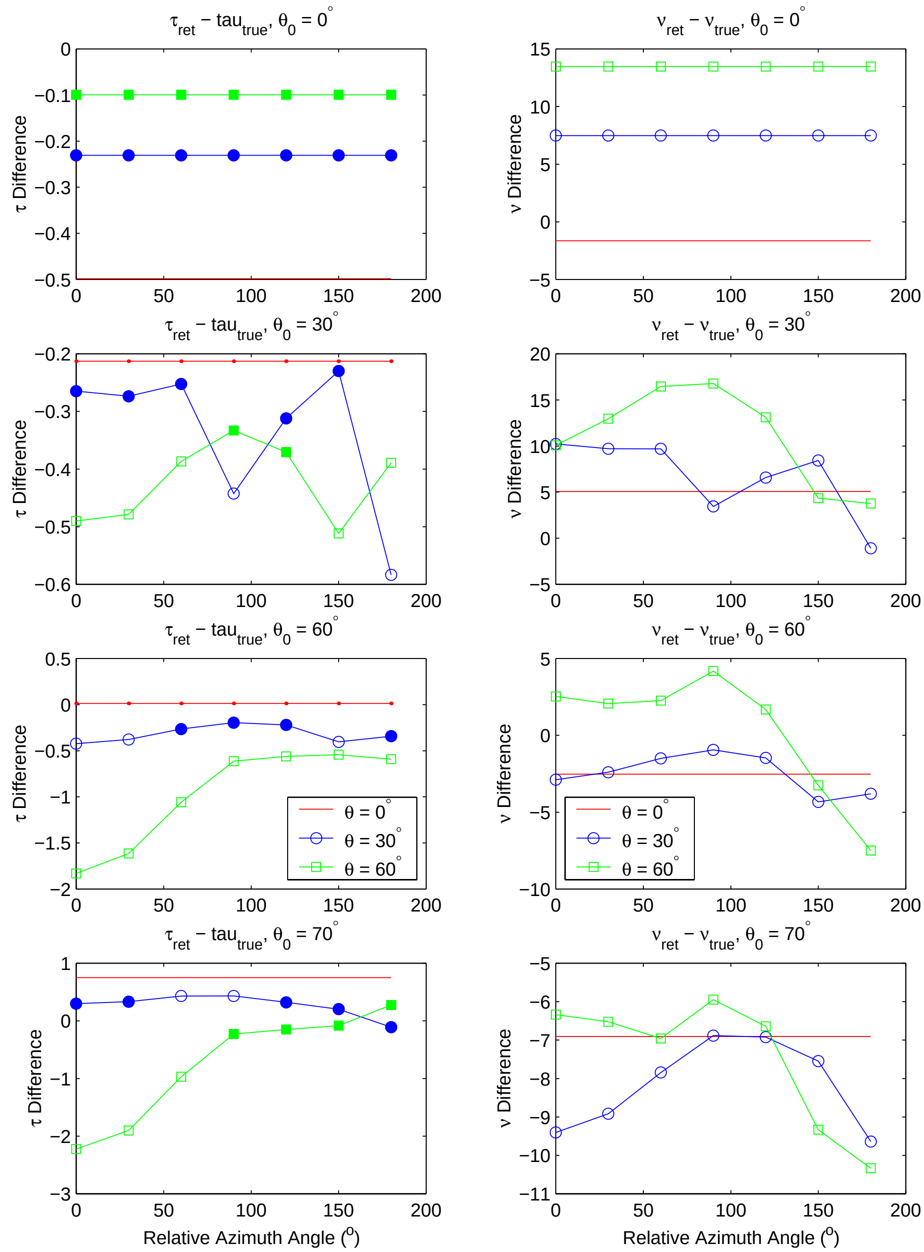

Figure 4 Left column) Difference of domain averaged retrieved optical thickness and the true domain averaged optical thickness ($\overline{\tau_{ret}} - \overline{\tau_{true}} = \overline{\Delta\tau}$). The relative azimuth angle is 0 when the imager looks into the sum. Optical thicknesses are retrieved with 1 km pixels. Solid symbols indicate the relative error less than 10%. Right column) Difference of the retrieved gamma distribution shape parameter and true shape parameter ($\nu_{ret} - \nu_{true}$). $\overline{\tau_{true}} = 3.75$ and $\overline{\nu_{true}} = 15$.

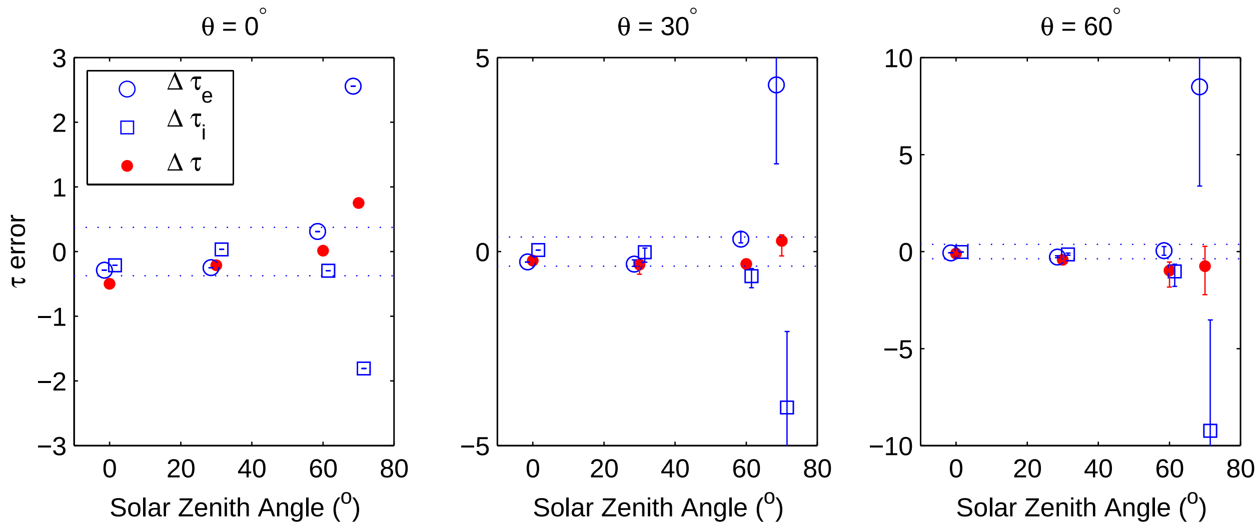

Figure 5 Domain averaged retrieved optical thickness error $\overline{\Delta\tau} = \overline{\Delta\tau_i} + \overline{\Delta\tau_e}$ as a function of solar zenith angle (closed circles). Open circles indicate the error due to external variability $\overline{\Delta\tau_e}$ and open squares indicate the error due to neglecting horizontal inhomogeneity within 1 km imager pixels $\overline{\Delta\tau_i}$ (internal variability). Error bars indicate the maximum and minimum errors among all simulated relative azimuth angles. Horizontal dotted lines indicate the plus and minus 10% errors.

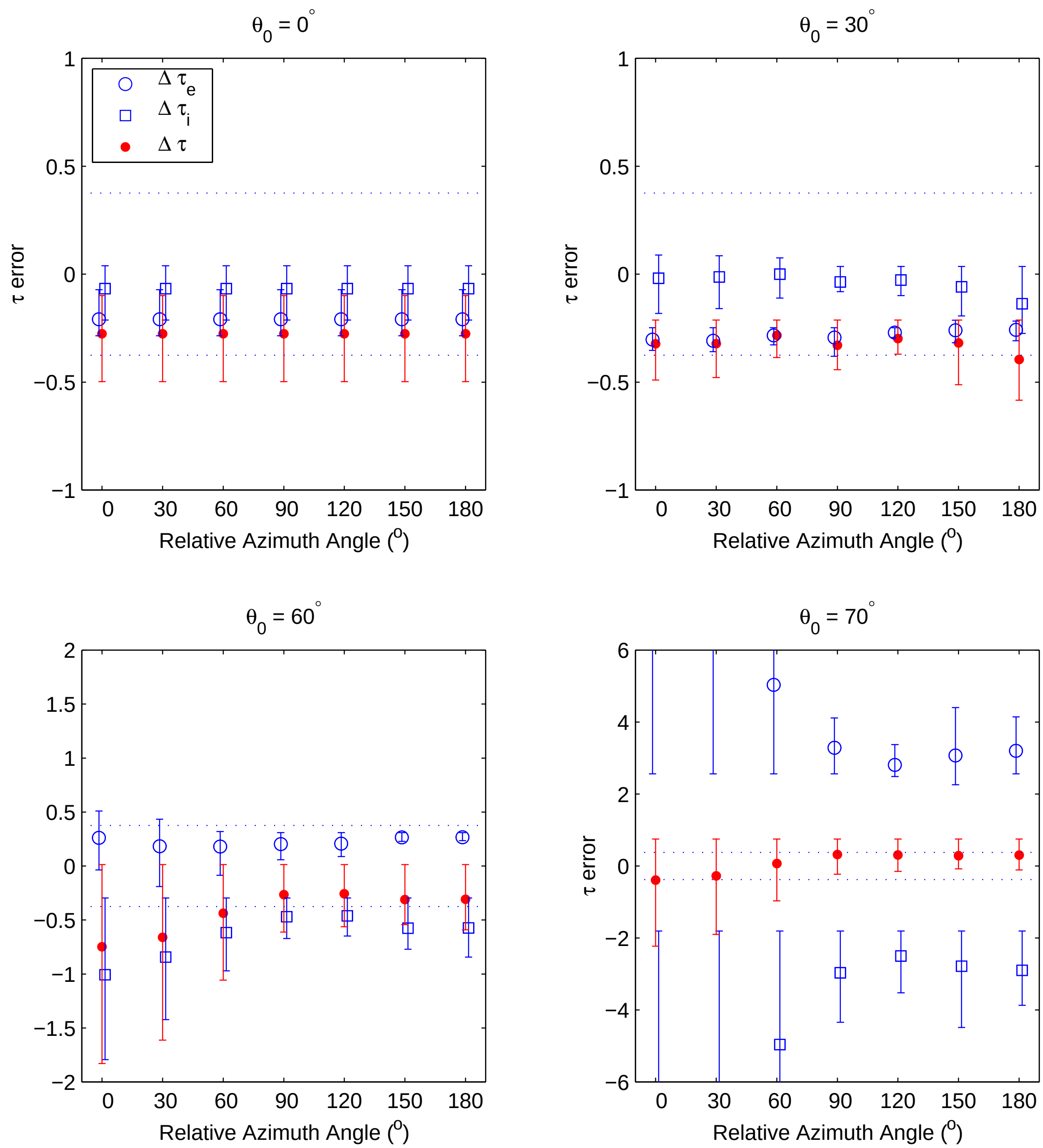

Figure 6 Domain averaged retrieved optical thickness error $\overline{\Delta\tau}$ as a function of solar zenith angle (closed circles). Open circles indicate the error due to external variability $\overline{\Delta\tau_e}$ and open squares indicate the error due to neglecting horizontal inhomogeneity within 1 km imager pixels $\overline{\Delta\tau_i}$ (internal variability). Error bars indicate the maximum and minimum errors among all simulated viewing zenith angles. Horizontal dotted lines indicate the plus and minus 10% errors.

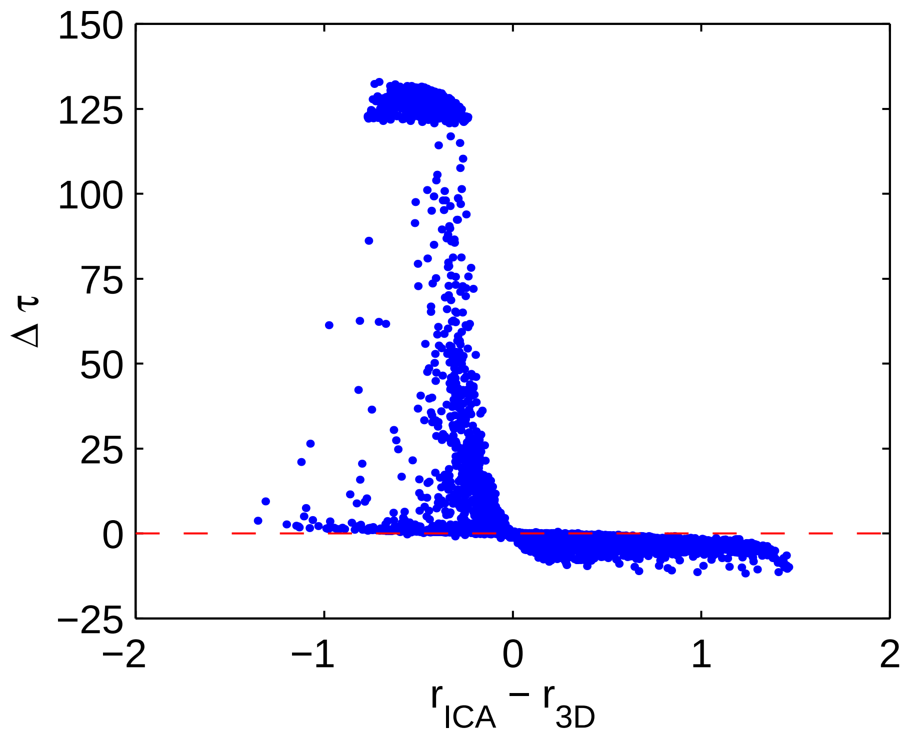

Figure 7 The error in the optical thickness retrieved from a 50 km pixel resolution $\Delta\tau_e$ as a function of the difference between ICA and 3D reflectances computed also with a 50 km resolution. The solar zenith angle is 70°, viewing and relative azimuth angles are 60° and 0°, respectively.

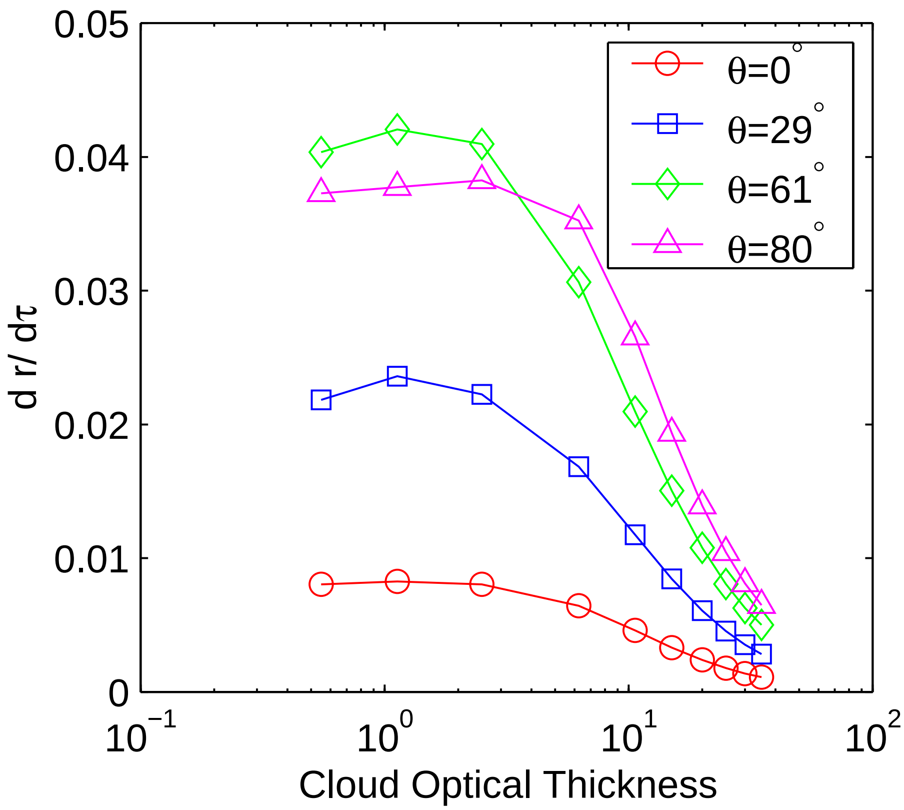

Figure 8 Derivative of the reflectance with respect to optical thickness as a function of optical thickness. The derivative is computed for overhead sun with a plane parallel non-absorbing cloud of which cloud top is 3 km that is placed in the mid-latitude summer atmosphere.

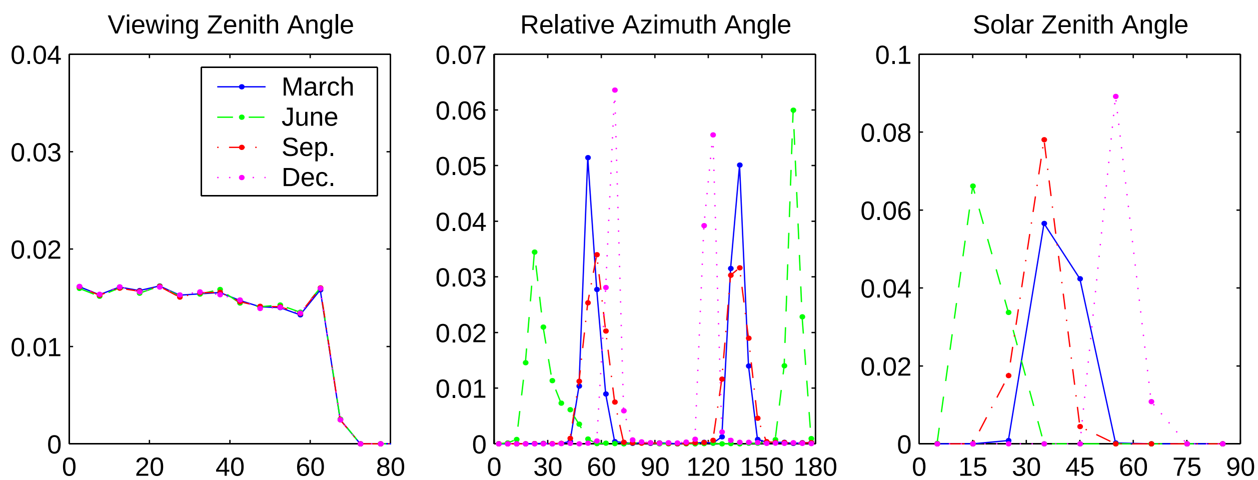

Figure 9 Probability of occurrence of viewing zenith and relative azimuth angles of MODIS on Terra, as well as probability of occurrence of the solar zenith angle at the time of the MODIS observation took place. Data are collected over a $1^\circ \times 1^\circ$ region centered at 32.5N and 134.5W.

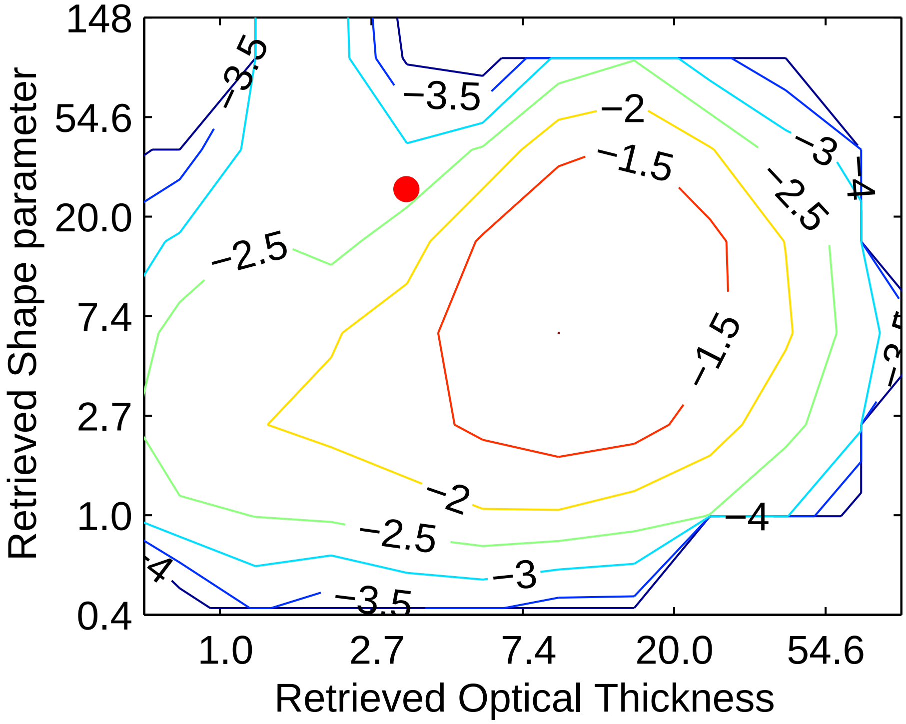

Figure 10 2D histogram of cloud properties derived from 1 km MODIS pixels over CERES footprints between 134W and 135W and 30N and 35N in March 2003. The shape parameter of a gamma distribution is derived from the difference between linear and logarithmic mean of optical thicknesses (Kato et al. 2005). Contour is the logarithmic (base 10) of the number of samples. The closed circle indicates properties of the cloud field used in this study.